Theory of Elementary Particles

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1. Introduction

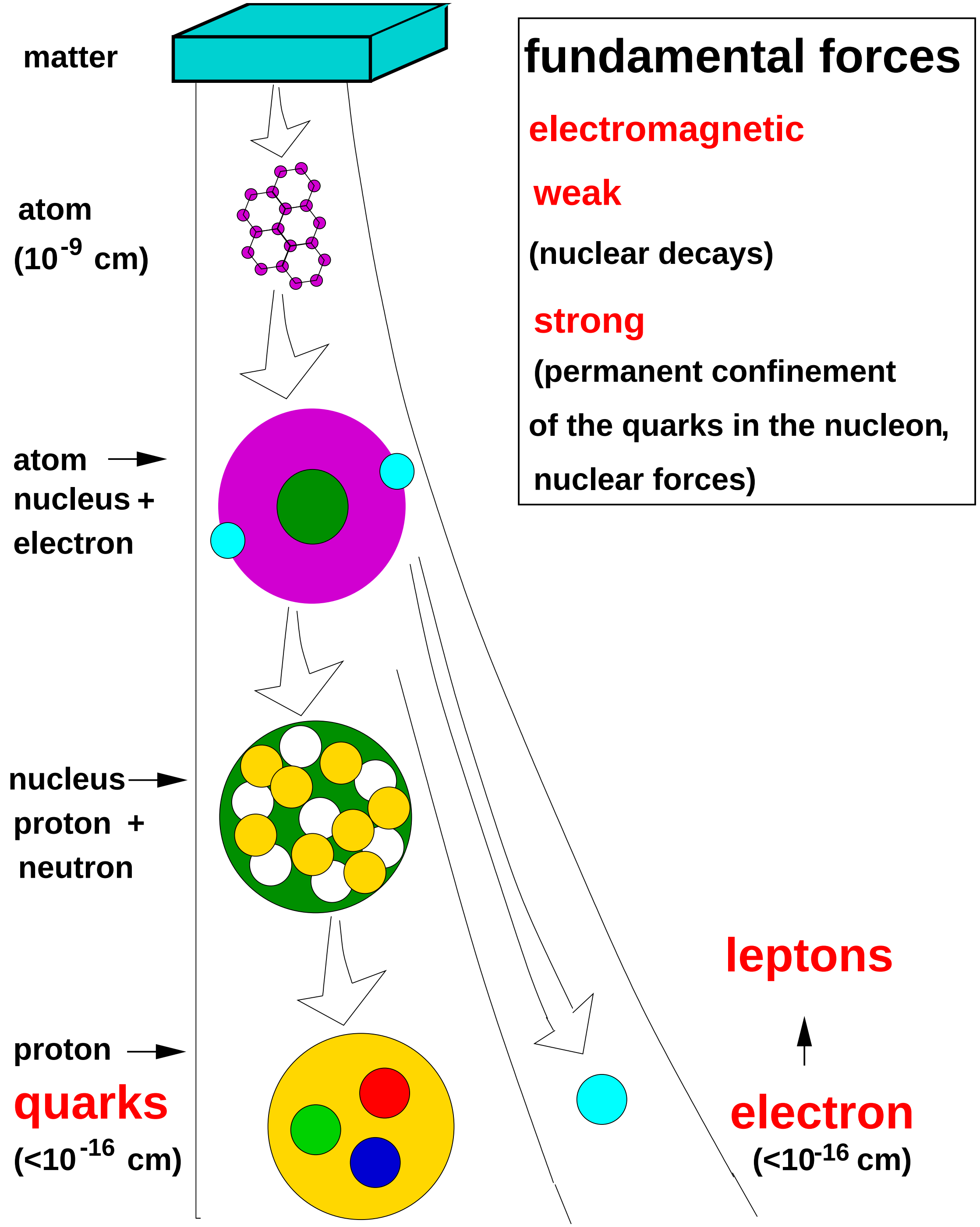
central questions in particle physics

elementary constituents of matter	**fundamental forces**

present status

STANDARD MODEL

quarks and leptons	**gauge forces** as a consequence of the **gauge principle=** **symmetrie requirement**



matter
fundamental forces
electromagnetic
weak
(nuclear decays)
strong
(permanent confinement
of the quarks in the nucleon,
nuclear forces)
atom
(10-9 cm)
atom
nucleus +
electron
nucleus
proton +
neutron
proton
quarks
(<10-16 cm)
leptons
electron
(<10-16 cm)

Elementary particles of the Standard Model

spin 1/2 matter particles, in three generations

				electric charge
leptons (l)	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
quarks (q)	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

spin 1 gauge bosons (mediators of the fundamental interactions)

photon (γ)

gluons (g)

$W^\pm$, Z bosons

- no free quarks and gluons
- confinement' in **hadrons**
- **indirect** evidence

spin 0

Higgs boson (H) ?

Fundamental forces = **gauge forces**

interaction	theory	participating matter particles	mediator	examples
electro-magnetic	**QED**	**electrically charged** l,q	photon (γ)	$e^+e^- \longrightarrow e^+e^-e^+e^-$ $e^+e^- \longrightarrow \mu^+\mu^-$ $\pi^0 \longrightarrow \gamma\gamma$
weak	**unified electroweak gauge theory**	all l, q **(in pairs)**	$W^\pm,\ Z,\ \gamma$	$\longrightarrow$ decay of nuclei $n \longrightarrow p\, e\, \bar{\nu}_e$ $\mu \longrightarrow \nu_\mu\, e\, \bar{\nu}_e$ $K^0 \longrightarrow \pi^+\pi^-$ $e^+e^- \longrightarrow Z$
strong resp. **colour**	**QCD**	**coloured** q	gluons	$\longrightarrow$ nuclear forces $\rho \longrightarrow \pi^+\pi^-$ in $e^\pm p \longrightarrow e^\pm +$ anything in $e^+ + e^- \longrightarrow$ three jets

(electromagnetic and weak are unified into the electroweak gauge theory; electromagnetic $\longrightarrow$ QED)

Plan of the lecture

2. quantum field theory (QFT) why? how formulated?

3. gauge interactions

- **all interactions** { electromagnetic (QED), weak, strong (QCD) } gained from a simple, elegant principle, the so-called

  gauge principle in form of gauge interactions

- – perturbation theory (for small couplings) $\longrightarrow$ **Feynman diagrams**

  – (lattice physics for large couplings)

- the Higgs boson, **spontaneous symmetry breakdown** and masses for $W^{\pm}$, Z, l, q

4. quantum effects, some applications and key precision tests

- quantum effects and precision tests in QED

- running couplings in QED and QCD $\longrightarrow$

qualitative understanding of **quark confinement** at large distances	**asymptotic freedom** of quarks at small distances

- test of **asymptotic freedom** and of **three colours**

- **HERA:** deep inelastic scattering and the nucleon structure functions **as test of perturbative QCD**

- quantum effects in electroweak interactions and the indirect determination of **m_t and m_H at LEP, SLD**

5. physics beyond the Standard Model

- some open questions in the Standard Model
- brief remark on neutrino masses
- composite quarks and leptons
- new particles, examples: leptoquarks and leptogluons
- new gauge interactions
- grand unification
- supersymmetry
- brief remarks on

  supergravity, superstrings, baryon asymmetry, cosmology, extra dimensions, non-commutative geometry

2. Quantum field theory

♣ QFT - why?

non-relativistic quantum mechanics

$\Delta x \searrow$ uncertainty principle $\Delta x \cdot \Delta p \gtrsim O(\hbar)$

$\Delta x \searrow \longrightarrow \Delta p \nearrow$ implying $p \nearrow$ implying $v \nearrow c$ ($c =$ speed of light)

special relativity

– $E = \sqrt{\vec{p}^2 c^2 + m^2 c^4}$ with $E_{\text{rest}} = mc^2$

– conservation of energy E

– **kinetic energy** $\overset{\text{transformation}}{\longleftrightarrow}$ **mass**

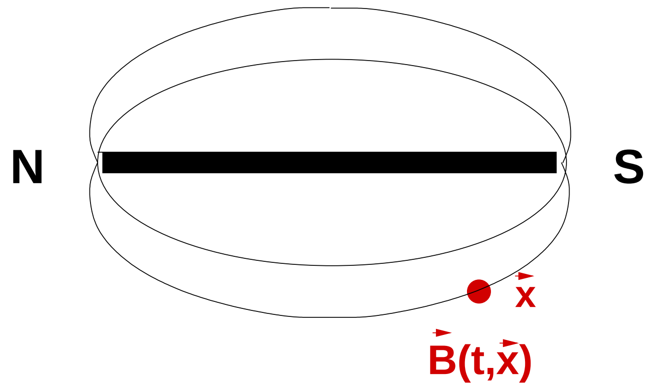
– **no conservation of particle number and particle species**

relativistic quantum mechanics is **insufficient**

quantum field theory allows description of **particle production and annihilation**

♣ QFT - formulation

step 1 **each particle species** $\overset{\text{association}}{\longleftrightarrow}$ **field $\Phi(t,\vec{x})$**



role model: electric and magnetic fields $\vec{E}(t,\vec{x}),\ \vec{B}(t,\vec{x})$

classical mechanics	classical field theory
1 mass point (in 1 dim.), described by 1 generalized coordinate	1 **field**, described by 1 generalized coordinate in **each space point $\vec{x}$**
$q(t),\ \dot{q}\ (t)$	$\Phi(t,\vec{x}),\ \dot{\Phi}\ (t,\vec{x}), \vec{\nabla}\Phi(t,\vec{x})$

Lagrangefunction $L = T - V$ (potential V contains interaction)

$L(q(t),\ \dot{q}\ (t))$	$L = \int \mathrm{d}^3x \underbrace{\mathcal{L}(\Phi(t,\vec{x}),\ \dot{\Phi}\ (t,\vec{x}), \vec{\nabla}\Phi(t,\vec{x}))}_{\text{Lagrange density}}$

action $S = \int \mathrm{d}tL,$ Hamilton principle of extremal action $\delta S = 0 \quad \rightarrow$

equation of motion	field equation
$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} = 0$	$\frac{\partial \mathcal{L}}{\partial \Phi} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{\Phi}} + \vec{\nabla}\frac{\partial \mathcal{L}}{\partial(\vec{\nabla}\phi)} = 0$

step 2

establish $\mathcal{L}$ for each of the **fundamental interactions** among the relevant **fields** (see sect.3)

For **free** fields, i.e. **no** interaction:

spin 0: $\Phi(t,\vec{x})$ field equation = Klein-Gordon equation

= relativistic generalization of the Schrődinger equation

$$(E^2 - c^2\vec{p}^2 - m^2c^4 = 0;\ E \to i\hbar\tfrac{\partial}{\partial t},\ \vec{p} \to -i\hbar\vec{\nabla})$$

$\hbar = c = 1$

$$(\Box + m^2)\Phi(t,\vec{x}) = 0$$

with $\Box = (\frac{\partial}{\partial t})^2 - \vec{\nabla}^2$

$$\to \quad \mathcal{L}^{\Phi}_{\text{free}} = \tfrac{1}{2}((\tfrac{\partial}{\partial t}\Phi)^2 - (\vec{\nabla}\Phi)^2) - \tfrac{1}{2}m^2\Phi^2$$

spin $\frac{1}{2}$: $\psi(t,\vec{x})$ field equation = Dirac equation

with $\partial_\mu = \begin{pmatrix} \frac{\partial}{\partial t} \\ \vec{\nabla} \end{pmatrix}$, μ=0,1,2,3

ψ =4-spinor= $(\psi_1, \psi_2, \psi_3, \psi_4)$,

γ_μ=4×4-matrices

$$(i\gamma^\mu \partial_\mu - m)\psi(t,\vec{x}) = 0 \quad \longrightarrow \quad \mathcal{L}^{\psi}_{\text{free}} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

spin 1, m=0: $A^\mu(t,\vec{x})$ $\mu = 0,1,2,3$ electromagnetic field

$$A^\mu(t,\vec{x}) = \begin{pmatrix} V(t,\vec{x}) = \text{scalar potential} \\ \vec{A}(t,\vec{x}) = \text{vector potential} \end{pmatrix} \qquad \vec{B} = \vec{\nabla}\times\vec{A}, \quad \vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$$

electromagnetic field strength tensor with components in terms of $\vec{E}$ and $\vec{B}$

$$F^{\mu\nu}(t,\vec{x}) = \partial^\mu A^\nu(t,\vec{x}) - \partial^\nu A^\mu(t,\vec{x}) \quad \text{with} \quad \partial_\mu = \begin{pmatrix} \frac{\partial}{\partial t} \\ \vec{\nabla} \end{pmatrix}$$

field equations = Maxwell equations (in absence of charge and current densities)

$$\partial_\mu F^{\mu\nu}(t,\vec{x}) = 0 \quad \longrightarrow \quad \mathcal{L}^{A^\mu}_{\text{free}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

step 3

field quantization

quantum mechanics	quantum field theory

conjugate coordinate

$$p(t) := \frac{\partial L}{\partial \dot{q}(t)}$$

spin 0: $\Phi(t, \vec{x})$

conjugate field

$$\pi(t, \vec{x}) := \frac{\partial \mathcal{L}}{\partial \dot{\Phi}(t, \vec{x})}$$

$$\left[q(t), p(t)\right] = i\hbar$$

$$\left[\Phi(t, \vec{x}),\ \pi(t, \vec{x}')\right] = i\hbar\delta^{(3)}(\vec{x} - \vec{x}')$$

scalar boson field quantization

$$[A, B] = A \cdot B - B \cdot A$$

spin 1/2:

$$\pi_\alpha(t, \vec{x}) := \frac{\partial \mathcal{L}}{\partial \dot{\psi}_\alpha(t, \vec{x})}$$

$\psi(t, \vec{x})$: $\psi_\alpha(t, \vec{x})$, $\alpha = 1, \ldots, 4$

$$\{A, B\} = A \cdot B + B \cdot A$$

$$\left\{\psi_\alpha(t, \vec{x}),\ \pi_\beta(t, \vec{x}')\right\} = \delta_{\alpha\beta}\ i\hbar\delta^{(3)}(\vec{x} - \vec{x}')$$

Dirac fermion field quantization

q, p $\longrightarrow$ operators

Φ, π resp. ψ_α, π_α $\longrightarrow$ field operators quantum mechanics

quantum field theory

spin 1, m=0: $A^\mu(t,\vec{x})$ $\mu = 0,1,2,3$

$$\pi^\mu(t,\vec{x}) := \frac{\partial \mathcal{L}}{\partial\ \dot{A}_\mu\ (t,\vec{x})}$$

$$\left[A^\mu(t,\vec{x}),\ \pi^\nu(t,\vec{x}')\right] = g^{\mu\nu}\ i\hbar\delta^{(3)}(\vec{x}-\vec{x}')$$

electromagnetic field quantization

(modulo complications due to gauge invariance)

$g^{\mu\nu}$=metric tensor

$A^\mu,\ \pi^\mu\ \longrightarrow$ field operators

♣ QFT for a free scalar field and particle interpretation

- **field equation** $(\Box + m^2)\Phi(t,\vec{x}) = 0$, with $\Box = (\frac{\partial}{\partial t})^2 - \vec{\nabla}^2$

general solution

$$\Phi(t,\vec{x}) \propto \int \mathrm{d}E\, \mathrm{d}^3p\, \delta(E^2 - \vec{p}^2 - m^2) \times \left(\boldsymbol{a(E,\vec{p})}\, e^{-i(Et-\vec{p}\vec{x})} + \boldsymbol{a^\dagger(E,\vec{p})}\, e^{+i(Et-\vec{p}\vec{x})}\right)$$

energy   3-momentum   relativistic energy-momentum relation

- **Lagrange density** $\mathcal{L}^{\Phi}_{\text{free}} = \frac{1}{2}((\frac{\partial}{\partial t}\Phi)^2 - (\vec{\nabla}\Phi)^2) - \frac{1}{2}m^2\Phi^2$

conjugate field $\pi(t,\vec{x}) := \dfrac{\partial \mathcal{L}}{\partial\, \dot{\Phi}\, (t,\vec{x})} = \dot{\Phi}\, (t,\vec{x})$

- **field quantization** $\left[\Phi(t,\vec{x}), \pi(t,\vec{x}\,')\right] = i\hbar\, \delta^{(3)}(\vec{x} - \vec{x}\,')$ $\longleftrightarrow$

$$[\boldsymbol{a}(p), \boldsymbol{a}^\dagger(p')] = 2E\,\hbar\,\delta^{(3)}(\vec{p}-\vec{p}\,'), \quad [a(p), a(p')] = 0, \quad [a^\dagger(p), a^\dagger(p')] = 0$$

with $p = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$

$a(p) = a(E,\vec{p})$ is field operator, $a^\dagger(p)$ is the hermitian conjugate field operator

- **Hamiltonoperator** (measures total energy in the field)

$$H = \int \mathrm{d}^3x(\pi\ \dot{\Phi}\ - \mathcal{L}) = \ldots + \int \mathrm{d}E\ \mathrm{d}^3p\ \delta(E^2 - \vec{p}^{\,2} - m^2) \boldsymbol{E}\ \underbrace{\boldsymbol{a^\dagger(p) a(p)}}_{\textbf{number operator } \boldsymbol{N(p)}}$$

- **eigenbasis of $N(p)$** (Fock space of multiparticle states)

$$\boxed{N(p)\,|n(p)> \,=\, \boldsymbol{n(p)}\,|n(p)>}$$

eigenvalue eigenstate

$n(p)$ = number of particles with spin 0, mass m with energy between E and $E + \mathrm{d}E$ and momentum between $\vec{p}$ and $\vec{p} + \mathrm{d}\vec{p}$, $E = +\sqrt{\vec{p}^{\,2} + m^2}$

$$N(p)\,\boldsymbol{a(p)}^{(\dagger)}\,|n(p)> \,=\, (n(p)\ \ (\mp)\ \ 1)\,\boldsymbol{a(p)}^{(\dagger)}\,|n(p)> \ \longleftrightarrow$$

$$\boldsymbol{a^\dagger(p)}\,|n(p)> \,\propto\, |(n\boldsymbol{+1})(p)> \qquad \boldsymbol{a^\dagger(p)} = \textbf{particle creation operator}$$

$$\boldsymbol{a(p)}\,|n(p)> \,\propto\, |(n\boldsymbol{-1})(p)> \qquad \boldsymbol{a(p)} = \textbf{particle annihilation operator}$$

provides basis for particle production and particle annihilation in QFT

Normalize the energy of the ground state $|0>$ to zero $\rightarrow$ eigenvalue spectrum $\boldsymbol{n(p) = 0, 1, 2, \ldots}$

consequence of the field quantization! **field $\longleftrightarrow$ particle**

- multiparticle states **Bose-Einstein statistics**

$$|n_1(p_1), ..., n_m(p_m)> \propto (a^\dagger(p_1))^{n_1} \cdot ... \cdot (a^\dagger(p_m))^{n_m}|0>$$

automatically: total **symmetry** with respect to the exchange of any two particles

♣ QFT for a free Dirac field

- $[,] \to \{,\}$ for a Dirac fermion field the arguments runs analogously, also leading to particle creation and annihilation operators. Due to the anticommutator $(\{a^\dagger(p), a^\dagger(p')\} = 0$, implying $(a^\dagger(p))^2 = 0)$ the multiparticle states obey

Fermi-Dirac statistics resp. the **Pauli principle**

$|p_1, ..., p_m> \propto a^\dagger(p_1) \cdot ... \cdot a^\dagger(p_m)|0>$ (for simplicity the spin degrees of freedom have been suppressed)

