QCD and Monte Carlo simulation II

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http://www.desy.de/~jung/qcd_and_mc_2015/

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- like if you read together with your colleagues a book, and mark places, which you think are important or which are not understandable.
- your comments, questions and remarks will be visible to others, they can also reply
- your comments, questions and remarks will be answered and replied by your lecturer
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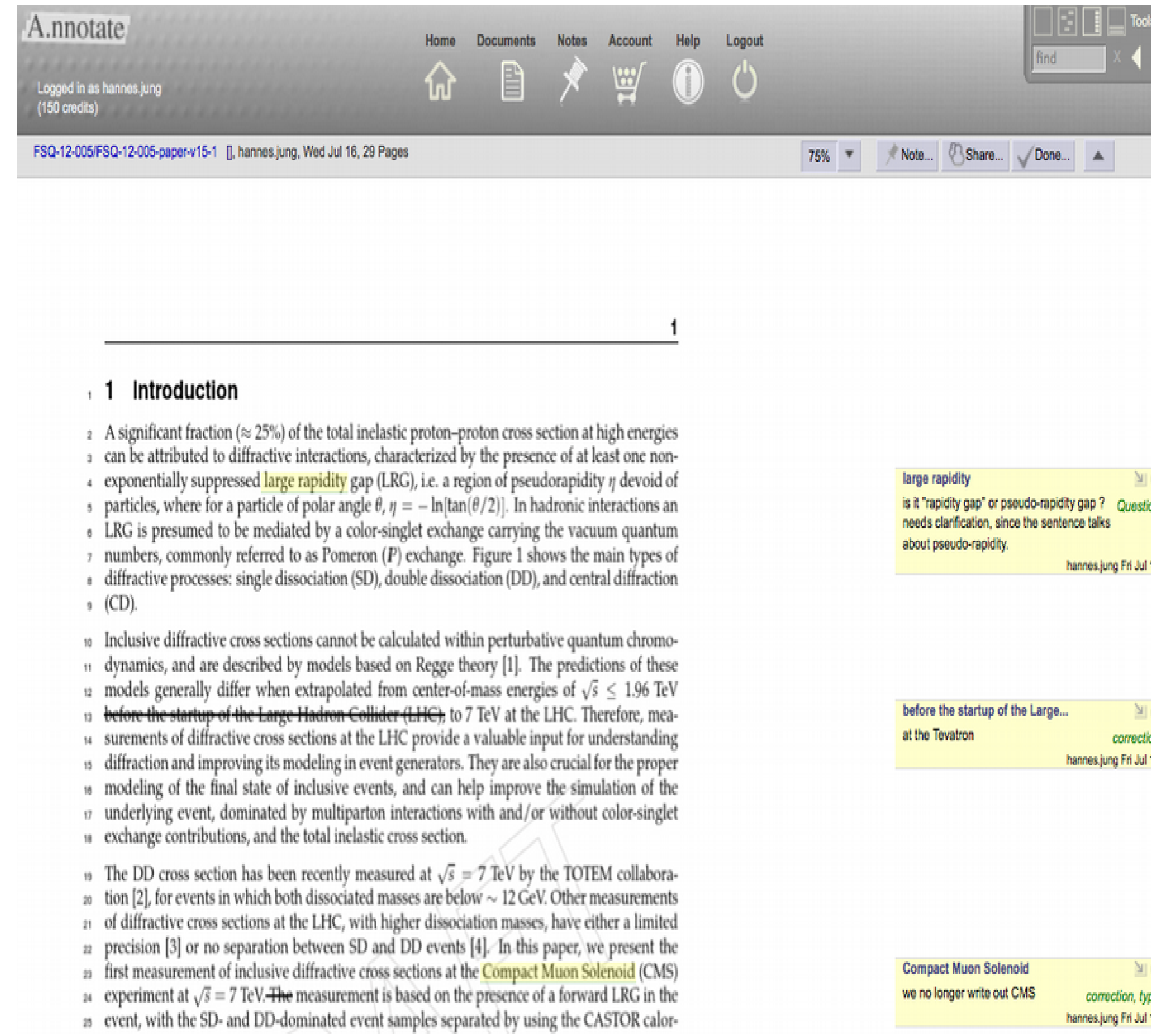

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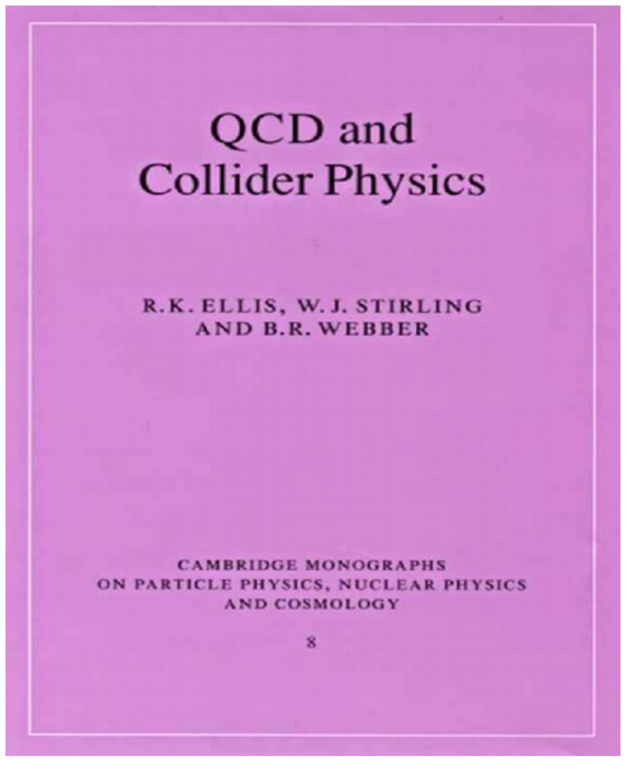
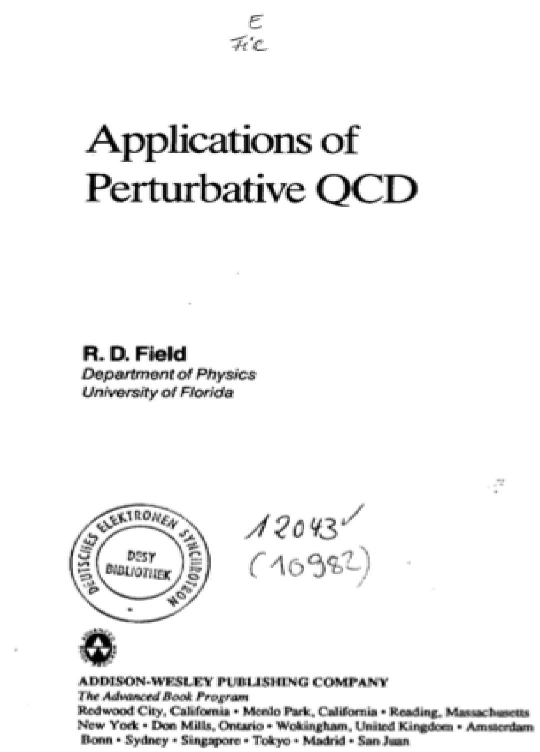
Literature





Applications of
Perturbative QCD

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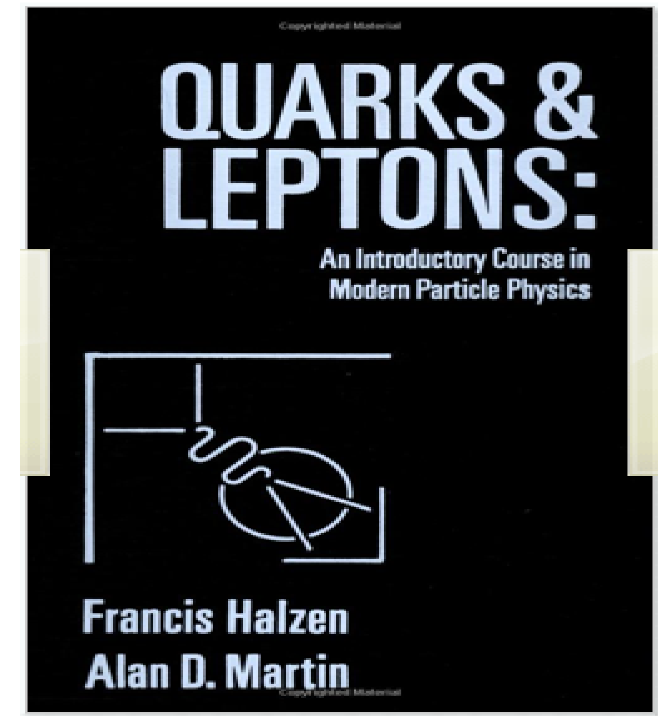

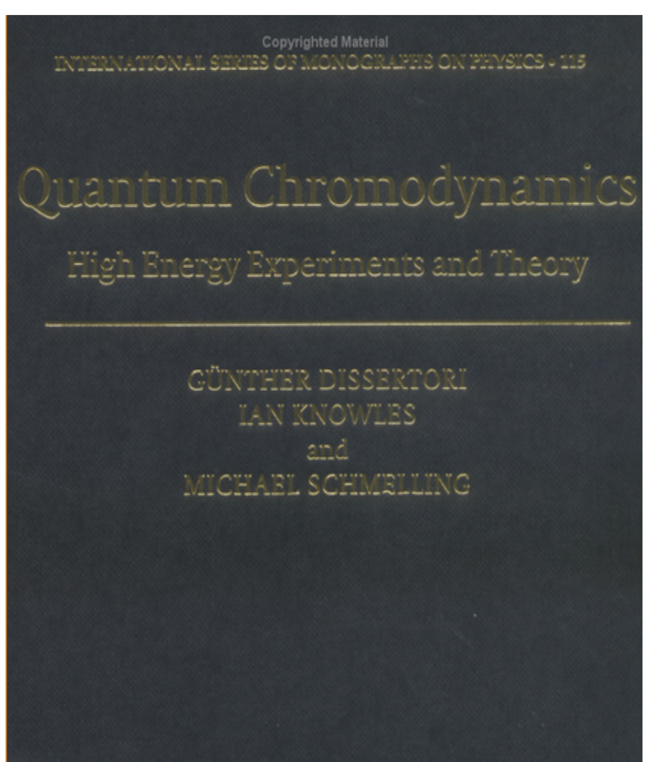

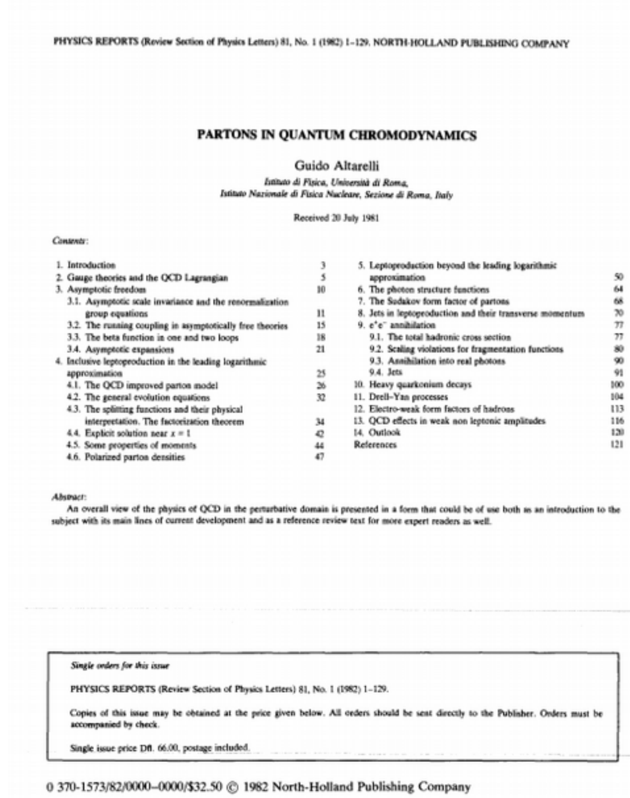

PHYSICS REPORTS (Review Section of Physics Letters) 81, No. 1 (1982) 1–129. NORTH-HOLLAND PUBLISHING COMPANY

PARTONS IN QUANTUM CHROMODYNAMICS

Guido Altarelli
Istituto di Fisica, Università di Roma,
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Received 20 July 1981

Contents:

Abstract:
An overall view of the physics of QCD in the perturbative domain is presented in a form that could be of use both as an introduction to the subject with its main lines of current development and as a reference review text for more expert readers as well.

