From Wave Propagation to Spin Dynamics: Mathematical and Computational Aspects

by

Oleksii Beznosov

B.S., Taras Shevchenko National University of Kyiv, 2012
M.S., Taras Shevchenko National University of Kyiv, 2014

Ph.D., Mathematics, University of New Mexico, 2021

Abstract

In this work we concentrate on two separate topics which pose certain numerical challenges. The first topic is the spin dynamics of electrons in high-energy circular accelerators. We introduce a stochastic differential equation framework to study spin depolarization and spin equilibrium. This framework allows the mathematical study of known equations and new equations modelling the spin distribution of an electron bunch. A spin distribution is governed by a so-called Bloch equation, which is a linear Fokker-Planck type PDE, in general posed in six dimensions. We propose three approaches to approximate solutions, using analytical and modern numerical techniques. We also present simple models that carry all computational difficulties of those modelling the realistic accelerators, to demonstrate the effectiveness of our framework and the approximations. In the second part of this work we present a high-order accurate numerical method for the wave equation posed on a domain

with complex boundary. The method combines efficient Hermite methods with the geometrically flexible Discontinuous Galerkin method by using overset grids. Near boundaries we use thin boundary-fitted curvilinear grids and inside the volume we use Cartesian grids so that the computational complexity of the method approaches that of a structured Cartesian Hermite method.