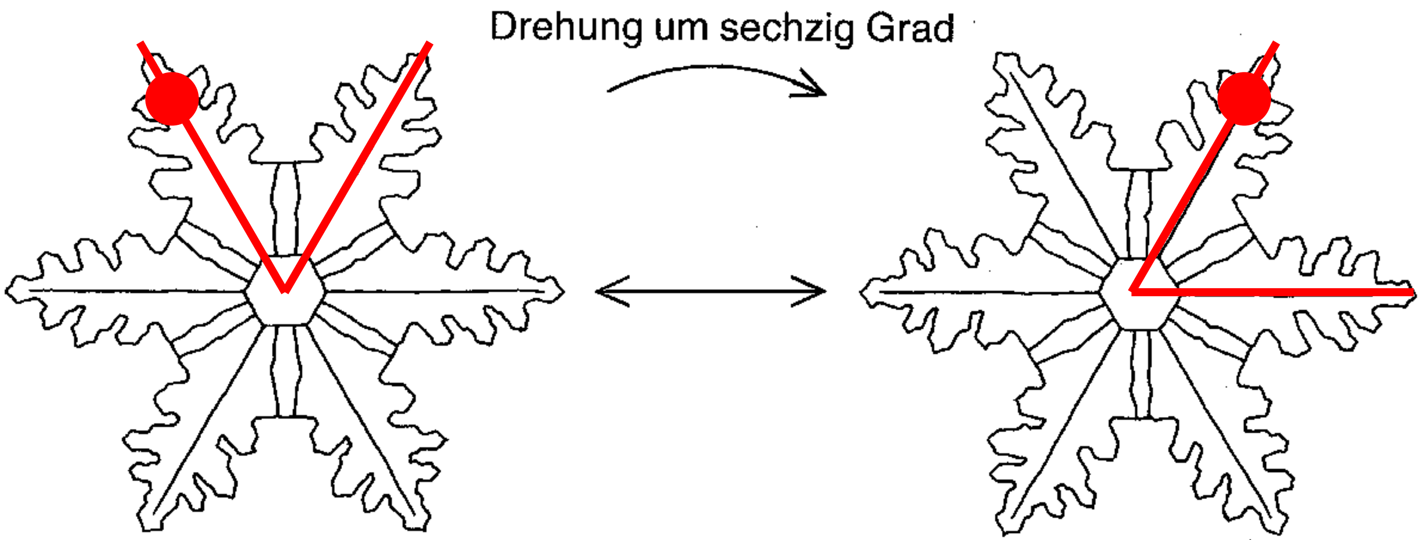
automatically: total **antisymmetry** with respect to the exchange of any two particles

- existence of antiparticles in QFT
- the field energy is bounded from below

3. Local gauge interactions

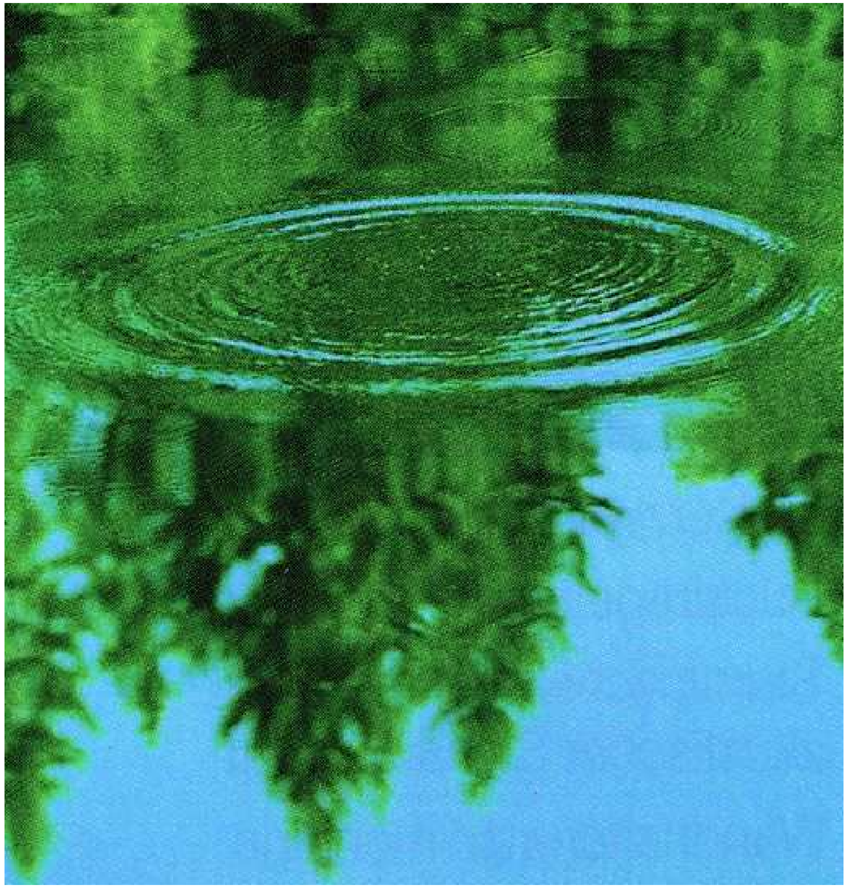
♣ preexercise in symmetries by means of examples from daily life

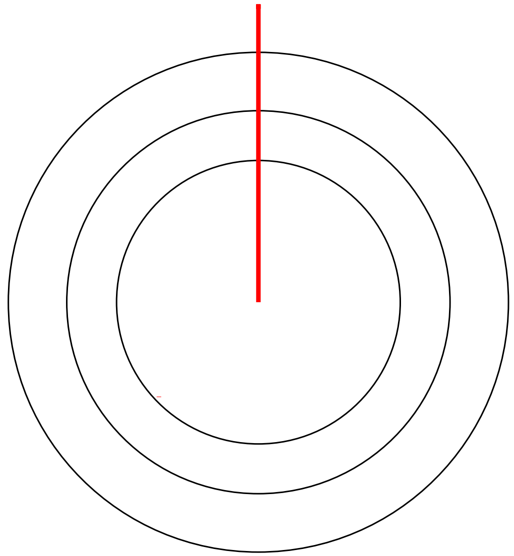
snowflake



- **invariance** with respect to common, i.e. **global** rotations by 60^{o}
- section can be chosen by convention

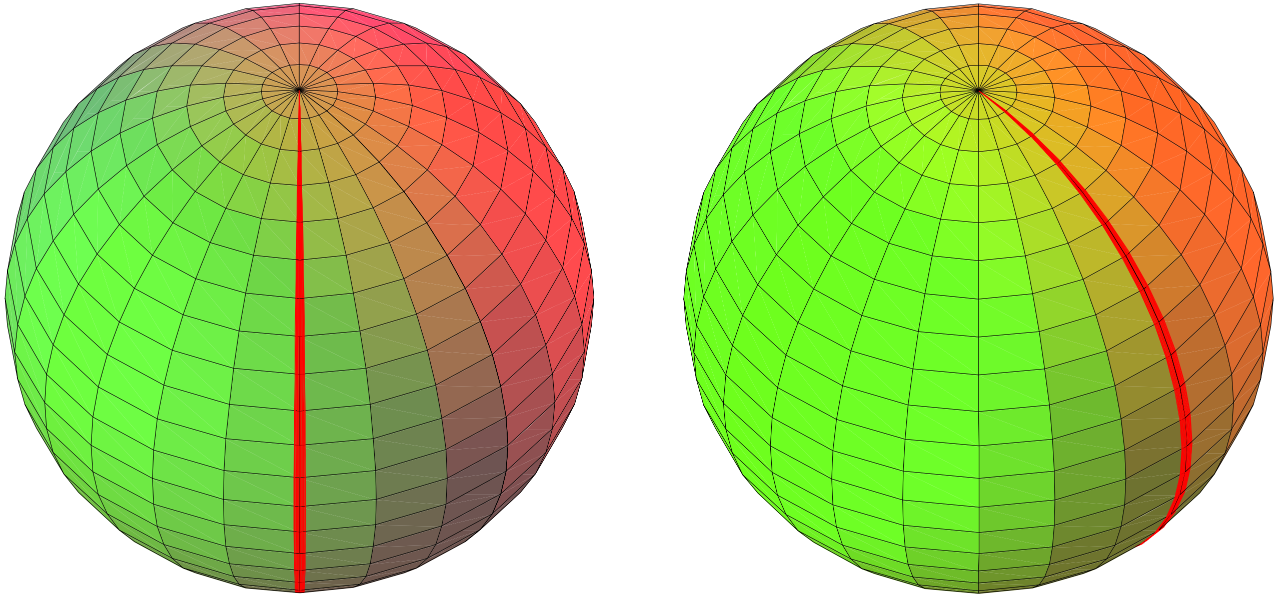
throwing a stone into the water





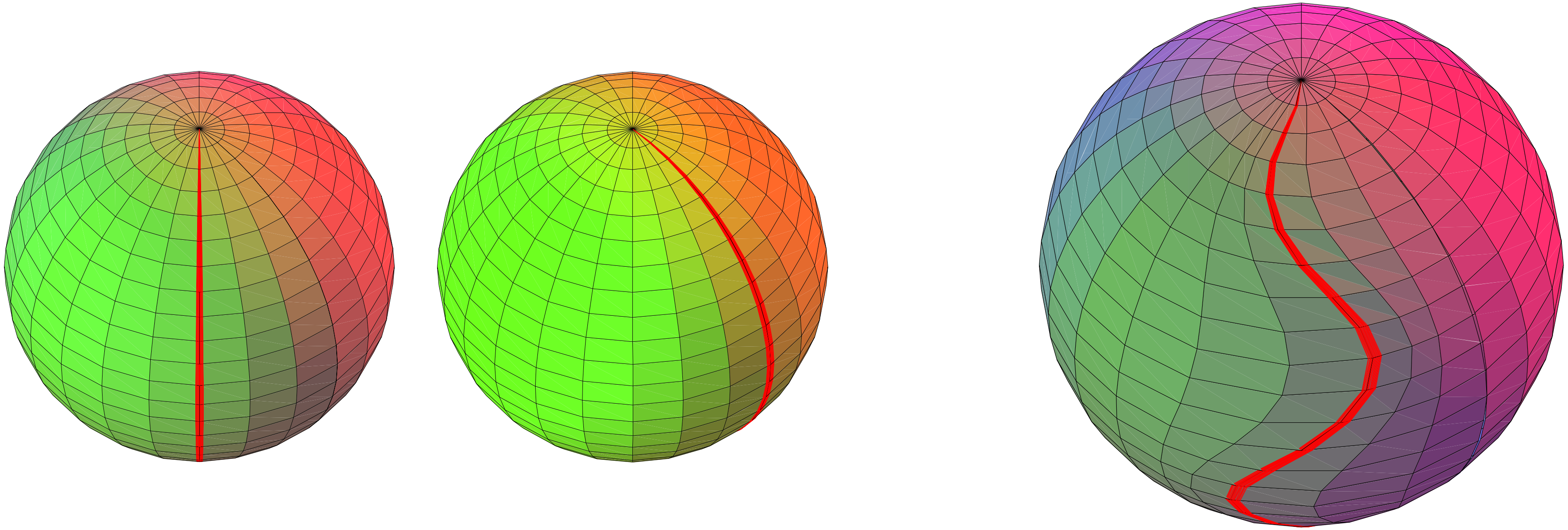
- **invariance** with respect to common, i.e. **global** rotations of all points by an arbitrary angle
- the line can be chosen by convention

balloon



- **invariance** with respect to common, i.e. **global** rotations of all points of the surface by an arbitrary angle around the given axis
- longitudinal circle can be chosen by convention

requirement of local symmetry



- the balloon is required to keep its form, if each point of the surface is allowed to be rotated by an arbitrary angle - independently of the other points - i.e. if the surface remains **invariant** with respect to **local** rotations
- angular convention can be chosen arbitrarily **for each point** of the surface

the local symmetry is only possible
in the presence of **forces**

'derivation' of quantum electrodynamics (QED) from the gauge principle

♣ history electromagnetic interactions (Maxwell equations, QED) have a local gauge invariance $\longrightarrow$ generalizeable $\longrightarrow$ put on the level of a principle $\longrightarrow$ access to the understanding of strong and weak interactions

♣ starting point **free matter particle** e.g. electron (with electric charge $Q_\psi = -1$),

described by the Dirac equation $(i\frac{\partial}{\partial t}\gamma^0 - i\vec{\nabla}\vec{\gamma} - m)\psi(t,\vec{x}) = 0$

♣ global symmetry the **absolute phase** of the field $\psi(t,\vec{x})$ is **not measurable**

invariance with respect to **global** phase transformations

$$\psi(t,\vec{x}) \quad \longrightarrow \quad e^{i\alpha} \; \psi(t,\vec{x}),$$

where α is an **arbitrary constant**.

The absolute phase can be fixed by **convention**. However, the convention has to be **identical** at all times and at all space points.

♣ requirement of local symmetry (conceptual analogy to general relativity)

GAUGE PRINCIPLE

- **invariance** with respect to **local** phase transformations

$$\psi(t,\vec{x}) \quad \longrightarrow \quad e^{i\alpha(t,\vec{x})}\ \psi(t,\vec{x}),$$

where α is an **arbitrary function** of $t, \vec{x}$.

- The **phase convention** can be chosen **arbitrarily** at **each** time and at **each** space point **without effect on observables**

♣ symmetry group the transformations $\psi(t,\vec{x}) \longrightarrow e^{i\alpha(t,\vec{x})}\psi(t,\vec{x})$ build a group of unitary transformations: $U(1)_{\mathrm{em}}$

♣ The requirement of local symmetry is **not** fulfilled for the **free** electron, since

$$\begin{pmatrix} \partial/\partial t \\ \vec{\nabla} \end{pmatrix} \left(e^{i\,\alpha(t,\vec{x})}\psi(t,\vec{x})\right) = e^{i\,\alpha(t,\vec{x})}\begin{pmatrix} \partial/\partial t \\ \vec{\nabla} \end{pmatrix}\psi(t,\vec{x}) + \underbrace{ie^{i\,\alpha(t,\vec{x})}\psi(t,\vec{x})\begin{pmatrix} \partial/\partial t \\ \vec{\nabla} \end{pmatrix}\alpha(t,\vec{x})}_{\textbf{additional term}}$$

♣ Force as a consequence of the gauge principle

in order to implement the local gauge invariance, the four additional terms require the introduction of **four** fields and , the so-called **gauge fields** with **spin 1, mass 0** and their interaction

$$\underbrace{\begin{pmatrix} \partial/\partial t \\ \vec{\nabla} \end{pmatrix}}_{\partial_\mu} \rightarrow \underbrace{\begin{pmatrix} \partial/\partial t \\ \vec{\nabla} \end{pmatrix}}_{\partial_\mu} + i\, e \underbrace{\begin{pmatrix} V(t,\vec{x}) \\ -\vec{A}(t,\vec{x}) \end{pmatrix}}_{A_\mu(t,\vec{x})} =: \mathcal{D}_\mu$$

covariant derivative

local gauge invariance with respect to the **simultaneous local gauge transformations**

$$\psi(t,\vec{x}) \quad \rightarrow \quad e^{i\alpha(t,\vec{x})}\ \psi(t,\vec{x})$$

$$\underbrace{\begin{pmatrix} V(t,\vec{x}) \\ -\vec{A}(t,\vec{x}) \end{pmatrix}}_{A_\mu} \rightarrow \underbrace{\begin{pmatrix} V(t,\vec{x}) \\ -\vec{A}(t,\vec{x}) \end{pmatrix}}_{A_\mu} - \frac{1}{e} \underbrace{\begin{pmatrix} \partial/\partial t \\ \vec{\nabla} \end{pmatrix}}_{\partial_\mu} \alpha(t,\vec{x})$$

$$(\partial_\mu + ieA_\mu)\,\psi \ \rightarrow \ \left(\partial_\mu + ie(A_\mu - \tfrac{1}{e}\,\partial_\mu\alpha)\right)(e^{i\alpha}\psi) = \partial_\mu(e^{i\alpha}\psi) + ie(A_\mu - \tfrac{1}{e}\,\partial_\mu\alpha)(e^{i\alpha}\psi) =$$

$$e^{i\alpha}\partial_\mu\psi + i(\partial_\mu\alpha)e^{i\alpha}\psi - i(\partial_\mu\alpha)e^{i\alpha}\psi + iee^{i\alpha}A_\mu\psi = e^{i\alpha}\,(\partial_\mu + ieA_\mu)\,\psi$$

♣ gauge field=electromagnetic field

♣ results: field equations and Lagrange density of QED

– **Dirac equation** field equation for electron field $\psi(t,\vec{x})$

$$(i\gamma^\mu\partial_\mu - m)\psi(t,\vec{x}) = e\,\gamma^\mu A_\mu(t,\vec{x})\psi(t,\vec{x})$$

interaction term

– **Maxwell equation** field equation for the gauge field = electromagnetic field $A_\mu(t,\vec{x})$

$$\partial_\mu F^{\mu\nu}(t,\vec{x}) = e\,\bar{\psi}(t,\vec{x})\gamma^\nu\psi(t,\vec{x}) = \begin{pmatrix} \rho(t,\vec{x}) \\ \vec{j}(t,\vec{x}) \end{pmatrix}$$

interaction term — charge density ρ, current density $\vec{j}$

– **Lagrange density** $\mathcal{L}_{\text{QED}} = \mathcal{L}^{\psi}_{\text{free}} + \mathcal{L}^{A_\mu}_{\text{free}} - e\bar{\psi}(t,\vec{x})\gamma^\mu\psi(t,\vec{x})A_\mu(t,\vec{x})$ local interaction

formulation of QED

♣

– $\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{free}} + \underbrace{e\sum_\psi Q_\psi\bar{\psi}(t,\vec{x})\gamma^\mu\psi(t,\vec{x})A_\mu(t,\vec{x})}_{\mathcal{L}_{\text{int}}}$

$\mathcal{L}_{\text{int}}$ for all matter fields ψ with electric charges Q_ψ

– satisfying **local gauge invariance** w.r. to $\psi \to e^{-iQ_\psi\alpha(t,\vec{x})}\psi$ for all ψ, $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha(t,\vec{x})$

– **quantization** of the fields ψ and A_μ

♣ **coupling** The gauge principle fixes the form of the electromagnetic interaction completely **except for** a constant **e** , the electromagnetic **coupling constant**, which is a measure for the interaction strength and is related to the

fine structure constant

$\boldsymbol{\alpha_{\text{em}} = e^2/(4\pi)}$ experimentally $\approx 1/137 \ll 1$

♣ **multiparticle states** matterfields $\psi \rightarrow$ multi-ψ and multi-$\bar{\psi}$ states, for $\psi = e, \mu, \tau$, quarks
electromagnetic field $A_\mu \rightarrow$ multi-photon states

♣ **formal solution of QED** scattering operator S, acting in the space of multi-ψ, multi-$\bar{\psi}$ and multi-photon states

$$|\, t = +\infty > = \boldsymbol{S}\, |\, t = -\infty >$$

scattering operator

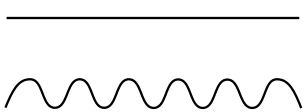
$$\boldsymbol{S} = \underset{\substack{\uparrow \\ \text{time ordering}}}{T} \left[1 + \underbrace{i \int \mathrm{d}t\, \mathrm{d}^3x\, \boldsymbol{\mathcal{L}}_{\text{int}}}_{\propto\, \boldsymbol{e}} + \underbrace{\frac{i^2}{2} \left(\int \mathrm{d}t\, \mathrm{d}^3x\, \boldsymbol{\mathcal{L}}_{\text{int}} \right)^2}_{\propto\, \boldsymbol{e^2}} + \ldots \right]$$