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Outline of the lectures

- 12. Oct Intro to Monte Carlo techniques and structure of matter
- 13. Oct parton evolution: DGLAP equations
- 26. Oct DGLAP/BFKL/CCFM: evolution for small x
- 27. Oct W/Z production in pp and soft gluon resummation
- 16. Nov Multiparton interactions
- 17. Nov Latest LHC results: small x, multiparton interactions,
  QCD in high luminosity phase: Higgs as a gluon trigger
- Exercises
- 14 & 15 Oct
- 28 & 29 Oct
- 18 & 19 Nov

Inelastic Scattering: QPM

Ellis,Webber, Stirling, p 90 ff

- Infinite momentum frame: $p^\mu = (P, 0, 0, P)$ with $P \gg M$
- Virtual photon scatters off point-like quark which moves parallel (collinear) to proton, with momentum fraction $p_q^\mu = \xi p^\mu$
- Using DIS variables gives for $eq \to eq$

$$|M|^2 = \frac{2e_q^2(4\pi\alpha)^2\hat{s}^2}{Q^4}\left(1+(1-y)^2\right)$$

Blackboard

- giving $$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left(1+(1-y)^2\right)$$
-
- Using mass shell condition for outgoing quark gives (with $\int_0^1 dx\delta(x-\xi) = 1$)

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(1+(1-y)^2\right)\frac{1}{2}e_q^2\delta(x-\xi)$$

- compare this with formula for DIS

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1+(1-y)^2\right)F_1 + \frac{1-y}{x}(F_2 - 2xF_1)\right]$$

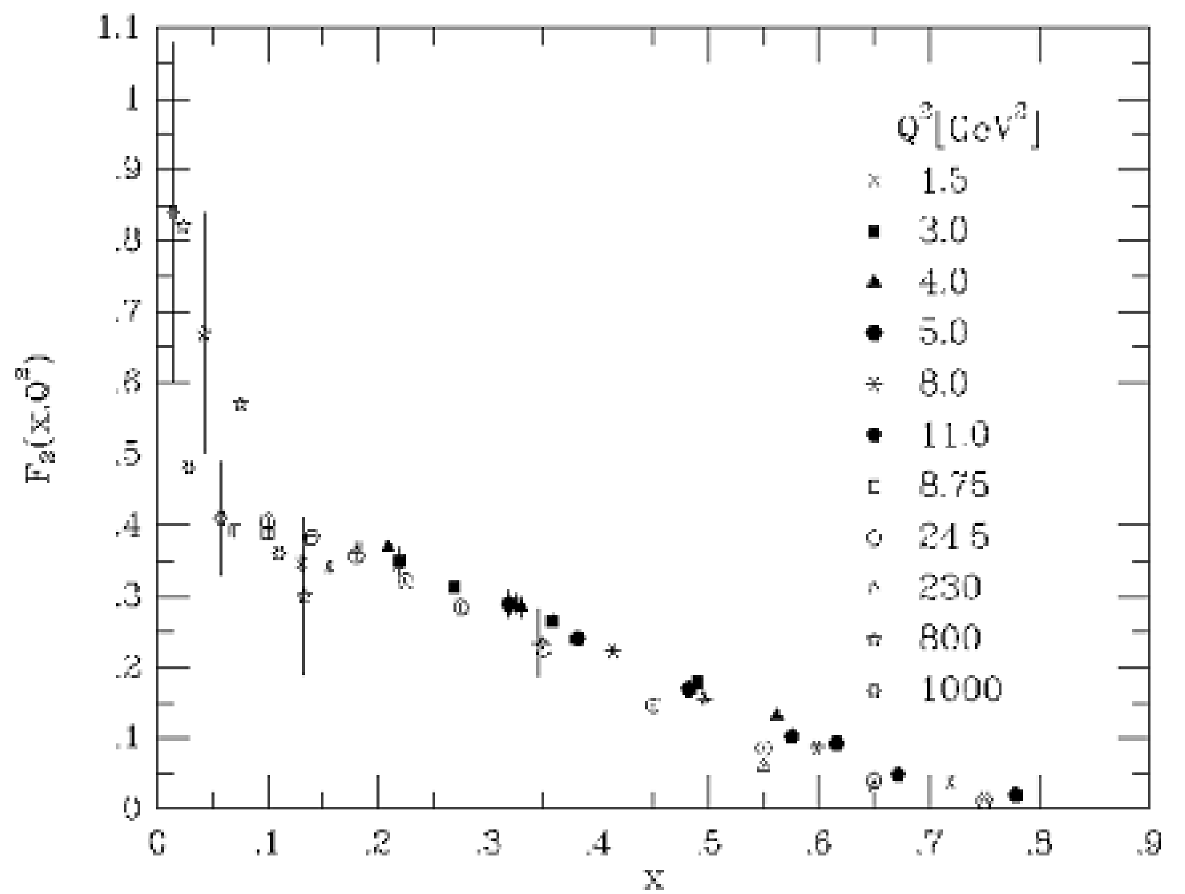
Is F_1 and F_2 a delta function ?

Inelastic Scattering QPM

- Simple model with

$$\tilde{F}_2 = x e_q^2 \delta(x-\xi) = 2x\tilde{F}_1$$

- BUT structure function is a distribution. F_2 is a function of *x*: **scaling, no Q^2 dependence**



- $q(\xi)d\xi$ is probability to find q with momentum fraction $\xi....\xi + d\xi$

$$F_2(x) = 2xF_1(x) = \sum_{q,\bar{q}} \int_0^1 d\xi q(\xi) x e_q^2 \delta(x-\xi) = \sum_{q,\bar{q}} e_q^2 x q(x)$$

- Proton structure function is:

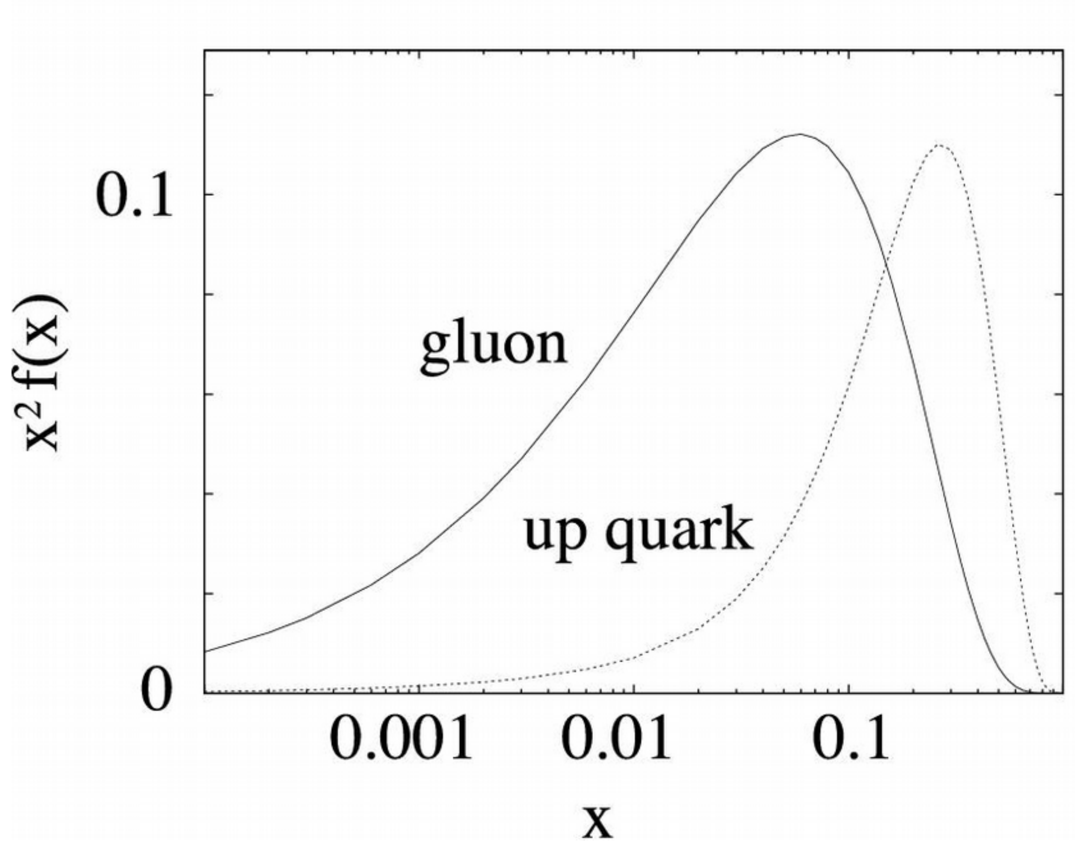
$$F_2^{em} = x\left[\frac{4}{9}\left(u(x)+\bar{u}(x)+c(x)+\bar{c}(x)\right)+\frac{1}{9}\left(d(x)+\bar{d}(x)+s(x)+\bar{s}(x)\right)\right]$$

Parton distribution functions (pdfs)

- $f_i(\xi)d\xi$ gives probability that parton *i* carries momentum fraction between ξ and $\xi + d\xi$ with $0 \leq \xi \leq 1$
- Number of partons i:

$$N_i = \int_0^1 d\xi \, f_i(\xi)$$

From D. Soper hep-ph/9609018

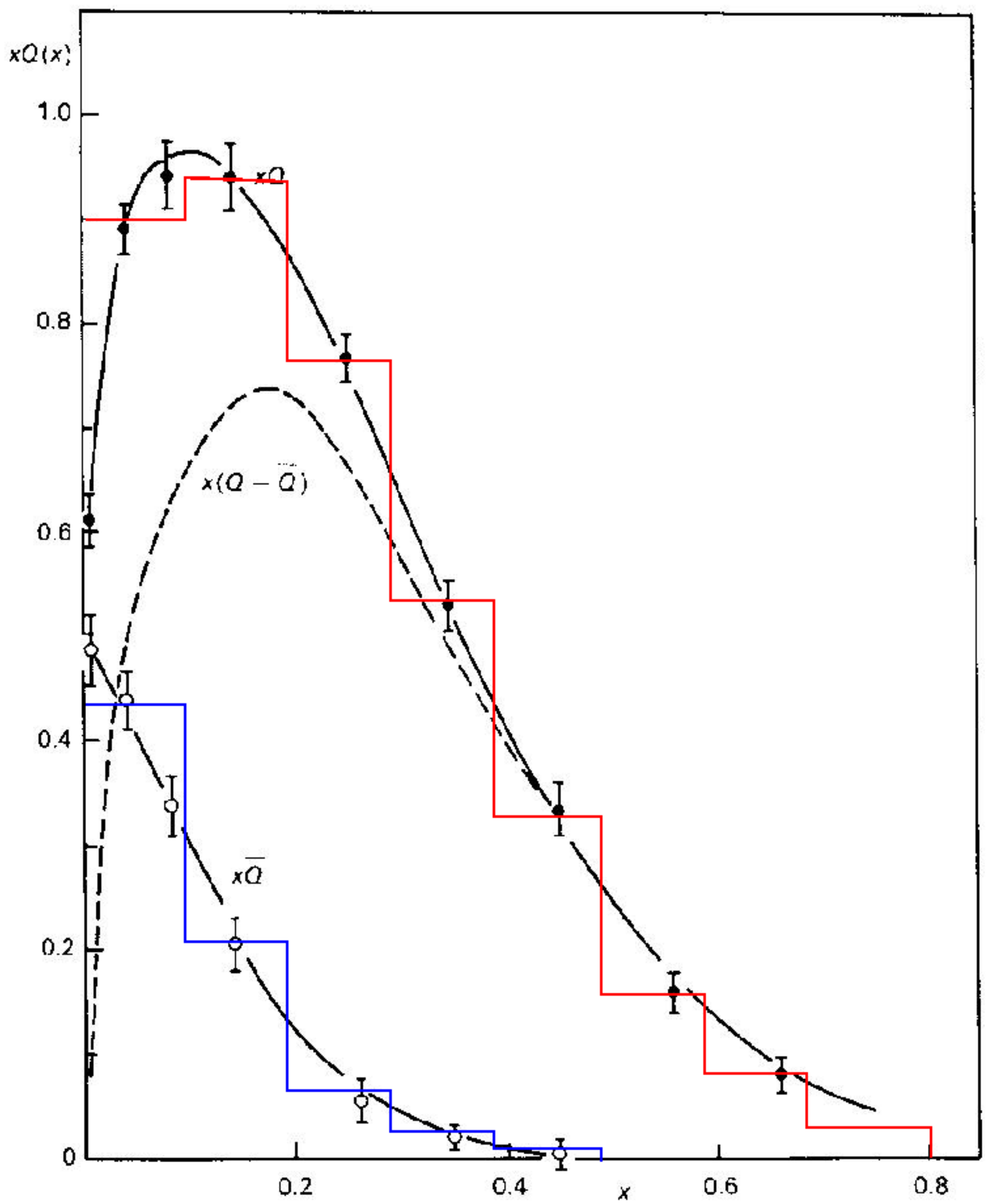


- Momentum fraction carried by partons i:

$$\frac{< p_i >}{P} = \int_0^1 d\xi \;\; \xi f_i(\xi) = \int_0^1 d \log \xi \;\; \xi^2 f_i(\xi)$$

- Define sum-rules for hadron target:
  - Number of valence partons
  - Momentum carried by partons
  - Flavor contents

Picture of the Proton



- Flavor sum rules for proton:

$$\int_0^1 dx u_V(x) = 2$$

$$\int_0^1 dx d_V(x) = 1$$

- Momentum sum of quarks:

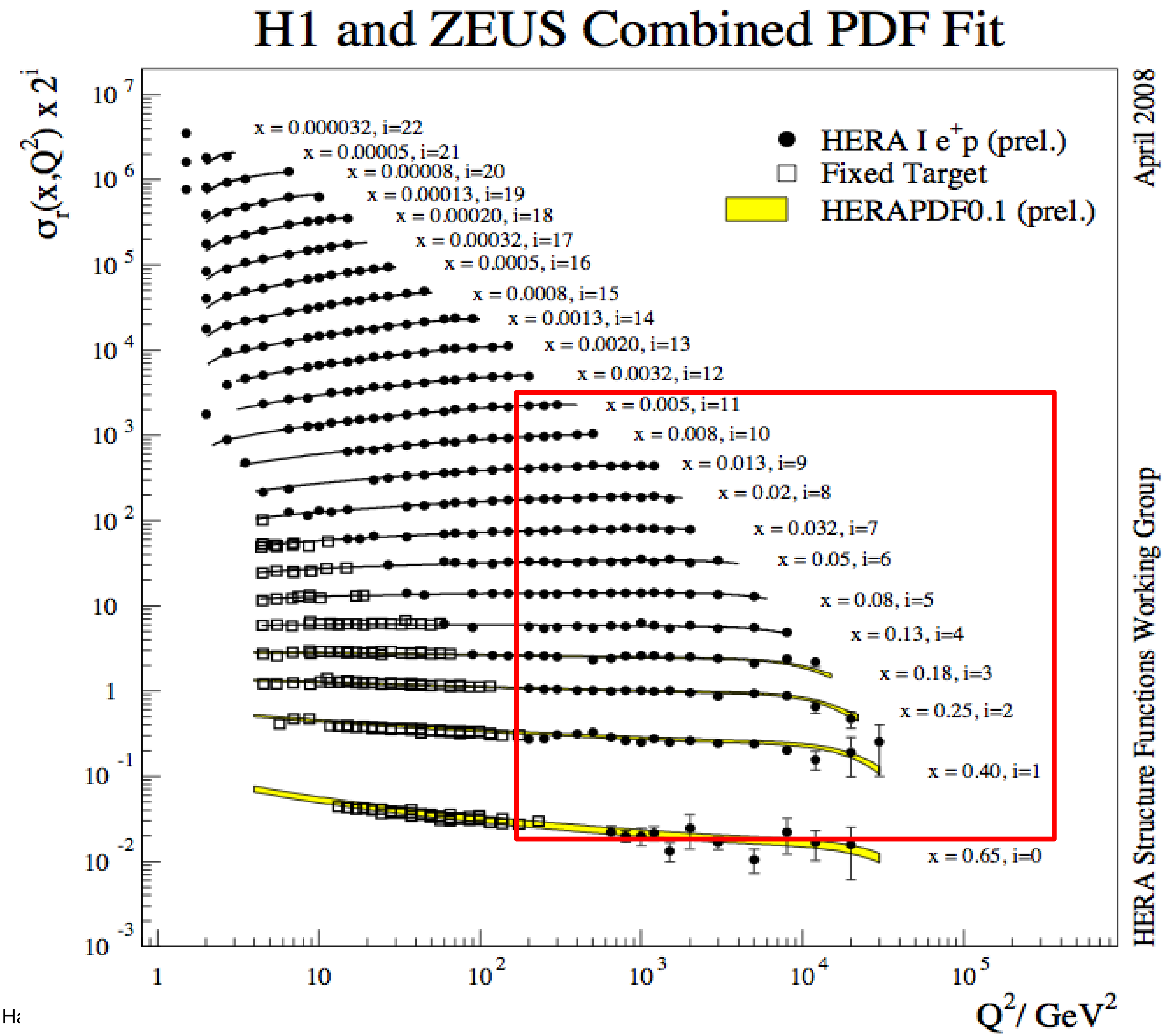
$$\sum_q \int_0^1 dx x \left[q(x) + \bar{q}(x) \right] \sim 0.5$$

