Hybrid Inflation and Tachyonic Preheating

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Hybrid inflation [1,2,3] combines chaotic inflation with spontaneous symmetry breaking. At its late stages it is driven by the energy density of the vacuum and it ends in a rather unusual waterfall transition which is triggered by a tachyonic instability in the effective potential. Preheating in this scenario is not based on parametric resonance [4] but occurs due to the spinodal growth of long-wavelength Higgs modes [5,6]. It is so efficient that symmetry breaking completes within a single oscillation of the scalar fields. Particles coupled to the inflaton sector may be produced non-adiabatically as well [7].

I. OVERALL PICTURE

The transition from the inflationary era to the standard thermal FLRW Universe may proceed in various different ways depending on the details of the underlying particle physics model. In our two talks in this seminar we wish to present the most important mechanisms on the market that may be responsible for the *reheating* of the Universe after the end of inflation.

Two weeks ago [4] we considered a simple chaotic scenario with a convex potential ($V_{,\phi\phi} > 0$) for a coherently oscillating classical scalar inflaton field ϕ,

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \tag{1}$$

and encountered a stage of *preheating* during which bosonic degrees of freedom coupled to the inflaton were excited due to *parametric resonance*. We, however, also noted that realistic theories of reheating require spontaneous symmetry breaking (SSB) in the inflaton sector in order to allow for inflaton two-body decays. But as preheating in combination with SSB turns out to be much more subtle we decided to postpone this issue.

Now we return to it and investigate preheating from a concave potential ($V_{,\sigma\sigma} < 0$) for a Higgs field σ that breaks some global symmetry of the vacuum:

$$V(\sigma) = \frac{\lambda}{4}\left(\sigma^2 - v^2\right)^2 \quad ; \quad M^2 = \lambda v^2 \tag{2}$$

In addition to changing the curvature of the effective scalar potential we choose completely different initial conditions. Last time we started with a homogeneous field at the Planck scale, $\phi \gtrsim M_p$. Now we take the homogeneous component σ_0 to be of the order of the initial quantum fluctuations $\delta\sigma$ which will render the usual perturbative picture of a classical field with small fluctuations on top of it inapplicable. Instead, we will find that SSB occurs due to the spinodal growth of the fluctuations such that any initial homogeneity soon is destroyed up to rather short scales. The purely classical description of the fields in the inflaton sector thus has to be traded for a semiclassical treatment of quantum fluctuations.

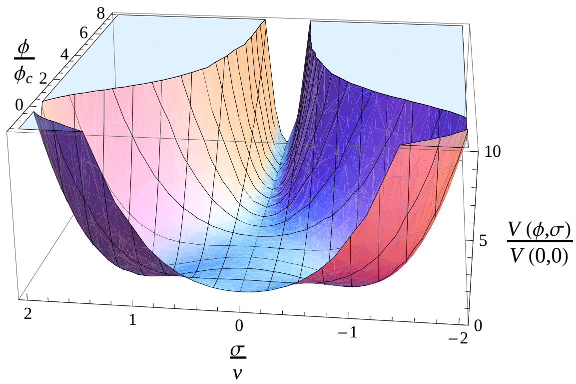

FIG. 1: Effective scalar potential $V(\phi, \sigma)$ as in Eq. (3) with $m = 100\,\mathrm{GeV}$, $M = 10^{11}\,\mathrm{GeV}$, $\lambda = 10^{-1}$ and $g = 1$.

One of the simplest cosmologically viable scenarios of inflation featuring the concave potential in Eq. (2) is the standard *hybrid inflation* model which was proposed by Andrei Linde in the early 90s [1] and which represents a mixture of the usual theory of SSB and chaotic inflation:

$$V(\phi, \sigma) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4}\left(\sigma^2 - v^2\right)^2 + \frac{1}{2} g^2 \phi^2 \sigma^2 \tag{3}$$

The first part of our talk outlines the characteristics of this inflationary model. The second part is devoted to the dynamics of SSB and the related preheating process.