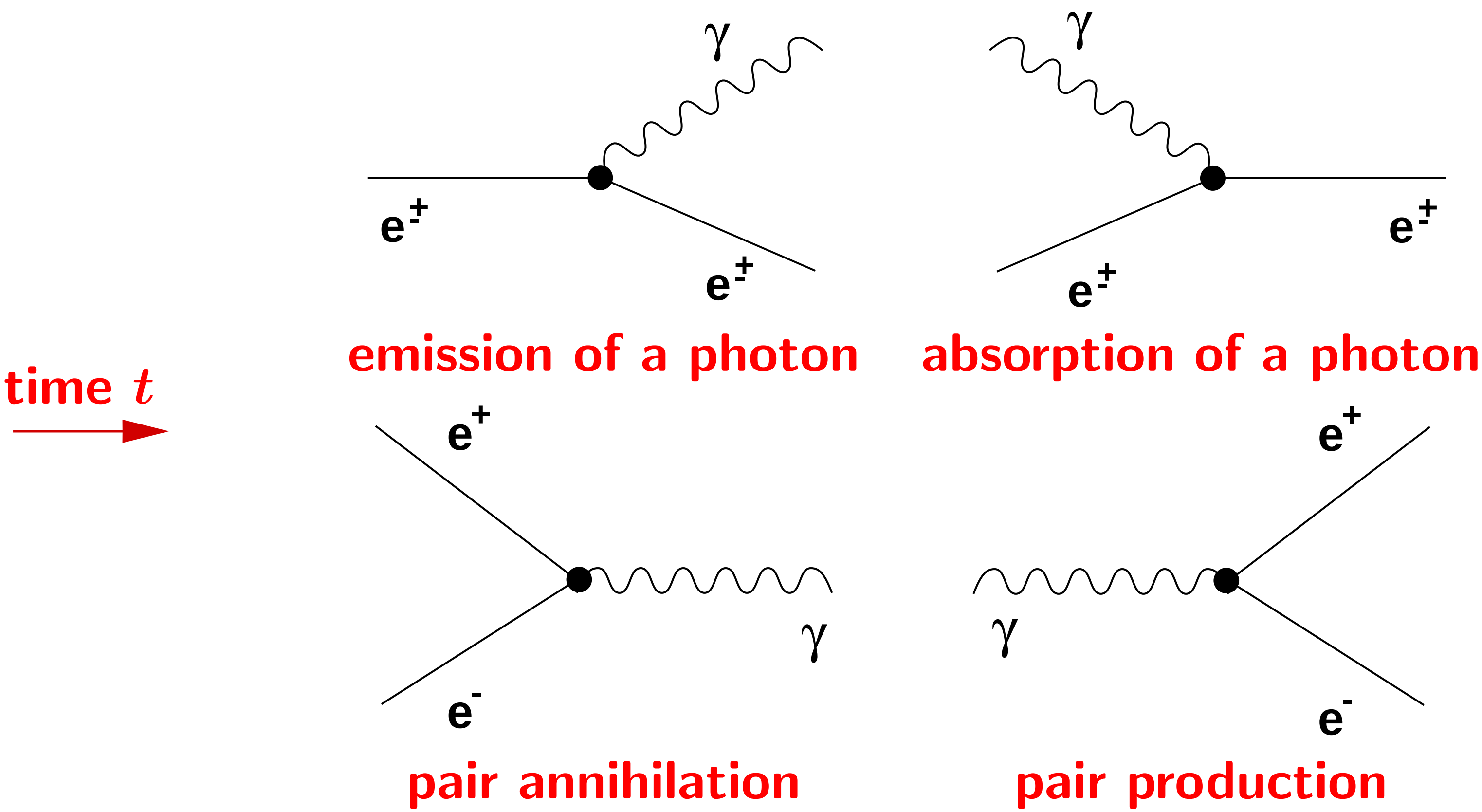
♣ **perturbation theory** cutting the series off after an appropriate number of terms

♣ Feynman diagrams

the transition probability for any QED reaction between electrically charged l, q, $\bar{l}$, $\bar{q}$ and/or photons can be calculated in perturbation theory. The contributions may be represented by Feynman diagrams with the basic building blocks (only electrons (e^-), positrons (e^+) and photons (γ) are considered)

- $e^{\pm}$ propagator
- photon propagator
- **interaction vertex** from $\mathcal{L}_{\mathrm{int}}$

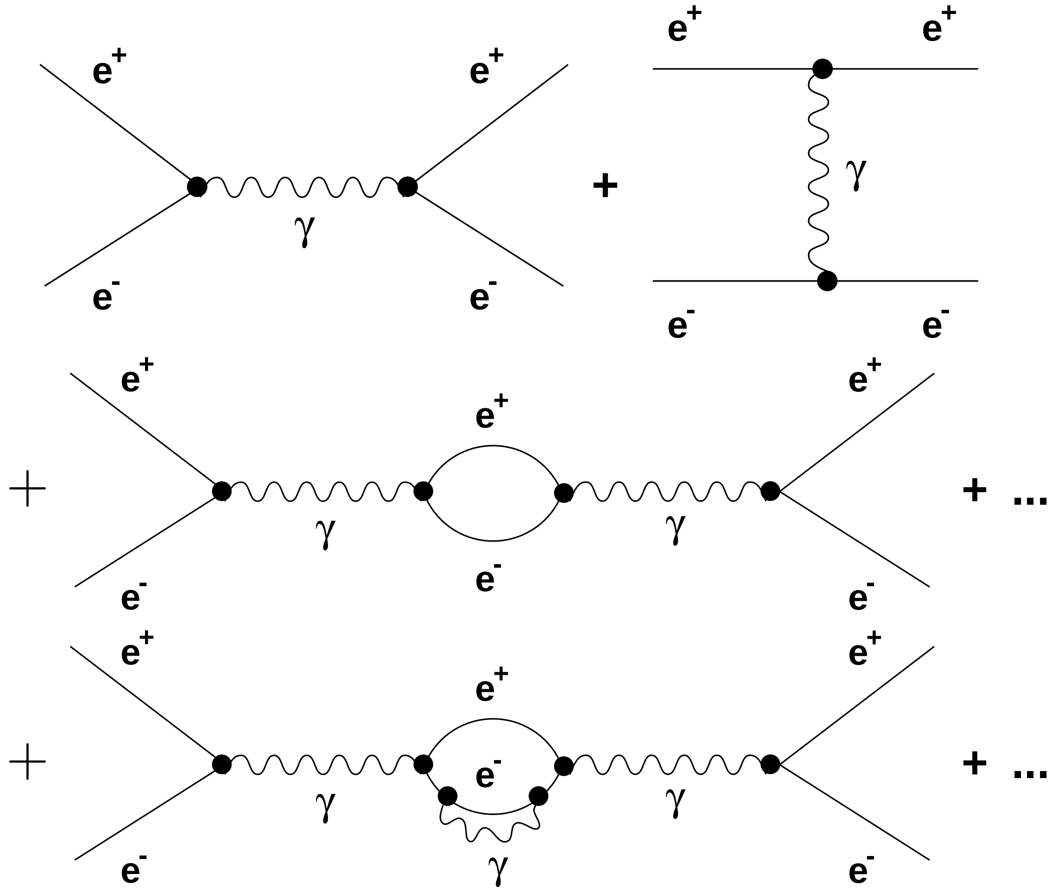




the same vertex with different orientation of its legs with repect to the time arrow; $e^{\pm} \rightarrow e^{\mp}$ if a line changes its direction with respect to the time arrow.

♣ examples

– $e^+e^- \longrightarrow e^+e^-$

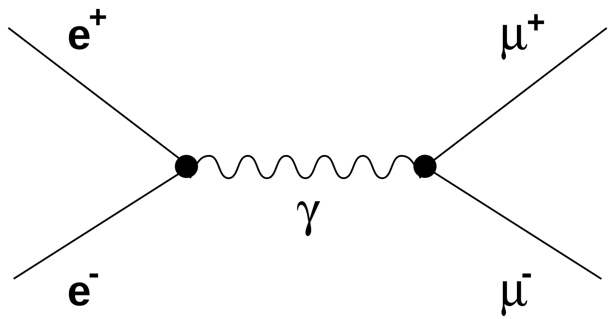


$\mathbf{O}(\alpha_{\rm em})$

$\mathbf{O}(\alpha_{\rm em}^{\mathbf{2}})$

$\mathbf{O}(\alpha_{\rm em}^{\mathbf{3}})$

– $e^+e^- \longrightarrow \mu^+\mu^-$ to $(\alpha_{\rm em})$



'derivation' of quantum chromodynamics (QCD) from the gauge principle

♣ global symmetry in analogy to QED

electric charge	color (charge)
	quarks appear in three different colour charges $q_{\mathbf{red}}$, , $q_{\mathbf{green}}$, $q_{\mathbf{blue}}$
	for each quark flavour $q = u,\ d,\ c,\ s,\ t,\ b$

starting point **free particles**

electrically charged particles	coloured quarks
field $\psi(t,\vec{x})$	fields $\psi_{\mathbf{red}}(t,\vec{x}), \psi_{\mathbf{green}}(t,\vec{x}), \psi_{\mathbf{blue}}(t,\vec{x})$

global symmetry with respect to the global transformations, which leave invariant

$\bar{\psi}\psi$	$\bar{\psi}_{\mathbf{red}}\psi_{\mathbf{red}} + \bar{\psi}_{\mathbf{green}}\psi_{\mathbf{green}} + \bar{\psi}_{\mathbf{blue}}\psi_{\mathbf{blue}}$
$\psi \longrightarrow e^{-i\alpha Q_\psi}\,\psi$	$\psi_{\boldsymbol{i}} \longrightarrow \sum_{j=1}^{3} \boldsymbol{U_{ij}}\,\psi_{\boldsymbol{j}} \quad i,\ j = \mathbf{red},\ \mathbf{green},\ \mathbf{blue}$
one real constant α	3×3 complex, unitary, **constant** matrix U with $UU^\dagger = U^\dagger U = \underline{1}$, det $U = 1$

♣ requirement of local symmetry

GAUGE PRINCIPLE

- **invariance** with respect to the **local** transformations

$$\psi_i \rightarrow \sum_{j=1}^{3} U_{ij}(t,\vec{x})\,\psi_j$$

$$i,\ j = \mathbf{red},\ \mathbf{green},\ \mathbf{blue},$$

with **arbitrary functions** $U_{ij}(t,\vec{x})$ of $t, \vec{x}$ satisfying $UU^\dagger = U^\dagger U = \underline{1},\ \det U = 1.$

QED	QCD
symmetry group	
$U(1)_{\mathrm{em}}$	$SU(3)_c$ group of special unitary transformations

QED	QCD

♣ The requirement of local symmetry is **not fulfilled for free particles** → interactions with

gauge fields resp. gauge particles

QED	QCD
1 electromagnetic field	$\mathbf{(3 \times 3 - 1)}$ **gluon fields**
$A^\mu(t,\vec{x}) = \begin{pmatrix} V(t,\vec{x}) \\ \vec{A}(t,\vec{x}) \end{pmatrix}$	$G^{\mu,\boldsymbol{A}}(t,\vec{x}),\ \boldsymbol{A = 1, ..., 8}$
photon spin 1, mass 0	**gluons** spin 1, mass 0
photons are electrically neutral	**gluons carry colour: decisive difference**

gluons carry colour: decisive difference

$\mathbf{r\,\bar{r}} \quad \mathbf{r\,\bar{g}} \quad \mathbf{r\,\bar{b}}$

$\mathbf{g\,\bar{r}} \quad \mathbf{g\,\bar{g}} \quad \mathbf{g\,\bar{b}}$

$\mathbf{b\,\bar{r}} \quad \mathbf{b\,\bar{g}} \quad \mathbf{b\,\bar{b}}$

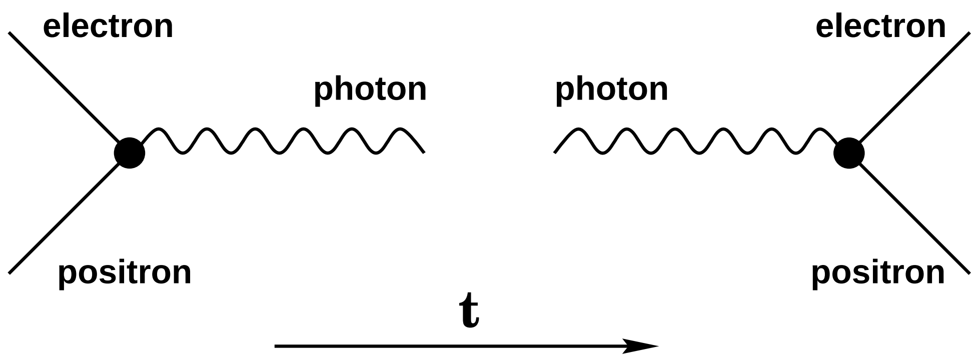
'minus' $\mathbf{r\,\bar{r} + g\,\bar{g} + b\,\bar{b}}$

♣ Local gauge invariance fixes all interactions in $\mathcal{L}_{\mathrm{QCD}}$ in terms of a single unknown

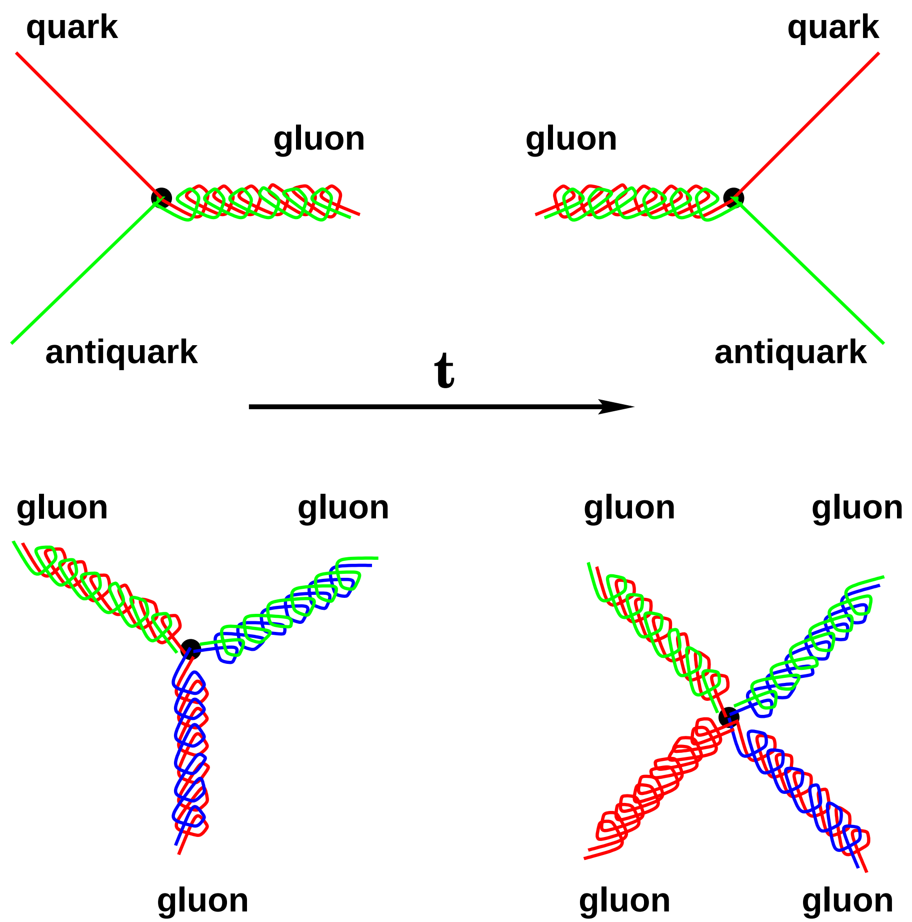
coupling constant $\boldsymbol{g_c}$ of QCD

♣ Feynman diagrams

QED



QCD



All couplings are completely determined in terms of a single unknown parameter, the QCD coupling constant g_c

unified electroweak gauge theory from the gauge principle

♣ parity violation in weak interactions

experimentally weak interaction processes violate the invariance with respect to

$$\text{space reflections} \quad \vec{x} \longrightarrow -\vec{x}$$

For each fermion $\psi(t,\vec{x}) = (\psi_L(t,\vec{x}), \psi_R(t,\vec{x}))$ with $\psi_L(t,\vec{x}) \to \psi_R(t,-\vec{x})$ and $\psi_R(t,\vec{x}) \to \psi_L(t,-\vec{x})$. Thus parity violation is implemented into the theory by treating differently the left-handed (ψ_L) and right-handed (ψ_R) components of the lepton and quark fields (see below). Since the handedness is only Lorentz invariant for massless fermions this implies as a

♣ starting point: massless leptons and quarks

♣ global symmetries to be gauged later on

The global symmetry of the system of **massless free** quarks and leptons is large (symmetry group $U(12)_L \times U(12)_R$). In nature only an $SU(2) \times U(1)$ subgroup appears to be gauged; the following selection leads to success

- $SU(2)_L$ weak isospin symmetry group

the left-handed leptons and quarks are arranged in doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

each described by a doublet of fields $\begin{pmatrix} \psi^L_{\boldsymbol{u.c.}}(t,\vec{x}) \\ \psi^L_{\boldsymbol{l.c.}}(t,\vec{x}) \end{pmatrix}$ $\quad \boldsymbol{I_3 = +1/2}$, $\boldsymbol{I_3 = -1/2}$

(u.c. for upper component, l.c. for lower component) with assigned quantum numbers I_3. The two quantum numbers $I_3 = \pm\frac{1}{2}$ play the role of generalized charges, in analogy to the three colours in QCD

invariance with respect to the **global $SU(2)_L$** transformations which leave invariant

$$\bar{\psi}^L_{\boldsymbol{u.c.}}\psi^L_{\boldsymbol{u.c.}} + \bar{\psi}^L_{\boldsymbol{l.c.}}\psi^L_{\boldsymbol{l.c.}}: \qquad \psi^L_i \rightarrow \sum_{j=1}^{2} U_{ij}\,\psi^L_j,\ \ i,j = u.c., l.c.$$

with $UU^\dagger = U^\dagger U = \underline{1}$, $\det U = 1$ for the 2×2 complex matrix U

Right-handed leptons and quarks

$e_R,\ \mu_R,\ \tau_R,\ u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$ are assigned zero weak isospin, $\boldsymbol{I_3 = 0}$

($\nu_{eR},\ \nu_{\mu R},\ \nu_{\tau R}$ do not exist in the SM)

- $U(1)_Y$ hypercharge symmetry group

Each l.-h. lepton and quark doublet and each r.-h. lepton and quark is assigned a so-called hypercharge quantum number $\boldsymbol{Y}$ with

$$\boldsymbol{Q = I_3 + Y/2}$$

where Q is the electric charge

invariance with respect to the **global $U(1)_Y$** transformations which leave $\bar{\psi}\psi$ invariant, i.e.

$$\psi(t, \vec{x}) \to e^{i\,\alpha\,Y_\psi}\,\psi(t, \vec{x})$$

ψ	Q_ψ	$I_{3\psi}$	Y_ψ
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	-1
$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	1/3
$e_R,\ \mu_R,\ \tau_R$	-1	0	-2
$u_R,\ c_R,\ t_R$	2/3	0	4/3
$d_R,\ s_R,\ b_R$	-1/3	0	-2/3

– **global $SU(2)_L \times U(1)_Y$ and $U(1)_{\text{em}}$ symmetries**

Because of $Q = I_3 + Y/2$, i.e. $e^{-i\alpha I_{3\psi}}\, e^{-i\alpha Y_\psi/2} = e^{-i\alpha Q_\psi}$