- Where are the other 50 % of the proton's momentum ?

$$\int dx x q(x) \sim 0.1 \left[0.9 + 0.95 + 0.85 + 0.7 + 0.35 + 0.15 + 0.1 + 0.05 \right] = 0.1 \cdot 4.05 = 0.405$$

$$\int dx x \bar{q}(x) \sim 0.1 \left[0.42 + 0.2 + 0.06 + 0.03 + 0.01 \right] = 0.1 \cdot 0.72 = 0.072$$

Structure functions from HERA



Proton structure function does not depend on Q^2 for large x

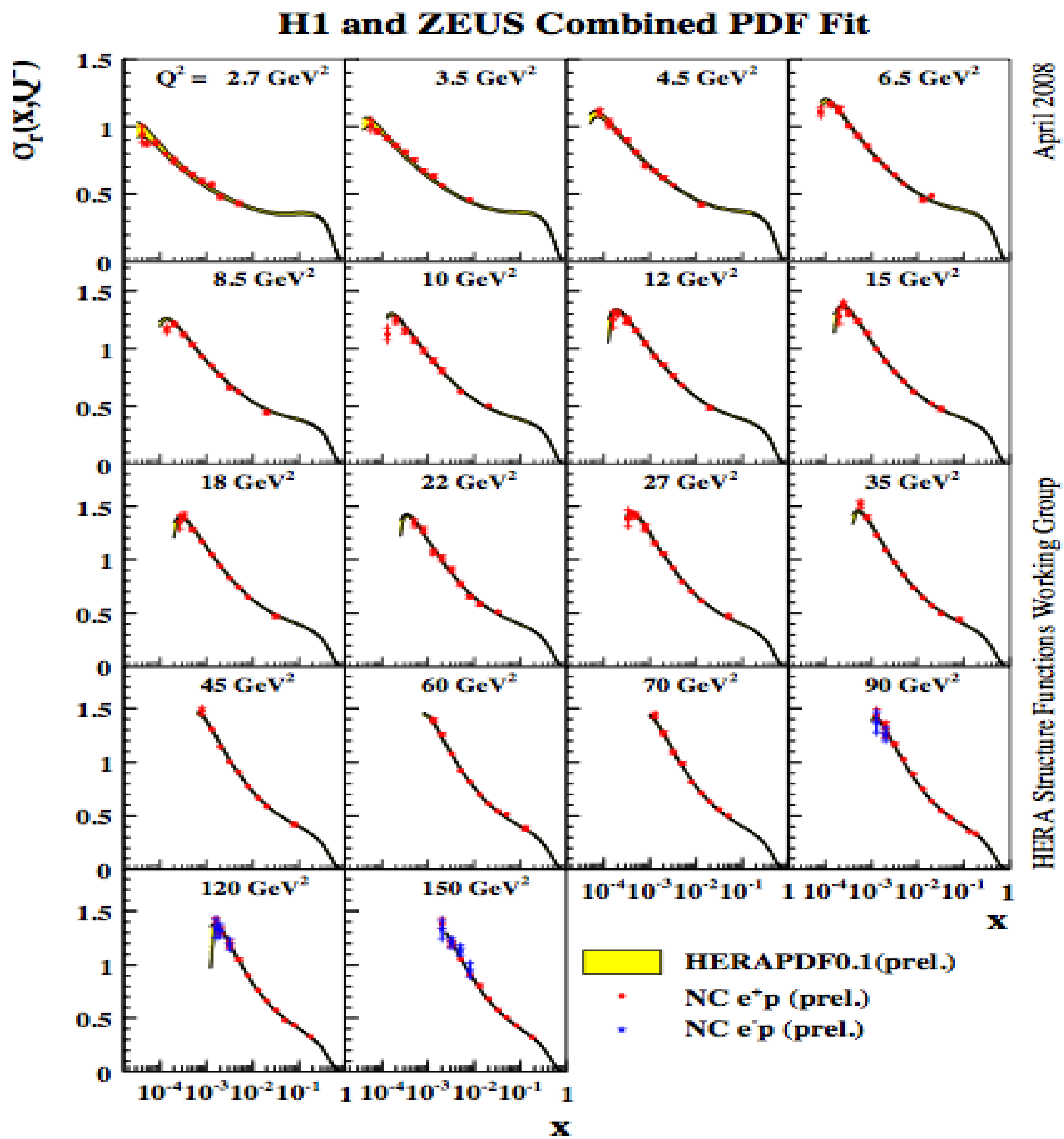
F_2 scales …

Quarks are pointlike constituents of proton

BUT things change at smaller x….. and smaller Q^2

Inelastic Scattering: main results

- F_2 scaling at large *x*
- ~ 50 % gluons
- F_2 rise at small x
    - How can rising F_2 be understood ?
    - Does rise continue forever ?
    - What limits F_2 ?



Inelastic Scattering: QPM (I)

- Key factor in QPM explanation is that over a short time in which the hard scattering takes place, the quarks behave as if they are free, i.e. no interaction between them.

- In the asymptotic limit ($Q^2 \to \infty$) the theory should describe quarks as free particles

- Equivalent demanding that effective charge in theory should vanish as smaller and smaller distances are probed.

Inelastic Scattering: QPM (II)

- Until 1973 in theories the reverse was true: because of screening of charge at larger distances coupling becomes smaller (QED)

- BREAK-THROUGH by 't Hooft (1972), Gross, Wilczek & Politzer (1973) non-Abelian theory describing asymptotic behaviour QCD

- As in QED there is screening at large distances by the color charge of quarks and gluons, but this is more than compensated by anti-screening (splitting) of gluons. Thus for $Q^2 \to \infty$ the effective coupling tends to vanish !

Deeper look to x-section:

separate leptonic from hadronic part

Separate $e\gamma$ part

- calculate $\gamma^* q \to q'$
- define: $$z = \frac{Q^2}{2 p_q . q}$$

Blackboard

- results $$\hat{\sigma} = \frac{4\pi^2\alpha}{2 p_q . q} e_q^2 \delta(1 - z)$$

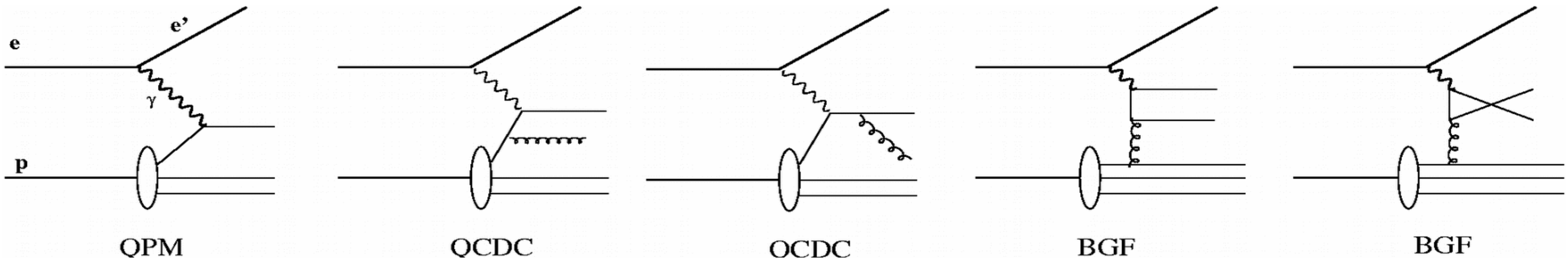
extract flux of virtual photons

Blackboard

$$\frac{d^2\sigma}{dydQ^2} = \frac{\alpha}{2\pi}\frac{1}{y}\frac{1}{Q^2}\left(1+(1-y)^2\right)\hat{\sigma}$$

- flux of virtual photons: different definitions exist....
- QCD is in $\hat{\sigma}$

Higher order corrections to DIS



- lowest order: $e + q \to e' + q' \quad \mathcal{O}(\alpha_s^0)$

$$\frac{d\sigma}{dy dQ^2} = \frac{4\pi\alpha^2}{Q^4}\frac{x}{y}\left(1 + (1-y)^2\right)\frac{1}{2}e_q^2\delta(x-\xi)$$

- higher order: $e + q \to e' + q' + g, \quad e + g \to e' + q + \bar{q} \quad \mathcal{O}(\alpha_s^1)$
- factorise electromagnetic vertex or calculate full $2 \to 3$ process
- use Weizsäcker(Z. Phys 88, 612 (1934)) -Williams (Phys Rev 45, 729 (1934)) (or Equivalent Photon(Budnev Phys Rep C15 , 181 (1974))) Approximation:

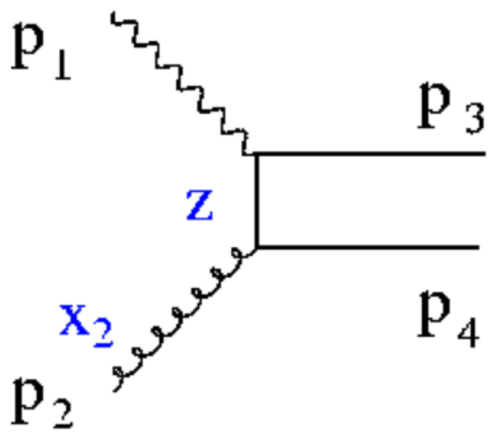
from:

$$\frac{d\sigma}{dy dQ^2} = \frac{\alpha}{2\pi}\frac{1}{yQ^2}\left(1 + (1-y)^2\right)\frac{4\pi^2\alpha}{Q^2}e_q^2 x\delta(x-\xi)$$

obtain:

$$\frac{d\sigma}{dy dQ^2} = F_{\gamma/e}(y, Q^2)\;\sigma(\gamma^* q \to q')$$

Kinematics



$$\hat{s} = (p_1 + p_2)^2 = Q^2 \frac{1-z}{z}$$

$$\hat{t} = k^2 = (p_1 - p_3)^2$$

$$\hat{u} = (p_2 - p_3)^2$$

Define:

$$\xi = \frac{p_2 k}{p_1 p_2} = 1 - \frac{p_2 p_3}{p_1 p_2}$$

$$z = \frac{Q^2}{2 p_1 p_2}$$

$$x_{bj} = z\xi$$

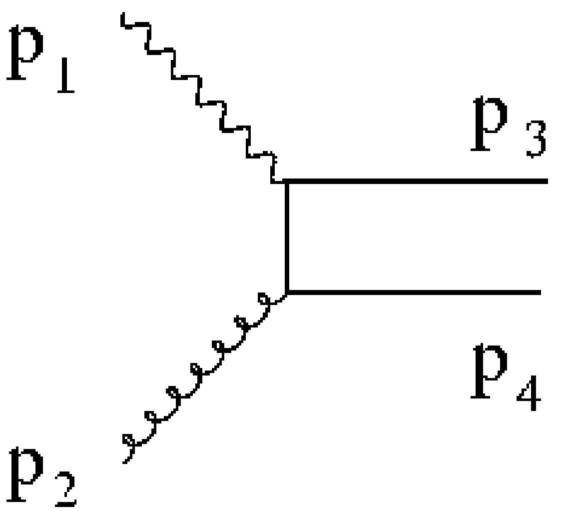
Using $s+t+u=-Q^2$ gives:

$$k_\perp^2 = \frac{\hat{t}\hat{u}\hat{s}}{(\hat{s}+Q^2)^2}$$

- and for $\hat{t} \ll \hat{s}$

$$k_\perp^2 = \frac{-\hat{t}\hat{s}}{\hat{s}+Q^2} = -t\,(1-z)$$

Partonic cross sections



$$\hat{s} = (p_1 + p_2)^2 = Q^2 \frac{1-z}{z}$$

$$\hat{t} = k^2 = (p_1 - p_3)^2$$

$$\hat{u} = (p_2 - p_3)^2$$

- Flux for virtual photons:

$$F = 4\sqrt{(p_1.p_2)^2 + m_1^2 m_2^2} = 2(\hat{s} + Q^2)$$

- x-section with virtual photons:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\frac{1}{\hat{s}^2}|M|^2 \to \frac{1}{16\pi}\frac{1}{\hat{s}+Q^2}\frac{1}{\hat{s}}|M|^2$$

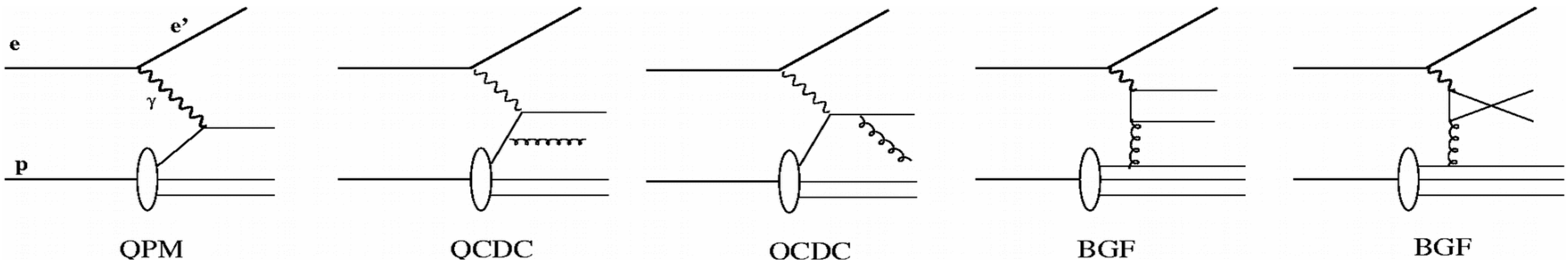
real photons

Isolate dominant parts

in the matrix elements:

region of small k_t !!!