II. HYBRID INFLATION

A. Motivation

From the perspective of particle physics hybrid inflation is very attractive as it provides a natural setting for SSB. Linde's original motivation, however, was to construct a model in which inflation ends differently compared to the standard scenarios of a first-order phase transition and slow-roll motion becoming faster and faster. Indeed, as we will show, hybrid inflation may end in a *waterfall regime* which is characterized by a fast rolling of the Higgs field σ triggered by the slow rolling of the inflaton field ϕ. Let us now formulate the conditions for a successful waterfall transition and confront them with observational data.

B. Evolution of the scalar fields

The evolution of the scalar fields is governed by the classical EOMs in an expanding background:

$$\ddot{\phi} + 3H\dot{\phi} + \left(m^2 + g^2\sigma^2\right)\phi = 0 \tag{4}$$

$$\ddot{\sigma} + 3H\dot{\sigma} - \frac{1}{a^2}\nabla^2\sigma + \left(g^2\phi^2 - M^2 + \lambda\sigma^2\right)\sigma = 0 \tag{5}$$

At the early stages of inflation ϕ is very large, $\phi \gtrsim M_p$, resulting in an effective Higgs mass $m_\sigma^{\text{eff.}}$ that is much larger than the corresponding effective inflaton mass $m_\phi^{\text{eff.}}$:

$$\left(m_\phi^{\text{eff.}}\right)^2 = V_{,\phi\phi} = m^2 + g^2\sigma^2 \tag{6}$$

$$\left(m_\sigma^{\text{eff.}}\right)^2 = V_{,\sigma\sigma} = g^2\phi^2 - M^2 + 3\lambda\sigma^2 \tag{7}$$

Because of that σ initially rolls down to $\sigma = 0$ and remains there for the most part of inflation. As initial conditions we can, hence, safely assume $\phi \gtrsim M_p$ and $\sigma = 0$. The end of inflation is initiated once ϕ has rolled down to the bifurcation point at the critical inflaton value $\phi_c = M/g$. For $\phi < \phi_c$ the Higgs potential exhibits a *tachyonic instability* at $\sigma = 0$, that is, the Higgs field possesses a negative effective mass squared, and inflation quickly ends due to false vacuum decay.

Before turning to the details of the waterfall regime let us first check how inflation in fact emerges in our model. Successful inflation requires a flat inflaton potential. We therefore assume the bare mass m to be small:

$$\boxed{m^2 \ll H^2} \tag{8}$$

At the beginning of inflation, when the inflaton energy density still dominates over the vacuum energy density,

$$\frac{1}{2}m\,\phi^2\big|_{\phi\gtrsim M_p} \gg \frac{M^4}{4\lambda} \tag{9}$$

we thus simply deal with ordinary chaotic inflation:

$$V\left(\phi \gtrsim M_p, \sigma = 0\right) \simeq \frac{1}{2}m^2\phi^2 \tag{10}$$

In his construction of the hybrid model Linde assumes in addition to Eq. (8) also that:

$$\boxed{m^2 \ll \frac{g^2}{\lambda}M^2} \tag{11}$$

such that eventually inflation is driven by the vacuum energy density:

$$V\left(\phi \simeq \phi_c, \sigma = 0\right) \simeq \frac{1}{2}m^2\left(\frac{M}{g}\right)^2 + \frac{M^4}{4\lambda} \tag{12}$$

$$= \frac{M^2}{2g^2}\left(m^2 + \frac{g^2}{2\lambda}M^2\right) \simeq \frac{M^4}{4\lambda} \tag{13}$$

The Hubble parameter can then be estimated as:

$$H^2 \simeq H_c^2 = \frac{8\pi}{3M_p^2}\cdot\frac{M^4}{4\lambda} = \frac{2\pi M^4}{3\lambda M_p^2} \tag{14}$$

Omitting the negligible acceleration term the EOM for the inflaton field turns into:

$$3H_c\dot{\phi} + m^2\phi \simeq 0 \quad\Rightarrow\quad \dot{\phi} \simeq -rH_c\phi\,;\; r = \frac{m^2}{3H_c^2} \tag{15}$$

Integration of Eq. (15) yields:

$$\phi(t) = \phi_c \exp\left[-rH_c\left(t - t_c\right)\right] \tag{16}$$

Given the time dependence of the scale factor, $a(t) \propto e^{H_c t}$, we may rewrite $\phi(t)$ as a function of the number of e-folds N to the phase transition:

$$N = H_c\left(t_c - t\right) \quad ; \quad \phi(N) = \phi_c e^{rN} \tag{17}$$

We find that for sufficiently small r the Universe also undergoes a stage of accelerated expansion when ϕ is close