$U(1)_{\text{em}}$ is subgroup: $SU(2)_L \times U(1)_Y \supset U(1)_{\text{em}}$

♣ **requirement of local symmetry**

GAUGE PRINCIPLE

- **invariance** with respect to **local** $SU(2)_L \times U(1)_Y$ transformations $\rightarrow$

unified electroweak gauge interactions $\supset$ **QED**

♣ **2 undetermined gauge couplings**

$$SU(2)_L \times U(1)_Y \supset U(1)_{\text{em}}$$
$$\updownarrow \quad \updownarrow \quad \updownarrow$$
$$g \quad g' \quad e \quad \longrightarrow \quad \boxed{\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}}$$

with $e = g \sin\theta_W = g' \cos\theta_W$, θ_W=Weinberg angle

> 2 parameters **not fixed by the gauge principle**
> $g,\ g' \quad \longleftrightarrow \quad \boldsymbol{e,\ \sin\theta_W}$

♣ **gauge fields resp. gauge bosons**

$$SU(2)_L \quad \times \quad U(1)_Y \quad \supset \quad U(1)_{\text{em}}$$
$$\updownarrow \quad \updownarrow \quad \updownarrow$$
$$\overbrace{W^{1,2,3}_\mu(t,\vec{x})} \quad B_\mu(t,\vec{x}) \quad A_\mu(t,\vec{x})$$

$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \pm i W^2_\mu\right)$ (with electric charge ± 1)

$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$ = electromagnetic field

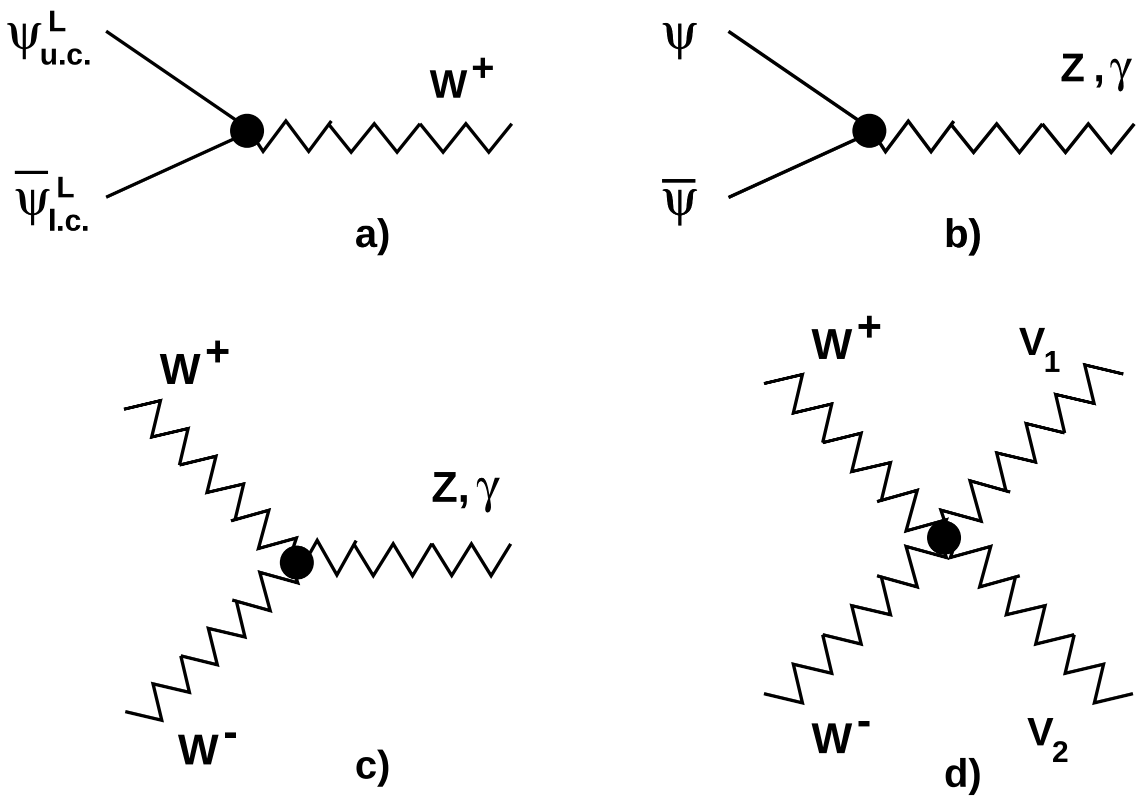
$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$ = orthogonal field combination

> gauge bosons $\boldsymbol{W^\pm,\ Z,\ \gamma}$

with spin 1, mass 0 (so far)

♣ interaction vertices of the electroweak gauge theory

all couplings are determined in terms of the two parameters $e,\ \sin\theta_W$



a) $\begin{pmatrix} \psi^L_{u.c.} \\ \bar{\psi}^L{}_{l.c.} \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e^+_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu^+_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau^+_L \end{pmatrix}, \begin{pmatrix} u_L \\ \bar{d}_L \end{pmatrix}, \begin{pmatrix} c_L \\ \bar{s}_L \end{pmatrix}, \begin{pmatrix} t_L \\ \bar{b}_L \end{pmatrix}$

b) $\psi = \nu_e,\ \nu_\mu,\ \nu_\tau,\ e,\ \mu,\ \tau,\ u,\ d,\ c,\ s,\ t,\ b$; no coupling of $\nu\,\bar{\nu}$ to γ

c) three gauge boson vertices

d) four gauge boson vertices
$V_1\,V_2 = W^+\,W^-,\ Z\,Z,\ Z\,\gamma,\ \gamma\,\gamma$

spontaneous symmetry breakdown

♣ aim **masses**

– for the gauge bosons $W^{\pm}$, Z (experimentally $m_W \approx 80$ GeV, $m_Z \approx 91$ GeV)

– for the quarks and charged leptons

without explicitly breaking the local $SU2_L \times U(1)_Y$ gauge symmetry
('explicit' $\longrightarrow$ on the level of the forces, i.e. of the Lagrange density)

♣ characteristics of spontaneous symmetry breakdown [SSB]

– symmetry is **unbroken** on the level of the forces

– **groundstate breaks** the symmetry

SSB appears in (classical and quantum) systems with infinitely many degrees of freedom

♣ classical example

– elastic rod, length l, radius r, Young elasticity modul E

– force $\vec{F}$ in direction of the rod axis
$\longrightarrow$ **cylindrical symmetry** with respect to the rod axis

– critical value of the force $F_{\text{crit}} = \frac{\frac{\pi^3}{4} r^4}{l^2} E$

$\boldsymbol{F} < \boldsymbol{F}_{\text{crit}}$ $\boldsymbol{F} > \boldsymbol{F}_{\text{crit}}$

♣ SSB in scalar field theory with global $U(1)$ symmetry

– global symmetry on the level of the Lagrange density

a complex scalar (spin 0) field $\Phi(t,\vec{x}) = e^{i\xi(t,\vec{x})}\rho(t,\vec{x})$ i.e. two real scalar fields $\xi,\ \rho$

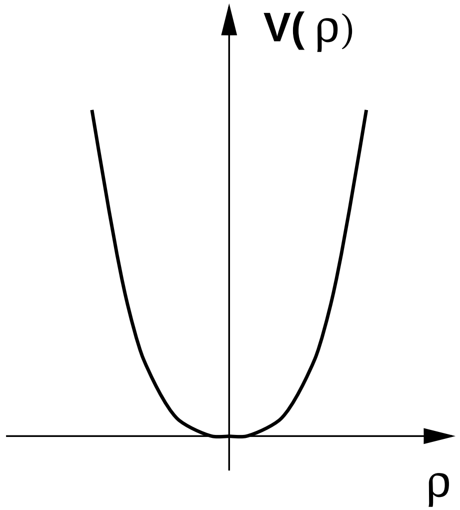
$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^\dagger \partial^\mu \Phi - \underbrace{\left(\frac{\mu^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}\Phi^\dagger\Phi\Phi^\dagger\Phi\right)}_{\text{potential } V(\Phi) = V(\rho) = \frac{\mu^2}{2}\rho^2 + \frac{\lambda}{4}\rho^4,\ \lambda > 0}$$

global $U(1)$ symmetry w.r. to $\Phi(t,\vec{x}) \to e^{i\alpha}\,\Phi(t,\vec{x})$

$\alpha =$ arbitrary constant

– ground state $\frac{\partial V(\rho)}{\partial\rho} = 0 \quad \longleftrightarrow \quad \mu^2\rho + \lambda\rho^3 = 0$

$\mu^2 > 0$

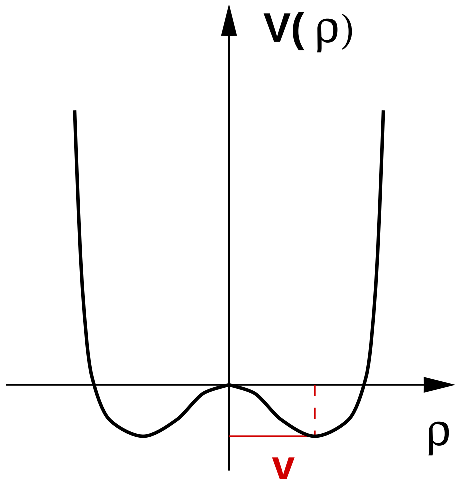


minimum at

$\rho(t,\vec{x}) = 0$

no SSB

$\mu^2 < 0$



minimum at

$\rho(t,\vec{x}) = \pm v$

$v = \sqrt{\frac{|\mu|^2}{\lambda}}$

SSB

expectation value of the field ρ in the ground state $|0>$ $\quad <0 \mid \rho(t,\vec{x}) \mid 0> = v \neq 0$

field shift $\rho(t,\vec{x}) = v + \eta(t,\vec{x}), \; <0 \mid \eta(t,\vec{x}) \mid 0> = 0$

♣ **SSB in scalar field theory with local $U(1)$ gauge symmetry**
Higgs mechanism

– **local gauge symmetry on the level of the Lagrange density**

$$\mathcal{L} = \frac{1}{2}\left(\mathcal{D}_\mu\Phi\right)^\dagger \mathcal{D}^\mu\Phi - V(\Phi) + \mathcal{L}^{A_\mu}_{\text{free}}$$

$$\mathcal{D}_\mu = \partial_\mu + i\,g\,A_\mu(t,\vec{x})$$

where $A_\mu(t,\vec{x})$ is the $U(1)$ gauge field and g the $U(1)$ gauge coupling

local $U(1)$ gauge symmetry with respect to

$$\Phi(t,\vec{x}) \;\rightarrow\; e^{i\alpha(t,\vec{x})}\,\Phi(t,\vec{x})$$

$\alpha(t,\vec{x})$ **= arbitrary function of** $(t,\vec{x})$

– **Higgs mechanism** $\mu^2 < 0$, i.e. SSB: $\Phi(t,\vec{x}) = e^{i\xi(t,\vec{x})}\,(v + \eta(t,\vec{x}))$

special gauge transformation to **unitary gauge**

$$\alpha(t,\vec{x}) = -\xi(t,\vec{x}) \;\rightarrow\; \Phi(t,\vec{x}) \rightarrow (v + \eta(t,\vec{x}))$$

$\frac{1}{2}\left(\mathcal{D}_\mu\Phi\right)^\dagger \mathcal{D}^\mu\Phi$ in $\mathcal{L}$ contains $\frac{1}{2}(vg)^2 A_\mu A^\mu =: \frac{1}{2}m_A^2 A_\mu A^\mu$

the ξ-field is **'eaten'** by A_μ

the gauge field A_μ aquires a mass $m_A = gv$

balance of number of fields

before field shift		after field shift, in unitary gauge	
A_μ, spin 1, mass=0	2	A_μ, spin 1, mass $\neq 0$	3
ξ, η	2	η	1

– the physical Higgs field $\boldsymbol{\eta(t, \vec{x})}$ with spin 0 and mass $\boldsymbol{m_H = \sqrt{2} \mid \mu \mid}$

♣ SSB in the Standard Model

– Higgs sector 4 scalar fields ($=$ 1 complex $SU(2)_L$ doublet field with hypercharge $Y = +1$)

– local $SU(2)_L \times U(1)_Y$ gauge invariance in $\mathcal{L}$

* including the Higgs sector
* including gauge invariant Yukawa couplings of l.h. and r.h. lepton and quark fields to the Higgs doublet field

– additional parameters

$*$ $\boldsymbol{\mu^2, \lambda}$ in V(scalar fields), **SSB** for $\mu^2 < 0$ $\longleftrightarrow$ $\boldsymbol{v} = \sqrt{\frac{|\mu|^2}{\lambda}}$, $\boldsymbol{m_H} = \sqrt{2}\,|\mu|$

$*$ $\boldsymbol{G_\psi}$, a Yukawa coupling for each quark and charged lepton

– spontaneous symmetry breakdown is arranged such that

$$\boldsymbol{SU(2)_L \times U(1)_Y} \stackrel{\text{spontaneously broken to}}{\longrightarrow} \boldsymbol{U(1)}_{\text{em}}$$

– Higgsmechanism $\rightarrow$ massive gauge fields $W^\pm$, Z

3 of the 4 scalar fields are eaten by the $W^\pm$, Z gauge fields $\rightarrow$

$$\boldsymbol{m_{W^\pm} = \frac{gv}{2}}, \quad \boldsymbol{m_Z = \frac{m_{W^\pm}}{\cos\theta_W}}$$

$m_\gamma = 0$ remains

– masses for quarks and charged leptons from the Yukawa couplings

$$m_\psi = \frac{v}{\sqrt{2}} G_\psi \quad \text{for } \psi = \text{quarks and charged leptons}$$

– one physical Higgs boson spin 0, $m_H = \sqrt{2}\,|\,\mu\,|$

– additional interactions

* Higgs boson selfinteractions
* gauge interactions of Higgs bosons with gauge bosons $W^\pm$, Z
* Yukawa interactions of Higgs bosons with quarks and leptons

– additional parameters

4 parameters for quark mass mixing
$\longrightarrow$ violation of invariance under time reversal $t \longrightarrow -t$

summary on the gauge theory of the Standard Model

the Standard Model is a **local gauge theory** with gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

and spontaneous symmetry breakdown

$$SU(2)_L \times U(1)_Y \overset{\text{spontaneously broken to}}{\longrightarrow} U(1)_{\text{em}}$$

All gauge interactions are fixed by the gauge principle in terms of the three parameters

- g_c, the gauge coupling of the $SU(3)_c$ colour gauge interactions
- e, the gauge coupling of the $U(1)_{\text{em}}$ electromagnetic gauge interactions
- $\sin\theta_W$, relating e by $e = g\sin\theta_W$ and $e = g'\cos\theta_W$ to the gauge couplings g and g' of the $SU(2)_L \times U(1)_Y$ unified electroweak gauge interactions

4. quantum effects, some applications and key precision tests

♣ quantum effects and precision tests in QED

– determination of α_{em} from quantum hall effect $\boldsymbol{\alpha}_{\text{em}} = \mathbf{1/137.03599911(46)}$

– precision test **magnetic moment of the electron**

the **electron** has spin = intrinsic angular momentum $(= \frac{1}{2}\hbar)$ and electric charge (-1)
$\longrightarrow$ it has a **magnetic moment**

$$\mu_e = (1 + \boldsymbol{a_e})\mu_B \qquad \mu_B = \text{Bohr magneton}$$

2002: $\boldsymbol{a_{e\ \text{exp}}} = \mathbf{0.0011596521859}(38)$

$\longrightarrow$ a second determination of

$\boldsymbol{\alpha}_{\text{em}} = \mathbf{1/137.0359988(5)}$

of equal precision and in perfect agreement

$$\boldsymbol{a_{e\ \text{theo}}} = \frac{1}{2}\frac{\boldsymbol{\alpha}_{\text{em}}}{\pi} + C_2\left(\frac{\boldsymbol{\alpha}_{\text{em}}}{\pi}\right)^2 + C_3\left(\frac{\boldsymbol{\alpha}_{\text{em}}}{\pi}\right)^3 + C_4\left(\frac{\boldsymbol{\alpha}_{\text{em}}}{\pi}\right)^4 + \ldots$$

$C_2,\ C_3,\ C_4$ calculated in QED with

m_e=0.510998918(44) MeV $\longrightarrow$

– precision test **magnetic moment of the muon**

2004 and 2006: $\boldsymbol{a_{\mu\,\text{exp}} = 0.00116592080(63)}$

with α_{em} and $m_e \longrightarrow m_\mu$=105.6583692(94) MeV up to $O((\alpha_{\text{em}}^5))$ as well as including weak and hadronic quantum corrections!

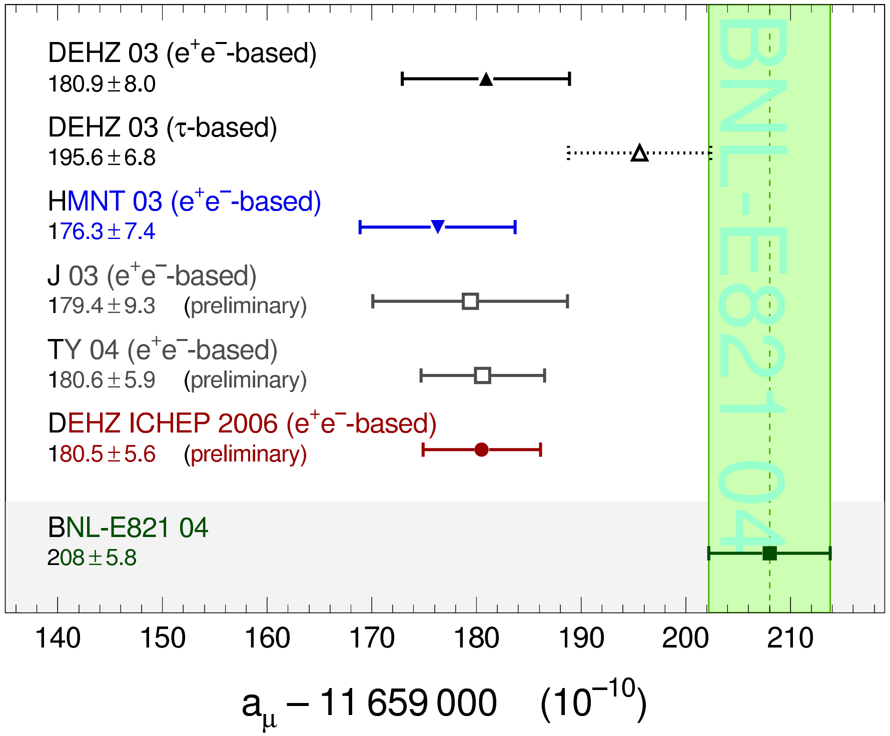
2005 and 2006: $\boldsymbol{a_{\mu\,\text{theor}} = 0.00116591805(56)}$

$$\boldsymbol{\Delta\, a_\mu = a_{\mu\,\text{exp}} - a_{\mu\,\text{theor}} = (27.5 \pm 8.4) \times 10^{-10}}$$

deviation of 3.3σ

signal for new physics beyond the standard model?