Higher order corrections to DIS



- lowest order: $e + q \rightarrow e' + q' \quad \mathcal{O}(\alpha_s^0)$
- higher order: $e + q \rightarrow e' + q' + g, \quad e + g \rightarrow e' + q + \bar{q} \quad \mathcal{O}(\alpha_s^1)$

- What is the dominant part of the x-section ?
  - Investigate full x-section of QCDC and BGF
  - dominant part comes from small transverse momenta ...
  - rewrite x-section in terms of $k_\perp$
  - use small t limit:

$$\frac{d\sigma}{dk_\perp^2} = \frac{d\sigma}{dt}\frac{1}{(1-z)} = \frac{1}{(1-z)}\frac{1}{F} dLips|ME|^2$$

$$= \frac{1}{(1-z)}\frac{1}{16\pi}\frac{1}{\hat{s}+Q^2}\frac{1}{\hat{s}}|ME|^2$$

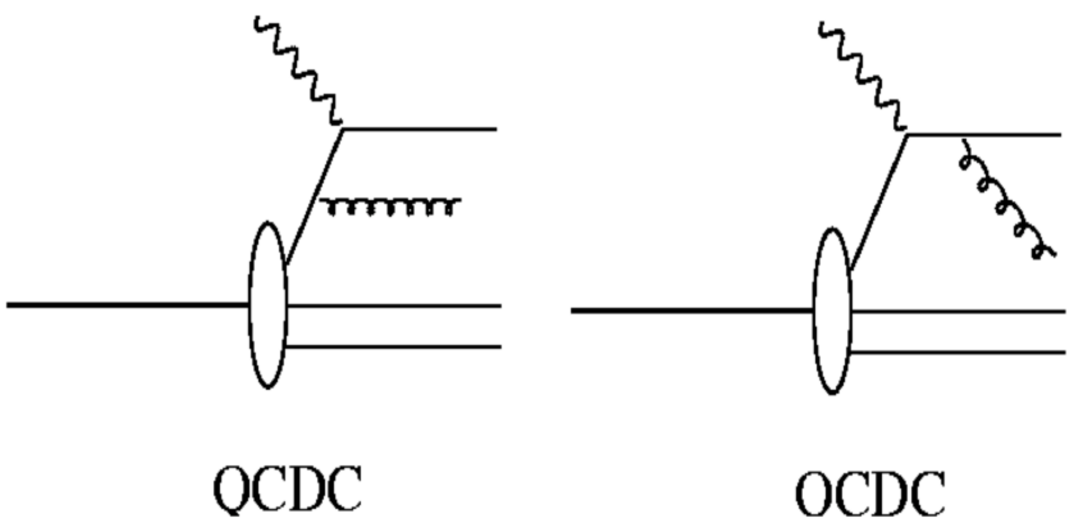
QCDC - contribution

$$|M|^2 = 32\pi^2 \left(e_q^2 \alpha\alpha_s\right) \frac{4}{3}\left[\frac{-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right]$$

$$= 32\pi^2 \left(e_q^2 \alpha\alpha_s\right) \frac{4}{3}\frac{-1}{t}\left[\frac{Q^2(1+z^2)}{z(1-z)} + \cdots\right]$$

Blackboard

integration over $k_\perp$ generates log BUT what is the lower limit

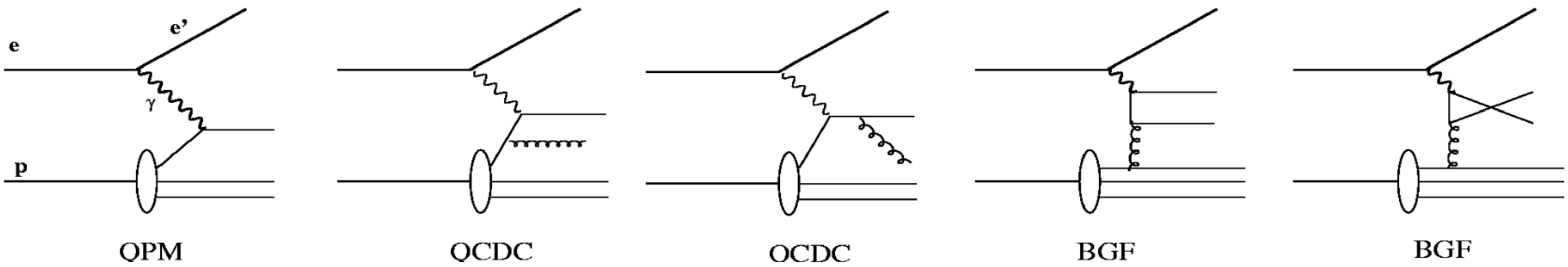


$$\frac{d\sigma}{dk_\perp^2} = \hat{\sigma}_0 e_q^2 \frac{\alpha_s}{2\pi}\frac{1}{k_\perp^2}\left[P_{qq}(z) + \cdots\right]$$

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \qquad \hat{\sigma}_0 = \frac{4\pi^2\alpha}{\hat{s}}$$

$$\sigma^{QCDC} = \hat{\sigma}_0 e_q^2 \frac{\alpha_s}{2\pi}\left[P_{qq}(z)\log\left(\frac{Q^2(1-z)}{\chi^2 z}\right) + \cdots\right]$$

Correction to cross section



- Connect with F_2:

$$\sigma^{\gamma * p} = \frac{4\pi^2\alpha}{Q^2}\left(F_2(x,Q^2) - F_L(x,Q^2)\right) \sim \frac{4\pi^2\alpha}{Q^2} F_2(x,Q^2) = \frac{4\pi^2\alpha}{2qP}\frac{F_2(x,Q^2)}{x}$$

$$\sigma^{\gamma * p} = \sigma_0 \frac{F_2(x,Q^2)}{x}$$

$$\sigma^{QPM} = \sigma_0 e_q^2 \delta\left(1 - \frac{x}{\xi}\right)$$

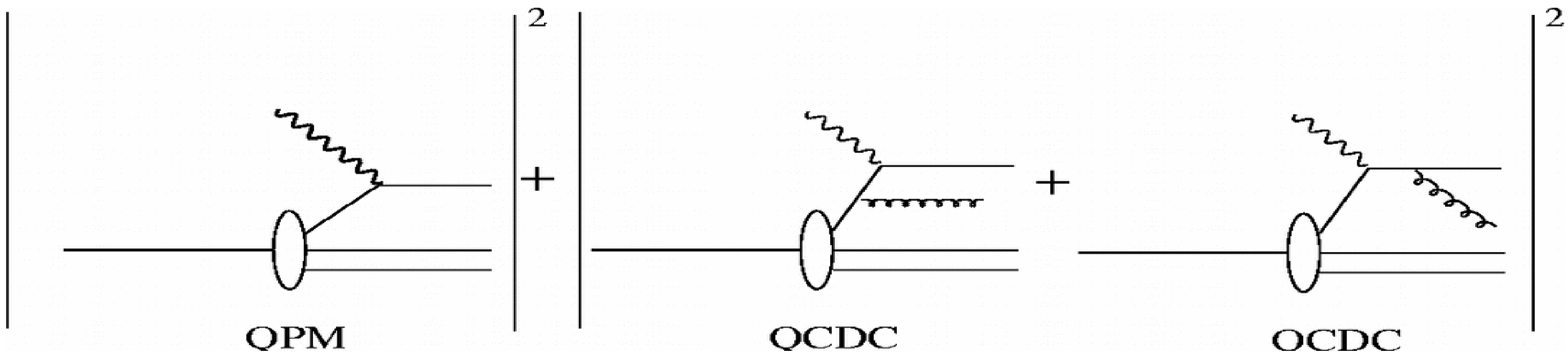
$$\sigma^{QCDC} = \hat{\sigma}_0 e_q^2 \otimes P_q(z) \otimes \log \ldots$$

ξ is parton momentum fraction

$$\sigma_0 = \frac{4\pi^2\alpha}{2qP}$$

QCDC contribution to F_2

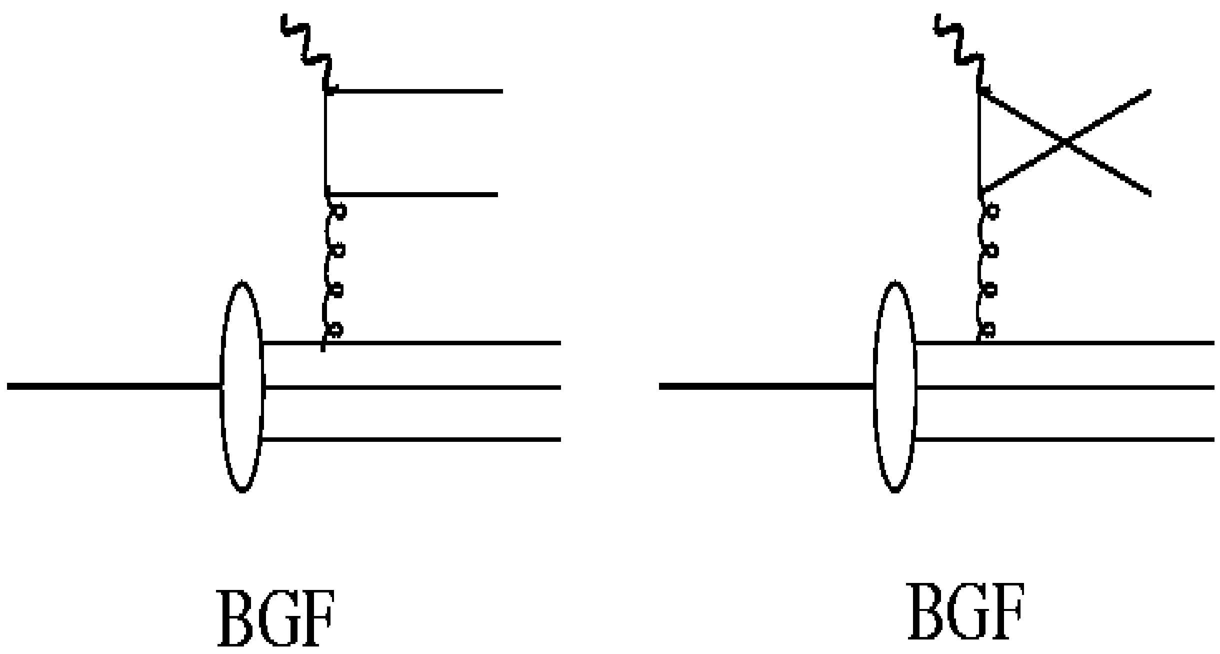
$$\frac{F_2}{x} =$$



again divergency for $k_\perp \to 0$ or $\chi \to 0$

- $$\frac{F_2}{x} = \sum e_q^2 \int \frac{d\xi}{\xi} f_q(\xi) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \left[\log\left(\frac{Q^2}{\chi^2}\right) + \log\left(\frac{1-z}{z}\right) + \ldots\right] + C_q(z, ..)\right]$$

Boson gluon fusion



$$|M|^2 = 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{1}{2} \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}} \right]$$

$$\frac{d\sigma}{dk_\perp^2} = \hat{\sigma}_0 e_q^2 \frac{\alpha_s}{2\pi} \frac{1}{k_\perp^2} \left[P_{qg}(z) + \cdots \right]$$

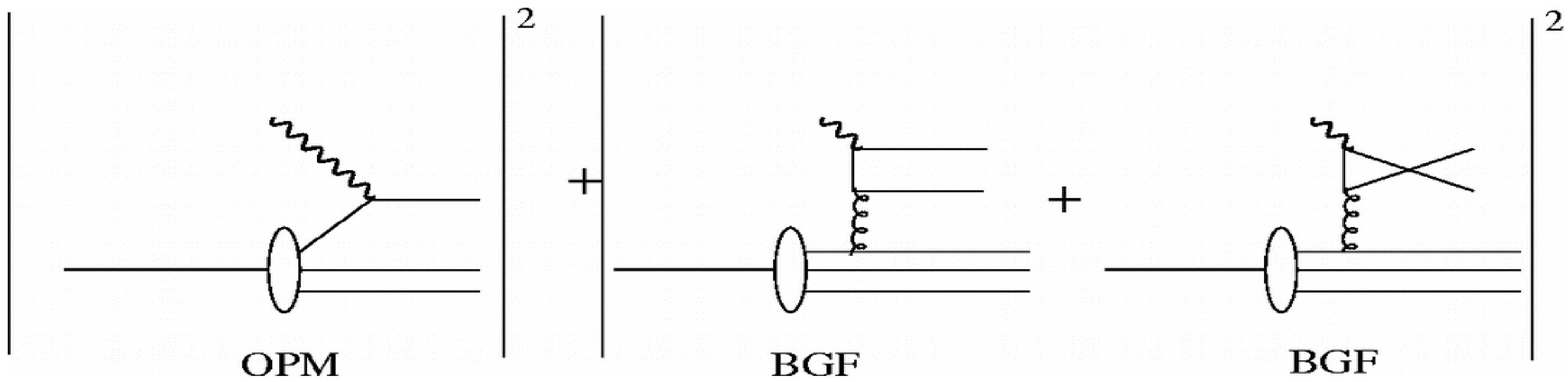
$$P_{qg}(z) = \frac{1}{2} \left(z^2 + (1-z)^2 \right)$$