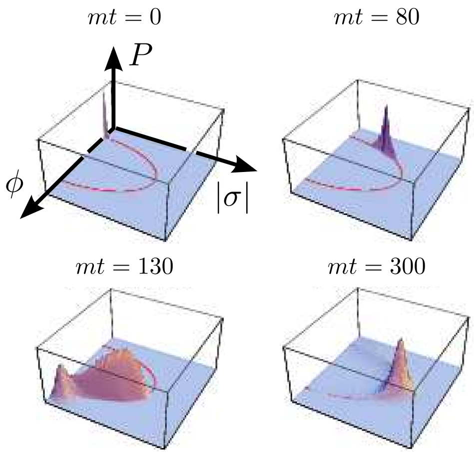

FIG. 2: Probability distribution $P(\phi, |\sigma|, t)$ for finding the field values ϕ and $|\sigma|$ at a given point in space at time t, extracted from numerical 3D lattice simulations with $M = 10^{15}$ GeV and $\lambda = g = 10^{-2}$. Figure taken from Ref. [5], an animated GIF version can be found on the arXiv.

to ϕ_c. For instance, with $r \lesssim 0.01$ more than 70 e-folds are generated during the period of vacuum domination. For comparision, recall that adiabatic density perturbations that we observe in the present Universe at galactic scales leave the Hubble horizon 50 to 60 e-folds before the end of inflation.

C. Waterfall conditions

In order to figure out for which parameter choices inflation ends in an almost instantaneous waterfall transition we have to determine two time scales. t_σ: The time it takes until the negative mass squared term becomes efficient such that SSB sets it. And t_ϕ: The time after which the inflaton field has settled at the minimum of the effective potential. We will speak of a successful waterfall transition once the following hierarchy is realized:

$$t_\sigma, t_\phi \ll H_c^{-1} \tag{18}$$

Given the initial conditions $\sigma_0 = 0$ and $\dot{\sigma}_0 = 0$ it is clear that SSB can only be accomplished by growing quantum fluctuations $\delta\sigma$. Neglecting the expansion of the Universe and the Higgs self-interaction for a moment the mode equations for these fluctuations read as follows:

$$\ddot{\sigma}_k + \left(k^2 - \mu_\sigma^2(\phi)\right)\sigma_k = 0 \tag{19}$$

Here $\mu_\sigma^2 = M^2 - g^2\phi^2$ denotes the negative mass squared of the Higgs field in the absence of any self-interaction. For momenta $k < \mu_\sigma$ the solutions of Eq. (19) are superpositions of exponentials that are either growing or decaying at rate ω_k:

$$\sigma_k(t) \sim \exp\left[\pm\omega_k(\phi)\, t\right] \; ; \; \omega_k(\phi) = \sqrt{\mu_\sigma^2(\phi) - k^2} \tag{20}$$

The $k = 0$ mode has the largest growth rate, $\omega_0 = \mu_\sigma$. We may, thus, estimate t_σ as the time scale on which the increase of this mode sets in [2]: $t_\sigma = \mu_\sigma^{-1}(t_\sigma)$. In other words: At time t_σ the negative mass squared has become that large that exactly the time μ_σ^{-1} has passed. Let us now estimate $\mu_\sigma^2(t_\sigma)$ (for convenience we set $t_c = 0$):

$$\mu_\sigma^2(t_\sigma) = -g^2\left(\phi^2(t_\sigma) - \phi_c^2\right) \simeq -g^2 \left.\frac{d}{dt}\right|_{t_c} \phi^2 \cdot t_\sigma \tag{21}$$

$$= -g^2 2\phi_c\dot{\phi}_c t_\sigma = g^2 2\phi_c \frac{m^2}{3H_c}\phi_c t_\sigma \tag{22}$$

$$= \frac{2}{3}g^2\frac{M^2}{g^2}m^2\sqrt{\frac{3\lambda}{2\pi}}\frac{M_p}{M^2}t_\sigma \tag{23}$$

