Problems for Cosmology

27.06.2014

1. The continuity equation for matter-energy density ρ and pressure p reads

$$\dot{\rho} + 3H(\rho + p) = 0 \quad .$$

Show that forms of energy with an equation of state $p = w\rho$ dilute with the scale factor a as

$$\rho \sim a^{-3(1+w)} \quad .$$

2. Today our universe appears to be dominated by vacuum energy density with a cosmological density parameter $\Omega_{\Lambda,0} \simeq 0.7$. For the total amount of non-relativistic matter, and for radiation one finds $\Omega_{m,0} \simeq 0.3$, and $\Omega_{\gamma,0} \simeq 8.4 \times 10^{-5}$, respectively, today. Further, spatial curvature is vanishing to within observational precision, so let us assume the universe as having been totally spatially flat always.

   Now determine the redshift z_Λ when the energy density in vacuum energy and in non-relativistic matter where equal in the past.

   Even earlier, the energy densities in matter and radiation have been equal. Determine z_{eq} of matter-radiation equality. How old was the universe at the time of matter-radiation equality? (Before matter-radiation equality, the universe was radiation dominated, and we can approximate the expansion law for a spatially flat universe before matter-radiation equality as $a(t) \sim \sqrt{t}$.)

   The temperature of a blackbody radiation field redshifts the same way as the individual photons. What was the temperature of the CMB at the time of matter-radiation equality? Compare your result with ionization energy of hydrogen ($E = 13.6$ eV).

3. The non-dark (baryonic) matter consists almost completely of protons and neutrons. Their average number density today is $n_{B,0} \simeq 0.23\, m^{-3}$. Calculate $\Omega_{B,0} = \rho_{B,0}/\rho_{cr,0}$ with $\rho_{cr,0} = 3H_0^2/(8\pi G)$.