- integration over k_t generates *log*, BUT what is the lower limit

$$\sigma^{BGF} = \hat{\sigma}_0 e_q^2 \frac{\alpha_s}{2\pi} \left[P_{qg}(z) \log \left(\frac{Q^2(1-z)}{\chi^2 z} \right) + \cdots \right]$$

BGF contribution to F_2

$$\frac{F_2}{x} =$$

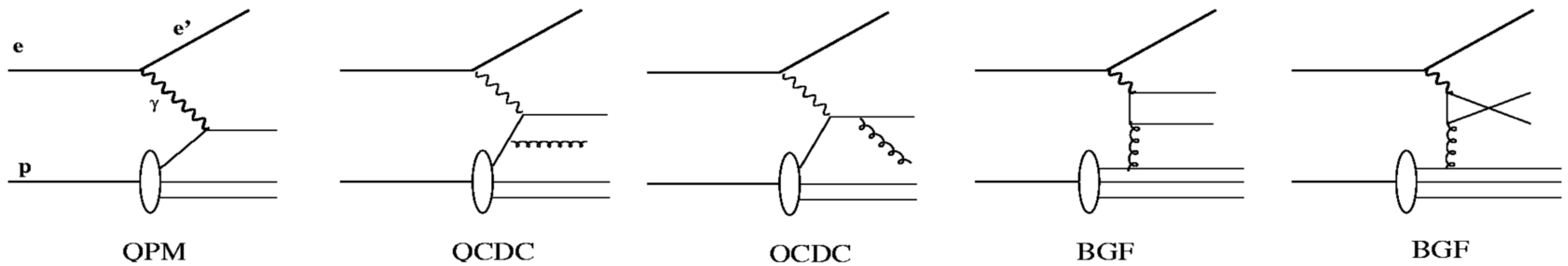


again divergency for $k_\perp \to 0$ or $\chi \to 0$

$$\frac{F_2}{x} = \sum e_q^2 \int \frac{dx_2}{x_2} g(x_2)$$

$$\frac{\alpha_s}{2\pi} \left(P_{qg}\left(\frac{x}{x_2}\right) \left[\log\left(\frac{Q^2}{\chi^2}\right) + \log\left(\frac{1-z}{z}\right) + \ldots \right] + C_g(z, \ldots) \right)$$

Adding up everything



$$\sigma^{\gamma * p} \sim \frac{4\pi^2\alpha}{Q^2} F_2(x, Q^2) = \frac{4\pi^2\alpha}{ys} \frac{F_2(x, Q^2)}{x}$$

$$\sigma^{\gamma * p} = \sigma_0 \frac{F_2(x, Q^2)}{x}$$

ξ is parton momentum fraction

$$\sigma_0 = \frac{4\pi^2\alpha}{2qP}$$

- Connect with F_2:

$$\begin{aligned} \sigma^{QPM} &= \sigma_0 e_q^2 \delta\left(1 - \frac{x}{\xi}\right) \\ \sigma^{QCDC} &= \hat{\sigma}_0 e_q^2 \otimes P_q(z) \otimes \log \ldots \\ \sigma^{BGF} &= \hat{\sigma}_0 e_q^2 \otimes P_g(z) \otimes \log \ldots \end{aligned}$$

Collinear factorization (part 1)

- bare distributions $q_0(x)$ are not measurable (like the bare charges ....)

$$F_2 = x \sum e_q^2 \left[q_0(x) + \int \frac{d\xi}{\xi} q_0(x) \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\left(\frac{Q^2}{\chi^2}\right) + C_q(z,..) \right]$$

- collinear singularities are absorbed into this bare distributions at a factorization scale $\mu^2 \gg \chi^2$, defining renormalized distributions

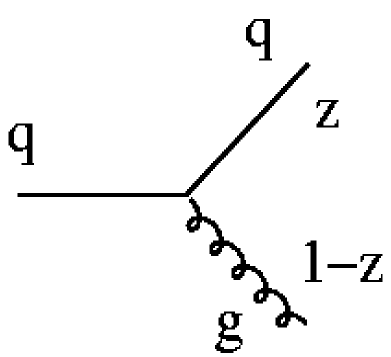
$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left[q_i^0(\xi) P_{qq}\left(\frac{x}{\xi}\right) \log\left(\frac{\mu^2}{\chi^2}\right) + C_q\left(\frac{x}{\xi}\right) \right] + \ldots$$

- now F_2 becomes:

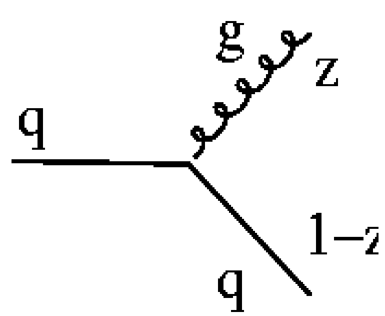
$$F_2 = x \sum e_q^2 \int \frac{d\xi}{\xi} q(\xi,\mu^2) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\left(\frac{Q^2}{\mu^2}\right) + C \right]$$

- separating or factorizing the long distance contributions to structure functions is a fundamental property of the theory
- factorization provides a description for dealing with the logarithmic singularities, there is arbitrariness in how the finite (non-logarithmic) parts are treated.

Splitting functions in lowest order

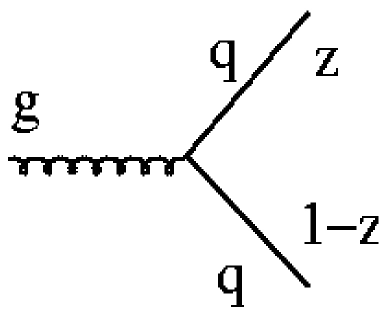


$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

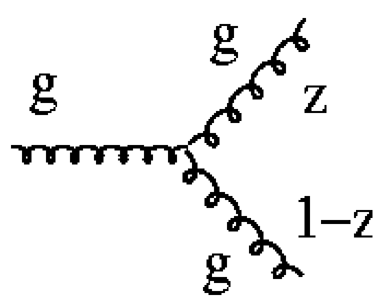


$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to EPA...



$$P_{qg} = \frac{1}{2}\left(z^2+(1-z)^2\right)$$



$$P_{gg} = 6\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

From factorization to DGLAP

BLACKBOARD

Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \chi^2$ and include soft, non-perturbative physics into renormalized parton density:

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\chi^2}\right)$$

- DokshitzerGribovLipatovAltarelliParisi equation:

V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20,
G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons....

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- DGLAP is the analogue to the beta function for running of the coupling

Collinear factorization

see handbook of pQCD, chapter IV, B

$$F_2^{(Vh)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_0^1 d\xi\, C_2^{(Vi)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \frac{\mu_f^2}{\mu^2}, \alpha_s(\mu^2)\right) \otimes f_{i/h}(\xi, \mu_f^2, \mu^2)$$

- Factorization Theorem in DIS (Collins, Soper, Sterman, (1989) in Pert. QCD, ed. A.H. Mueller, Wold Scientific, Singapore, p1.)
  - hard-scattering function $C_2^{(Vi)}$ is infrared finite and calculable in pQCD, depending only on vector boson *V*, parton *i*, and renormalization and factorization scales. It is independent of the identity of hadron *h*.
  - pdf $f_{i/h}(\xi, \mu_f^2, \mu^2)$ contains all the infrared sensitivity of cross section, and is specific to hadron h, and depends on factorization scale.
- Generalization: applies to any DIS cross section defined by a sum over hadronic final states .... but be careful what it really means....
- explicit factorization theorems exist for:
  - diffractive DIS (... see above....)
  - Drell Yan (in hadron hadron collisions)
  - single particle inclusive cross sections (fragmentation functions)

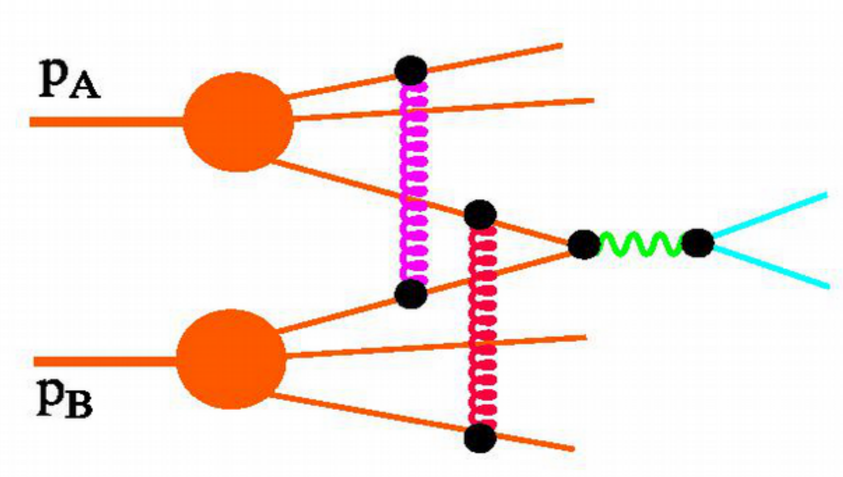
Factorization proofs and all that ...

- About factorization proofs (Wu-Ki Tung, pQCD and the parton structure of the nucleon, 2001, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 2* 887-971)

tions $F_a^{\wedge}(x, \frac{\omega}{m}, \alpha_s(\mu))$ (a = all parton flavors). Although the underlying physical ideas are relatively simple, as emphasized in the last two sections, the mathematical proofs are technically very demanding.[7,15,19] For this reason, actual proofs of factorization only exist for a few hard processes; and certain proofs (e.g. that for the Drell-Yan process) stayed controversial for some time before a consensus were reached.[15] Because of the general character of the physical ideas and the mathematical methods involved, however, it is generally *assumed* that the attractive *quark-parton model does apply to all high energy interactions* with at least one large energy scale.

$$\frac{d\sigma}{dy} = \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B f_A^a(\xi_A,\mu) f_B^b(\xi_B,\mu) \frac{d\hat{\sigma}_{ab}(\mu)}{dy} + \mathcal{O}\left(\left(\frac{m}{P}\right)^p\right)$$

- The problem with Drell-Yan: initial state interactions...
- factorization here does not hold graph-by-graph but only for all ....

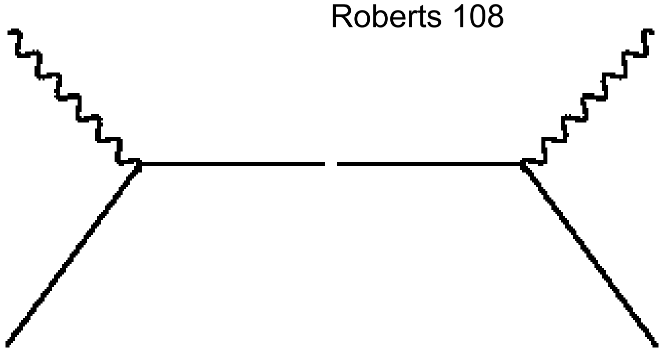


Collinear factorization ....

- So far considered only *"leading twist"*

  twist = dimension (spin) of operators in Operator Product Expansion (OPE)

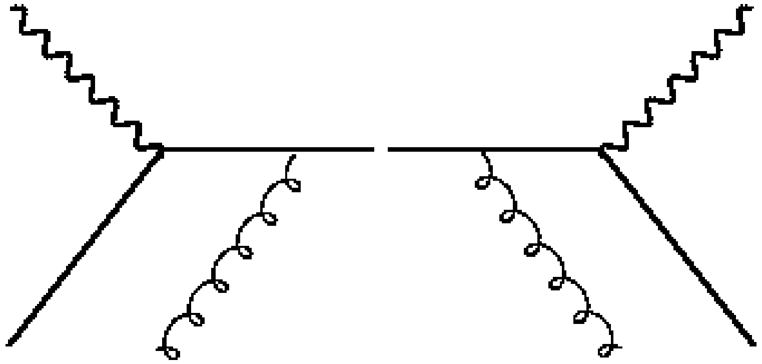
Ellis, Webber, Stirling, 123
Roberts 108

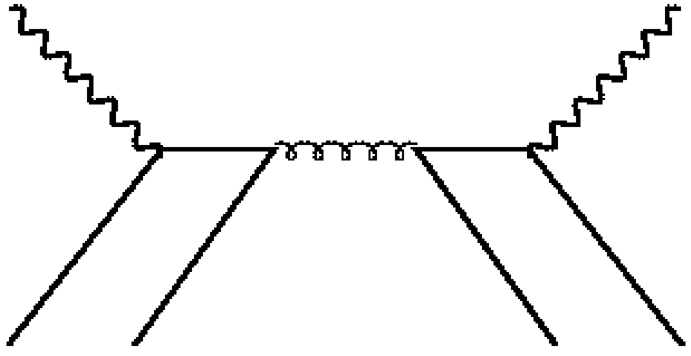


- Factorization theorem (Collins hep-ph/9709499):

$$F_2(x,Q^2) = \sum_i C_{2\,i} \otimes f_i + \text{non-leading power of } Q$$

- in general:





$$F_2(x,Q^2) = \sum_n \frac{B_n(x,Q^2)}{Q^{2n}}$$

n>0 higher twists
non-leading powers ...

- NOT covered by factorization theorem.... but contributions can be large ?!?

