Multiple Scattering at HERA and at LHC - Remarks on the AGK Rules

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Abstract
We summarize the present status of the AGK cutting rules in perturbative QCD. Particular attention is given to the application of the AGK analysis to multiple scattering in DIS at HERA and in pp collisions at the LHC

1 Introduction

Multiple parton interactions play an important role both in electron proton scattering at HERA and in high energy proton proton collisions at the LHC. At HERA, the linear QCD evolution equations provides, for not too small Q^2, a good description of the F_2 data (and of the total $\gamma^* p$ cross section, $\sigma_{tot}^{\gamma^* p}$). This description corresponds to the emission of partons from a single chain (Fig.1a). However, at low Q^2 where the transition to nonperturbative strong interaction physics starts, this simple picture has to supplemented with corrections. First, there exists a class of models [1] which successfully describe this transition region; these models are based upon the idea of parton saturation: they assume the existence of multiple parton chains (Fig.1b) which interact with each other, and they naturally explain the observed scaling behavior, $F_2(Q^2, x) \approx F_2(Q^2/Q_s^2(x))$ with $Q_s^2(x) = Q_0^2(1/x)^\lambda$. Next, in the photoproduction region, $Q^2 \approx 0$, direct evidence for the presence of multiple interactions also comes from the analysis of final states [2]. A further strong hint at the presence of multi-chain configurations comes from the observation of a large fraction of diffractive final states in deep inelastic scattering at HERA. In the final states analysis of the linear QCD evolution equations, it is expected that the produced partons are not likely to come with large rapidity intervals between them. In the momentum-ordered single chain picture (Fig.1a), therefore, diffractive final states should be part of the initial conditions (inside the lower blob in Fig.1a), i.e. they should lie below the scale Q_0^2 which seperates the parton description from the nonperturbative strong interactions. This assignment of diffractive final states, however, cannot be complete. First, data have shown that the Pomeron which generates the rapidity gap in DIS diffraction is harder than in hadron - hadron scattering; furthermore, there are specific diffractive final states with momentum scales larger than Q_0^2, e.g. vector mesons built from heavy quarks and diffractive dijets (illustrated in Fig.2): the presence of such final states naturally requires corrections to the single chain picture (Fig.2b). From a t-channel point of view, both Fig.1b and Fig.2b belong to the same class of corrections, characterized by four gluon states in the t-channel.

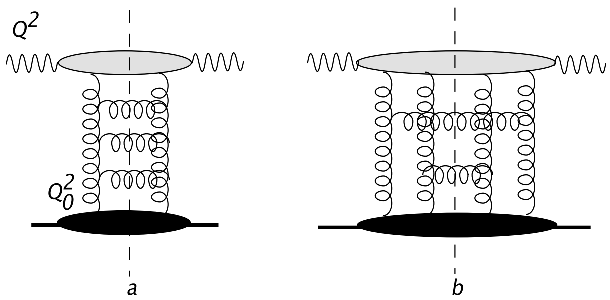

Fig. 1: Contributions to the total cross section $\sigma_{tot}^{\gamma^* p}$: (a) the single chain representing the linear QCD evolution equations; (b) gluon production from two different gluon chains.

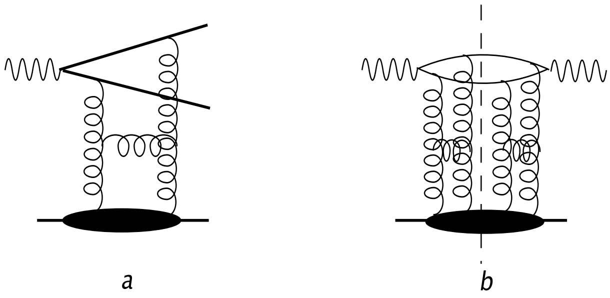

Fig. 2: Hard diffractive final states.(a) dijet production; (b) the diffractive cross section as s-channel discontinuity of a two-ladder diagram.

