From HERA to the LHC

H. Jung (DESY)

From HERA to the LHC

H. Jung (DESY)

or

Why HERA physics is important for discoveries at LHC !

"…The mechanic, who wishes to do his work well, must first sharpen his tools …"

—Chapter15, "**The Analects**" attributed to Confucius, translated by James Legge.
(from X. Zu talk at DIS05)

From HERA to the LHC

Many thanks to all conveners and authors !

CERN–2005–014
14 December 2005

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

HERA AND THE LHC

A workshop on the implications of HERA for LHC physics

March 2004 — March 2005

hep-ph/0601012
hep-ph/0601013

Proceedings

Editors: A. De Roeck and H. Jung

Available on request from CERN/DESY libs

>650 pages

GENEVA
2005

Next working group week:
29. Oct. - 2. Nov. 2007, DESY

next workshop
May 2008 CERN

From HERA to the LHC

H. Jung (DESY)

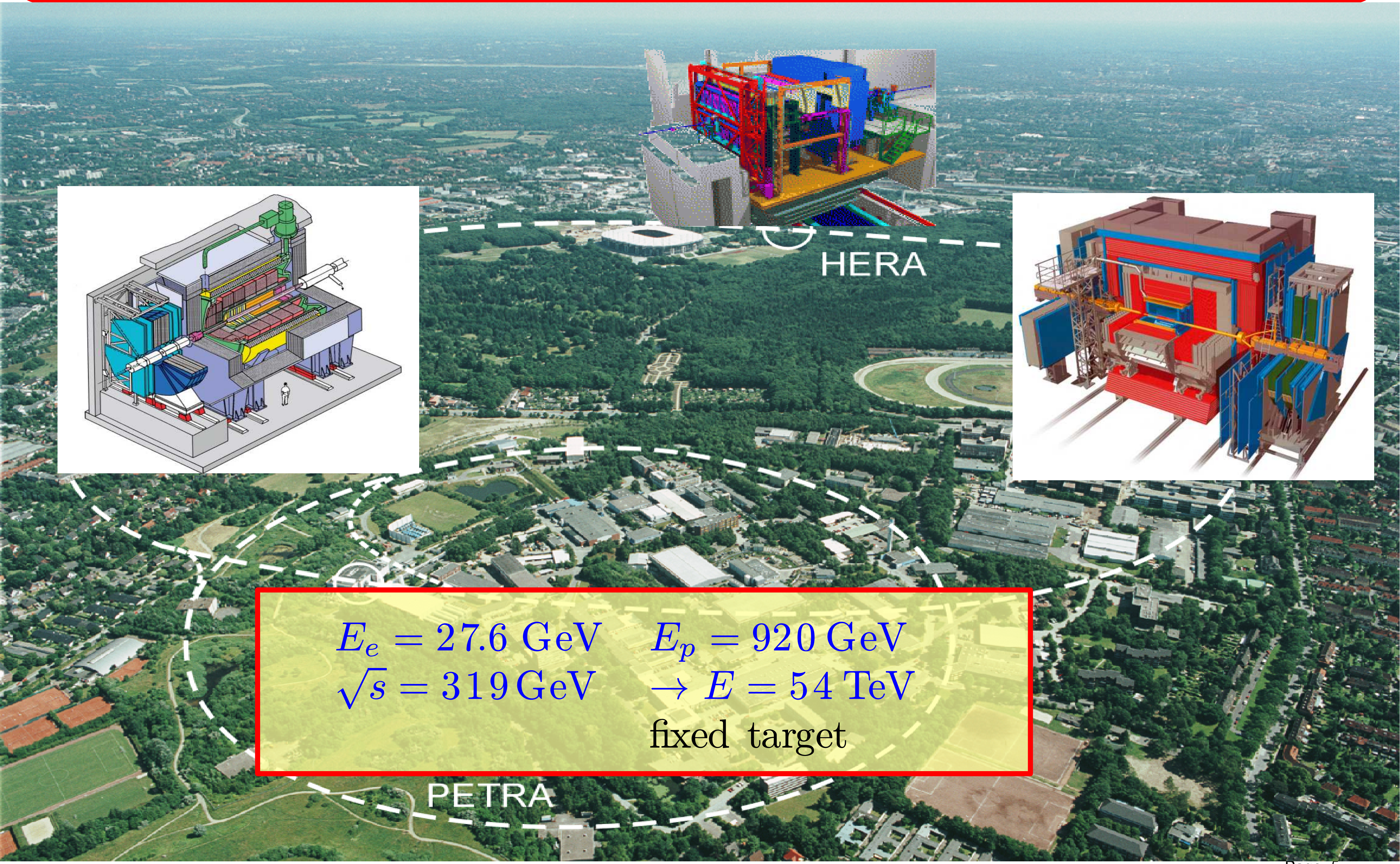
- HERA and the structure of the proton
- QCD is challenging
  - from inclusive x-section measurements to detailed investigations of QCD
    - measurements of hadronic final states:
    - ➔ lead to a detailed understanding of QCD
    - ➔ jets, heavy flavours
- next lecture:
  - ➔ implications and applications for LHC
  - ➔ PDFs, multiparton interactions, etc

lectures based on lecture series:

"QCD & collider physics" H.Jung,J. Bartels University HH, 2005 -2007

Contributions to "HERA and the LHC" workshops: www,desy.de/~heralhc

HERA collider and experiments



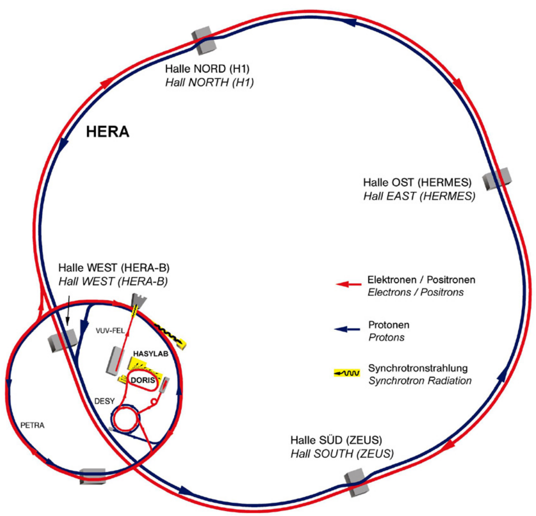
$E_e = 27.6\,\mathrm{GeV} \quad E_p = 920\,\mathrm{GeV}$

$\sqrt{s} = 319\,\mathrm{GeV} \quad \to E = 54\,\mathrm{TeV}$

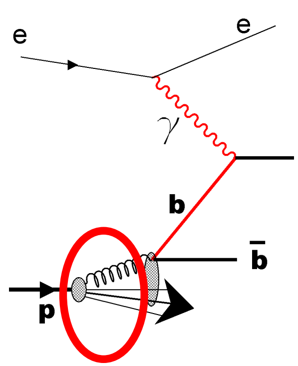
fixed target

What is HERA doing in Hamburg ?

electron proton collider HERA
√s = 320 GeV



HERA: QCD
structure of the proton

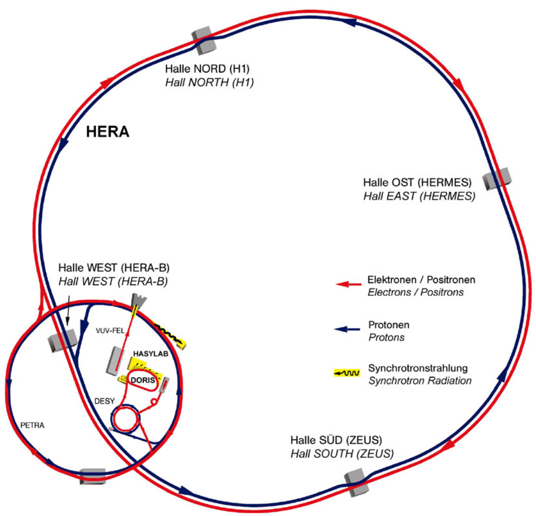


$E_e = 27.6\,\mathrm{GeV}$ $E_p = 920\,\mathrm{GeV}$
$\sqrt{s} = 319\,\mathrm{GeV}$ $\rightarrow E = 54\,\mathrm{TeV}$
fixed target

- Physics Program:
  - structure functions, parton density functions
  - jets
  - heavy quarks
  - diffraction in QCD
- high energy behavior of QCD
- precision machine for QCD, like LEP was for electroweak...
- running until mid 2007

How was HERA performing?

electron proton collider HERA
$\sqrt{s}$ = 320 GeV



HERA: QCD
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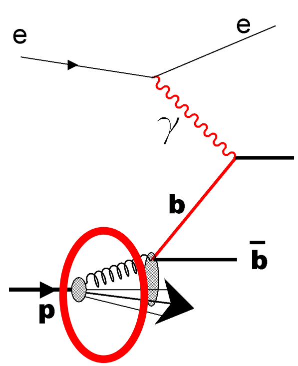
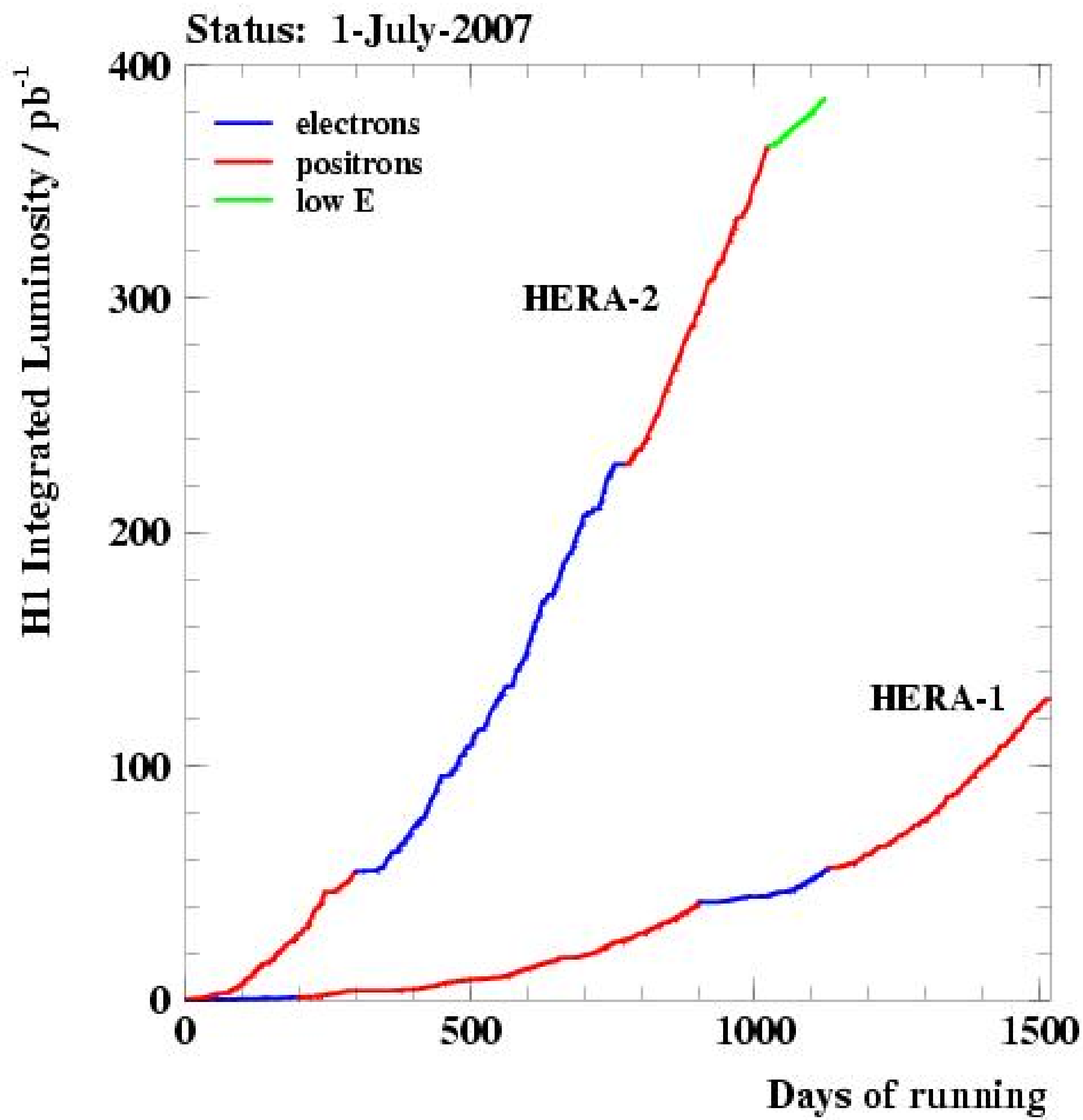
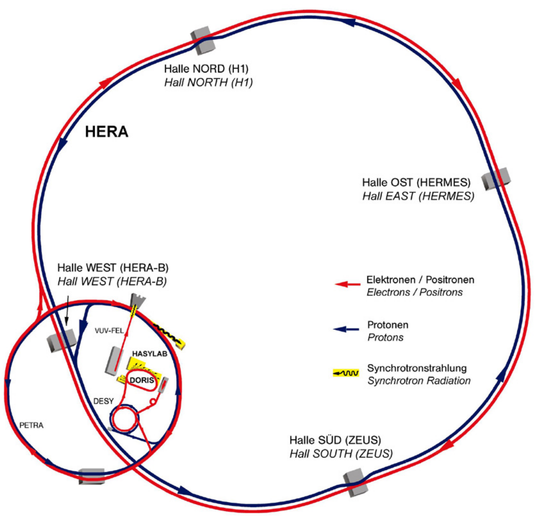


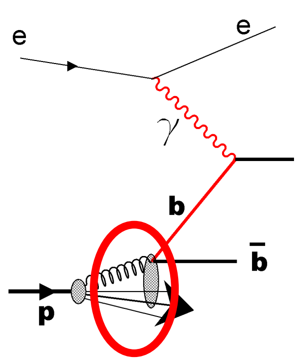


Why HERA and LHC ?