Warning on Factorization:

- The limits are factorization (i.e., the universality) of h h $\rightarrow$ h + X is not yet fully explored!
- You must surely sum over (i.e., not ask questions about) the soft stuff (as we do with jets)
- Some limits are becoming "clear" in h h $\rightarrow$ h h (b-to-b) + X
  See, e.g., J. Collins, hep-ph/0708.4410
- The INTRO discussion in
  G. Sterman, hep-ph/0807.5118
- The application of SCET (Soft Collinear Effective Theory)
  C. W. Bauer, et al., hep-ph/0808.2191
- See also, M. Seymour, et al., hep-ph/0808.1269

But even this is not the full story...

- factorization breaking in $pp \to j_1 j_2 X$

  J. Collins, J.W. Qiu hep-ph 0705.2141

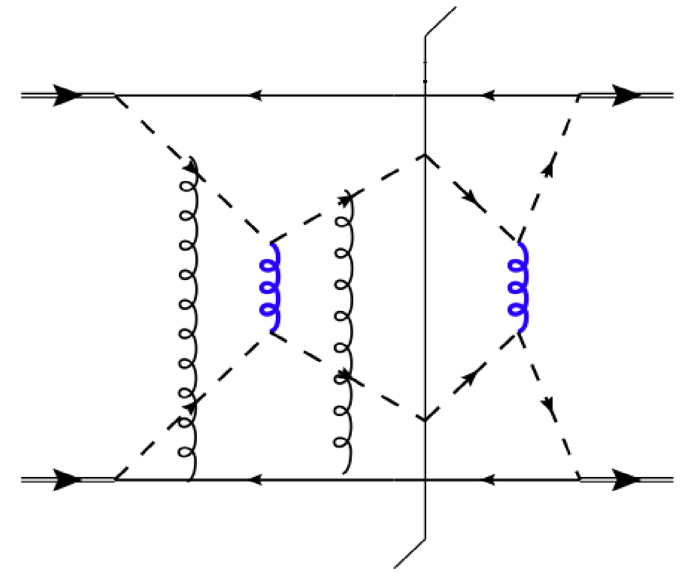
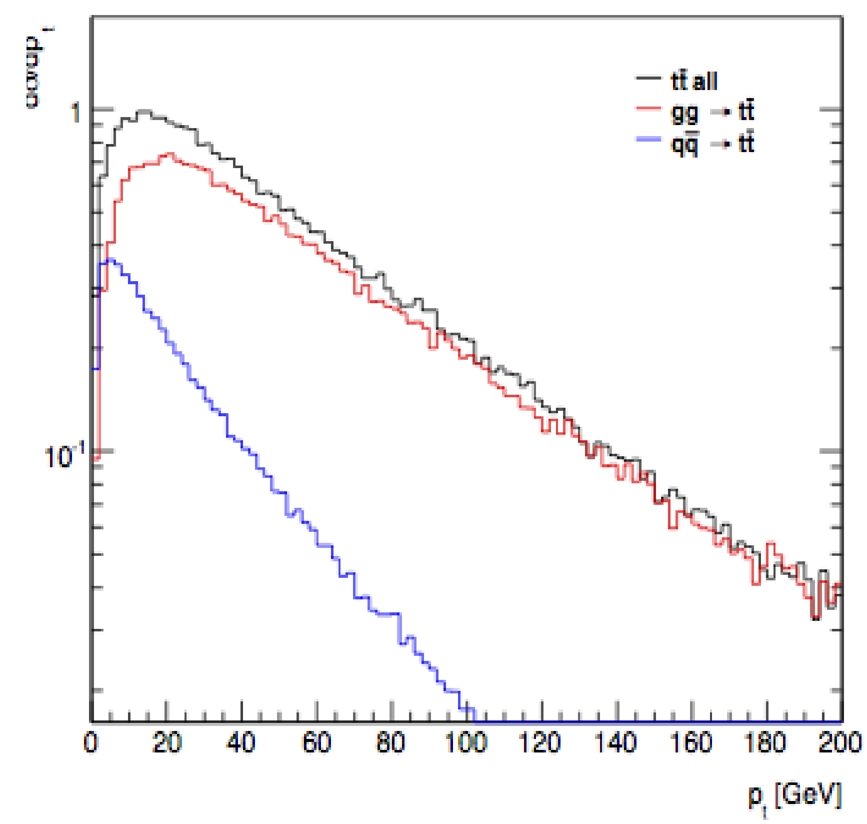


FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

- factorization breaking also in tt production at large $p_t{}^{top}$

  S. Catani, M. Grazzini, and A. Torre. Transverse-momentum resummation for heavy-quark hadropro- duction. arXiv 1408.4564



Collinear factorization schemes

- DIS scheme: absorbing all finite contributions C_q into quark densities, with no finite $\mathcal{O}(\alpha_s)$ corrections:

$$F_2^{DIS}(x,Q^2) = x \sum e_q^2 q(x,Q^2)$$

- $\overline{MS}$ scheme: only minimal contributions from the finite parts are absorbed in the quark distributions:

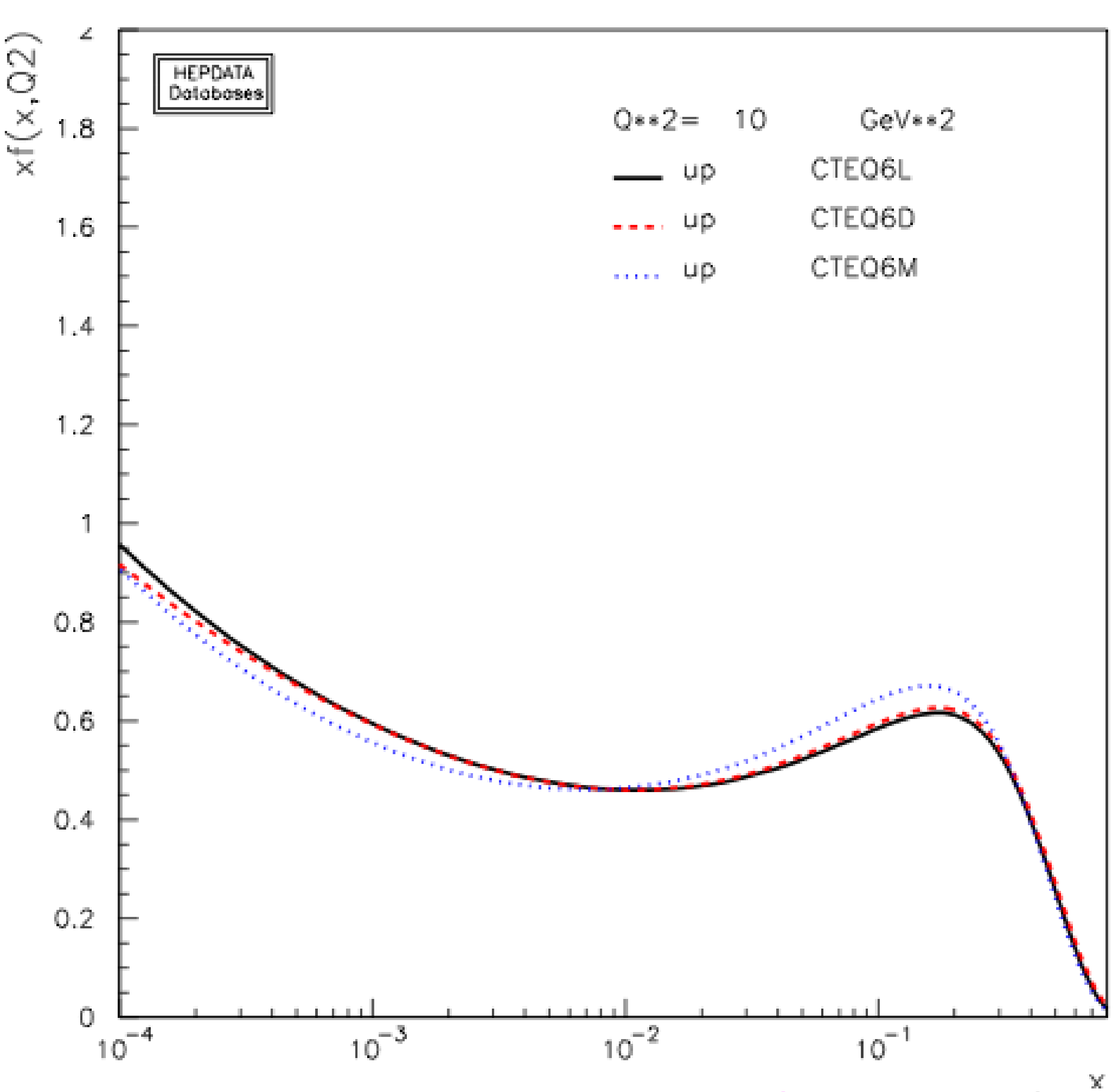
$$F_2^{\overline{MS}}(x,Q^2) = x \sum e_q^2 \int \frac{dx_2}{x_2} q^{\overline{MS}}(x,Q^2) \left[\delta\left(1 - \frac{x}{x_2}\right) + \frac{\alpha_s}{2\pi} C^{\overline{MS}}\left(\frac{x}{x_2}\right) + \ldots \right]$$

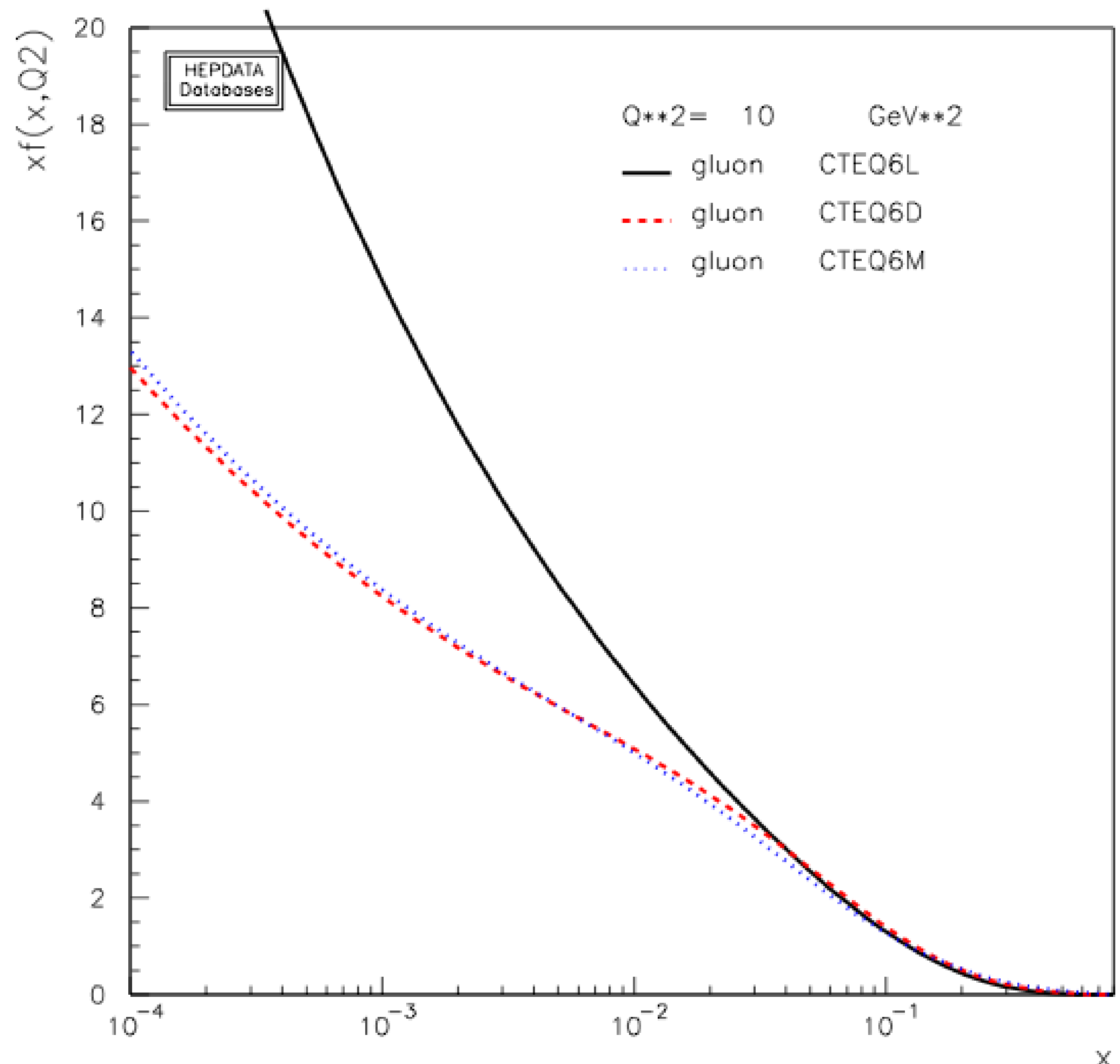
- once the scheme is chosen, it MUST be used in all other cross section calculations

- higher order corrections will of course depend on the chosen scheme...