Theory vs Experiment – II

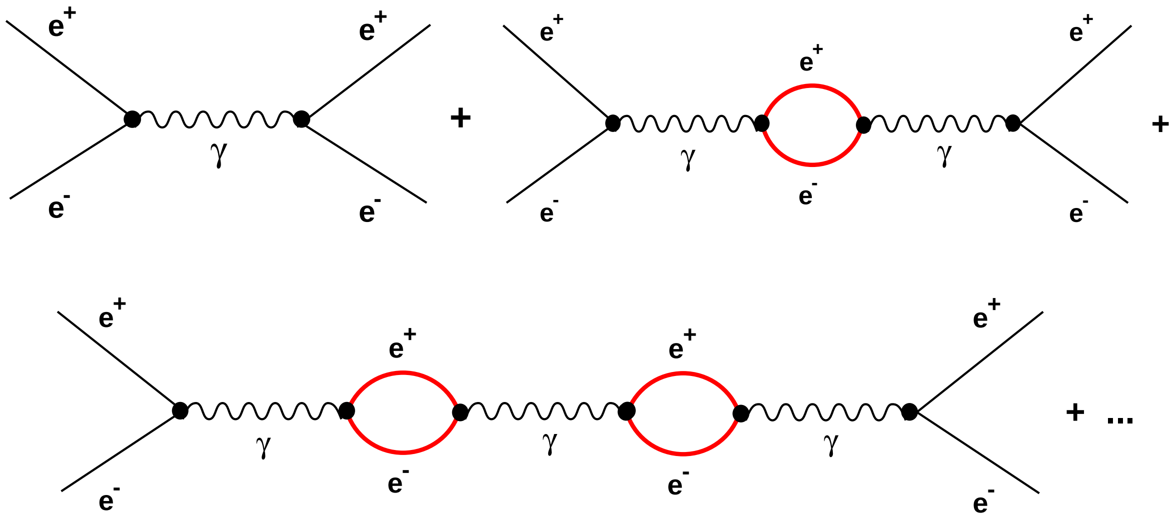


The difference between experiment and theory is 3.3σ!

♣ running couplings in QED and QCD – confinement and asymptotic freedom

– QED   for simplicity for electrons only

evaluation of an important class of diagrams to all orders in perturbation theory of QED leads to



running coupling of QED

$$\alpha_{em}(Q^2) = \frac{\alpha_{em}(Q_0^2)}{1 - \frac{\alpha_{em}(Q_0^2)}{3\pi} \log \frac{Q^2}{Q_0^2}}$$

$\sqrt{Q^2} =$ momentum transfer

uncertainty principle:

$\sqrt{Q^2}\,\Delta x \gtrsim \hbar$

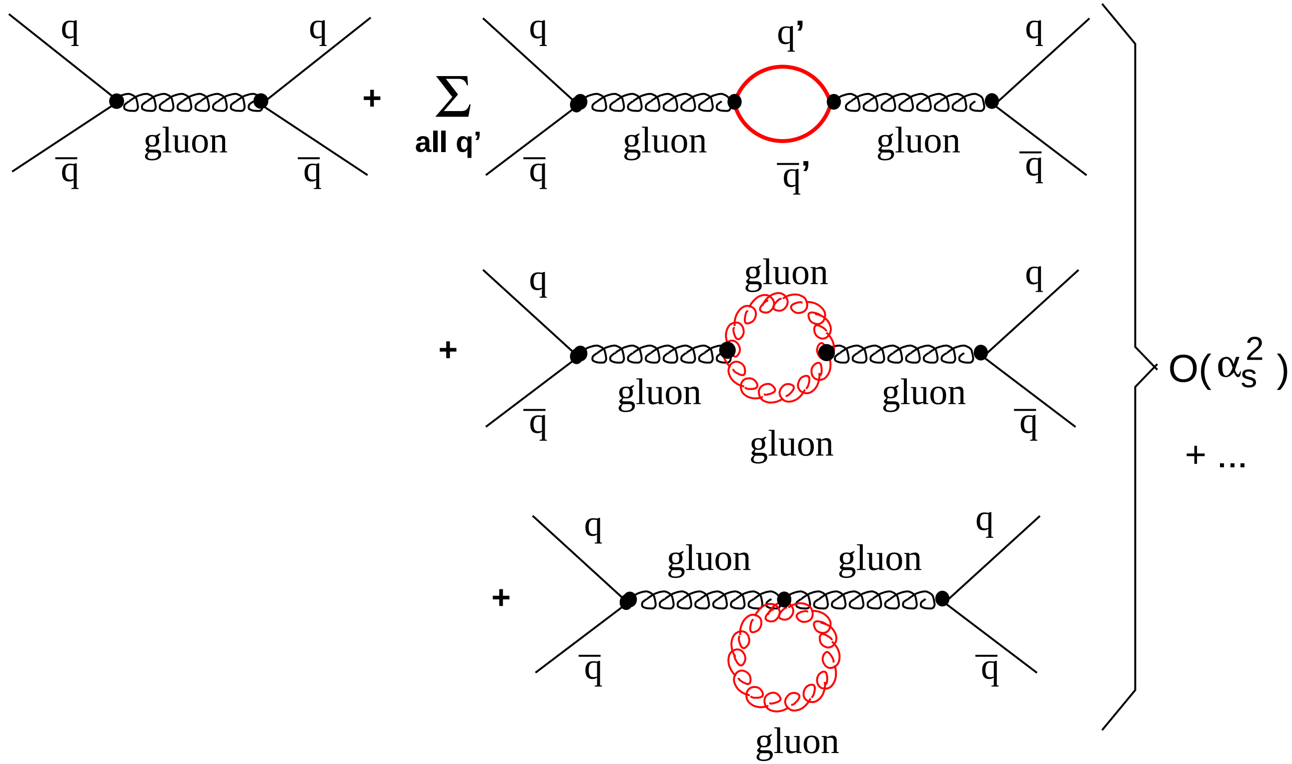
$\alpha_{em}(0) = 1/137.03599911(46)$

increasing distance Δx ↗, i.e. Q^2 ↘: $\alpha_{em}(Q^2)$ ↘

screening of electric charge

– QCD

evaluation of the corresponding class of diagrams to all orders in perturbation theory of QCD – provided $\alpha_s = g_c^2/(4\pi) \ll 1$ – leads to



valid for $\alpha_s \ll 1$

$n_q =$ number of quark flavours, $11 - \frac{2}{3}n_q > 0$ for $n_q \leq 6$

running coupling of QCD

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + (11 - \frac{2}{3}n_q)\frac{\alpha_s(Q_0^2)}{4\pi}\log\frac{Q^2}{Q_0^2}}$$

antiscreening **screening**

the antiscreening is the **consequence of the gluon selfinteraction**, which in turn is the consequence of the gauge principle!

$$\alpha_s(\boldsymbol{Q^2}) = \frac{\alpha_s(Q_0^2)}{1 + (\mathbf{11} - \frac{2}{3} n_q) \frac{\alpha_s(Q_0^2)}{4\pi} \log \frac{\boldsymbol{Q^2}}{Q_0^2}}$$

increasing distance $\Delta x \nearrow$, i.e. $Q^2 \searrow$: $\alpha_s(Q^2) \nearrow$

antiscreening of colour

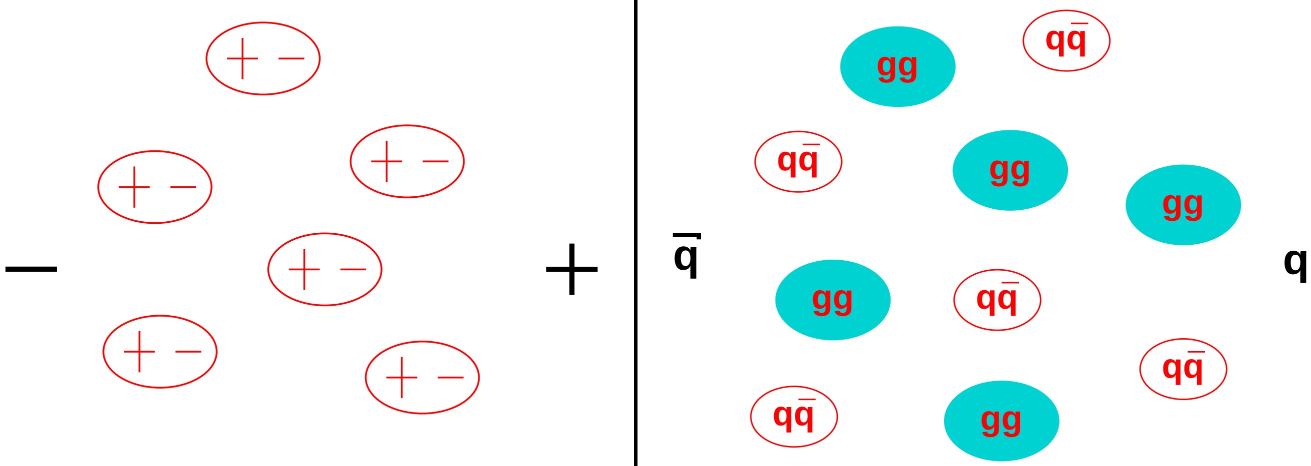
$\longrightarrow$ suggests **confinement**

decreasing distance $\Delta x \searrow$, i.e. $Q^2 \nearrow$: $\alpha_s(Q^2) \searrow \mathbf{0}$

$\longrightarrow$ **asymptotic freedom** (no interaction for $Q^2 \rightarrow \infty$)

– qualitative discussion

QED	QCD



vacuum polarisation

the +- pairs effect	the $q\bar{q}$ pairs effect screening the gg pairs antiscreening

altogether

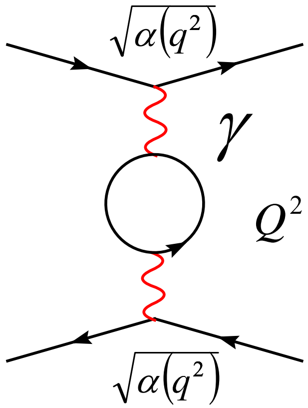
screening	**antiscreening**

2006: running $\alpha_{\rm em}(Q^2)$ – on the right in a plot $1/\alpha_{\rm em}(Q^2)$ – versus Q^2 from LEP

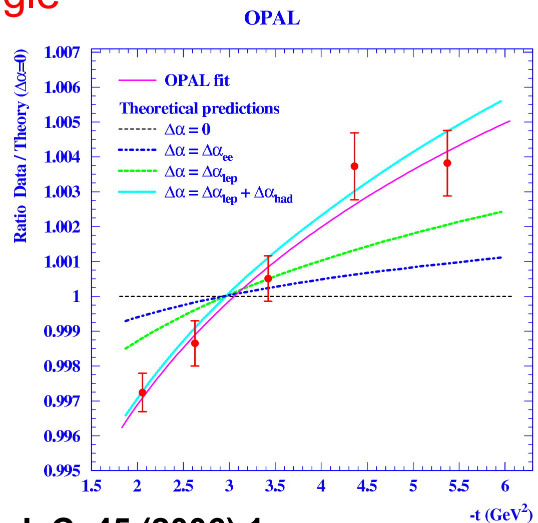
γee couplings: α_{EM} at LEP

- Running of α_{EM} measured at LEP
- Bhabha scattering at low angle dominated by t-channel
  - $q^2<0$
  - q^2 measured by scattering angle

$$\alpha(q^2) = \frac{\alpha_0}{1-\Delta\alpha(q^2)}$$



$$Q^2 = -q^2 = -t$$



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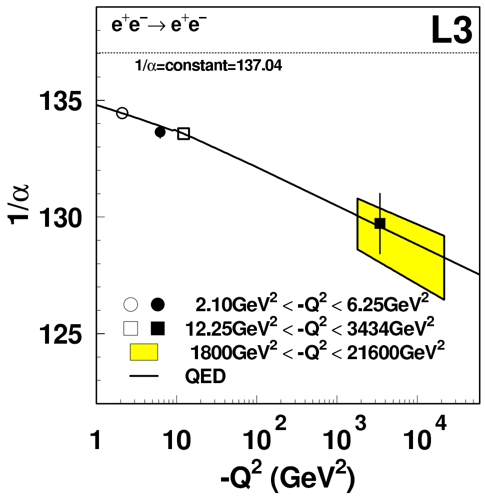

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3 ICHEP 2006 World Average

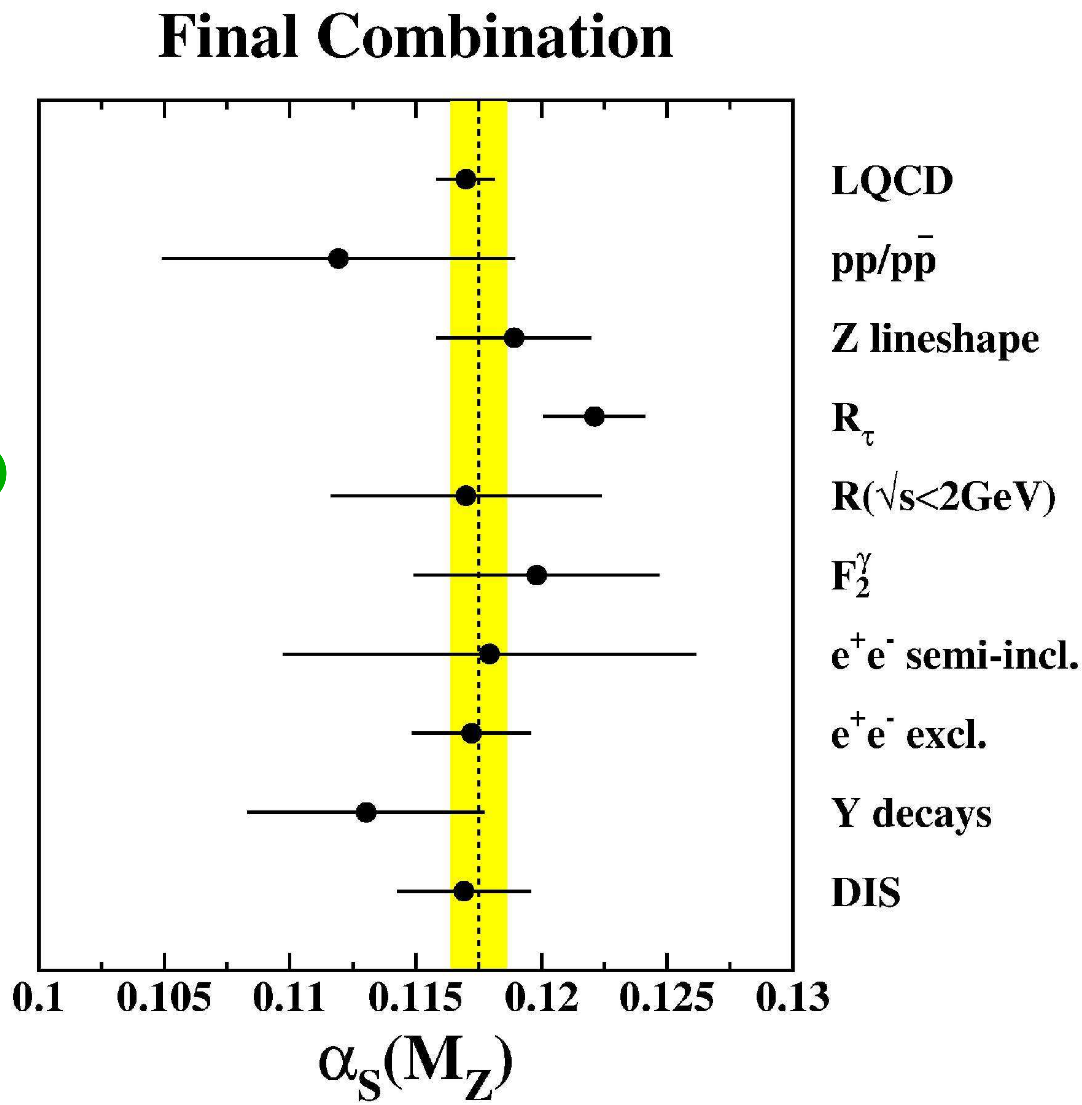
Final average (uuup):

$\alpha_s(m_z) = 0.1175 \pm 0.0006(\text{stat.})$
$\pm 0.0001(\text{exp.})$
$\pm 0.0002(\text{soft})$
$\pm 0.0009(\text{hard})$

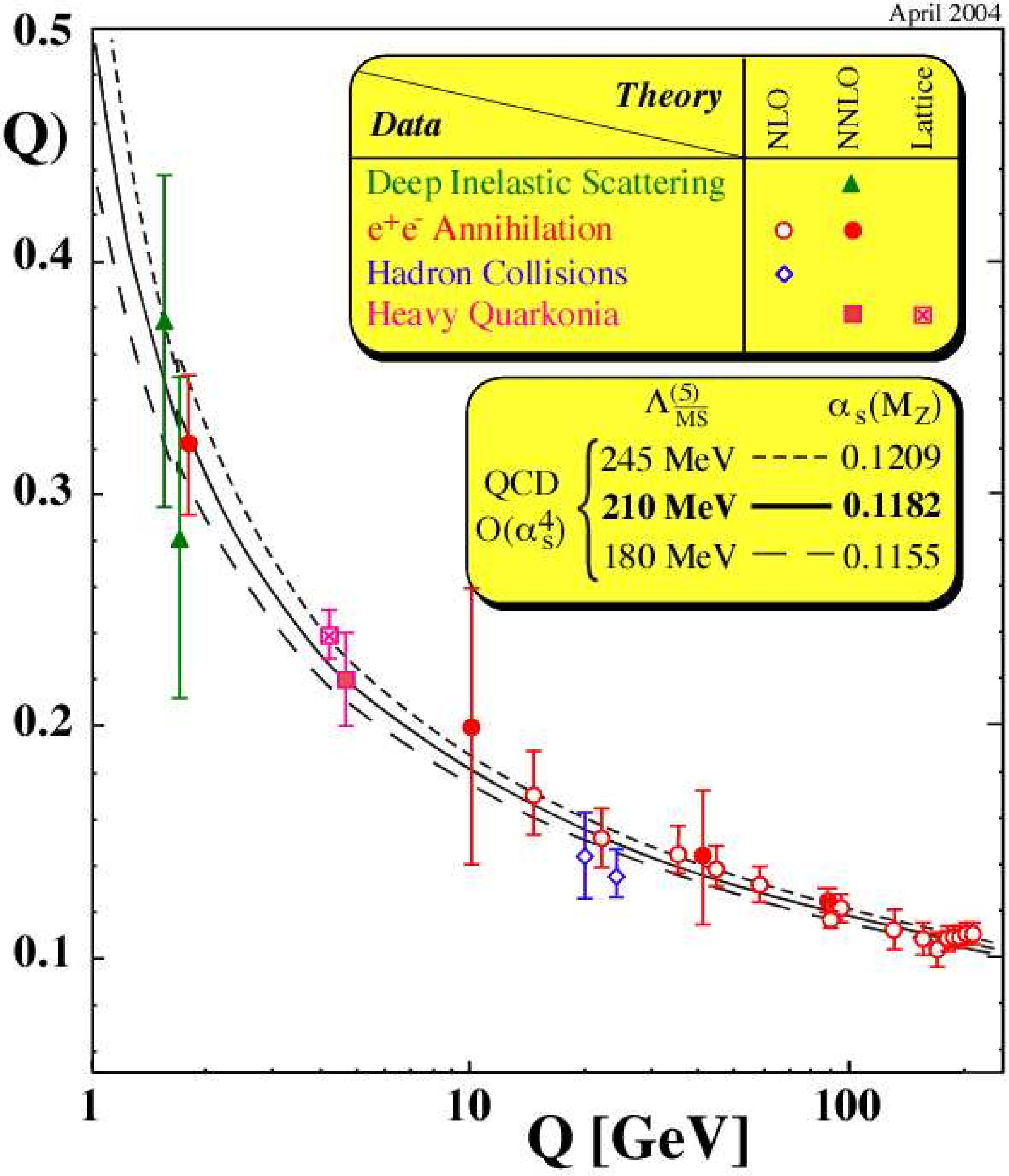
$\alpha_s(m_z) = 0.1175 \pm 0.0011(\text{tot.})$