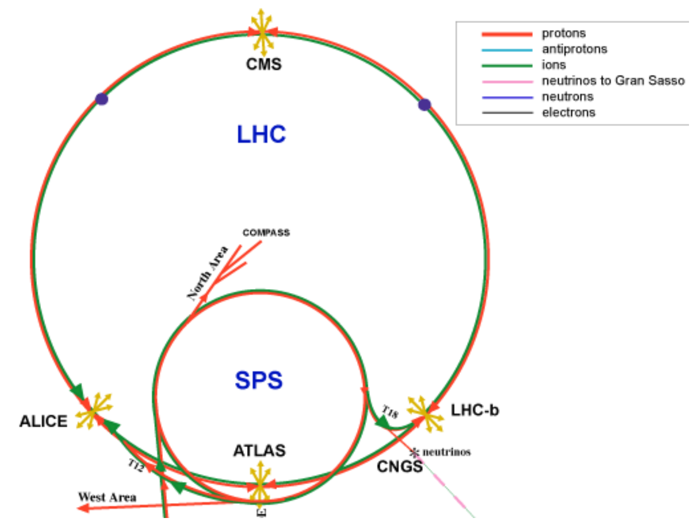
electron proton collider HERA
$\sqrt{s}$ = 320 GeV



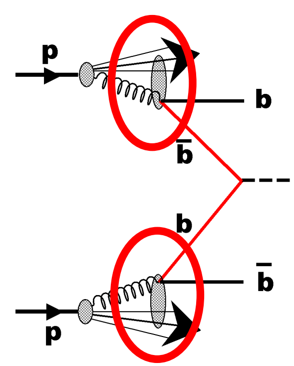
HERA: QCD
structure of the proton



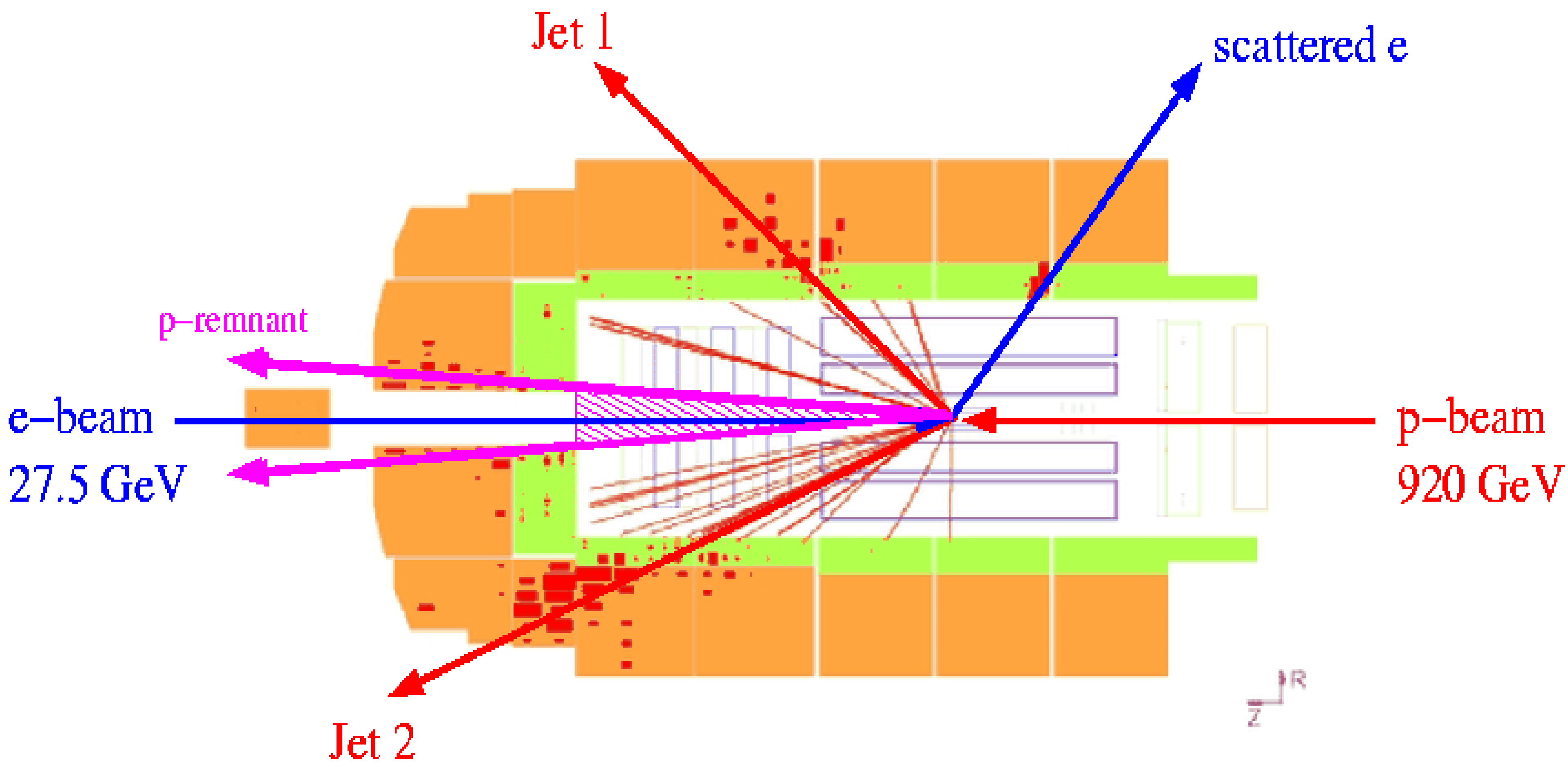
proton proton collider LHC
$\sqrt{s}$ = 14 TeV



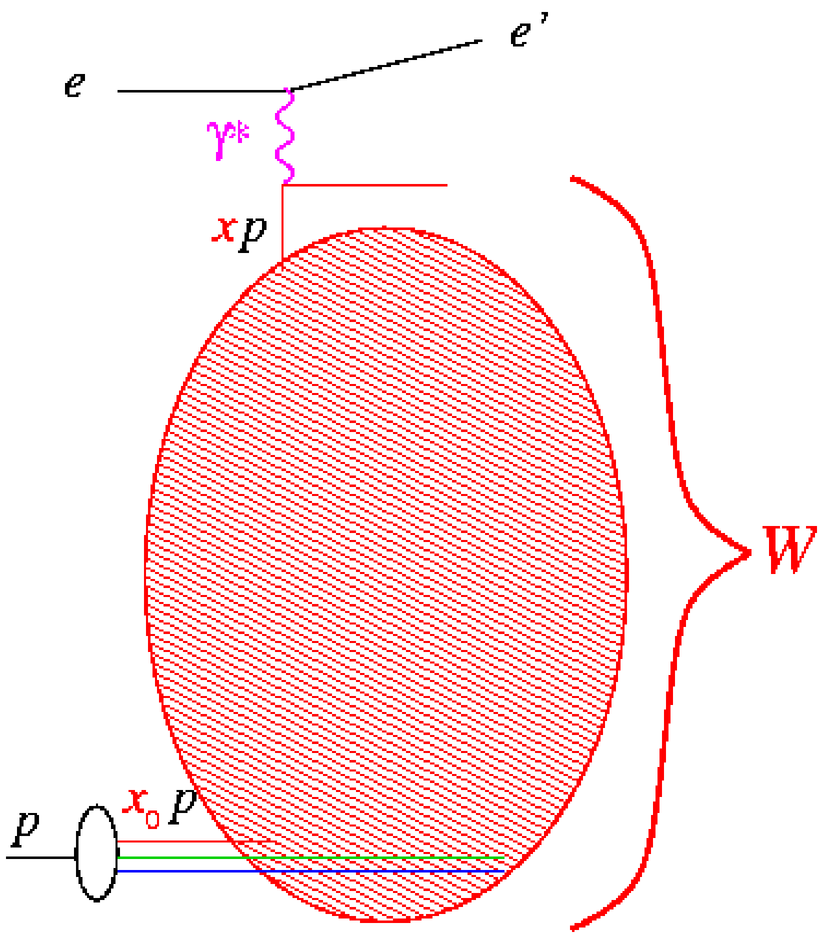
LHC: Higgs, SUSY etc.,
but mostly QCD...



A typical ep event at HERA



Kinematics



$$s = (p_e + p_p)^2$$
$$Q^2 = -(p_e - p_{e'})^2$$
$$y = \text{scaled } \gamma \text{ energy}$$
$$W^2 = (p_\gamma + p_p)^2 = Q^2 + ys$$
$$x = \frac{Q^2}{ys} = \frac{Q^2}{W^2+Q^2}$$

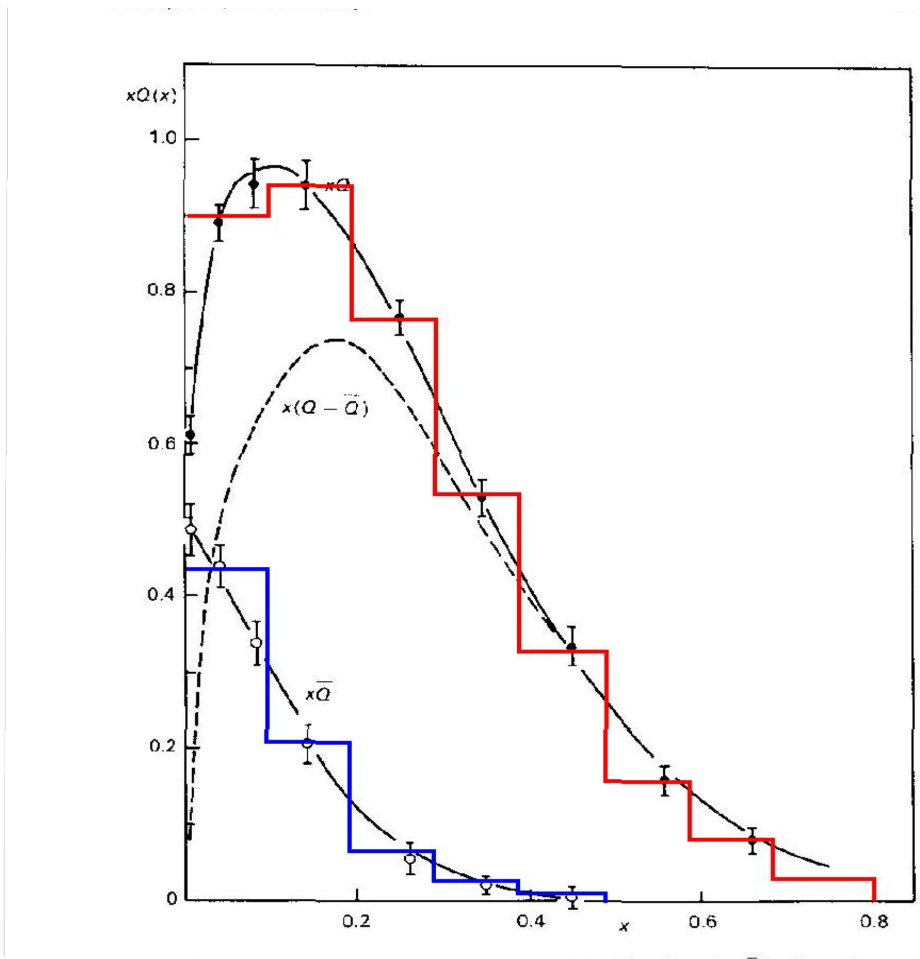
$$\frac{d\sigma^{ep}}{dydQ^2} = F_{\gamma/e}(y, Q^2)\sigma^{\gamma^* p}(W, Q^2)$$