- **BUT....** there are still other contributions to be included... gluon induced processes

PDFs in different fact. schemes





- differences between LO and NLO DIS, MS scheme in quark and gluon densities

- can make significant effects for x-sections

But back to the evolution equation

Splitting functions at higher orders

S. Moch, HERA-LHC workshop, June 2004

The calculation (in a nut shell)

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

– Calculate anomalous dimensions (Mellin moments of splitting functions)
$\longrightarrow$ divergence of Feynman diagrams in dimensional regularization $D = 4 - 2\varepsilon$

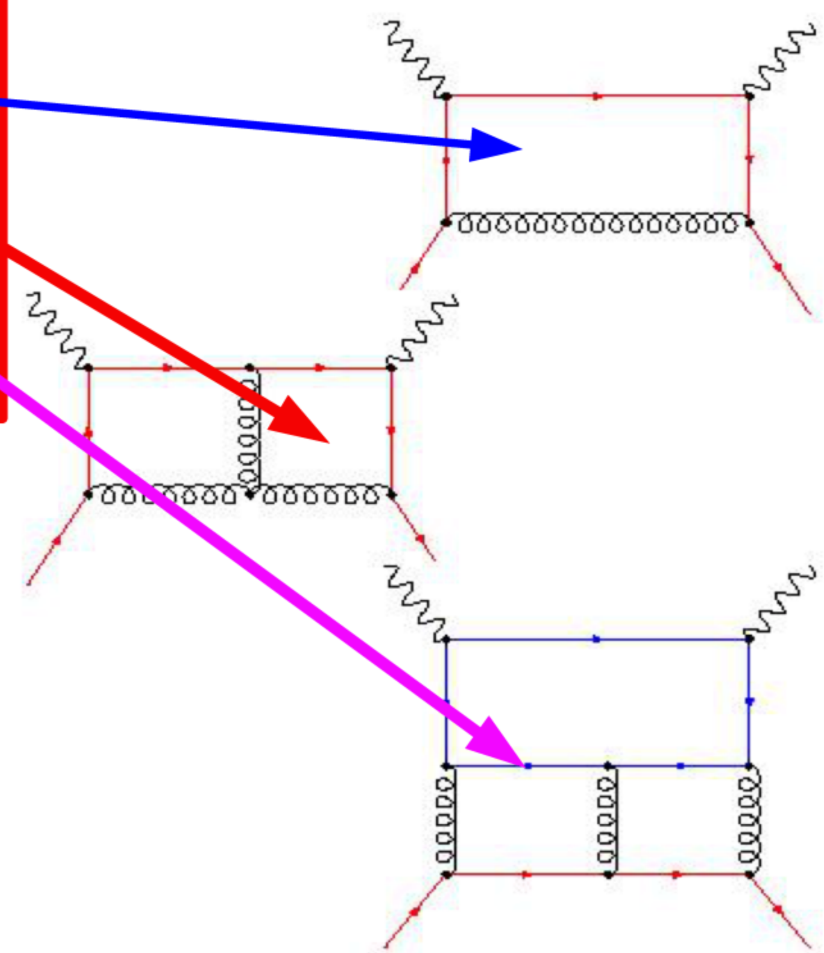
$$\gamma_{ij}^{(n)}(N) = -\int_0^1 dx\, x^{N-1} P_{ij}^{(n)}(x)$$

loops again:
1-loop
2-loops
3-loops

– **One-loop** Feynman diagrams
$\longrightarrow$ in total 18 for $\gamma_{ij}^{(0)}$ / $P_{ij}^{(0)}$
(pencil + paper)

– **Two-loop** Feynman diagrams
$\longrightarrow$ in total 350 for $\gamma_{ij}^{(1)}$ / $P_{ij}^{(1)}$
(simple computer algebra)

– **Three-loop** Feynman diagrams
$\longrightarrow$ in total 9607 for $\gamma_{ij}^{(2)}$ / $P_{ij}^{(2)}$
(cutting edge technology $\longrightarrow$ computer algebra system FORM Vermaseren '89-'04)



Splitting functions (cont'd)

S. Moch, HERA-LHC workshop, June 2004

LO and NLO singlet splitting functions

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

$$
\begin{aligned}
P^{(0)}_{\rm ps}(x) &= 0 \\
P^{(0)}_{\rm qg}(x) &= 2 n_f \, p_{\rm qg}(x) \\
P^{(0)}_{\rm gq}(x) &= 2 C_F \, p_{\rm gq}(x) \\
P^{(0)}_{\rm gg}(x) &= C_A \Big(4 p_{\rm gg}(x) + \frac{11}{3}\delta(1-x) \Big) - \frac{2}{3} n_f \, \delta(1-x)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm ps}(x) &= 4 C_F n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2 \Big[\frac{8}{3}{\rm H}_0 - \frac{56}{9} \Big] + (1+x) \Big[5{\rm H}_0 - 2{\rm H}_{0,0} \Big] \Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4 C_A n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2 p_{\rm qg}(-x){\rm H}_{-1,0} - 2 p_{\rm qg}(x){\rm H}_{1,1} + x^2 \Big[\frac{44}{3}{\rm H}_0 - \frac{218}{9} \Big] \\
&+ 4(1-x) \Big[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1 \Big] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0 \Big) + 4 C_F n_f \Big(2 p_{\rm qg}(x) \Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 \\
&- \zeta_2 \Big] + 4x^2 \Big[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2} \Big] + 2(1-x) \Big[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4} \Big] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0 \Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4 C_A C_F \Big(\frac{1}{x} + 2 p_{\rm gq}(x) \Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1 \Big] - x^2 \Big[\frac{8}{3}{\rm H}_0 - \frac{44}{9} \Big] + 4\zeta_2 - 2 \\
&- 7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x) \Big[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9} \Big] - 2 p_{\rm gq}(-x){\rm H}_{-1,0} \Big) - 4 C_F n_f \Big(\frac{2}{3}x \\
&- p_{\rm gq}(x) \Big[\frac{2}{3}{\rm H}_1 - \frac{10}{9} \Big] \Big) + 4 C_F^2 \Big(p_{\rm gq}(x) \Big[3{\rm H}_1 - 2{\rm H}_{1,1} \Big] + (1+x) \Big[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0 \Big] - 3{\rm H}_{0,0} \\
&+ 1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x \Big)
\end{aligned}
$$

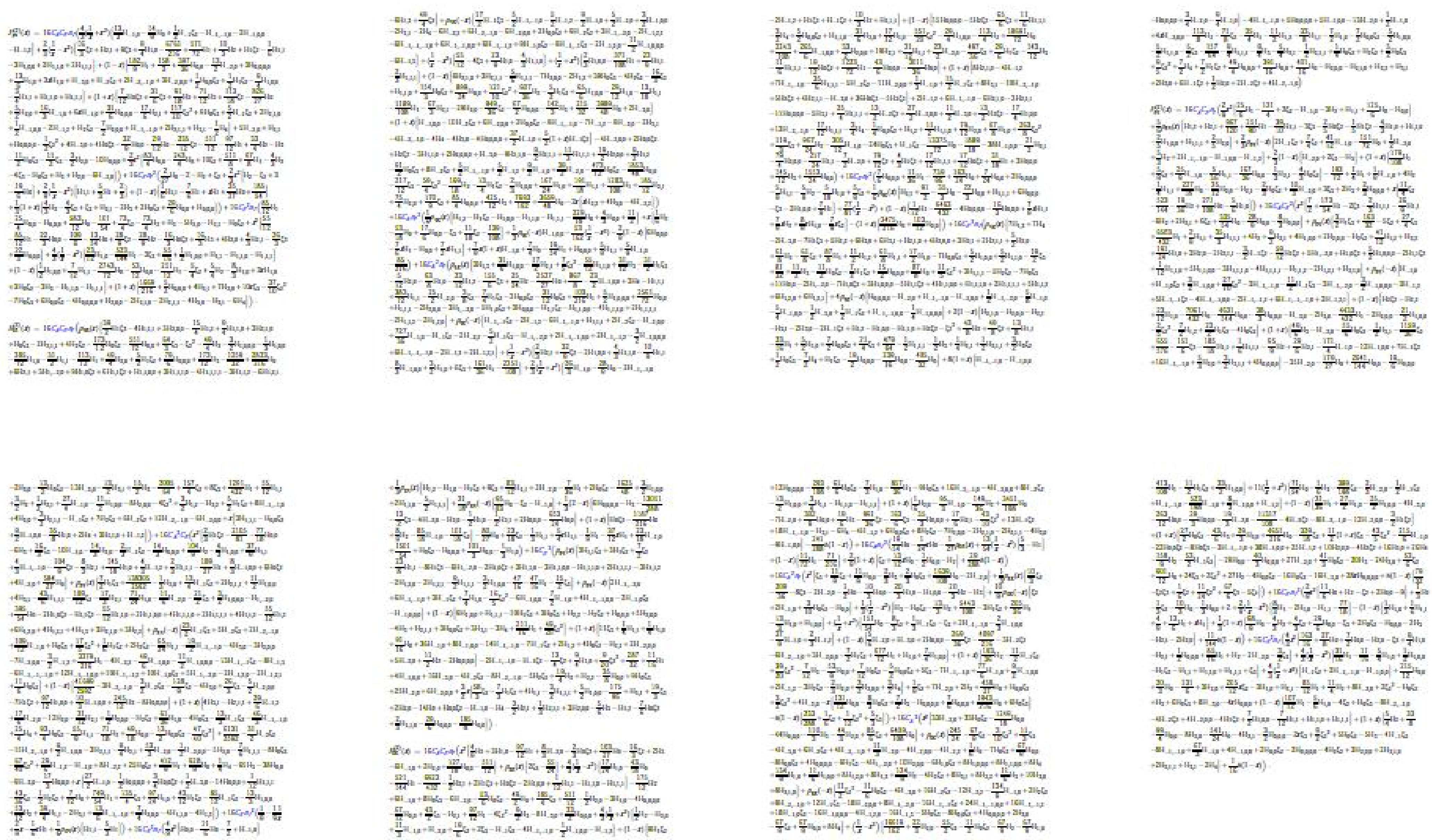
$$
\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4 C_A n_f \Big(1 - x - \frac{10}{9} p_{\rm gg}(x) - \frac{13}{9} \Big(\frac{1}{x} - x^2 \Big) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x) \Big) + 4 C_A^2 \Big(27 \\
&+ (1+x) \Big[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2} \Big] + 2 p_{\rm gg}(-x) \Big[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2 \Big] - \frac{67}{9} \Big(\frac{1}{x} - x^2 \Big) - 12{\rm H}_0 \\
&- \frac{44}{3} x^2 {\rm H}_0 + 2 p_{\rm gg}(x) \Big[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2 \Big] + \delta(1-x) \Big[\frac{8}{3} + 3\zeta_3 \Big] \Big) + 4 C_F n_f \Big(2{\rm H}_0 \\
&+ \frac{2}{3}\frac{1}{x} + \frac{10}{3} x^2 - 12 + (1+x) \Big[4 - 5{\rm H}_0 - 2{\rm H}_{0,0} \Big] - \frac{1}{2}\delta(1-x) \Big) .
\end{aligned}
$$

Splitting functions (cont'd)

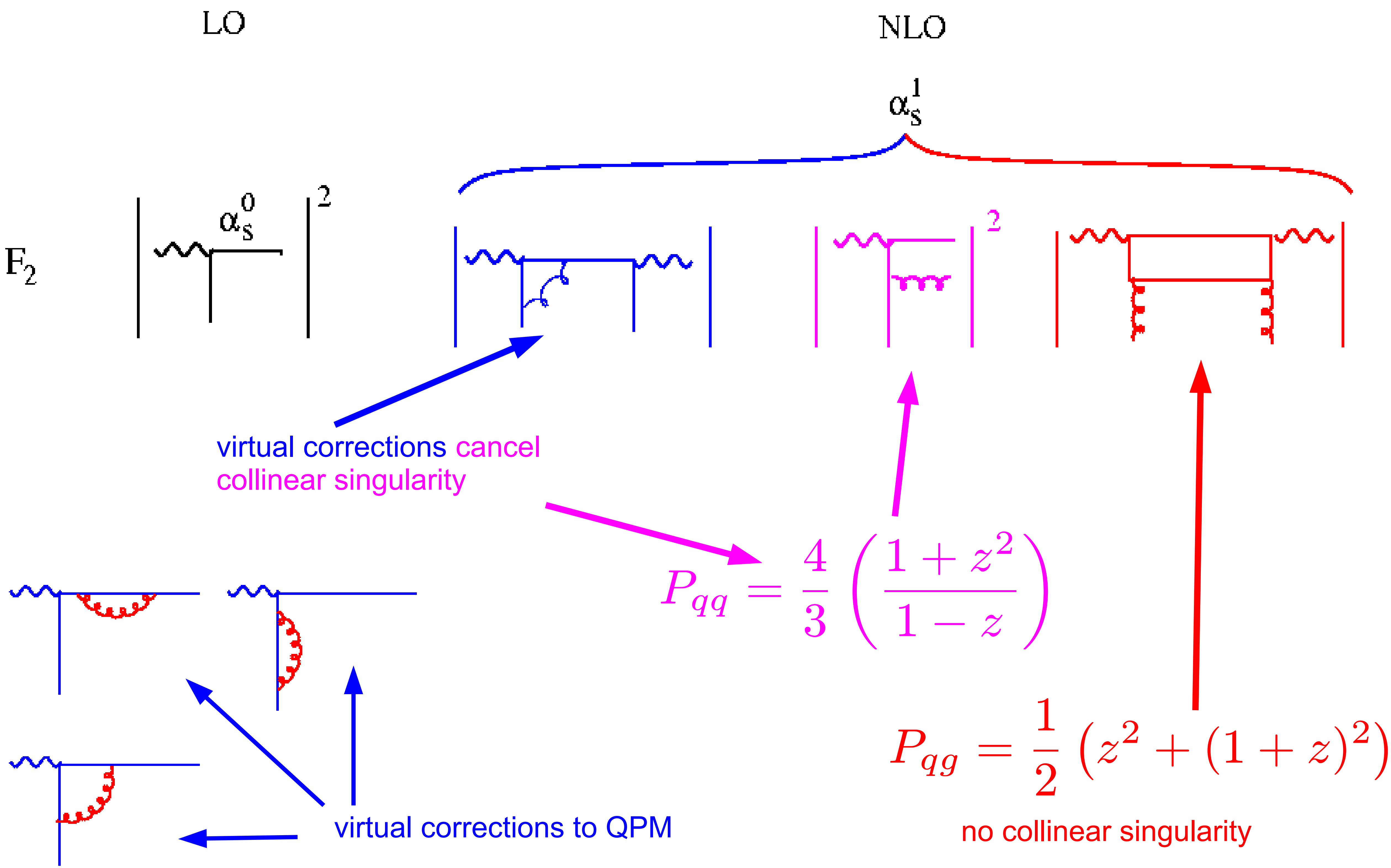
$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{(2)}(z) + \ldots$$

S. Moch, HERA-LHC workshop, June 2004

NNLO singlet splitting functions



NLO contributions to $F_2(x,Q^2)$



Evolution kernels – splitting fcts

- some of the splitting functions are also divergent...