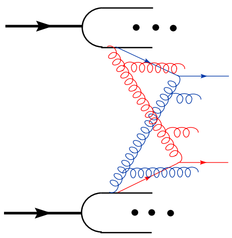

Fig. 3: Jet production in pp collisions from two different parton chains

In proton-proton collisions corrections due to multiple interactions should be important in those kinematic regions where parton densities for small momentum fractions values and for not too large momentum scales are being probed, e.g. jet production near the forward direction. Another place could be the production of multijet final states (Fig.3): multiple jets may come from different parton chains, and these contributions may very well affect the background to new physics beyond the standard model. Moreover, the modelling of multijet configurations will be necessary for understanding the underlying event structure in pp collidions [3].

From the point of view of collinear factorization, multiple interactions with momentum ordered parton chains are higher-twist effects, i.e they are suppressed by powers of the hard momentum scale. At small x, however, this suppression is compensated by powers of the large logarithms, $\ln 1/x$: multiple interactions, therefore, are mainly part of small-x physics. In this kinematic region the Abramovsky-Gribov-Kanchelli (AGK) [4] rules can be applied to the analysis of multi-gluon chains, and it is the aim of this article to present a brief overview about the current status of the AGK rules in pQCD.

As we will discuss below, in the analysis of multiple parton chains the couplings of n gluons to the proton play an essential role. Regge factorization suggests that these coupling should be universal, i.e. the couplings in $\gamma^* p$ collisions at HERA are the same as those in pp scattering at the LHC. Therefore, a thorough analysis of the role of multiple interactions in deep inelastic electron-proton scattering at HERA should be useful for a solid understanding of the structure of events at the LHC.

2 Basics of the AGK cutting rules

The original AGK paper [4], which was written before the advent QCD, addresses the question how, in the optical theorem,

$$\sigma_{tot}^{pp} = \frac{1}{s} Im\, T_{2\to 2} \;=\; \sum_f \int d\Omega_f |T_{i\to f}|^2 \tag{1}$$

the presence of multi-Pomeron exchanges (Fig.4) in the total hadron-hadron cross section leads to observable effects in the final states (rhs of eq.(1)). Based upon a few model-independent assumptions

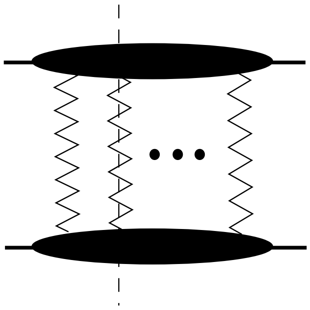

Fig. 4: s-cut through a multi-Pomeron exchange: the zig-zag lines stand for nonperturbative Pomerons.

on the couplings of multi-Pomeron exchanges to the proton, the authors derived simple 'cutting rules': different contributions to the imaginary part belong to different cuts across the multi-Pomeron diagrams, and each cut has its own, quite distinct, final state characteristics. As a result, the authors found counting rules for final states with different particle multiplicities, and they proved cancellations among rescattering corrections to single-particle and double-particle inclusive cross sections.

In the QCD description of hard (or semihard) final states a close analogy appears between (color singlet) gluon ladders and the nonperturbative Pomeron: multiple parton chains (for example, the two chains in Fig.1b) can be viewed as cuts through two perturbative BFKL Pomerons. In the same way as in the original AGK paper, the question arises how different cuts through a QCD multi-ladder diagram can be related to each other. In the following we briefly describe how AGK cutting rules can be derived in pQCD [5,6]. In the subsequent section we will present a few new results which come out from pQCD calculations, going beyond the original AGK rules.