with

$$\sigma^{\gamma^* p}(W, Q^2) = \frac{4\pi\alpha}{Q^2} F_2(x, Q^2)$$

$$F_2(x, Q^2) = \sum_i e_i^2 x q_i(x, Q^2)$$

Picture of the Proton



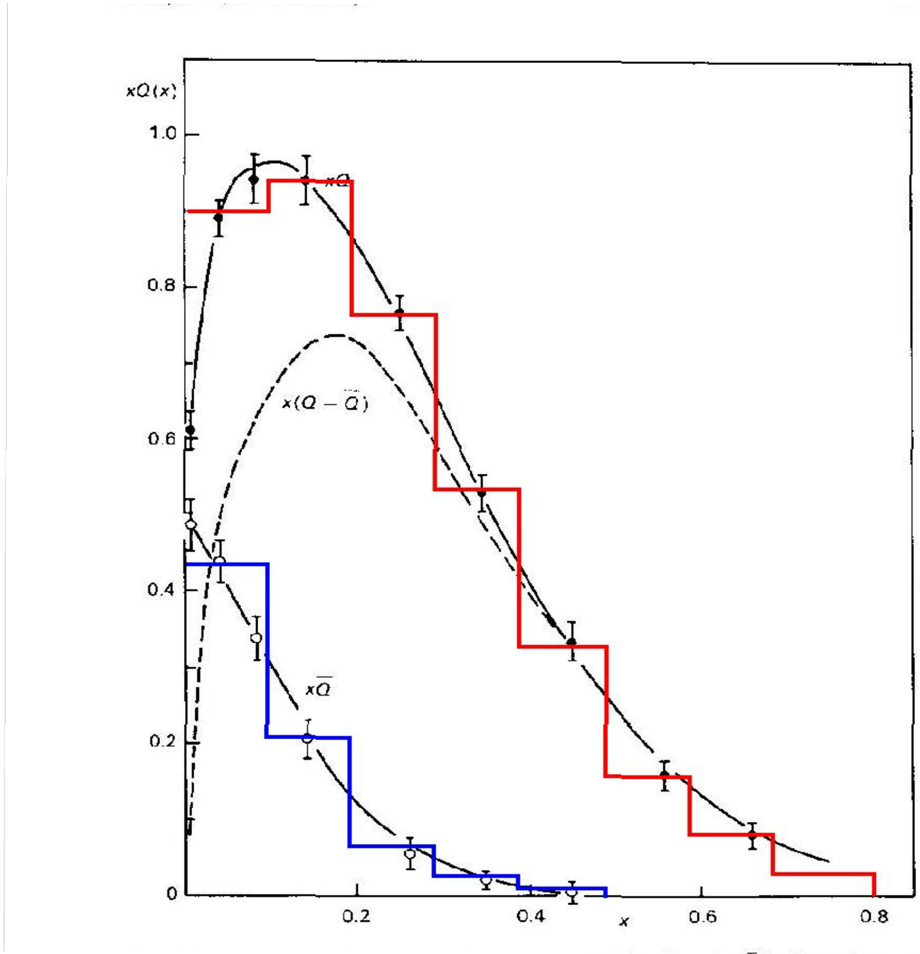
- Flavor sum rules for proton:

$$\left.\begin{aligned}\int_0^1 dx\, u_V(x) &= 2\\ \int_0^1 dx\, d_V(x) &= 1\end{aligned}\right\}\ \text{p=(uud)}$$

$$\int dx\, x\, q(x) \sim 0.1\,[0.9 + 0.95 + 0.85 + 0.7 + 0.35 + 0.15 + 0.1 + 0.05] = 0.1\cdot 4.05 = 0.405$$

$$\int dx\, x\bar{q}(x) \sim 0.1\,[0.42 + 0.2 + 0.06 + 0.03 + 0.01] = 0.1 \cdot 0.72 = 0.072$$

Picture of the Proton



- Flavor sum rules for proton:

$$\int_0^1 dx\, u_V(x) = 2$$

$$\int_0^1 dx\, d_V(x) = 1$$

p=(uud)

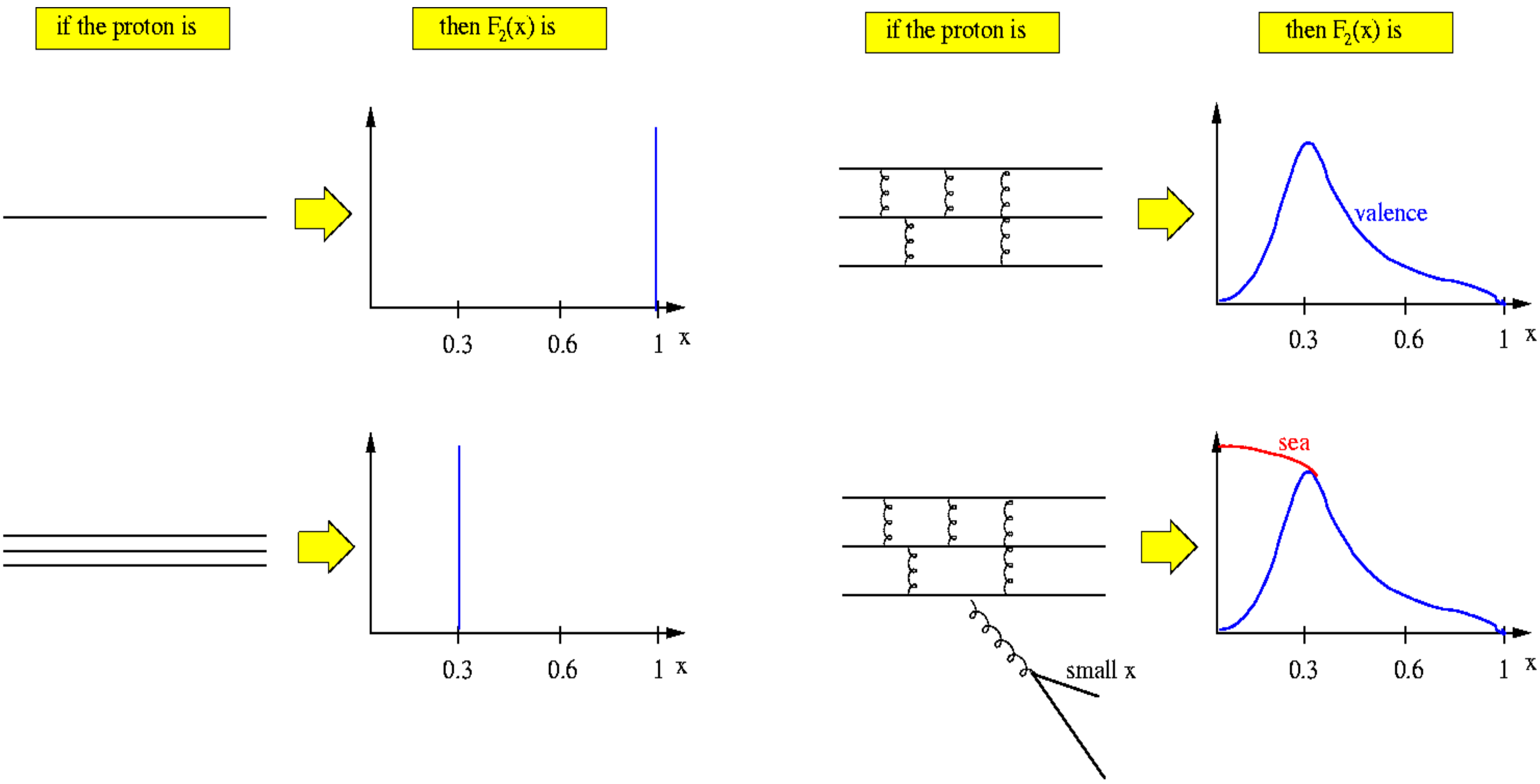
- Momentum sum of quarks:

$$\sum_q \int_0^1 dx\, x\,[q(x) + \bar{q}(x)] \sim 0.5$$

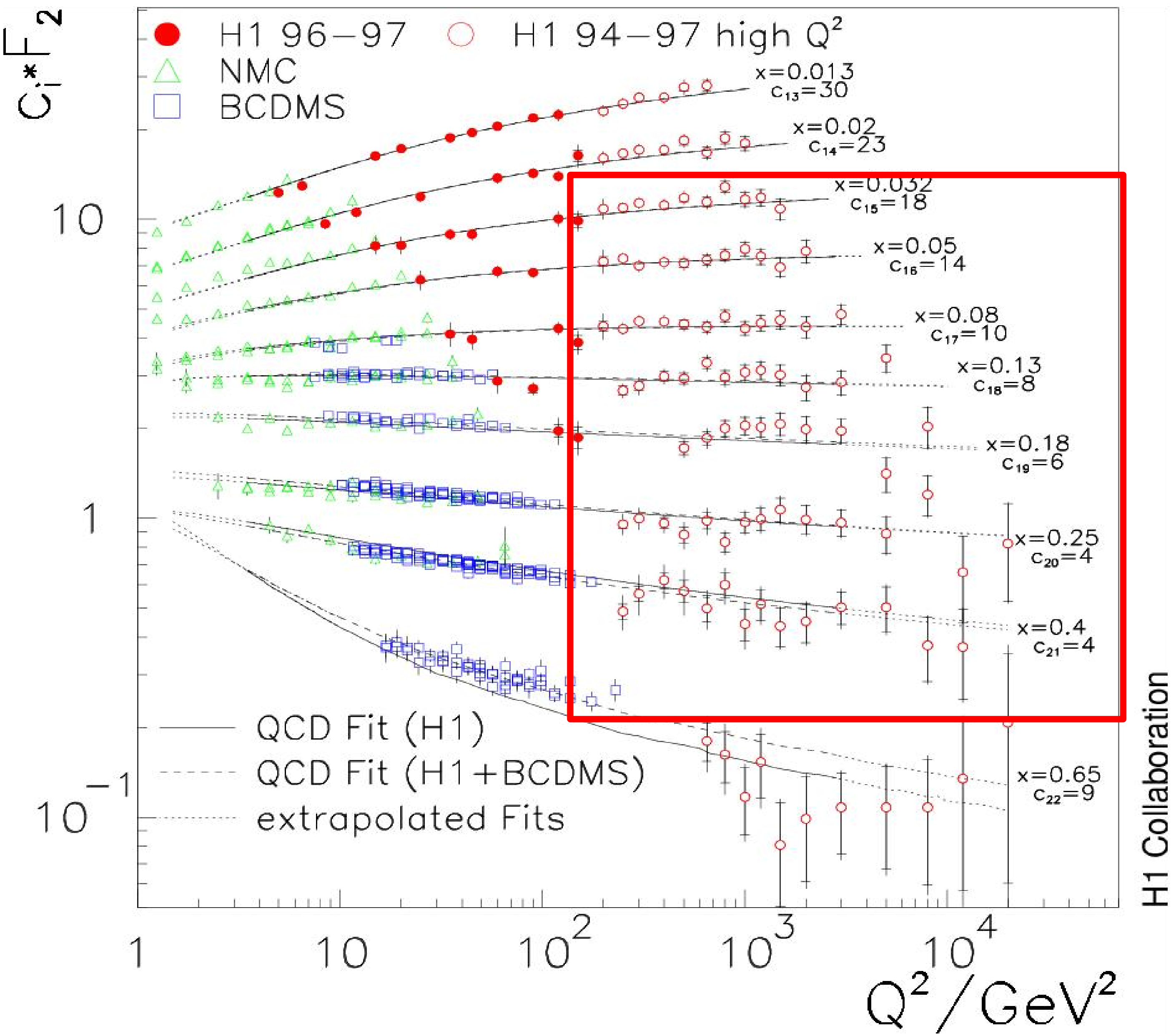
- Where are the other 50 % of the proton's momentum ?

Naive picture of the proton: F_2

From Halzen & Martin: Quarks & Leptons, p201



Structure functions from HERA

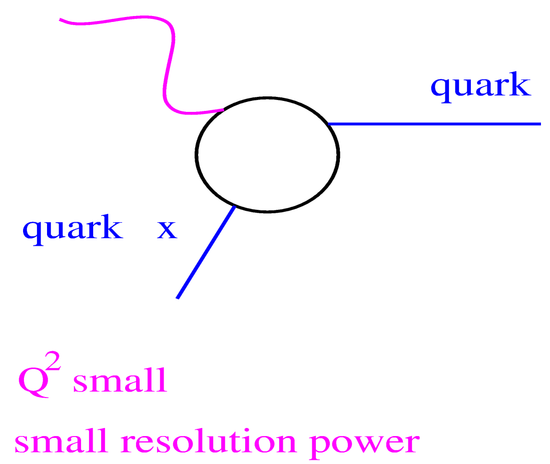


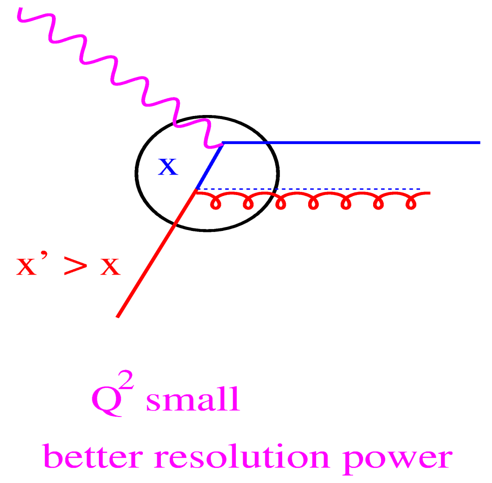
- Proton structure function does not depend on Q^2 for large x
- F_2 scales ...
- Quarks are pointlike constituents of proton
- BUT things change at smaller x..... and smaller Q^2

What about the scaling violations ?