$\chi^2/\text{d.o.f.} = 17/10,\ P(\chi^2) = 0.07$

no LQCD:
$\alpha_s(m_z) = 0.1197 \pm 0.0018(\text{tot.})$

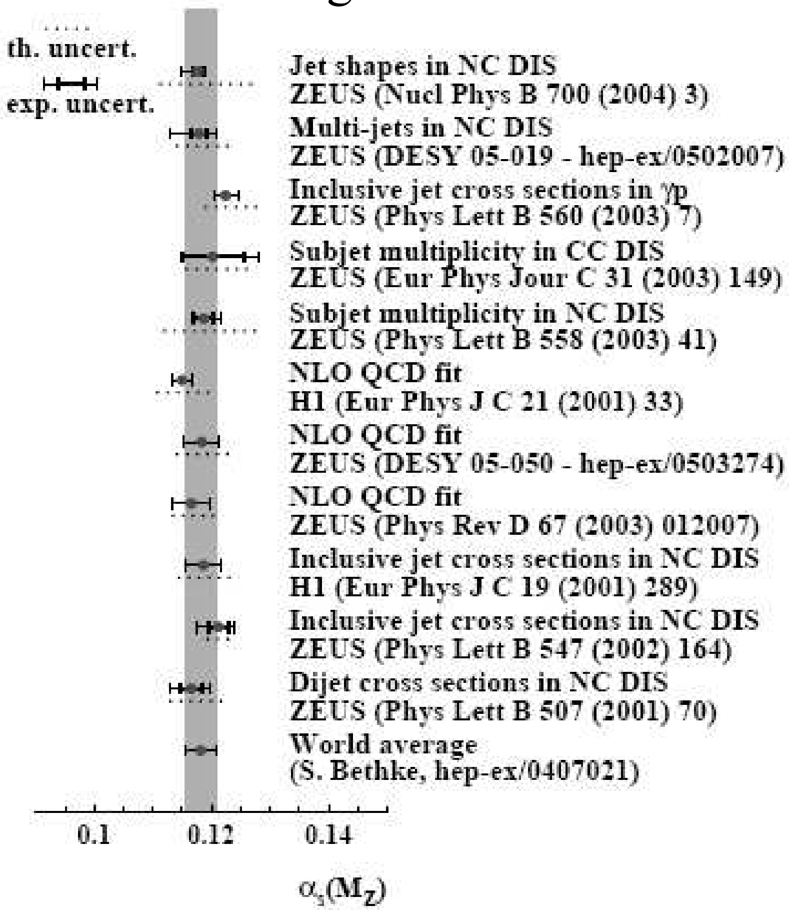


running $\alpha_s(Q)$

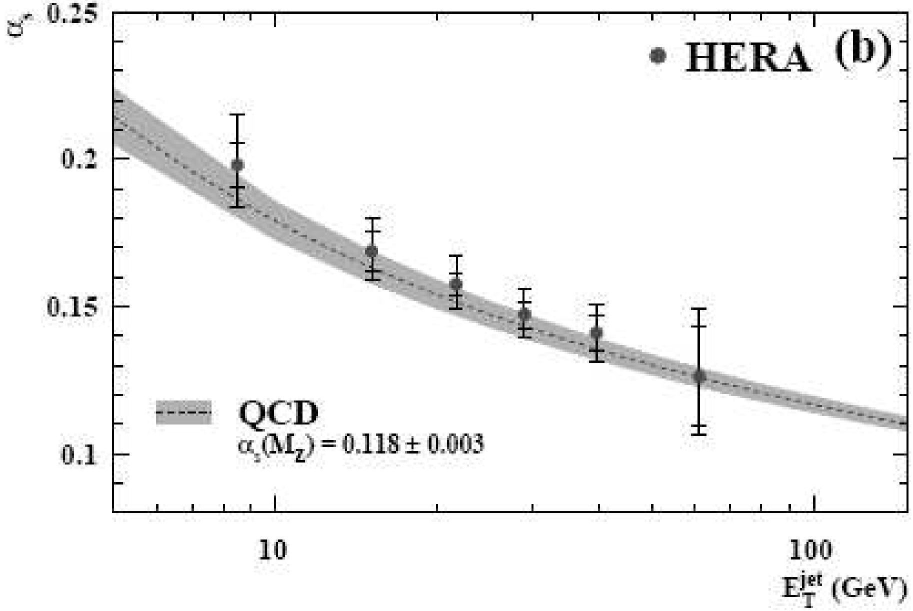


HERA α_s summary

- C.f. HERA α_s measurements and world average:



- α_s measurements from jets.



Glasman

- Preliminary HERA average:

$$\alpha_s(m_Z^2) = 0.1186 \pm 0.0011(\text{exp.})$$
$$\pm 0.0050(\text{th.})$$

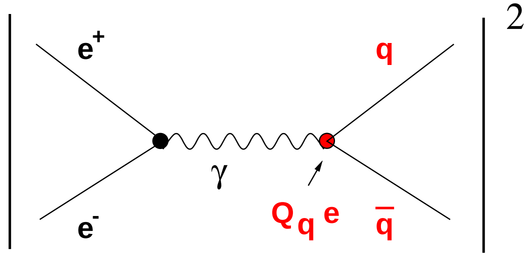
♣ test of asymptotic freedom and of three colours   process of interest:

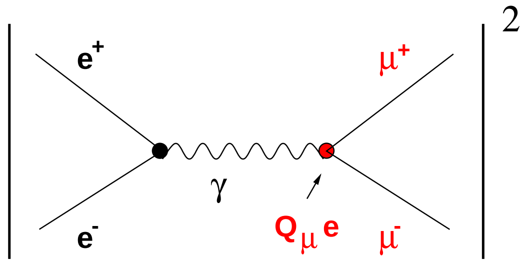
$e^+e^- \longrightarrow$ **all hadronic final states** at small distances, i.e. at **large Q^2** $(m_b^2 \ll Q^2 \ll m_Z^2)$

> parton model = 'zeroth order' QCD, asymptotic freedom approximated by **$\alpha_s = 0$: no colour interaction** between the q and $\bar{q}$, i.e. no exchange or radiation of gluons, etc.

$$R_{e^+e^-} = \frac{\sum_{\text{all hadronic final states}} |\, e^+e^- \longrightarrow \text{hadrons}\,|^2}{|\, e^+e^- \longrightarrow \mu^+\mu^-\,|^2} =$$

$\sum$ all quarks q



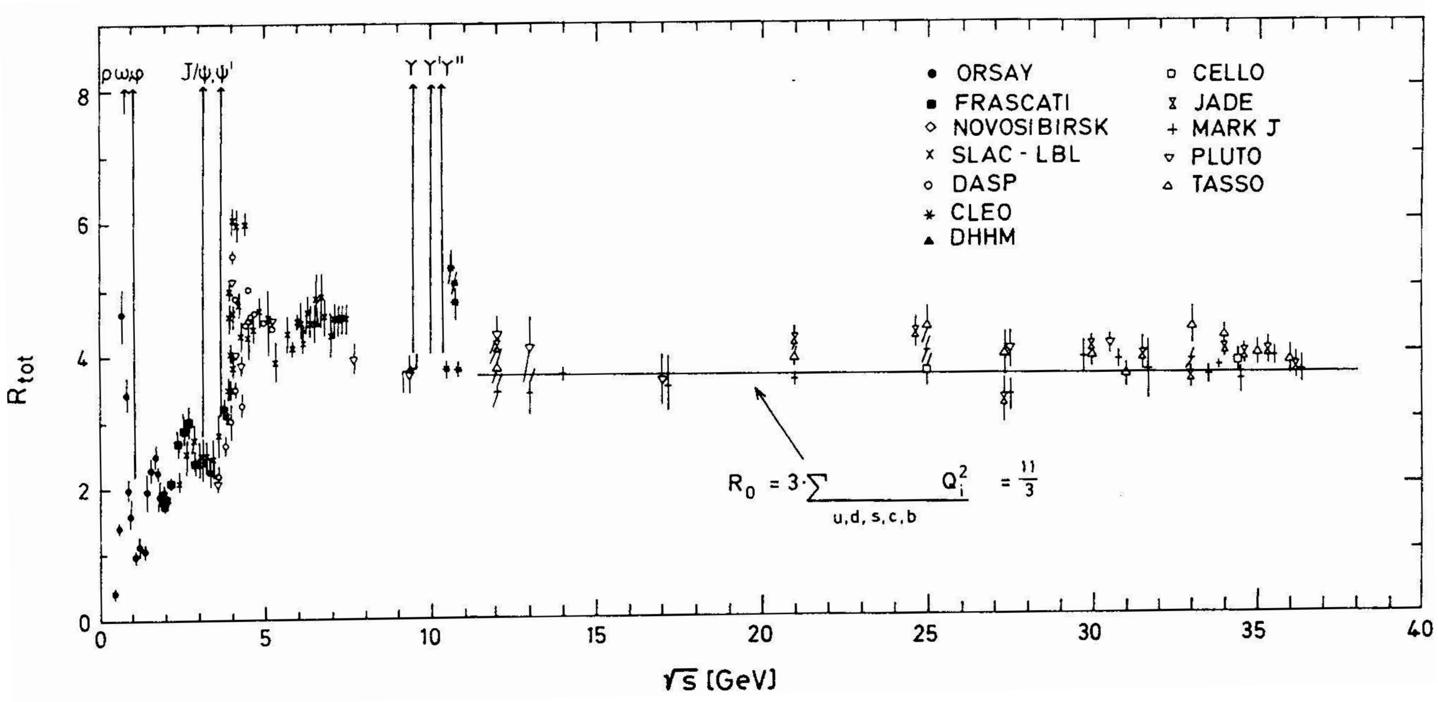


$\times$ { probability that the qq̄ final state turns at large distances into some hadronic final state }

$\equiv$ 1 due to confinement

$$= \sum_{\text{all quarks q}} Q_q^2 = \underset{\uparrow}{3} \sum_{q=u,d,s,c,b} Q_q^2 = 3\left(\underset{u}{\tfrac{4}{9}} + \underset{d}{\tfrac{1}{9}} + \underset{s}{\tfrac{1}{9}} + \underset{c}{\tfrac{4}{9}} + \underset{b}{\tfrac{1}{9}}\right) = \frac{11}{3} = 3.67$$

colour

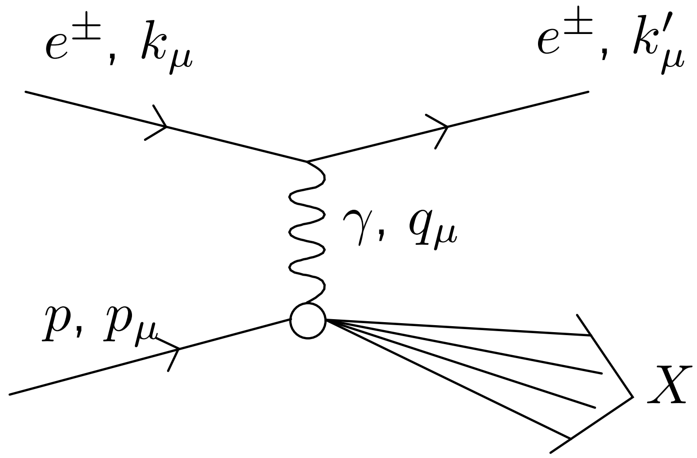


sensitive to

– asymptotic freedom
– number of colours
– electric charges of the quarks

♣ deep inelastic scattering and proton structure functions as test of perturbative QCD

HERA!



– process

$e^{\pm}p \;\rightarrow\; e^{\pm}$ **all hadronic final states (X)**

at small distances, i.e. at **large Q^2**

– two variables

* $\boxed{Q^2} = -q^2 = -(\text{momentum transfer})^2$ carried by the photon

  $Q^2 \nearrow$:

  · the **resolution increases** with which the photon probes the (electrically charged) constituents of the proton, i.e. the quarks

  · $\alpha_S(Q^2) \searrow$, which allows to treat the interactions between the quarks and gluons in the proton within the framework of QCD perturbation theory

* $\boxed{x} = \frac{Q^2}{2p\cdot q} =$ fraction of the proton momentum carried by the quark interacting with the photon ($0 \leq x \leq 1$)

– **quark distribution functions** $q_i(x, Q^2)$ = probability to find the quark q_i in the proton with proton momentum fraction x, probed by the photon carrying Q^2.

– **parton model = 'zeroth' order QCD** asymptotic freedom approximated by $\alpha_s = 0$

$$\rightarrow \quad q_i(x, Q^2) = q_i(x), \quad Q^2 \text{ independence} \quad \rightarrow \quad \textbf{scaling}$$

– **first order QCD** DGLAP equations (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi)

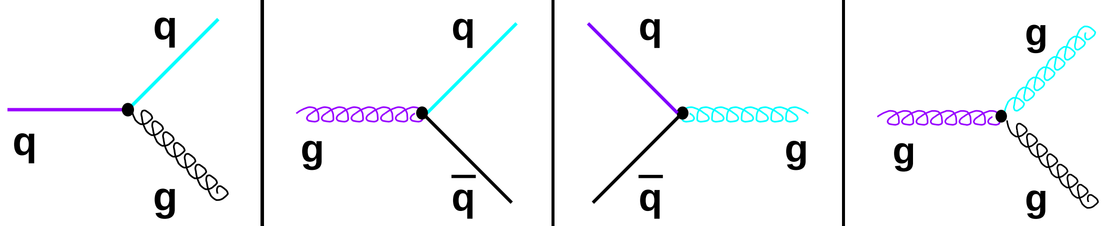
coupled integro-differential equations for the quark distribution functions $q_i(x, Q^2)$ and the gluon distribution function $g(x, Q^2)$

$$Q^2 \frac{\partial q_i(x, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} \left(q_i(y, Q^2) P_{qq}(\frac{x}{y}) + g(y, Q^2) P_{qg}(\frac{x}{y})\right)$$

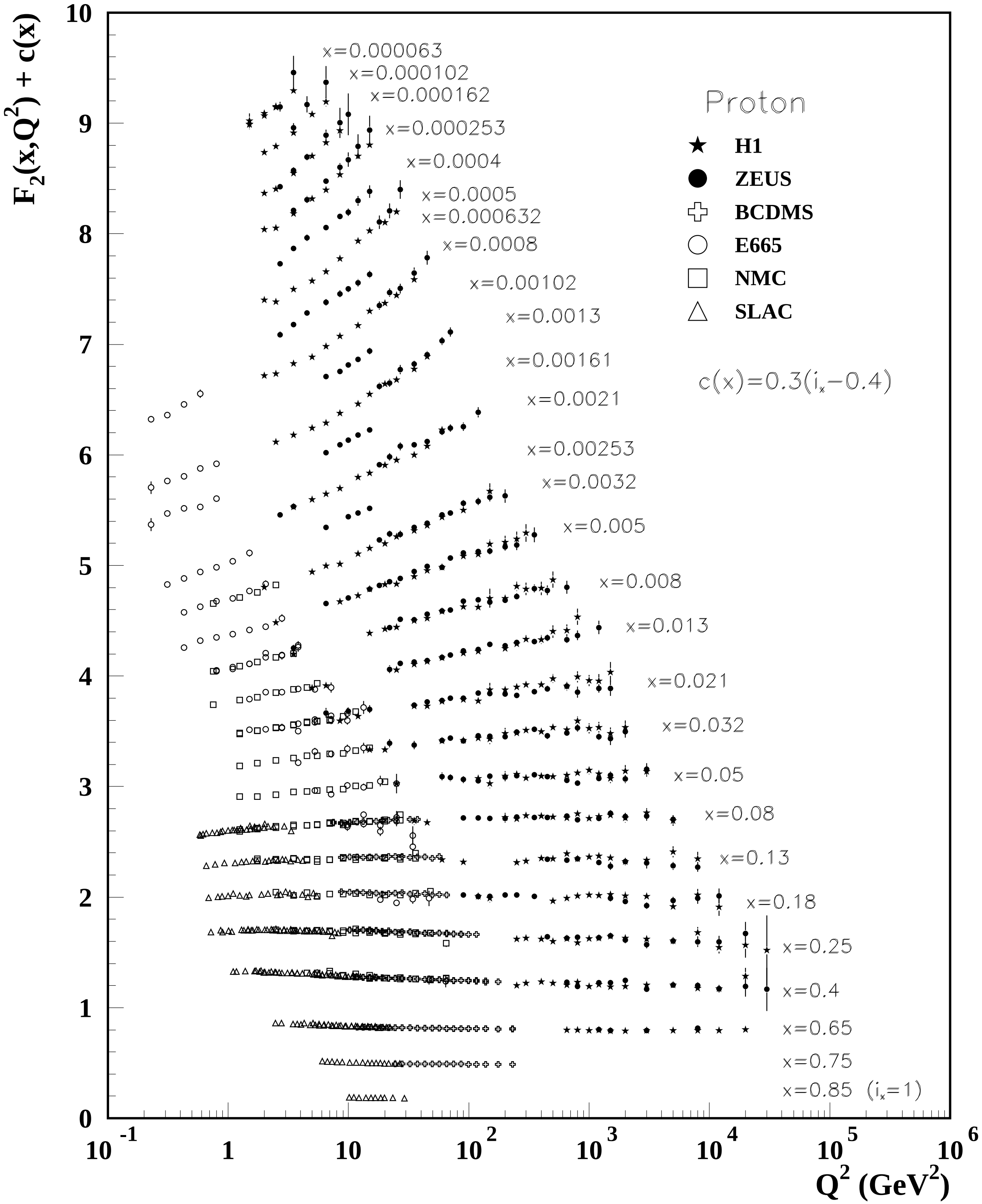
$$Q^2 \frac{\partial g(x, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{\mathrm{d}y}{y} \left(\sum_i q_i(y, Q^2) P_{gq}(\frac{x}{y}) + g(y, Q^2) P_{gg}(\frac{x}{y})\right)$$

the splitting functions P are known from QCD

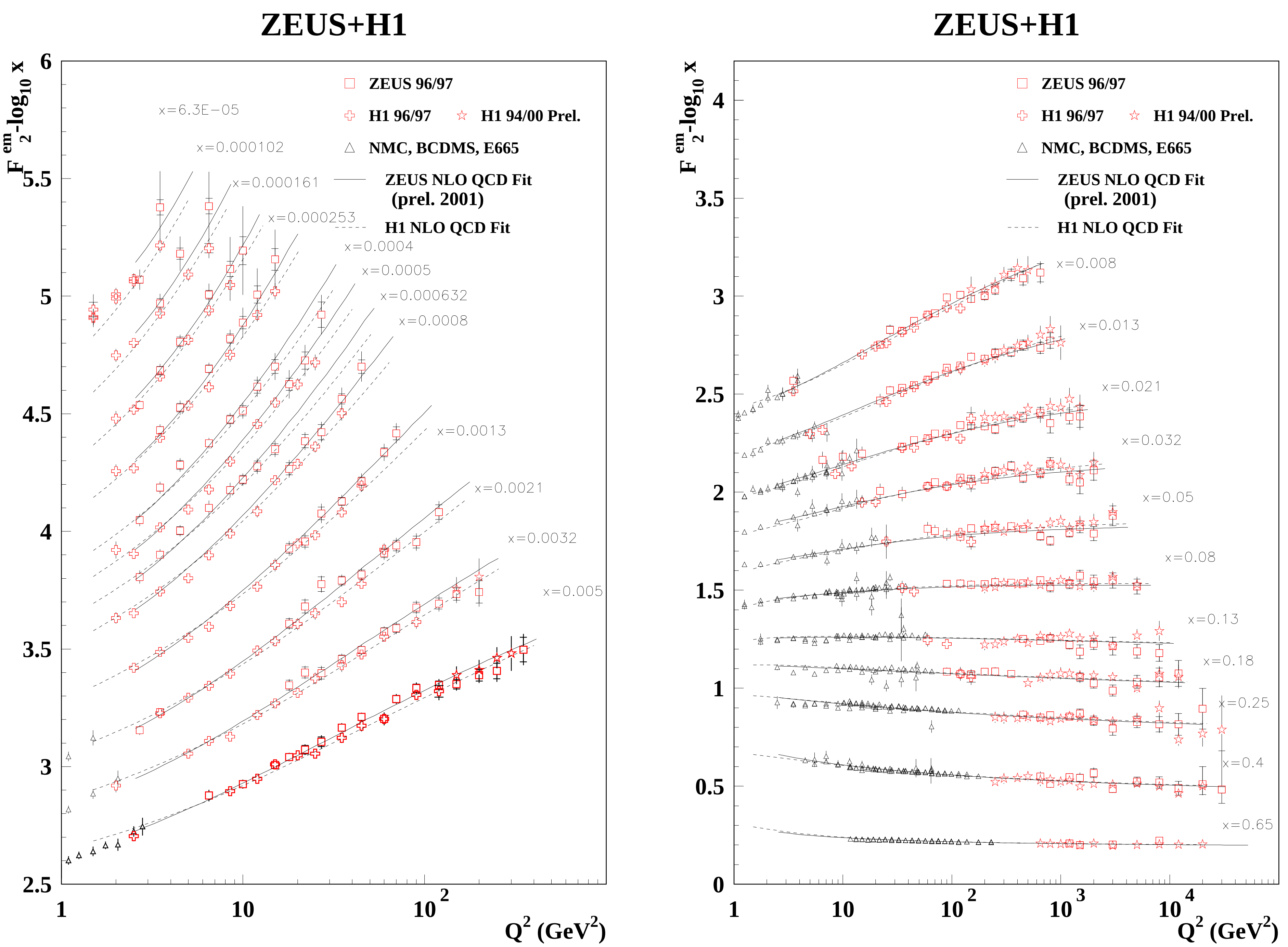
$P_{qq}(z)$	$P_{qg}(z)$	$P_{gq}(z)$	$P_{gg}(z)$



prediction of **scaling violation** as function of Q^2



Proton
H1
ZEUS
BCDMS
E665
NMC
SLAC
c(x)=0.3(i_x−0.4)
x=0.000063
x=0.000102
x=0.000162
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.00102
x=0.0013
x=0.00161
x=0.0021
x=0.00253
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
x=0.75
x=0.85 (i_x=1)
F_2(x,Q^2) + c(x)
Q^2 (GeV^2)