One of the few assumptions made in the original AGK paper states that the coupling of the Pomerons to the external particle are (i) symmetric under the exchange of the Pomerons (Bose symmetry), and (ii) that they remain unchanged if some of the Pomerons are beeing cut. These properties also hold in pQCD, but they have to be reformulated: (i') the coupling of (reggeized) gluons to external particles is symmetric under the exchange of reggeized gluons, and (ii') it remains unchanged if we introduce cutting lines between the gluons. In QCD, however, the color degree of freedom also allows for another possibility: inside the n-gluon state (with total color zero), a subsystem of two gluons can form an antisymmetric color octet state: in this case the two gluons form a bound state of a reggeized gluon (bootstrap property). For the case of $\gamma^*\gamma^*$ scattering, explicit calculations [7] have shown that the coupling of n gluons to virtual photons can be written as a sum of several pieces: the fully symmetric ('irreducible') one which satisfies (i') and (ii'), and other pieces which, by using the bootstrap property, can be reduced to symmetric couplings of a smaller number of gluons ('cut reggeons'). This decomposition is illustrated in Fig.5.

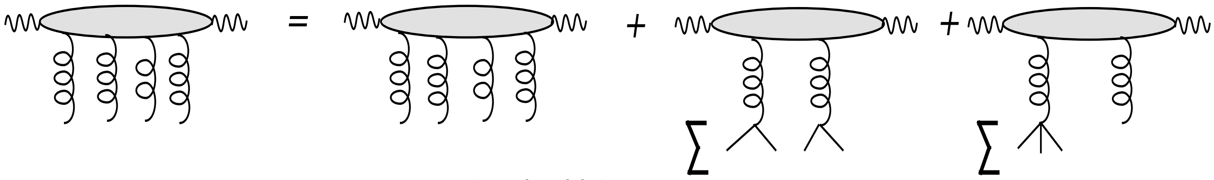

Fig.5 Decomposition of the coupling of four gluons to a virtual photon. In the last two terms on the rhs it is understood that we have to sum over different pairings of gluons at the lower end.

Since the bootstrap property is related to the regeization of the gluon and, therefore, is expected to be valid to all orders perturbation theory, also these properties of the couplings of multi-gluon states to external particles should be of general validity. In this short review we will mainly concentrate on the symmetric couplings.

As an illustrative example, we consider the coupling of four gluons to a proton. The simplest model of a symmetric coupling is a sum of three pieces, each of which contains only the simplest color structure:

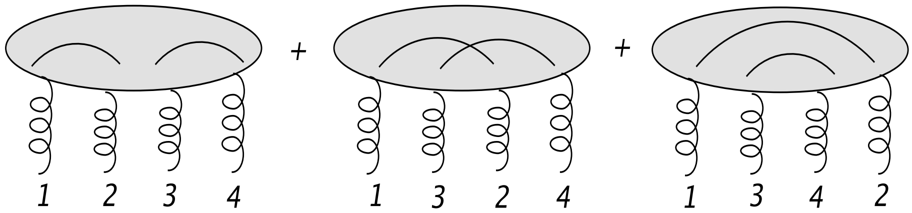

Fig.6 The symmetric coupling of four gluons to an external particle. The lines inside the blob denote the color connection, e.g. the first term has the color structure $\delta_{a_1 a_2}\delta_{a_3 a_4}$.

The best-known cutting rule for the four gluon exchange which follows [5,6] from this symmetry requirement is the ratio between the three different pairings of lines:

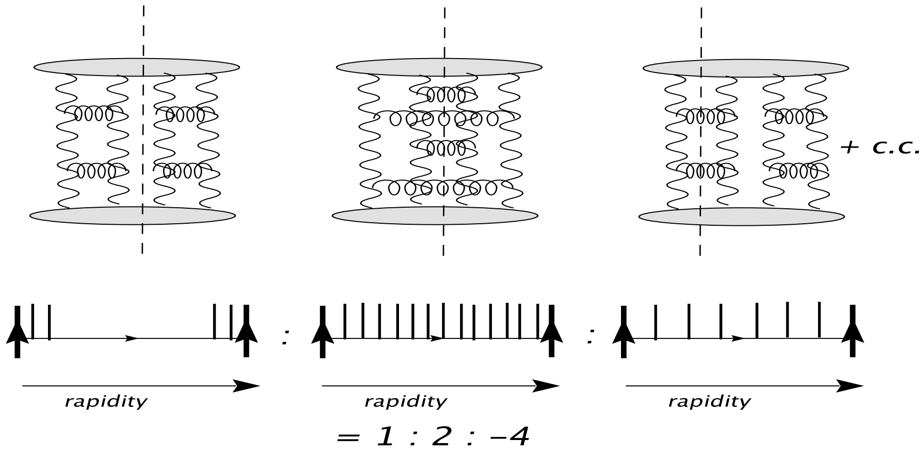

Fig 7: different cutting lines in the four-gluon exchange.

Each term, on the partonic level, corresponds to a certain multiplicity structure of the final state: a rapidity gap ('zero multiplicity'), double multiplicity, and single multiplicity. Simple combinatorics then leads to the ratio

$$1 : 2 : -4. \tag{2}$$

In order to be able to generalize and to sum over an arbitrary number of gluon chains, it is convenient to use an eikonal ansatz:

$$N_{2n}^{A}(\boldsymbol{k}_1, a_1; \ldots; \boldsymbol{k}_{2n}, a_{2n}; \omega) =$$

$$\frac{1}{\sqrt{(N_c^2-1)^n}}\left(\sum_{Pairings} \phi^A(\boldsymbol{k}_1, \boldsymbol{k}_2; \omega_{12})\delta_{a_1 a_2} \cdot \ldots \cdot \phi^A(\boldsymbol{k}_{2n-1}, \boldsymbol{k}_{2n}; \omega_{2n-1,2n})\delta_{a_{2n-1} a_{2n}}\right). \tag{3}$$

Inserting this ansatz into the hadron - hadron scattering amplitude, using the large-N_c approximation, and switching to the impact parameter representation, one obtains, for the contribution of k cut gluon ladders, the well-known formula:

$$ImA_k = 4s \int d^2\boldsymbol{b} e^{i\boldsymbol{q}\boldsymbol{b}} P(s, \boldsymbol{b}) \tag{4}$$

where

$$P(s, \boldsymbol{b}) = \frac{[\Omega(s, \boldsymbol{b})]^k}{k!} e^{-\Omega(s,\boldsymbol{b})}, \tag{5}$$

and Ω stands for the (cut) two-gluon ladder.

Another result [6] which follows from the symmetry properties of the n gluon-particle coupling is the cancellation of rescattering effects in single and double inclusive cross sections. In analogy with the AGK results on the rescattering of soft Pomerons, it can be shown that the sum over multi-chain contributions and rescattering corrections cancels (Fig.8),

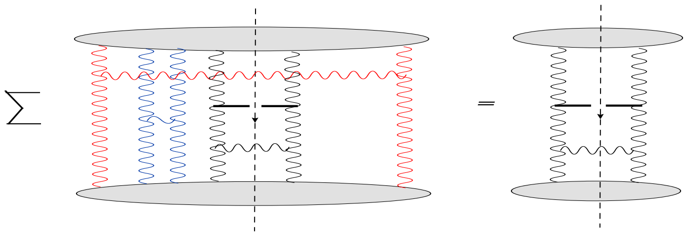

Fig 8: AGK cancellations in the one-jet inclusive cross section.

leaving only the single-chain contribution (in agreement with the factorization obtained in the collinear analysis). This statement, however, holds only for rescattering between the two projectile: it does not affect the multiple exchanges between the tagged jet and the projectile (Fig.9) which require a seperate discussion (see below).

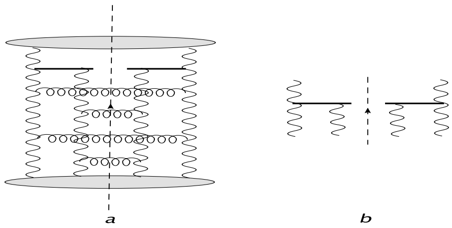

Fig 9: (a) Nonvanishing rescattering corrections in the one-jet inclusive cross section; (b) a new vertex: $g + 2g \to jet$.