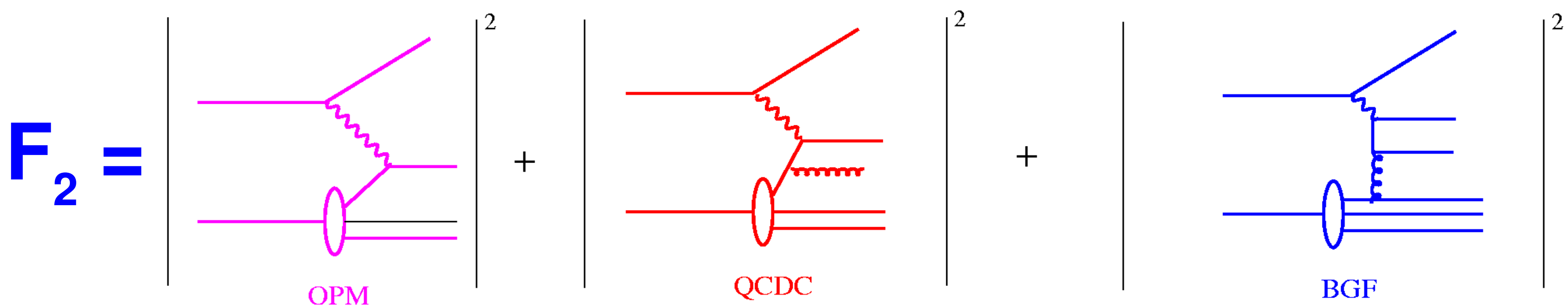
$F_2(x,Q^2)$: DGLAP evolution equation

- QPM: F_2 is independent of Q^2
- Q^2 dependence of structure function: **D**okshitzer**G**ribov**L**ipatov**A**ltarelli**P**arisi





➔ Probability to find parton at small x increases with Q^2



➔ **Test of theory: Q^2 evolution of $F_2(x,Q^2)$!!!!!**

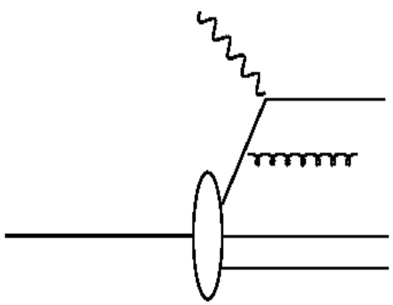
DGLAP, collinear factorization and P_{qq}

$$|ME|^2 = 32\pi^2\left(e_q^2\alpha\alpha_s\right)\frac{4}{3}\left[\frac{-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right]$$

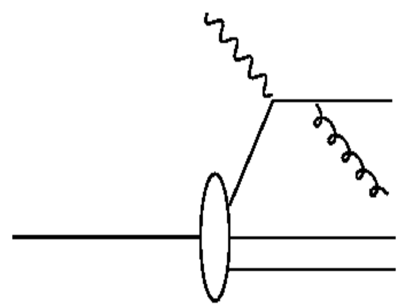
$$= 32\pi^2\left(e_q^2\alpha\alpha_s\right)\frac{4}{3}\frac{-1}{\hat{t}}\left[\frac{Q^2(1+z^2)}{z(1-z)} + \cdots\right]$$

$$z = \frac{Q^2}{\hat{s} + Q^2}$$

- integral over k_t generates log, BUT what is the lower limit



QCDC



OCDC

$$\frac{d\sigma}{dk_\perp^2} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi}\frac{1}{k_\perp^2}\left[P_{qq}(z) + \cdots\right]$$

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \qquad \sigma_0 = \frac{4\pi^2\alpha}{\hat{s}}$$

$$\sigma^{QCDC} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi}\left[P_{qq}(z)\log\left(\frac{Q^2(1-z)}{\chi^2 z}\right) + \cdots\right]$$

Collinear factorization

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi) P_{qq}\left(\frac{x}{\xi}\right)\log\left(\frac{\mu^2}{\chi^2}\right) + C_q\left(\frac{x}{\xi}\right)\right] + \ldots$$

- bare distributions $q_0(x)$ are not measurable (like the bare charges ....)
- collinear singularities are absorbed into this bare distributions at a factorization scale $\mu^2 \gg \chi^2$, defining renormalized distributions

$$F_2 = x\sum e_q^2\left[q_0(x) + \int \frac{dx_2}{x_2} q_0(x)\frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{x_2}\right)\log\left(\frac{Q^2}{\chi^2}\right) + C_q(z,..)\right]$$

- now F_2 becomes:

$$F_2 = x\sum e_q^2 \int \frac{dx_2}{x_2} q(x_2,\mu^2)\left[\delta\left(1-\frac{x}{x_2}\right) + \frac{\alpha_s}{2\pi}P_{qq}\left(\frac{x}{x_2}\right)\log\left(\frac{Q^2}{\mu^2}\right) + C\right]$$

- separating or factorizing the long distance contributions to structure functions is a fundamental property of the theory
- factorization provides a description for dealing with the logarithmic singularities, there is arbitrariness in how the finite (non-logarithmic) parts are treated.
- **Be aware that factorization is just an approximation to the full story**

Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \chi^2$ and include soft, non-perturbative physics into renormalised parton density:

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\chi^2}\right)$$

- **D**okshitzer **G**ribov **L**ipatov **A**ltarelli **P**arisi equation (take derivative of the above eq):

  V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20,
  G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons....

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

What the hell is factorization ?

Collinear factorization

see handbook of pQCD, chapter IV, B

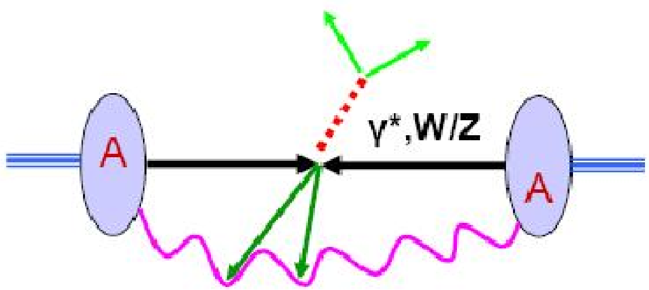
$$F_2^{(Vh)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_0^1 d\xi C_2^{(Vi)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \frac{\mu_f^2}{\mu^2}, \alpha_s(\mu^2)\right) \otimes f_{i/h}(\xi, \mu_f^2, \mu^2)$$

- Factorization Theorem in DIS (Collins, Soper, Sterman, (1989) in Pert. QCD, ed. A.H. Mueller, Wold Scientific, Singapore, p1.)
  - ➔ generalization of the parton model result
  - hard-scattering function $C_2^{(Vi)}$ is infrared finite and calculable in pQCD, depending only on vector boson *V*, parton *i*, and renormalization and factorization scales. It is independent of the identity of hadron *h*.
  - pdf $f_{i/h}(\xi, \mu_f^2, \mu^2)$ contains all the infrared sensitivity of cross section, and is specific to hadron h, and depends on factorization scale. It is universal and independent of hard scattering process.
- Generalization: applies to any DIS cross section defined by a sum over hadronic final states .... but be careful what it really means....
- explicit factorization theorems exist for:
  - diffractive DIS (... see above....)
  - Drell Yan (in hadron hadron collisions)
  - single particle inclusive cross sections (fragmentation functions)

Factorization is an approximation !!!

Factorization is an approximation

❑ Drell-Yan cross section is NOT completely factorized!



- There is always soft gluon interaction between two hadrons!
- Gluon field strength is one power more Lorentz contracted than ruler

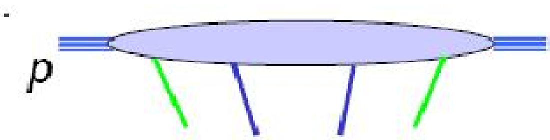
$$\frac{d\sigma}{dQ^2} = f^{(2)} \otimes f^{(2)} \otimes \frac{d\hat{\sigma}^{(2)}}{dQ^2} + \frac{1}{Q^2} f^{(2)} \otimes f^{(4)} \otimes \frac{d\hat{\sigma}^{(4)}}{dQ^2} + \frac{1}{Q^4} F\left(\frac{Q^2}{S}\right) + \ldots$$

Not factorized!

$$f^{(2)} \propto \langle p | \bar{\psi}(0) \gamma^+ \psi(y^-) | p \rangle, \quad \langle p | F^{+\alpha}(0) F_\alpha^{\ +}(y^-) | p \rangle$$

$$f^{(4)} \propto \langle p | \bar{\psi}(0) \gamma^+ F^{+\alpha}(y_1^-) F_\alpha^{\ +}(y_2^-) \psi(y^-) | p \rangle$$

...



September 28, 2005 4 Jianwei Qiu, ISU

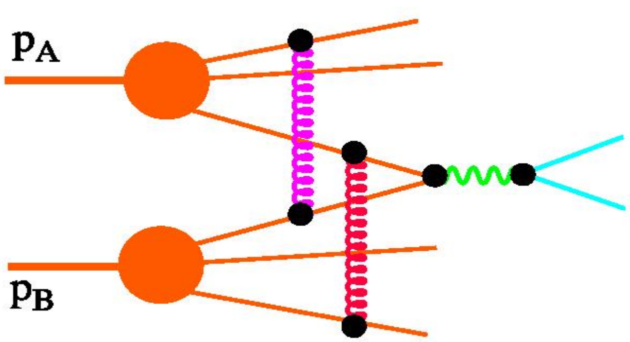
Factorization proofs and all that ...

- About factorization proofs (Wu-Ki Tung, pQCD and the parton structure of the nucleon, 2001, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 2* 887-971)

tions $F_a^\lambda(x, \frac{Q}{m}, \alpha_s(\mu))$ (a = all parton flavors). Although the underlying physical ideas are relatively simple, as emphasized in the last two sections, the mathematical proofs are technically very demanding.[7,15,19] For this reason, actual proofs of factorization only exist for a few hard processes; and certain proofs (e.g. that for the Drell-Yan process) stayed controversial for some time before a consensus were reached.[15] Because of the general character of the physical ideas and the mathematical methods involved, however, it is generally *assumed* that the attractive *quark-parton model does apply to all high energy interactions* with at least one large energy scale.

$$\frac{d\sigma}{dy} = \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \, f_A^a(\xi_A,\mu) f_B^b(\xi_B,\mu) \frac{d\hat{\sigma}_{ab}(\mu)}{dy} + \mathcal{O}\left(\left(\frac{m}{P}\right)^p\right)$$

- The problem with Drell-Yan: initial state interactions...
- factorization here does not hold graph-by-graph but only for all ....



Factorization is violated ...

arXiv:0705.2141v1 [hep-ph]

ANL-HEP-PR-07-25

Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

John Collins*
Physics Department, Penn State University, 104 Davey Laboratory, University Park PA 16802, U.S.A.

Jian-Wei Qiu†
Department of Physics and Astronomy, Iowa State University, Ames IA 50011, U.S.A. and High Energy Physics Division, Argonne National Laboratory, Argonne IL 60439, U.S.A.

(Dated: 15 May 2007)

We show that hard-scattering factorization is violated in the production of high-p_T hadrons in hadron-hadron collisions, in the case that the hadrons are back-to-back, so that k_T factorization is to be used. The explicit counterexample that we construct is for the single-spin asymmetry with one beam transversely polarized. The Sivers function needed here has particular sensitivity to the Wilson lines in the parton densities. We use a greatly simplified model theory to make the breakdown of factorization easy to check explicitly. But the counterexample implies that standard arguments for factorization fail not just for the single-spin asymmetry but for the unpolarized cross section for back-to-back hadron production in QCD in hadron-hadron collisions. This is unlike corresponding cases in e^+e^- annihilation, Drell-Yan, and deeply inelastic scattering. Moreover, the result endangers factorization for more general hadroproduction processes.

We come to that point later ....

Solving the DGLAP equations

Solving DGLAP equations ...

- Different methods to solve integro-differential equations
  - brute-force (BF) method (M. Miyama, S. Kumano CPC 94 (1996) 185)

$$\frac{df(x)}{dx} = \frac{f(x)_{m+1} - f(x)_m}{\Delta x_m} \qquad \int f(x)dx = \sum f(x)_m \Delta x_m$$

  - Laguerre method (S. Kumano J.T. Londergan CPC 69 (1992) 373, and L. Schoeffel Nucl.Instrum.Meth.A423:439-445,1999)
  - Mellin transforms (M. Glueck, E. Reya, A. Vogt Z. Phys. C48 (1990) 471)
  - QCDNUM: calculation in a grid in x,Q2 space (M. Botje Eur.Phys.J. C14 (2000) 285-297)
  - CTEQ evolution program in x,Q2 space: http://www.phys.psu.edu/~cteq/
  - QCDFIT program in x,Q2 space (C. Pascaud, F. Zomer, LAL preprint LAL/94-02, H1-09/94-404,H1-09/94-376)
  - MC method using Markov chains (S. Jadach, M. Skrzypek hep-ph/0504205)
  - Monte Carlo method from iterative procedure
- brute-force method and MC method are best suited for detailed studies of branching processes !!!