$$\frac{1}{1-z}$$

- use *plus-distribution* to avoid dangerous region:

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{1-z}$$

Blackboard

- divergence cancelled by virtual corrections ...
- use splitting functions with plus-distribution

Conservation rules with DGLAP

$$\int_0^1 dxx \left[\sum_{i=-6}^{6} q(x,\mu^2) + g(x,\mu^2)\right] = 1$$

- use DGLAP

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

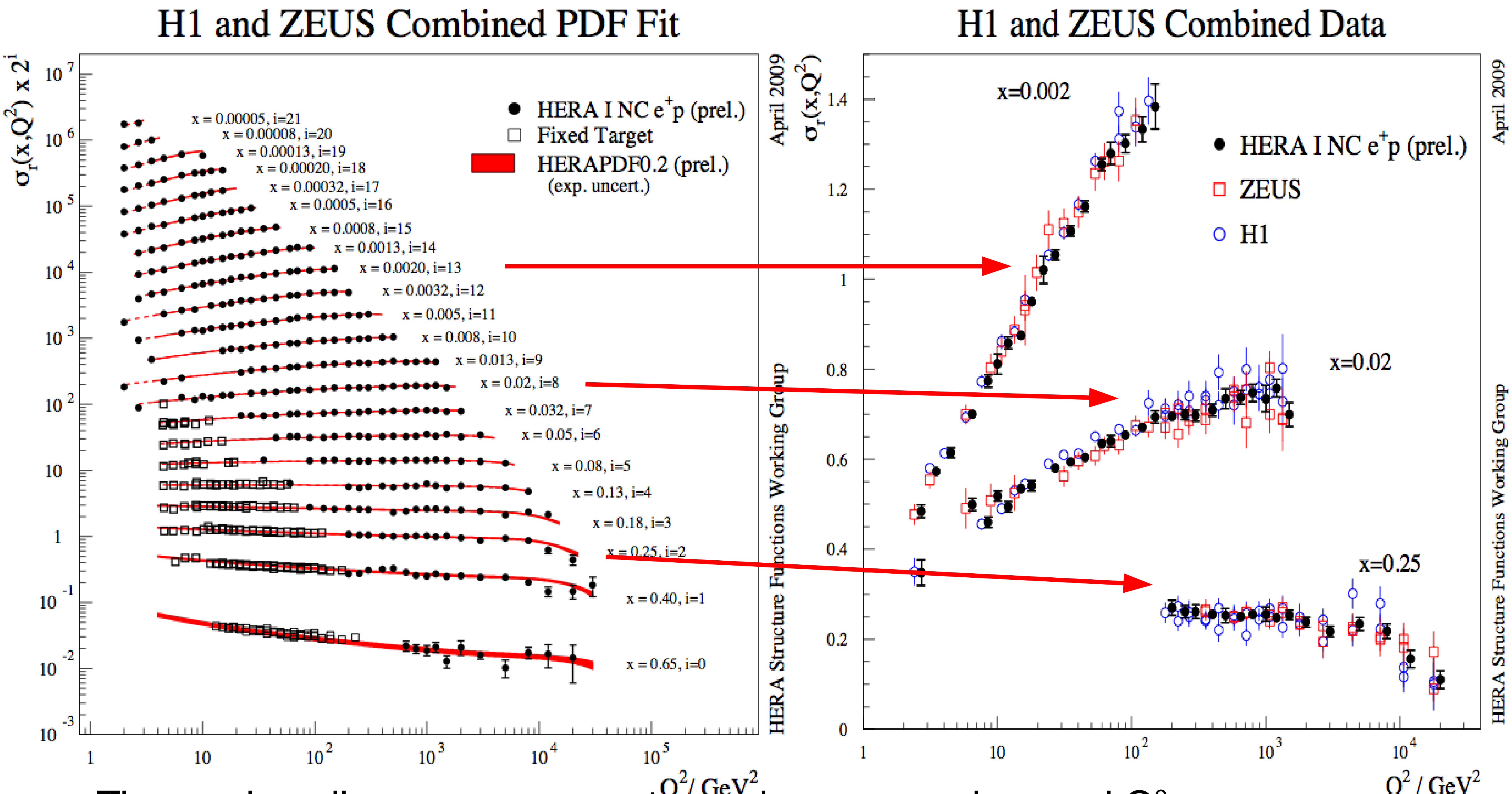
➔ to obtain:

$$\int_0^1 dx\, x\,[P_{qq}(x) + P_{gq}(x)] \quad = \quad 0$$

$$\int_0^1 dx\, x\,[P_{gg}(x) + 2n_f P_{qg}(x)] \quad = \quad 0$$

How to apply these results

Applying DGLAP to DIS data ...

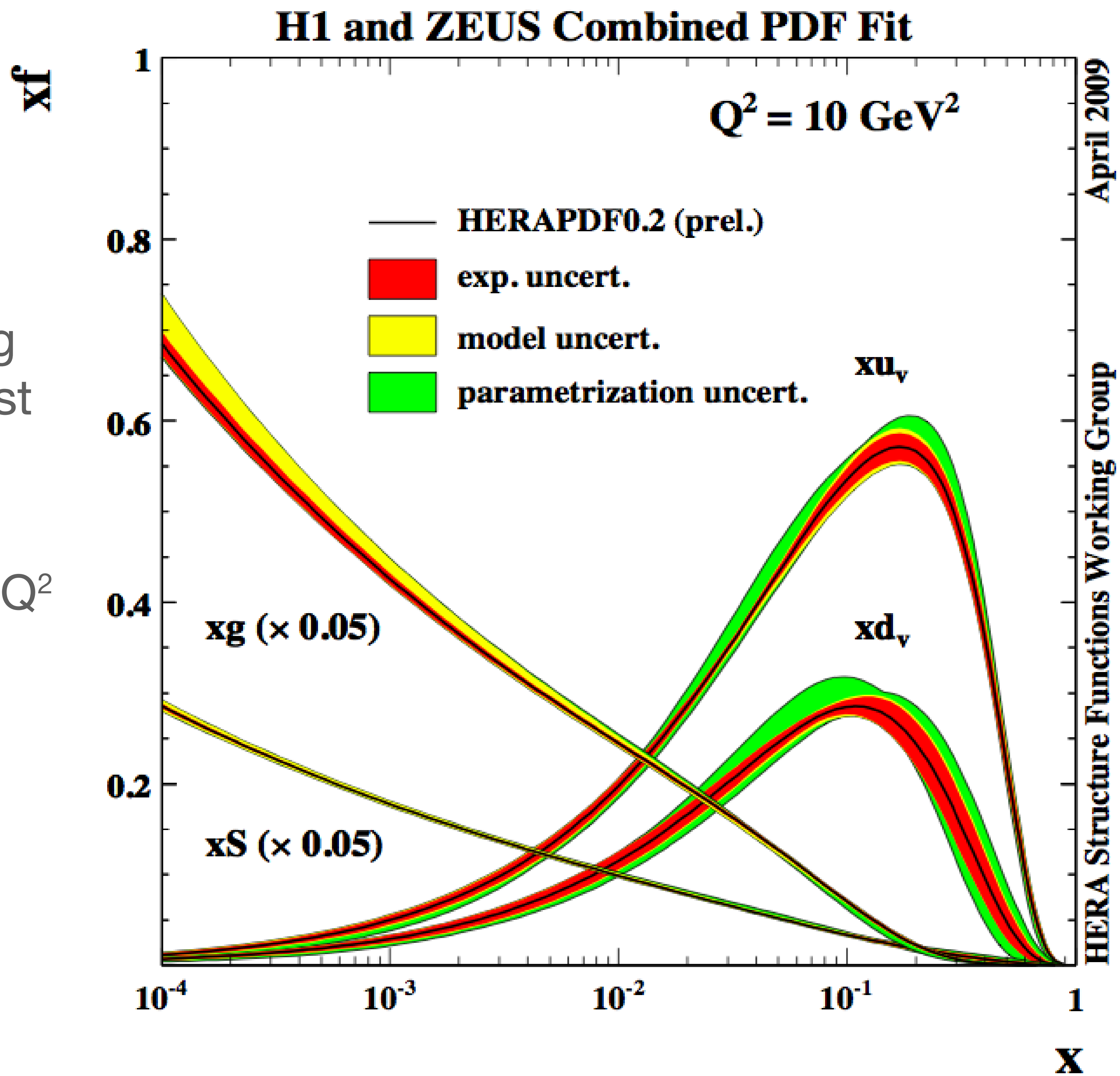


- Theory describes measurement over huge range in x and Q^2
- Success of theory (DGLAP)

Extraction of PDFs from DGLAP fits

- Sum rules are essential to constrain starting distributions

- Solve DGLAP equations
- adjust input parameters (starting distributions) such that F2 is best described
- extract PDFs as fct of x
- then DGLAP gives PDFs at any Q^2



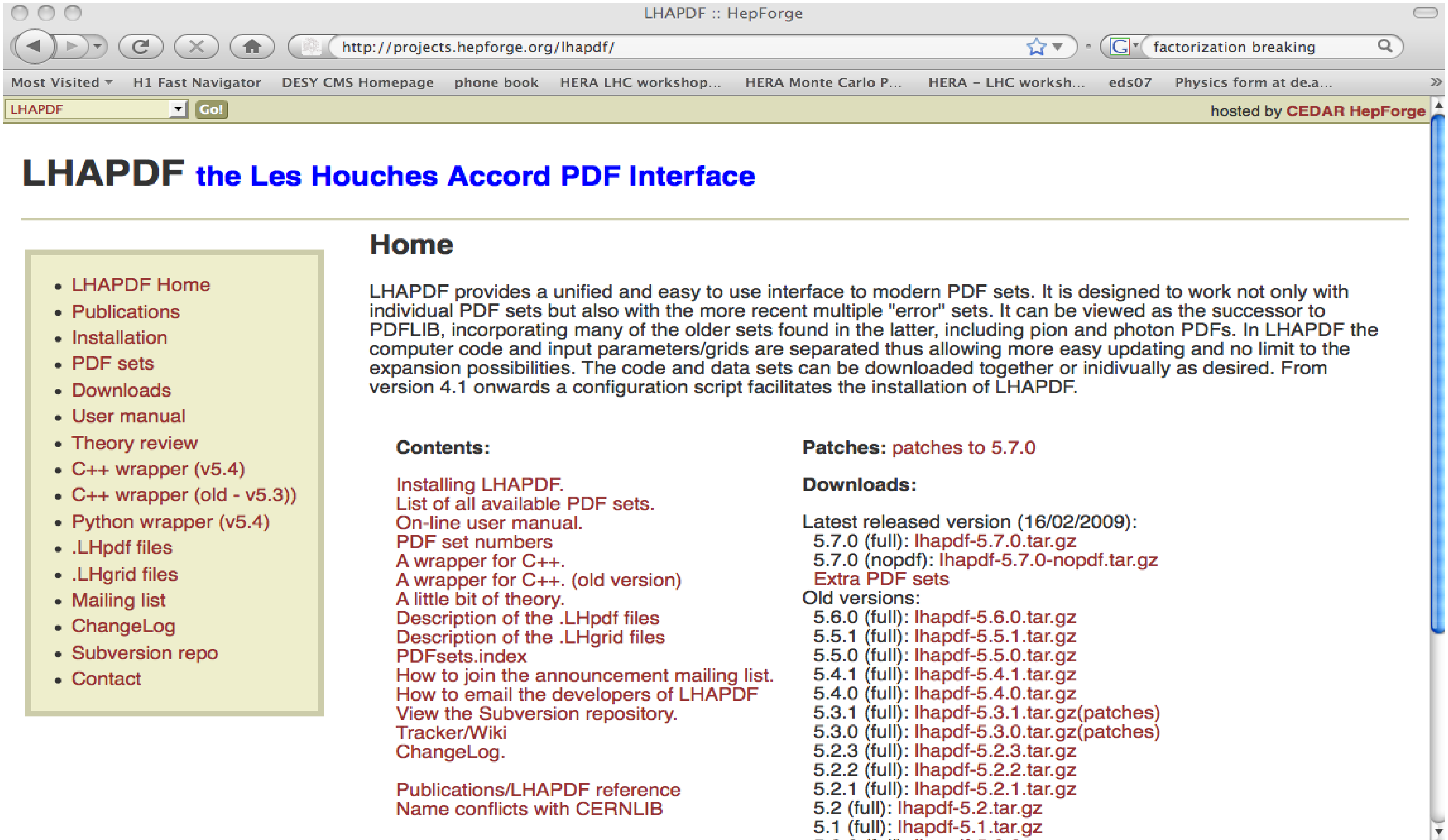
Solving DGLAP equations ...

- Different methods to solve integro-differential equations
  - brute-force (BF) method (M. Miyama, S. Kumano CPC 94 (1996) 185)

$$\frac{df(x)}{dx} = \frac{f(x)_{m+1} - f(x)_m}{\Delta x_m} \qquad \int f(x)dx = \sum f(x)_m \Delta x_m$$

  - Laguerre method (S. Kumano J.T. Londergan CPC 69 (1992) 373, and L. Schoeffel Nucl.Instrum.Meth.A423:439-445,1999)
  - Mellin transforms (M. Glueck, E. Reya, A. Vogt Z. Phys. C48 (1990) 471)
  - QCDNUM: calculation in a grid in x,Q2 space (M. Botje Eur.Phys.J. C14 (2000) 285-297)
  - CTEQ evolution program in x,Q2 space: http://www.phys.psu.edu/~cteq/
  - QCDFIT program in x,Q2 space (C. Pascaud, F. Zomer, LAL preprint LAL/94-02, H1-09/94-404,H1-09/94-376)
  - MC method using Markov chains (S. Jadach, M. Skrzypek hep-ph/0504205)
  - Monte Carlo method from iterative procedure
- brute-force method and MC method are best suited for detailed studies of branching processes !!!

Evolution code in LHAPDF



Can use LHAPDF to evolve starting distribution to any Q^2 with

- CTEQ, QCDNUM, and other evolution packages...