All these results can be generalized to include also the soft Pomeron: all one needs to assume is that the couplings of soft Pomerons and reggeized gluons are symmetric under interchanges, and they are not altered if cutting lines are introduced.

3 New results

Explicit calculations in QCD lead to futher results on multiple interactions. First, in the four gluon exchange there are other configurations than those shown in Fig.7; one example is depicted in Fig.10. Here the pairing of gluon chains switches from $(14)(23)$ in the upper part (= left rapidity interval) to $(12)(34)$ in the lower part (= right rapidity interval).

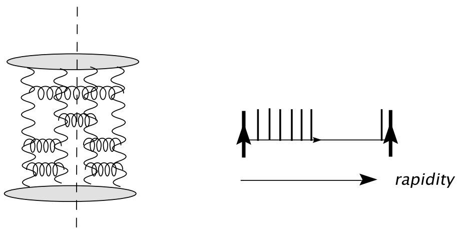

Fig 10: Decomposition into two rapidity intervals: the upper (left) interval has double multiplicity, the lower (right) one corresponds to a rapidity gap.

One can show that the ratio $1 : 2 : -4$ holds for each rapidity interval. In [6] this has been generalized to an arbitrary number of exchanged gluon lines.

Another remark applies to the applicability of the cutting rules to rescattering corrections in the single jet inclusive cross section (Fig.9). Below the jet vertex we, again, have an exchange of four gluon lines, similar to the diagram in the middle of Fig.7. As to the cutting rules, however, there is an important difference between the two situations. In Fig.7, the blob above the four gluons is totally inclusive, i.e. it contains an unrestricted sum over s-channel intermediate states, whereas in Fig.9 the part above the four gluon state is semi-inclusive , i.e. it contains the tagged jet. This 'semi-inclusive' nature destroys the symmetry above the four gluon states, and the cutting rules have to be modified [8, 9]. In particular, eqs.(3) - (4) are not applicable to the rescattering corrections between the jet and projectile. A further investigation of these questions is in progress [10].

Finally a few comments on reggeization and cut reggeons. Clearly there are more complicated configurations than those which we have discussed so far; an example appears in $\gamma^* p$ scattering (deep inelastic electron proton scattering). In contrast to pp scattering, the coupling of multi-gluon chains to the virtual photon can be computed in pQCD, and the LO results, for the case of $n = 4$ gluons, are illustrated in Fig.11.

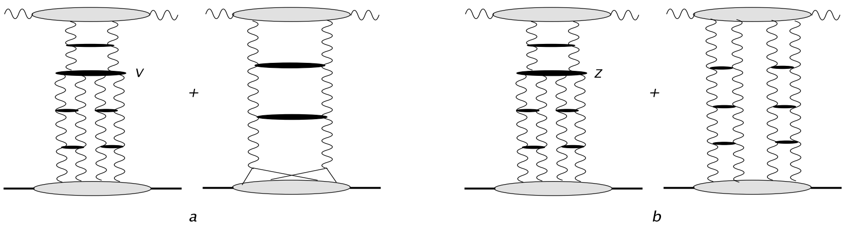

Fig.11: Four-gluon contributions to $\gamma^* p$ proton scattering: two equivalent ways of summing over all contributions. (a) the decomposition of Fig.5 with the pQCD triple Pomeron vertex. (b) an alternative way of summation which explicitly shows the coupling of two Pomerons to the photon vertex and which leads to a new vertex Z.