We have to deal with divergencies....

- collinear divergencies factored into renormalized parton distributions
- what about soft divergencies ? $z \to 1$

treated with "plus" prescription

$$\frac{1}{1-z} \to \frac{1}{1-z}_+ \quad \text{with} \quad \int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$$

- soft divergency treated with Sudakov form factor:

$$\Delta(t) = \exp\left[- \int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z) \right]$$

resulting in

$$t \frac{\partial}{\partial t}\left(\frac{f}{\Delta}\right) = \frac{1}{\Delta} \int^{z_{max}} \frac{dz}{z} \frac{\alpha_s}{2\pi} \tilde{P}(z) f(x/z, t)$$

and

$$f(x,t) = \Delta(t) f(x,t_0) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int^{z_{max}} \frac{dz}{z} \frac{\alpha_s}{2\pi} \tilde{P}(z) f(x/z, t')$$

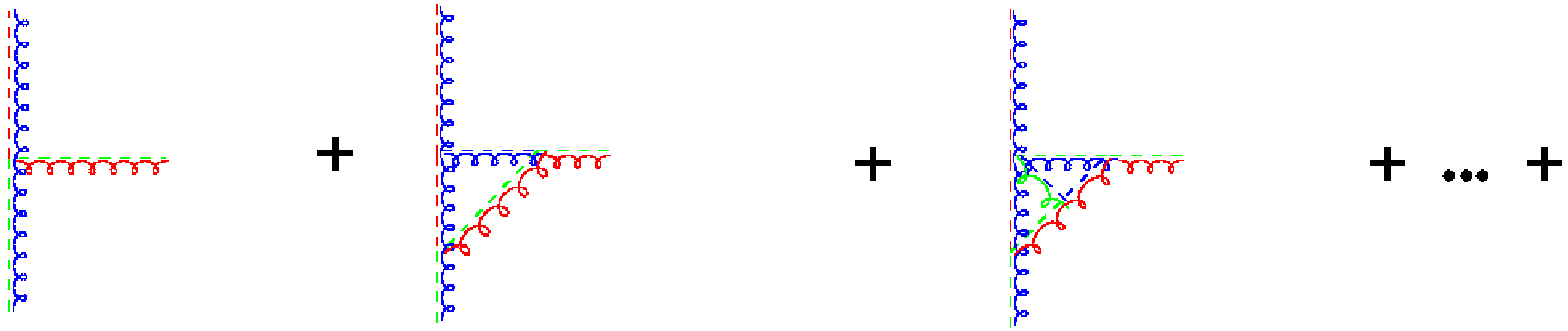
Sudakov form factor: all loop resum...

$g \to gg$ Splitting Fct $\tilde{P}(z) = \frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z} + \ldots$

- Sudakov form factor .... all loop resummation

$$\Delta_S = \exp\left(- \int dz \int \frac{dq'}{q'} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)$$

$$\Delta_S = 1 + \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^1 + \frac{1}{2!} \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 \ldots$$



$$\tilde{P}(z) \left[1 - \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) + \frac{1}{2!} \left(- \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 + \ldots + \right]$$

and again DGLAP evolution ....

- differential form: $$t\frac{\partial}{\partial t}f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}P_+(z)\,f\left(\frac{x}{z},t\right)$$

- differential form using f/Δ_s with

$$\Delta_s(t) = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}(z)\right)$$

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)} = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)}f\left(\frac{x}{z},t\right)$$

- integral form

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z)f\left(\frac{x}{z},t'\right)$$

no - branching probability from t_0 to t

and again DGLAP evolution ....

- differential form: $$t\frac{\partial}{\partial t}f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}P_+(z)\, f\left(\frac{x}{z},t\right)$$

- differential form using f/Δ_s with

$$\Delta_s(t) = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^{t}\frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}(z)\right)$$

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)} = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)}\, f\left(\frac{x}{z},t\right)$$

We need only:

$$P(z) \to \frac{1}{1-z}$$

- integral form

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

no – branching probability from to

Solving integral equations

- Integral equation of Fredholm type: $\phi(x) = f(x) + \lambda \int_a^b K(x,y)\phi(y)dy$
- solve it by iteration (Neumann series):

$$
\begin{aligned}
\phi_0(x) &= f(x) \\
\phi_1(x) &= f(x) + \lambda \int_a^b K(x,y) f(y) dy \\
\phi_2(x) &= f(x) + \lambda \int_a^b K(x,y_1) f(y_1) dy_1 + \lambda^2 \int_a^b \int_a^b K(x,y_1) K(y_1,y_2) f(y_2) dy_2 dy_1 \\
\phi_n(x) &= \sum_{i=0}^{n} \lambda^i u_i(x) \\
u_0(x) &= f(x) \\
u_1(x) &= \int_a^b K(x,y) f(y) dy \\
u_n(x) &= \int_a^b \int_a^b K(x,y_1) K(y_1,y_2) \cdots K(y_{n-1},y_n) f(y_n) dy_2 \cdots dy_n
\end{aligned}
$$

with the solution: $\phi(x) = \lim_{n\to\infty} q_n(x) = \lim_{n\to\infty} \sum_{i=0}^{n} \lambda^i u_i(x)$

DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

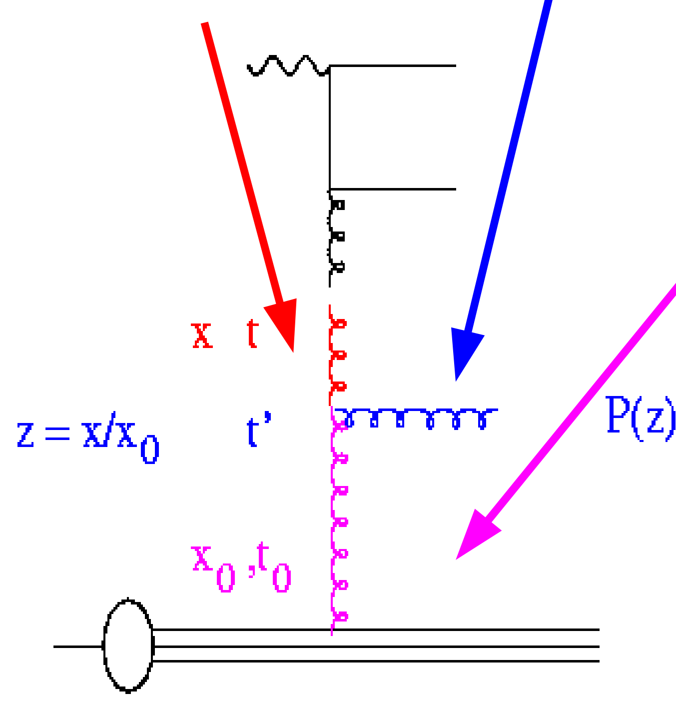
from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t')$$



DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t')$$

$$= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z, t_0)$$

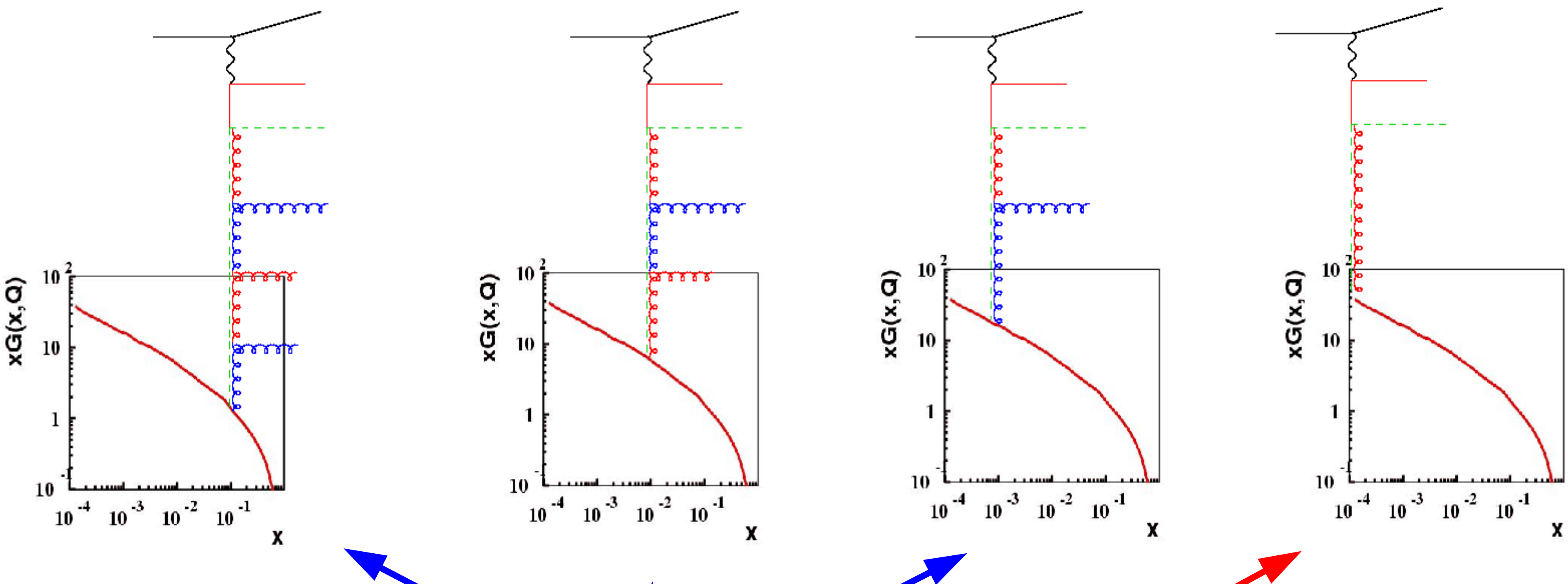
$$f_2(x,t) = f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z, t_0) + \frac{1}{2}\log^2\frac{t}{t_0} A \otimes A \otimes \Delta(t) f(x/z, t_0)$$

$$f(x,t) = \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty} \sum_n \frac{1}{n!} \log^n\left(\frac{t}{t_0}\right) A^n \otimes \Delta(t) f(x/z, t_0)$$

DGLAP re-sums $\log t$ **to all orders !!!!!!!!!!!!!!!!!**

DGLAP evolution equation... again...

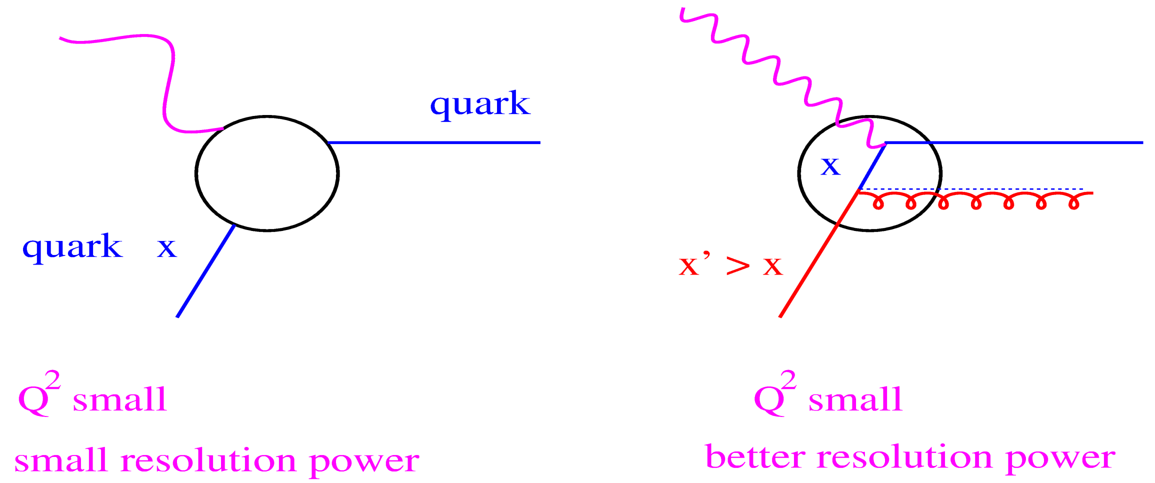
- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, spacelike parton showering