ZEUS+H1
ZEUS 96/97
H1 96/97
H1 94/00 Prel.
NMC, BCDMS, E665
ZEUS NLO QCD Fit
(prel. 2001)
H1 NLO QCD Fit
x=6.3E-05
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
ZEUS+H1
ZEUS 96/97
H1 96/97
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H1 NLO QCD Fit
x=0.008
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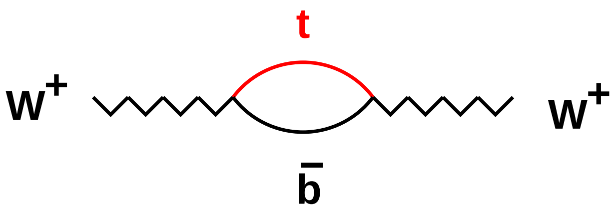
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65

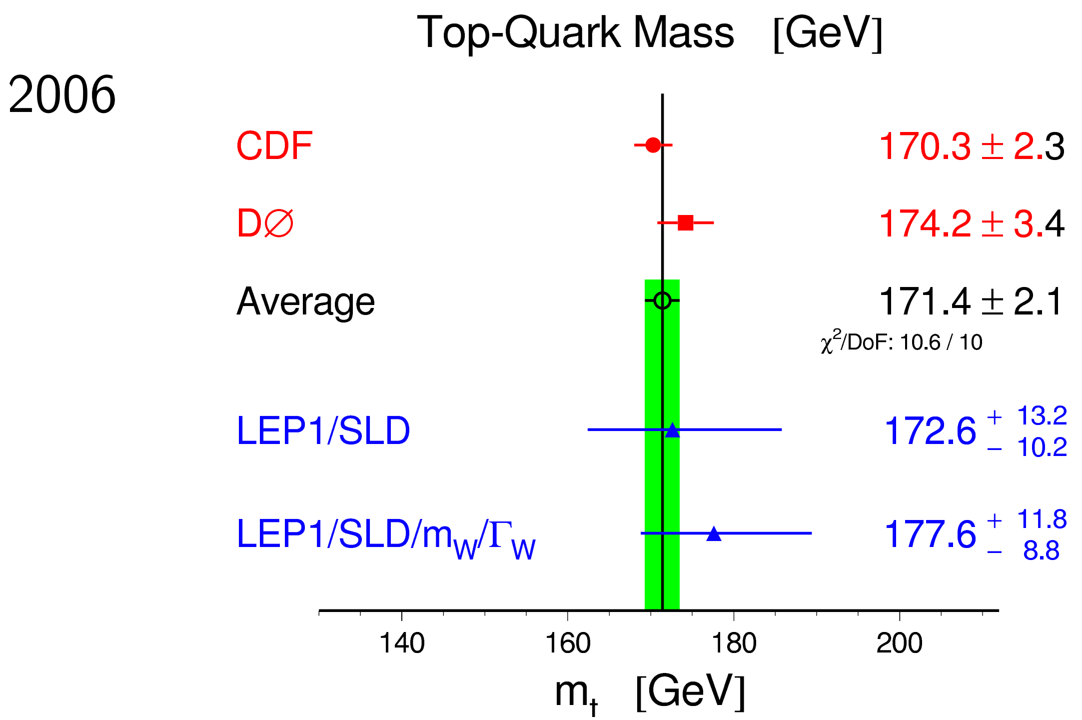
♣ quantum effects in electroweak interactions

– indirect determination of m_t through its effect in loops at LEP and SLD, eg.



assumption: no effects from new physics beyond the Standard Model

– comparison with direct measurement of m_t at Tevatron (D0 and CDF)



indirect determination of m_H

lower bound on Higgs mass $m_H > 114.4$, resp. 117 GeV

at 95% CL from LEP resp. Tevatron

best fit for the Higgs mass $m_H = 85 \; {}^{+39}_{-28}$ GeV

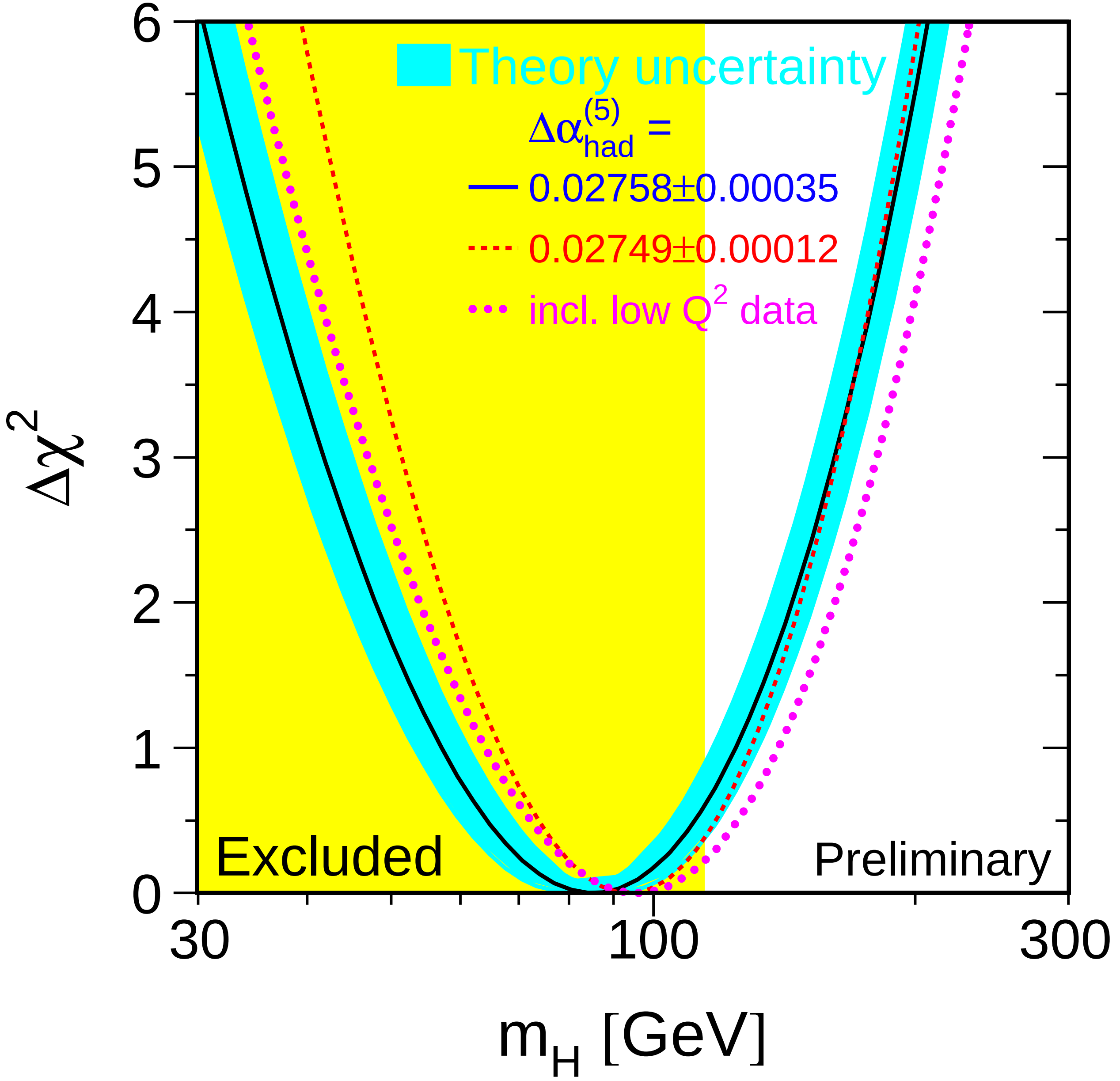
at 68% CL from LEP, using the Tevatron value for m_t

upper bound at 95% CL on Higgs mass

$m_H < 166$ GeV ignoring the direct lower bound of 114 GeV

$m_H < 199$ GeV including the direct lower bound of 114 GeV

Constraints on SM Higgs mass – July 06



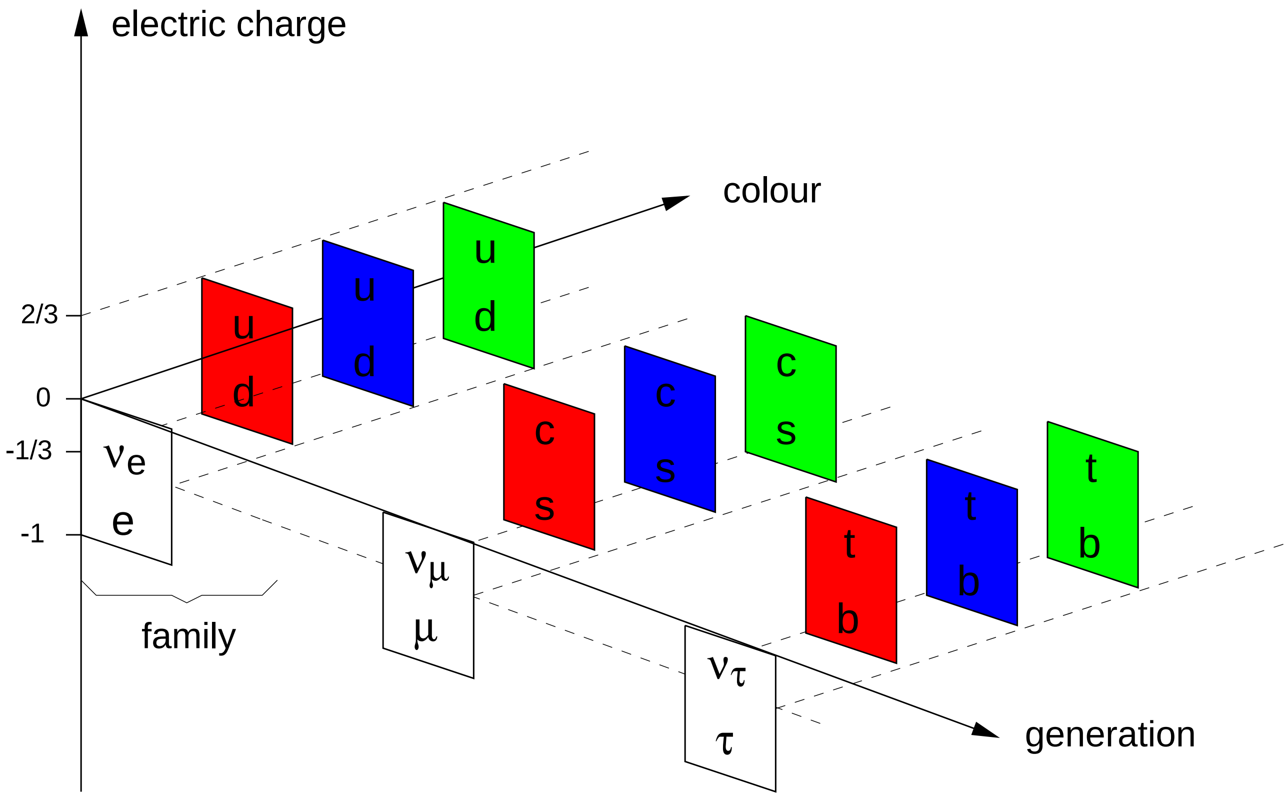
Data	$\widehat{s}_Z^2$	s_W^2	$\alpha_s(M_Z)$	M_H
All data	0.23120(15)	0.2228(4)	0.1213(18)	113^{+56}_{-40}
All indirect (no m_t)	0.23116(17)	0.2229(4)	0.1213(18)	79^{+95}_{-38}
Z pole (no m_t)	0.23118(17)	0.2231(6)	0.1197(28)	79^{+94}_{-38}
LEP 1 (no m_t)	0.23148(20)	0.2237(7)	0.1210(29)	140^{+192}_{-74}
SLD + M_Z	0.23067(28)	0.2217(6)	0.1213 (†)	43^{+38}_{-23}
$A_{FB}^{(b,c)}$ + M_Z	0.23185(28)	0.2244(8)	0.1213 (†)	408^{+317}_{-179}
M_W + M_Z	0.23089(37)	0.2221(8)	0.1213 (†)	67^{+77}_{-45}
M_Z	0.23117(15)	0.2227(5)	0.1213 (†)	117 (†)
DIS (isoscalar)	0.2359(16)	0.2274(16)	0.1213 (†)	117 (†)
Q_W (APV)	0.2292(19)	0.2207(19)	0.1213 (†)	117 (†)
polarized Møller	0.2292(42)	0.2207(43)	0.1213 (†)	117 (†)
elastic $\nu_\mu(\overline{\nu_\mu})e$	0.2305(77)	0.2220(77)	0.1213 (†)	117 (†)
SLAC eD	0.222(18)	0.213(19)	0.1213 (†)	117 (†)
elastic $\nu_\mu(\overline{\nu_\mu})p$	0.211(33)	0.203(33)	0.1213 (†)	117 (†)

$\sin^2\theta_W$ in two next to leading order variants

5. Physics beyond the Standard Model

♣ open questions in the Standard Model

– 'periodic system' of elementary particles

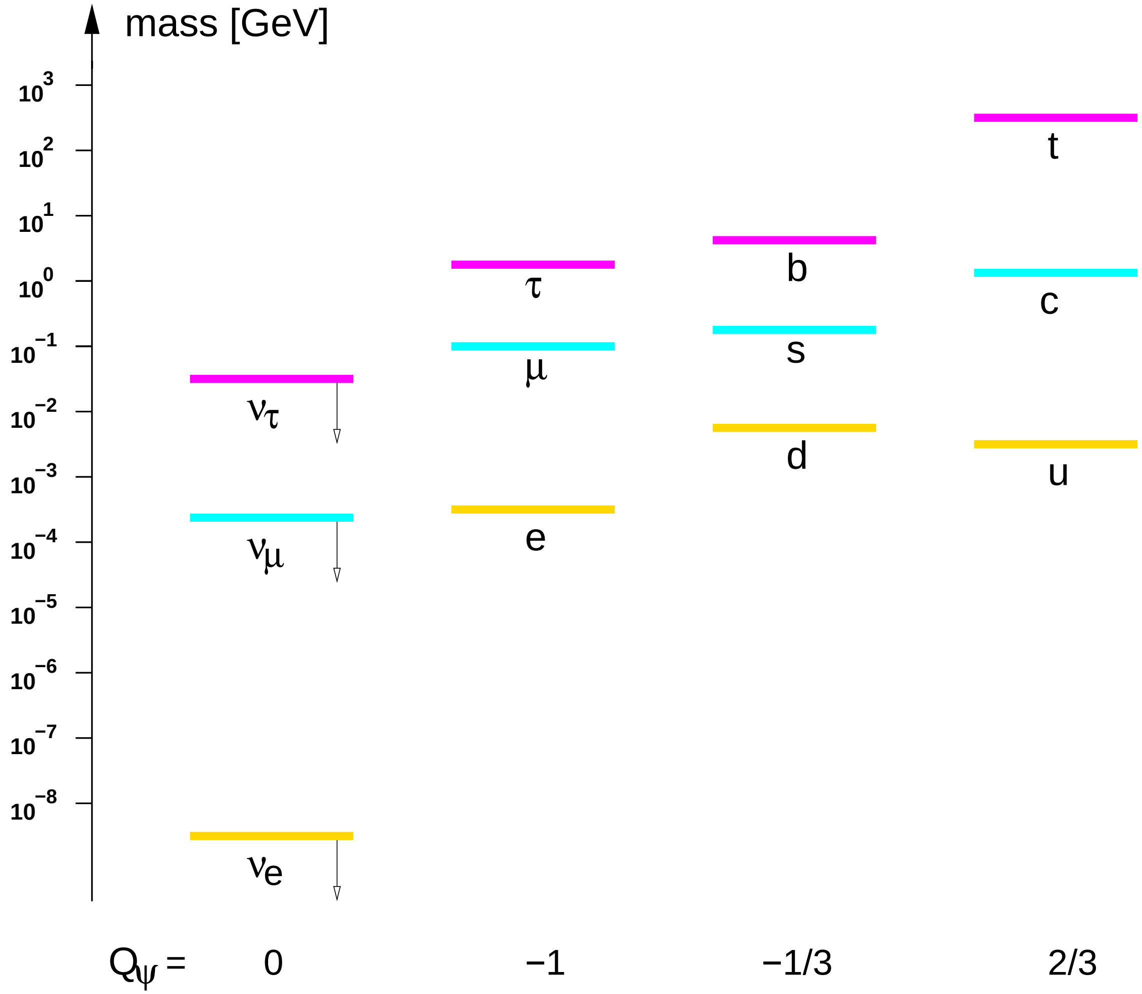


∗ more than 3 generations?

no, if they have neutrinos lighter than $m_Z/2$

∗ if 3 generations, why 3?