It turns out that we have two alternative possibilities: in the completely inclusive case (total cross section), it is convenient to chose Fig.11a, i.e. the sum of all contributions can be decomposed into two sets of diagrams. In the first set, at the top of the diagram two gluons couple to the quark-antiquark pair, and the subsequent transition to the four-gluon state goes via the pQCD triple Pomeron vertex. This vertex, as a function of the 4 gluons below, has the symmetry properties described above. As a result, we can apply the cutting rules to the four gluon state, as discussed before. However, there is also the second term in Fig.11a, which consists of a two gluon state only: this is the reggeizing contribution we have mentioned before. As indicated in the figure, the splitting of the reggized gluons at the bottom amounts to a change in the (nonperturbative) coupling. We want to stress that, because of the inclusive nature of this set of diagrams, the triple Pomeron vertex V in Fig.11a, similar to the BFKL kernel, contains both real and virtual contributions. For this reason, the decomposition in Fig.11a is applicable to inclusive cross sections, and it is not convenient for investigating specific final states such as, for example, diffractive final states with a fixed number of quarks and gluons in the final state.

There exists an alternative way of summing all contributions (Fig.11b) which is completely equivalent to Fig.11a but allows to keep track of diffractive $q\bar{q}$, $q\bar{q}g$,... final states: this form is illustrated in Fig.11b. One recognizes the 'elastic intermediate state' which was not visible in Fig.11a, and the new triple Pomeron vertex Z which contains only real gluon production. This vertex Z, as discussed in [11] is no longer symmetric under permutations of the gluons at the lower end; consequently, we cannot apply the AGK cutting rules to the four gluon states below.. These findings for multiple scattering effects in DIS imply, strictly speaking, that cross sections for diffractive $q\bar{q}$ or $q\bar{q}g$ states cannot directly be inserted into the counting rules (2).

Also pp scattering will contain corrections due to multiple interactions which are more complex. There are, for example, graphs which contain the $2 \to 4$ gluon vertex V, leading to a change of the number of gluon lines (Fig.12).

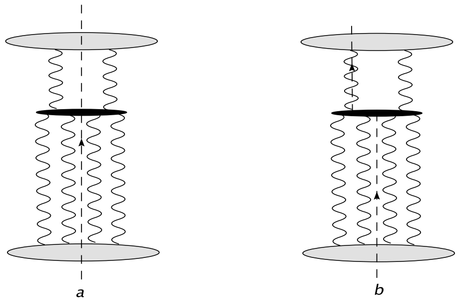

Fig 12: A correction in which the number of lines changes.
The black vertex denotes the $2 \to 4$ gluon vertex.

Since this $2 \to 4$ gluon vertex, as a function of the four gluons below the vertex, satisfies the symmetry requirements listed above, we can apply our previous analysis to the cutting lines below the vertex. In addition, however, one can ask how the lines continue above the $2 \to 4$ gluon vertex: we show two examples, one of them containing a cut (reggeized) gluon. Concentrating on this two-gluon state (i.e. we imagine that we have already summed over all possible cutting lines below the vertex V), the counting rules are quite different: in contrast to the even-signature Pomeron, the gluon is a odd-signature reggeon. Consequently, the cut gluon is suppressed w.r.t. the uncut gluon by one power in α_s, and this suppression leads to the following hierarchy of cutting lines: the cut between the gluons belongs to leading order, the cut through one of the two reggeized gluons is supressed by one power in α_s, the cut through both reggeized gluons is double suppressed (order α_s^2). A closer analysis of this question is under investigation [10].

4 Conclusions

Corrections due to multiple interactions seem to be important in DIS at small x and low Q^2; they are expected to play a significant role also in multijet production in pp scattering. The study of the AGK rules to pQCD provides help in understanding the systematics of multiple gluon chains. Results described in this review represent the beginning of a systematic analysis. We have listed a few questions which require further work.

As an immediate application, we believe that a quantitative analysis of multiple scattering at HERA will provide a useful input to the modelling of final states at the LHC.

A question of practical importance which we have not addressed at all is the hadronization of partonic final states. All statements on ratios of 'particle densities in the final states' made in this paper refer to the parton (gluon) level. However, the hadronization of events which, for example, belong to a double-cut ladder configuration may be quite different from the one obtained by applying just the normal single-chain hadronization to each chain seperately. The answer to this question [1] goes beyond the AGK analysis discussed in this paper.

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