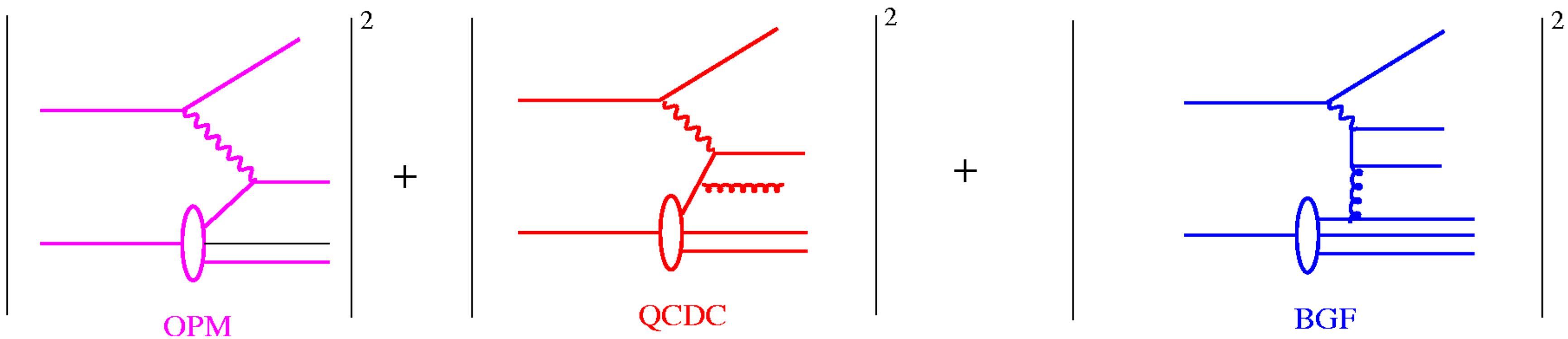
- $$f(x,t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0)\Delta_s(t)$$

$F_2(x,Q^2)$: DGLAP evolution equation

- QPM: F_2 is independent of Q^2
- Q^2 dependence of structure function: **D**okshitzer**G**ribov**L**ipatov**A**ltarelli**P**arisi



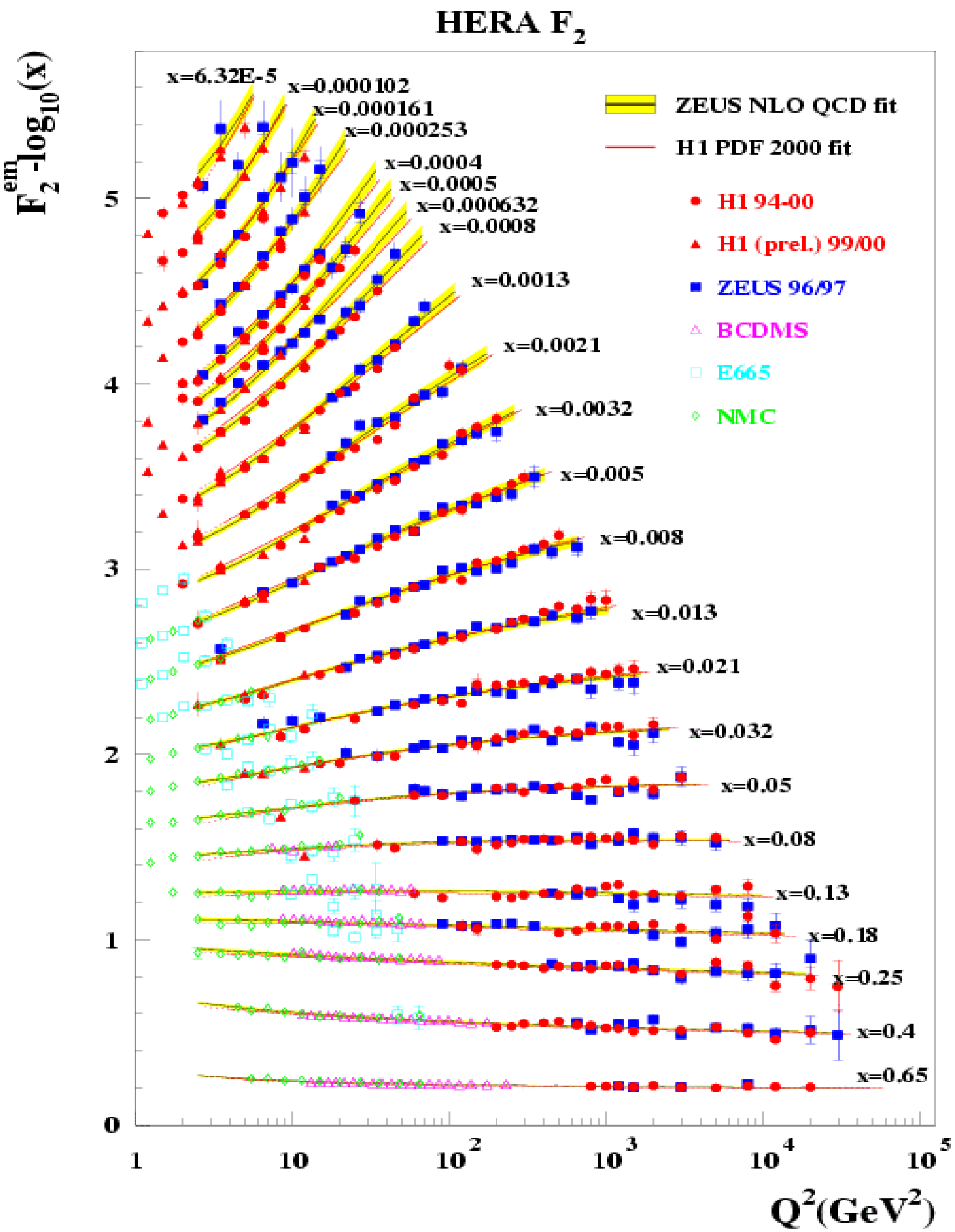
→ Probability to find parton at small x increases with Q^2

$F_2 =$


→ **Test of theory: Q^2 evolution of $F_2(x,Q^2)$!!!!!**

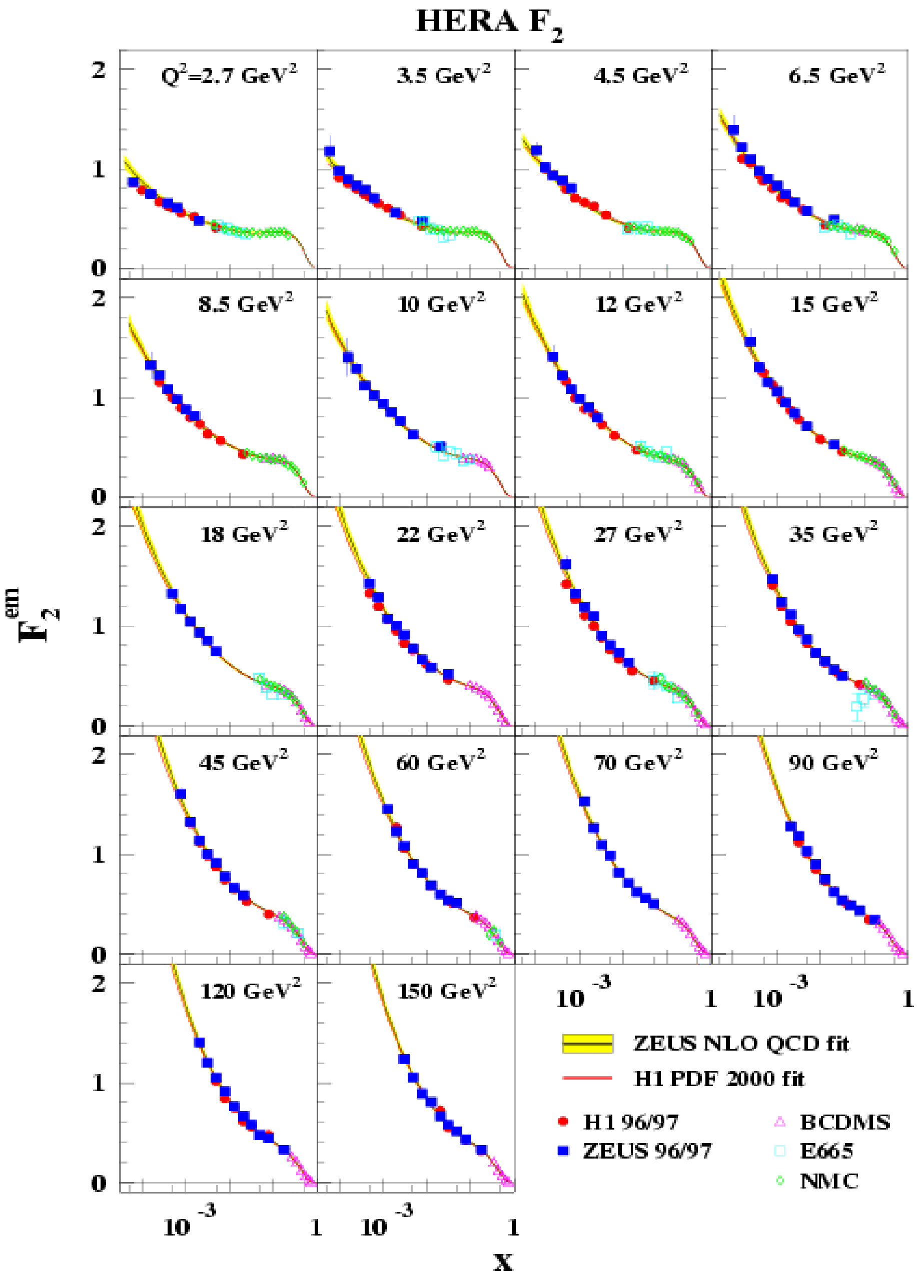
Is our theory working at all ?

Q^2 dependence of $F_2(x,Q^2)$



- ➔ F_2 is rising with Q^2 at small x
- ➔ Scaling violations !!!!!
- ➔ Well described by theory ...

x dependence of $F_2(x,Q^2)$



- new level of precision reached: ~ 1 %
- DGLAP fits data well even at low Q^2
- strong rise towards small x
- $F_2 \sim xq(x,Q^2)$
  - ➔ probability to find parton at small x increases
  - ➔ How can rising F_2 be understood ?
  - ➔ Does rise continue forever ?
  - ➔ What limits F_2 ?

What is happening at small x ?

- For $x \to 0$ only gluon splitting function matters:

$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right) = 6\left(\frac{1}{z} - 2 + z(1-z) + \frac{1}{1-z}\right)$$

$$P_{gg} \sim 6\frac{1}{z} \text{ for } z \to 0$$

- evolution equation is then:

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')}\frac{\alpha_s}{2\pi}P(z)f\left(\frac{x}{z},t'\right)$$

- at small *z*: $\Delta_s(t) \to 1$

$$x\,g(x,t) = \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t')$$ with $t = \mu^2$

- when $f(x,t_0)$ is neglected (compared to evolved piece ....)

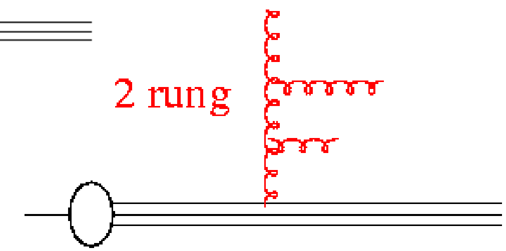
Estimates at small x: DLL

$$x\,g(x,t) = \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t') \qquad \text{with} \quad t = \mu^2$$

- use constant starting distribution at small t: $xg_0(x) = C$

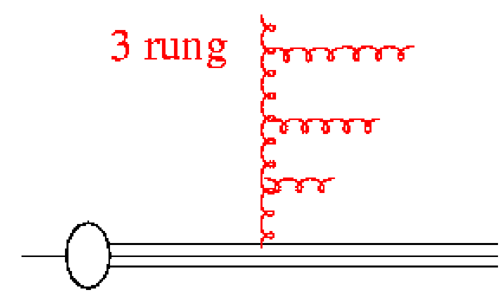
$$x\,g_1(x,t) = \frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}C$$

$$x\,g_2(x,t) = \left(\frac{3\alpha_s}{\pi}\frac{1}{2}\log\frac{t}{t_0}\frac{1}{2}\log\frac{1}{x}\right)^2 C$$



$$\vdots$$

$$x\,g_n(x,t) = \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$



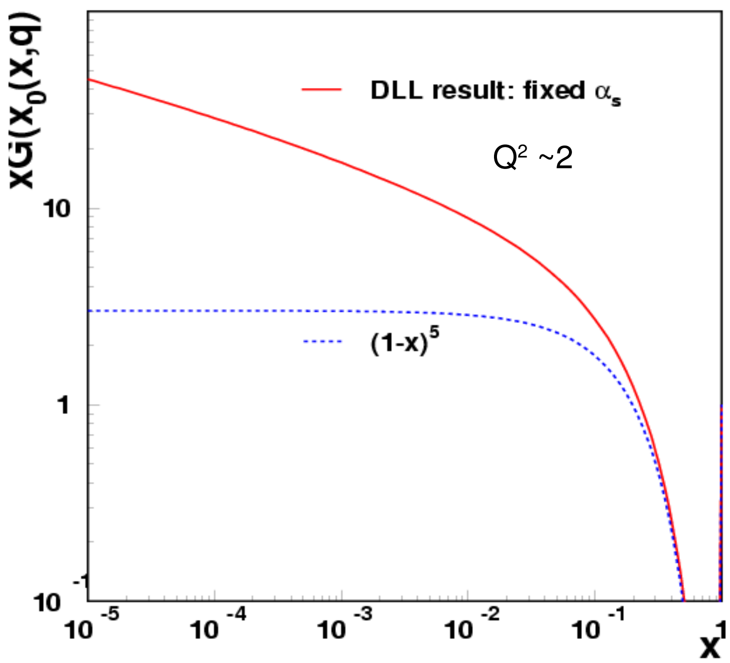
$$x\,g(x,t) = \sum_n \left(\frac{1}{n!}\right)^2 \left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$