– unknown parameters



∗ why is $m_1 \ll m_2 \ll m_3$? (1,2,3 denote generation indices)

∗ why is $m_{\frac{2}{3}} > m_{-\frac{1}{3}} > m_{-1} > m_0$ for each generation except for $m_u < m_d$
(the indices denote the electric charge)

- further questions

  * where is the Higgs Boson?

  * why three gauge forces
    ($\longrightarrow$ three undetermined gauge couplings)?
    why the gauge groups $SU(3)_c \times SU(2)_L \times U(1)_Y$?

  * origin of parity violation?

- expectation

  answers to these questions from measurements at smaller distances, i.e. at higher momenta.

♣ experimental signatures for neutrino masses

Experimental signatures suggesting neutrino masses, neutrino mass mixing, neutrino oscillations.

This issue leads beyond the SM; it is discussed in a separate DESY summerschool lecture.

♣ substructure

– hypothesis

increasing heresy ↓

Higgs boson
leptons and quarks
(($W^{\pm}$, Z bosons))

} **are composite** particles, built from smaller **common constituents = preons**

– Standard Model charges

electroweak and colour forces **remain gauge forces**

if preons carry appropriate electroweak and colour charges

– model building

remember:

atoms are electrically neutral, but bound states of the electrically charged electrons and nucleus

protons and all **hadrons are colour neutral**, but bound states of coloured quarks

– basic assumptions

* **preons carry hypercolour**, a new conserved quantum number $\longrightarrow$
  bound states of preons (among them quarks and leptons) are hypercolour neutral
* there exists a **local hypercolour gauge theory** leading to confinement of preons in their bound states

– basic question and constraint

* radius of quarks and leptons $\lesssim 10^{-16}$ cm $\longrightarrow$ expected from uncertainty principle

mass of bound states of preons $\gtrsim$ $\mathrm{O}(200\,\mathrm{GeV})$

* theory has to provide a natural explanation, why the composite quarks and leptons are so light in comparison to this scale $\longrightarrow$

chiral symmetry, a strong constraint on model building

– prediction of new exotic particles

suitable combinations of preons lead to the bound state quarks and leptons etc.

depending on the specific model, further allowed bound states of preons lead to the

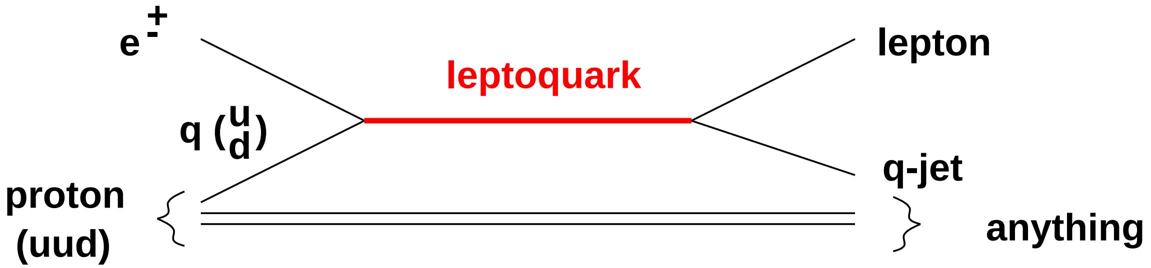
prediction of **new particles**

with exotic electroweak and colour charges and

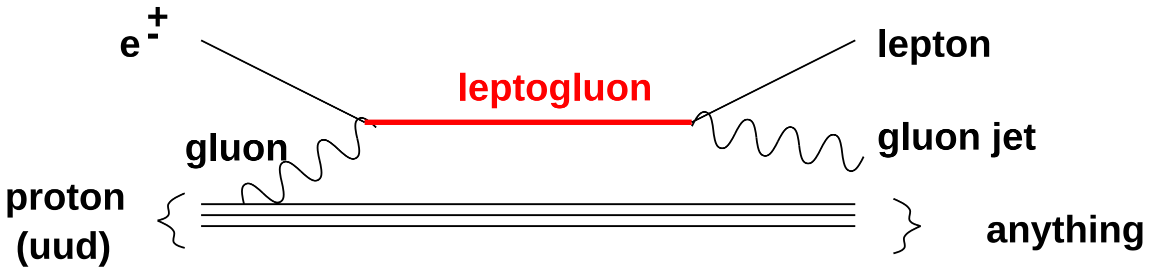
masses $\gtrsim$ $\mathrm{O}(200\,\mathrm{GeV})$

– examples of new (composite) particles **HERA**

∗ leptoquarks (bosons)



∗ leptogluons (fermions)



♣ Additional gauge groups

– an additional $U(1)$ gauge group

$$\underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}_{\text{standard model}} \times \boldsymbol{U(1)}$$ simplest example

SSB of $U(1)$ $\rightarrow$ **massive Z' gauge boson**

– a left-right symmetric gauge theory above $p \gtrsim m_{W_R}$: **parity conserving theory**

$$SU(3)_c \times SU(2)_L \times \underbrace{SU(2)_R \times U(1)_{B-L}}$$

SSB to $U(1)_Y$ $\rightarrow$ **massive W_R, Z_R gauge bosons**

♣ grand unification of the electroweak and colour forces

– assume the "grand desert", i.e. no new physics for

$$10^{-16}\,\text{cm} \;\gtrsim\; \text{distance } d \;\gtrsim\; \mathbf{10^{-29}\,cm},$$

i.e. according to the uncertainty principle for

$$10^{2}\,\text{GeV} \;\lesssim\; \text{momentum } p \;\lesssim\; \mathbf{10^{15}\,GeV}$$

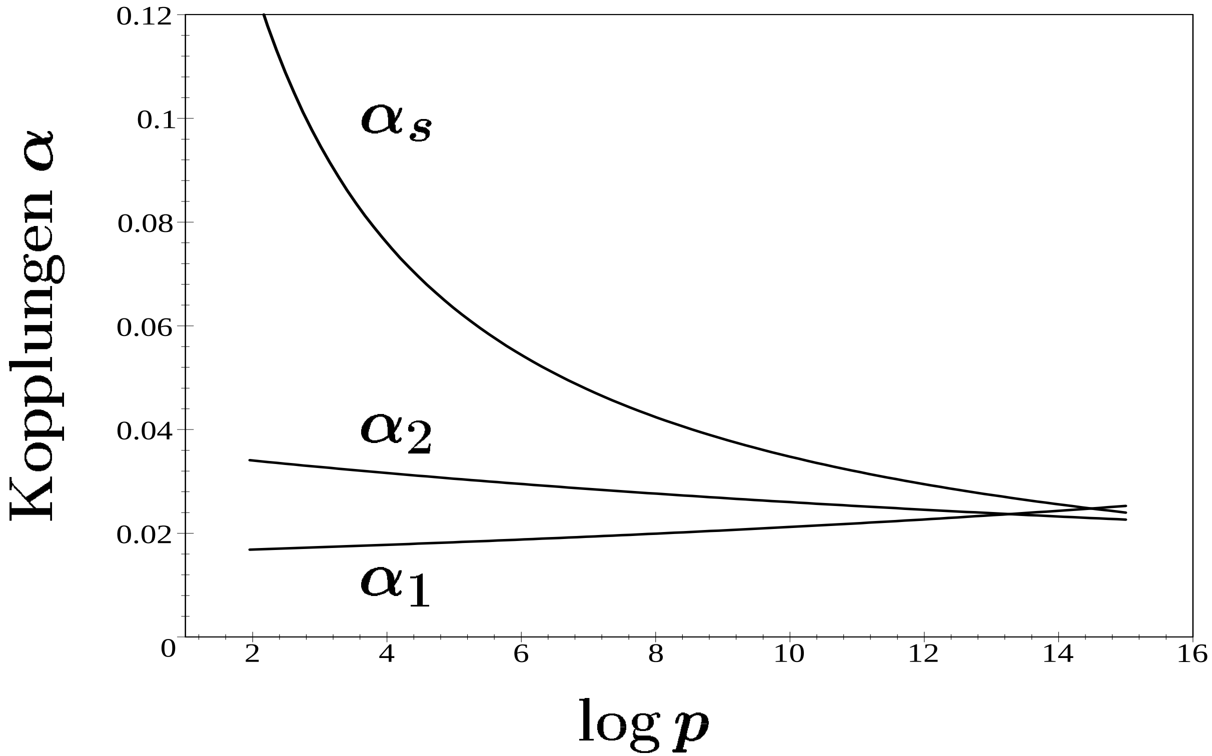
– extrapolation of the running couplings to higher momenta p from experimentally determined initial values at $p = m_Z$

$$\alpha_s(p) = g_c^2/4\pi,\;\; \alpha_1(p) = (5/3)\,g'^2/4\pi,\;\; \alpha_2(p) = g^2/4\pi,$$

– unification of gauge couplings

$$\boldsymbol{\alpha_s(p) \approx \alpha_1(p) \approx \alpha_2(p)}$$
at $p \approx 10^{15}$ GeV (i.e. $d \approx 10^{-29}\,\mathrm{cm}$)

with slight mismatch



– unification of gauge forces suggesting

one **single fundamental force, unifying** the electroweak and colour forces in terms of a single (undetermined) coupling

– model scenario

* single fundamental force = **gauge force**

* gauge group contains $SU(3)_c \times SU(2)_L \times U(1)_Y$ smallest group: $\boldsymbol{SU(5)}$

  number of gauge bosons: $5 \times 5 - 1 = 24$, among them 8 gluons, 3 $W^{\pm}$, Z, 1 γ.

  $\rightarrow$ 12 of the 24 $SU(5)$ gauge bosons have to be heavy

* via **spontaneous symmetry breakdown** $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$

  $$\text{at } \boldsymbol{p \approx 10^{15}} \text{ GeV} \quad \rightarrow \quad \boldsymbol{m_{\text{gauge boson}} \approx 10^{15}} \text{ GeV}$$

* heavy gauge bosons mediate **proton decay**, e.g. $p \rightarrow e^+\pi^0$ **problem!**

  predicted lifetime $\tau_p \approx 10^{31}$ years, experiment for $p \rightarrow e^+\pi^0$: $\tau_p > 1.6 \cdot 10^{33}$ years.

* possible solution: grand unification with supersymmetry

♣ unification with gravity?

- gravitational forces are described by general relativity (classical theory)
- gravitational forces become of comparable size as electroweak and color forces at the

Planck scale $p \approx 10^{19}$ GeV $\hat{=}$ $d \approx 10^{-33}$ cm

- problem: **NO** renormalizeable quantum field theory
- substantial amelioration by **supersymmetry** an extended space-time symmetry
- leading to particle multiplets (**fermion**,**boson**) with Δ spin $= \frac{1}{2}$

each standard model particle has a
supersymmetric partner

leptons	$\longrightarrow$	**sleptons**	(spin 0)
quarks	$\longrightarrow$	**squarks**	(spin 0)
gauge bosons	$\longrightarrow$	**gauginos**	(spin 1/2)
Higgs boson	$\longrightarrow$	**higgsino**	(spin 1/2)

- → **supergravity**
- **superstring theory** (mainly relevant for $p \geq 10^{19}$ GeV)
  - elementary fields → **strings with length 10^{-33} cm**
  - leptons, quarks, gauge bosons, Higgs boson are lowest string excitations

♣ grand unification with supersymmetry

- Implement

supersymmetry into the Standard Model

→ minimal supersymmetric Standard Model

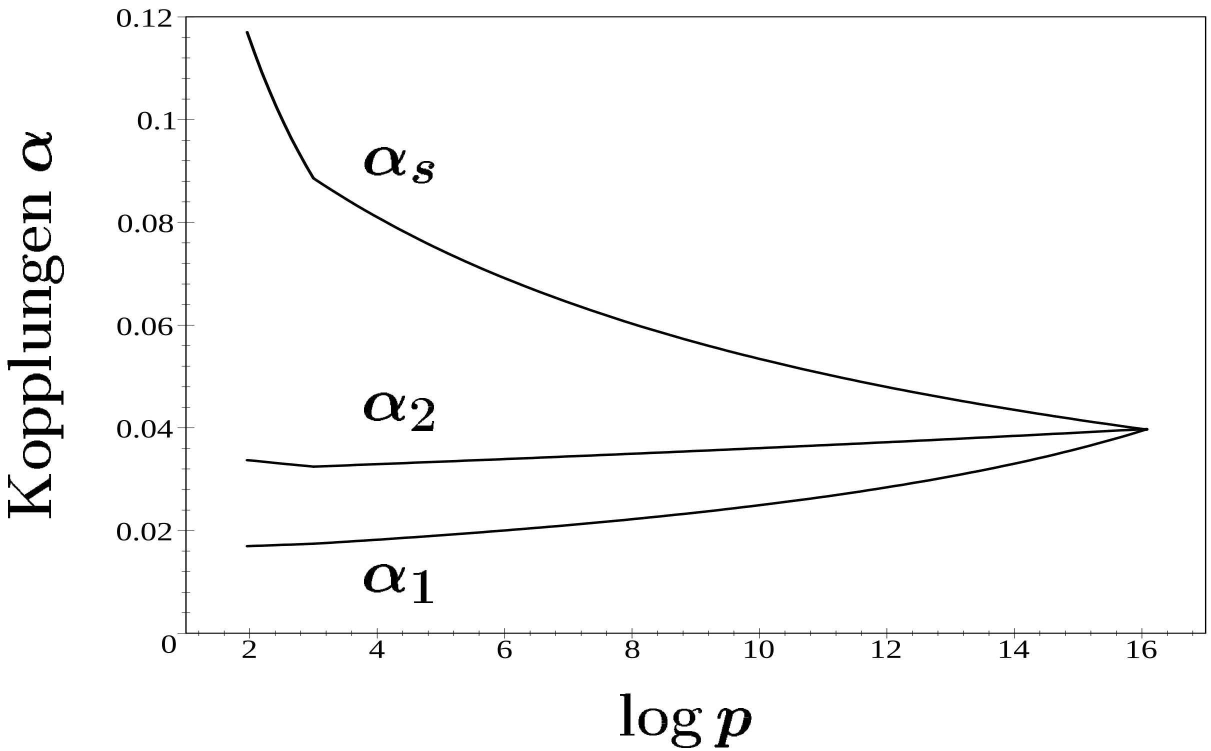
→ improved renormalizeability properties

- soft supersymmetry breaking (necessary since $m_{\text{particle}} \neq m_{\text{sparticle}}$) at scale $M_{SUSY} \approx 200\text{-}1000$ GeV →

$$m_{\text{sparticles}} \approx 200\text{-}1000 \text{ GeV}$$

– grand unification in the supersymmetric framework (e.g. with $SU(5)$ unifying gauge group)

unification of gauge couplings at $p \approx 2 \cdot 10^{16}$ GeV

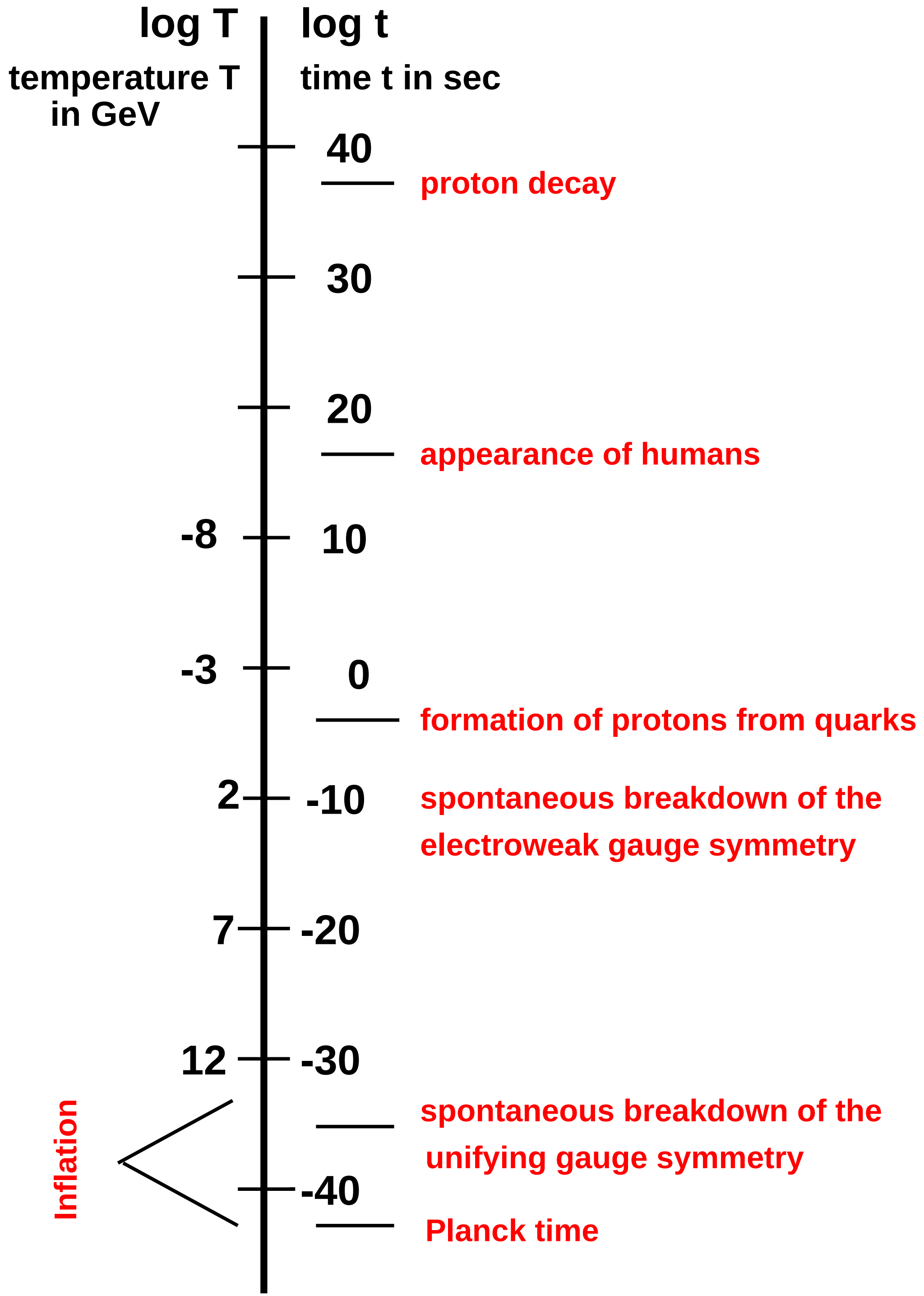


(marginally) no problem with proton decay

The unifying theory implies violation of baryon number and of time reversal invariance which – together with thermal inequilibrium – allows to explain

the baryon asymmetry of the universe

$\longrightarrow$ of interest for the cosmolgy of the early universe


log T
log t
temperature T in GeV
time t in sec
40
proton decay
30
20
appearance of humans
-8
10
-3
0
formation of protons from quarks
2
-10
spontaneous breakdown of the electroweak gauge symmetry
7
-20
12
-30
spontaneous breakdown of the unifying gauge symmetry
Inflation
-40
Planck time

♣ Extra dimensions

- Theoretical developments based on the idea that there are extra dimensions in addition to the 4 space-time dimensions.

- Idea with the most immediate implications for future experiments:

  while the Standard Model gauge interactions "live" in our habitual four dimensions, gravitational forces act in a higher dimensional space with the result that gravitational forces become comparable in strength to the Standard Model gauge forces at a momentum scale as low as

$$\boxed{\mu \approx 1000\,\text{GeV}.}$$

- Grand unification as discussed above has then to be reconsidered under the new circumstances; it is not straightforwardly recovered.

♣ Noncommutative Geometry

Noncommuting space-time coordinates are assumed. Effects can be looked for at future Accelerators.