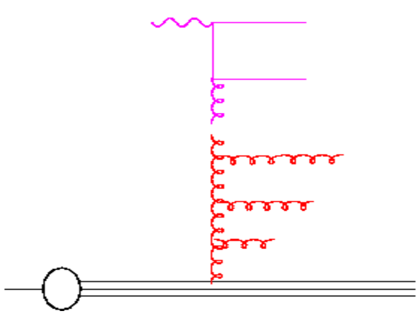
$$x\,g(x,t) \sim C\exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$$

double leading log approximation (DLL)

Results from DLL approximation

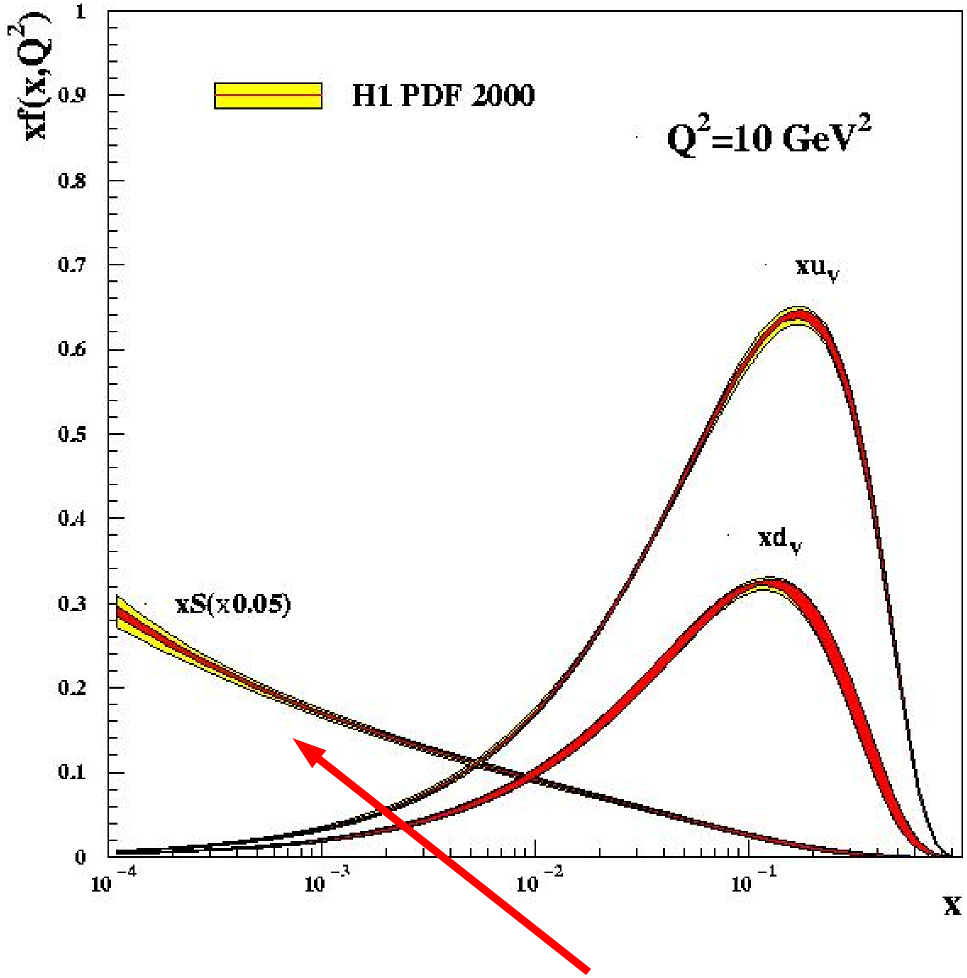
- DLL arise from taking small x limit of splitting fct: $x\,g(x,t) \sim C \exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$
  - $\log 1/x$ from small x limit of splitting fct
  - $\log t/t_0$ from t integration... gives evolution length... α_s softer for running
- DLL gives rapid increase of gluon density from a flat starting distribution
- gluons are coupled to F_2... strong rise of F_2 at small x:

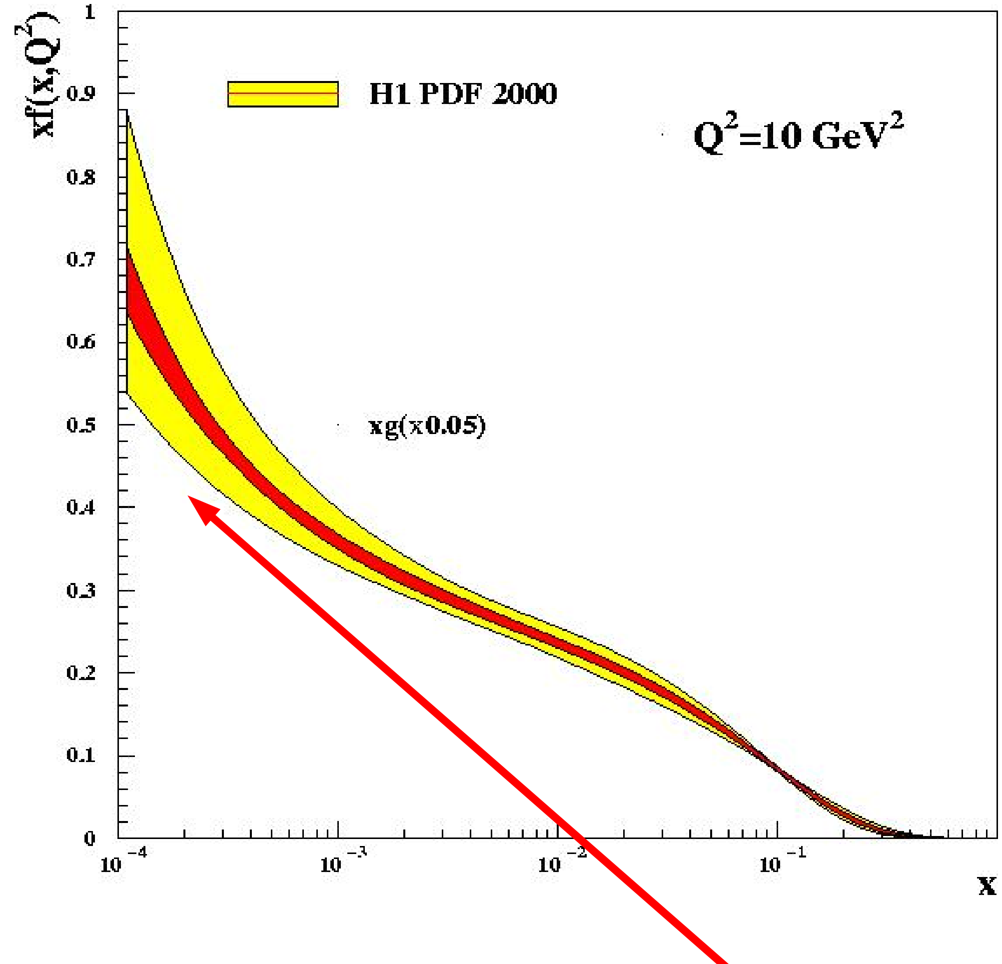




- consequences:
- rise continues forever ???
- what happens when too high gluon density ?
-

Parton Distribution Functions





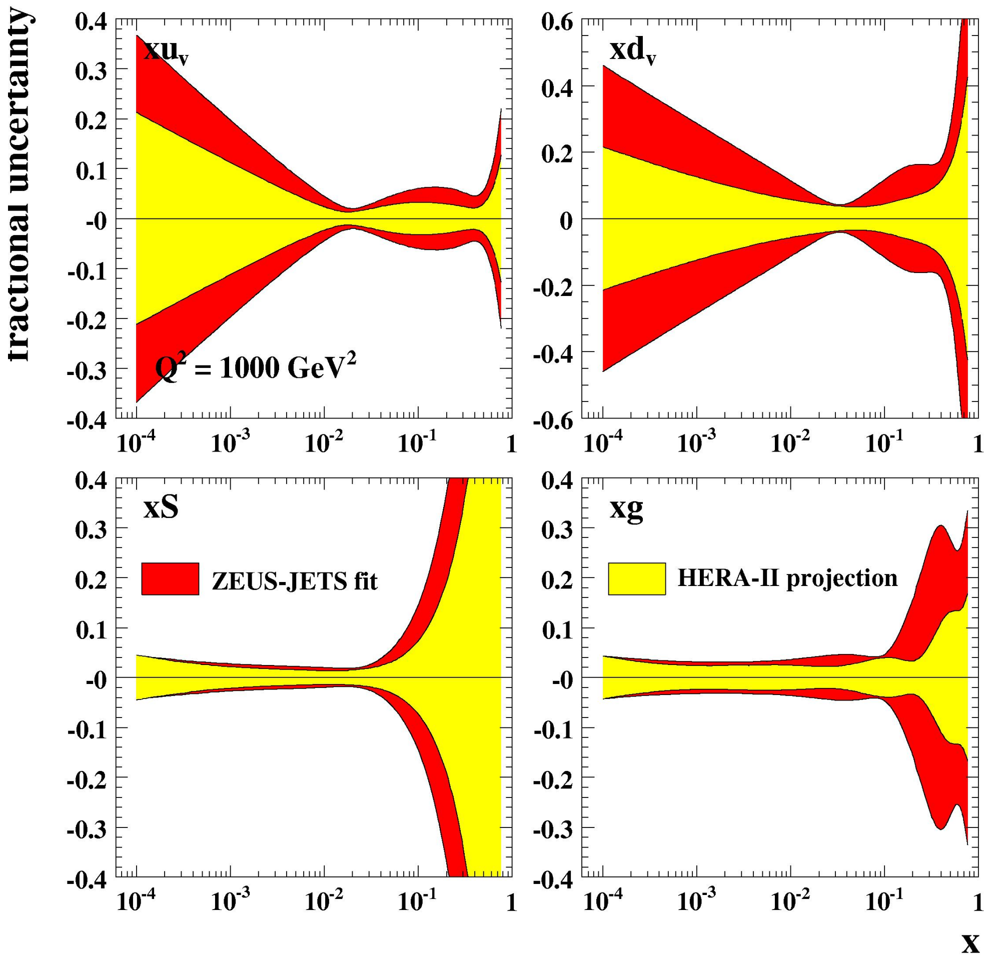
- at small x sea quarks dominate
- gluon density is very large

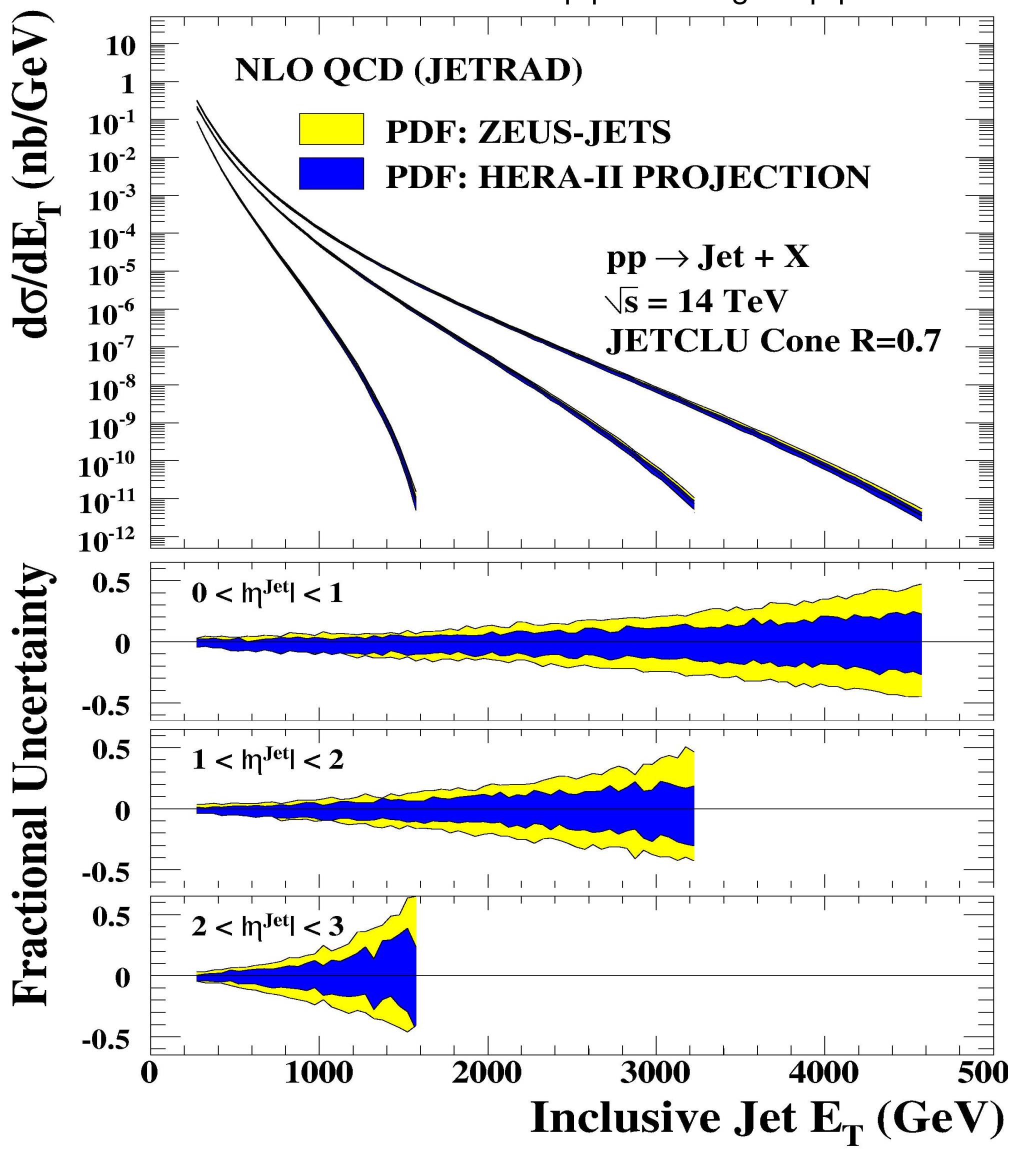
PDF uncertainty: improvements

Using jets together with F_2 (at large Q^2)

quark and gluon uncertainties

from C. Gwenlan, A. Cooper-Sarkar, C. Targett-Adams
in HERA – LHC workshop proceedings hep-ph/0601012





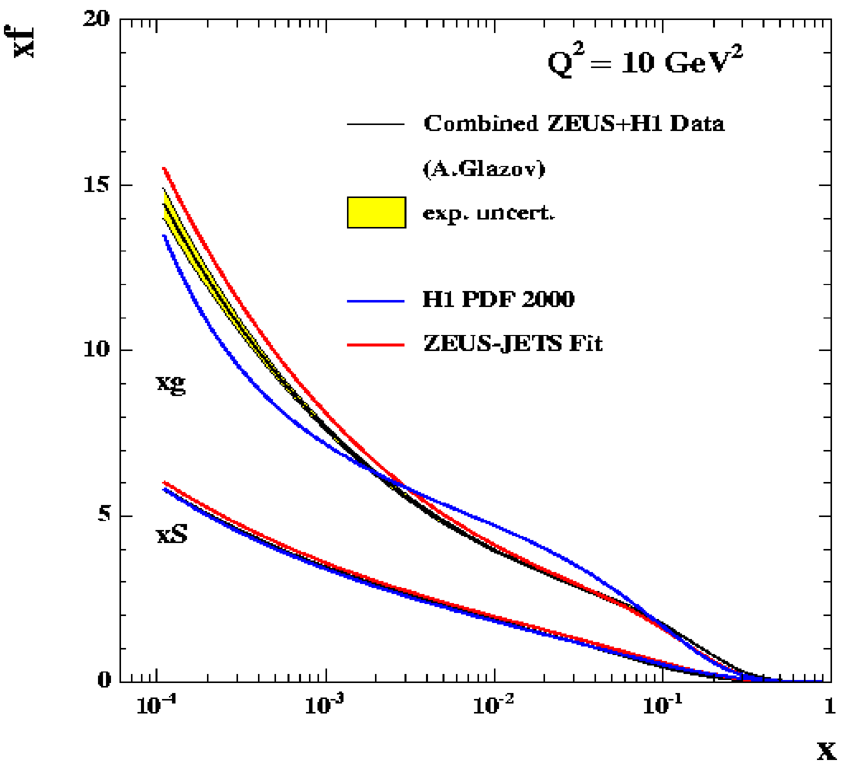
high statistics from HERA II is important
(assumed 700 pb^{-1})

Error on LHC jet xsection reduced !!!

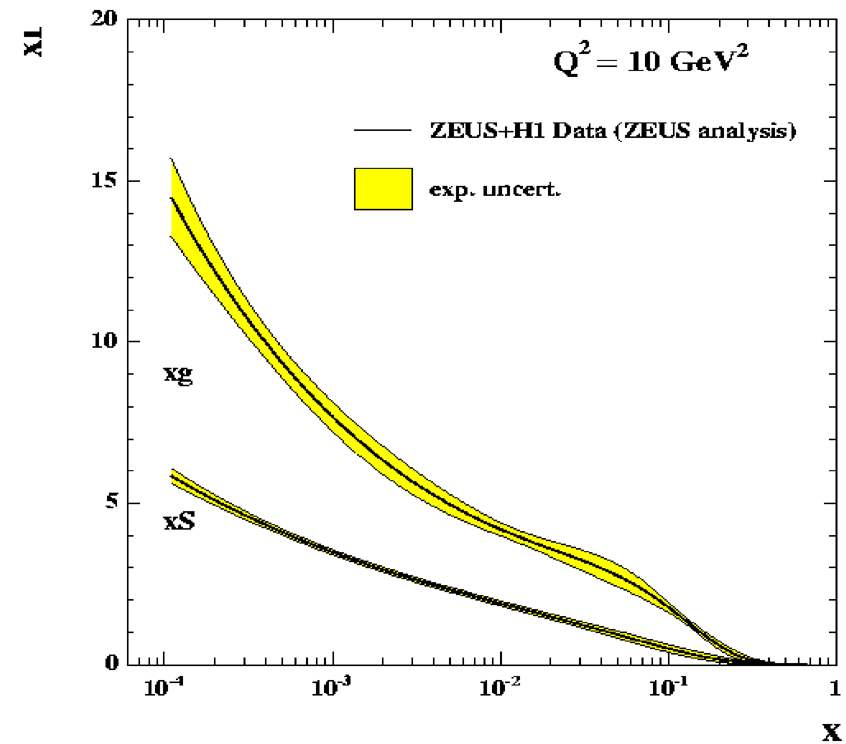
Average of HERA data

From M. Cooper-Sakar, C. Gwenlan and S. Glazov

- Average H1&ZEUS data sets



- Combined PDF fit to H1& ZEUS



Much reduced uncertainties ....

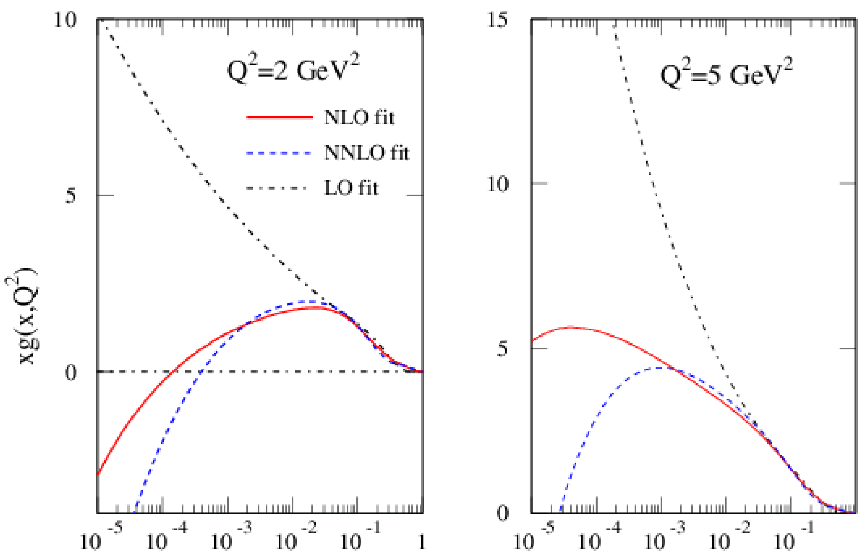
Model independent analysis of data desirable

Activities started to get HERA - PDF !!!!!

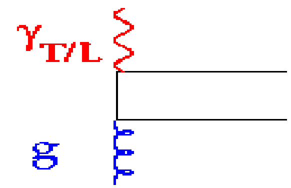
HERA measurements: F_L

The gluon distribution

R. Thorne, hep-ph/0511351

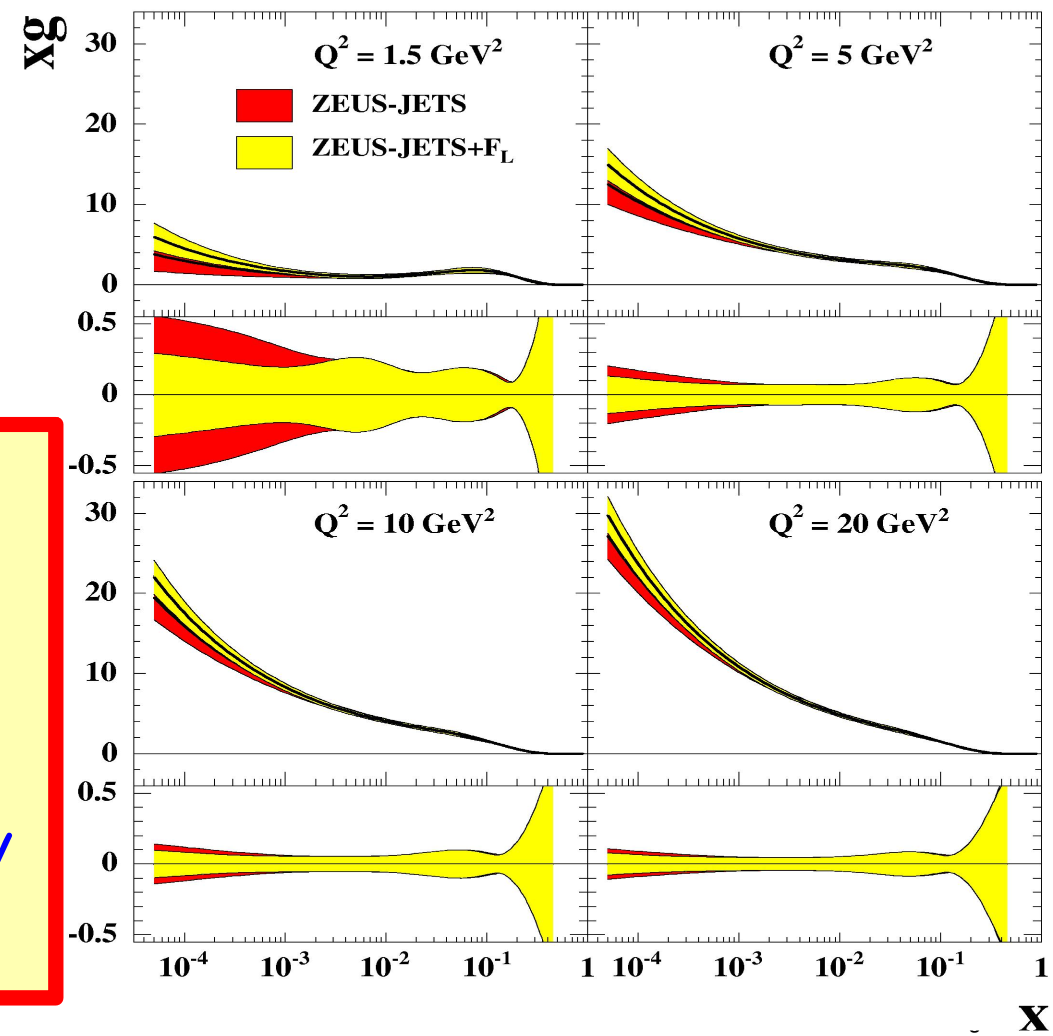


$\sigma_L(\gamma g \longrightarrow q\bar{q}) \longrightarrow F_L$



From J. Feltesse, C. Gwenlan, S. Glazov, M. Klein, S. Moch

$$F_L \propto \alpha_s x\, g(x, Q^2)$$



- Precision measurement of F_L at
- E_p = 460 GeV and 5-10 pb^{-1}
- cleanest for gluon
- crucial test of QCD at higher orders and consistency of theory

➔ Is measured at HERA !!

Structure Functions at HERA

Structure Functions at HERA are well described by

NLO DGLAP

Structure Functions at HERA

Structure Functions at HERA are well described by

NLO DGLAP

is that all we can learn ?

Structure Functions at HERA

Structure Functions at HERA are well described by

NLO DGLAP

HERA and QCD is more and much richer !!!

Conclusions

- HERA physics is very rich:
  - from inclusive x-section measurements to detailed investigations of QCD
  - measurements of hadronic final states:
    - jets, heavy flavors
    - ➔ lead to a detailed understanding of QCD
    - ➔ what about saturation, diffraction multi-parton interactions ?
- next lecture
  - ➔ HERA implications for LHC
  - ➔ PDFs, small x, multiple interactions, diffraction

Understanding of QCD at high energies is still challenging !