Solving for $t_\sigma^{-1} = \mu_\sigma(t_\sigma)$ leads us to:

$$t_\sigma^{-3} \sim \sqrt{\lambda}m^2 M_p \tag{24}$$

We arrive at our first waterfall condition:

$$t_\sigma^{-6} \sim \lambda m^4 M_p^2 \gg H^6 \sim \frac{M^{12}}{\lambda^3 M_p^6} \tag{25}$$

which is equivalent to:

$$\boxed{M^3 \ll \lambda m M_p^2} \tag{26}$$

In order to estimate t_ϕ suppose now that one Hubble time $H_c^{-1} \gg t_\sigma$ has passed since t_c. At this time symmetry breaking has already occured and the Higgs field distribution has reached the minimum of the effective potential. To a good approximation the peak of the distribution $\bar{\sigma} \sim \left\langle\delta\sigma^2\right\rangle^{1/2}$ minimizes $V(\phi, \sigma)$ at fixed ϕ:

$$V_{,\sigma}(\phi, \bar{\sigma}) = 0 \quad \Rightarrow \quad g^2\phi^2 - M^2 + \lambda\bar{\sigma}^2 = 0 \tag{27}$$

$$\Rightarrow \quad \bar{\sigma}^2 = \frac{1}{\lambda}\left(M^2 - g^2\phi^2\right) \tag{28}$$

The distribution of the fields ϕ and σ evidently moves along an ellipse in the (ϕ, σ)-plane:

$$\phi^2 + \frac{\lambda}{g^2}\bar{\sigma}^2 = \phi_c^2 \tag{29}$$

After one Hubble time the inflaton field value has decreased by $\Delta\phi$. From the inflaton EOM we infer:

$$\frac{\Delta\phi}{H_c^{-1}} \simeq -rH_c\phi_c \quad \Rightarrow \quad \phi\left(H_c^{-1}\right) \simeq \phi_c\left(1-r\right) \tag{30}$$

Within the first Hubble time after t_c the effective mass squared of the inflaton $m_\phi^{\text{eff.}}$ grows significantly and it is reasonable to estimate the time scale on which inflation will end t_ϕ by $1/m_\phi^{\text{eff.}}$ at $t = H_c^{-1}$ [1]. According to Eq. (6) we then have:

$$\begin{aligned}
t_\phi^{-2} &= m^2 + g^2\bar{\sigma}^2\left(H_c^{-1}\right) && (31)\\
&= m^2 + \frac{g^2}{\lambda}\left(M^2 - g^2\phi^2\left(H_c^{-1}\right)\right) && (32)\\
&= m^2 + \frac{g^2}{\lambda}\left(M^2 - g^2\phi_c^2\left(1-r\right)^2\right) && (33)\\
&\simeq m^2 + \frac{g^2}{\lambda}\cdot 2M^2 r && (34)\\
&= m^2 + \frac{2g^2M^2}{\lambda}\cdot\frac{m^2}{3H_c^2} && (35)\\
&= m^2\left(1 + \frac{2g^2M^2}{3\lambda}\cdot\frac{3\lambda M_p^2}{2\pi M^4}\right) && (36)\\
&= m^2\left(1 + \frac{g^2M_p^2}{\pi M^2}\right) \simeq \frac{g^2m^2M_p^2}{\pi M^2} && (37)
\end{aligned}$$

The second waterfall condition demands that t_ϕ be much smaller than H_c^{-1}:

$$t_\phi^{-2} \sim \frac{g^2m^2M_p^2}{M^2} \gg H_c^2 \sim \frac{M^4}{\lambda M_p^2} \tag{38}$$

which is equivalent to:

$$\boxed{M^3 \ll \sqrt{\lambda}gmM_p^2} \tag{39}$$

D. Cosmological implications

Confronting the power spectrum of adiabatic energy density perturbations predicted by hybrid inflation with observational data (CMB anisotropies, galaxy surveys, gravitional lensing, Ly-α forest) gives preference to certain regions in parameter space. We will now show that in these favored regions the end of inflation typically is described by the waterfall transition.

First, let us write the cosmic density field ρ as:

$$\rho\left(t,\vec{x}\right) = \rho_0(t) + \delta\rho\left(t,\vec{x}\right) \tag{40}$$

with $\rho_0 = \langle\rho\rangle$. The power spectrum $\delta\rho_k^2$ follows from a decomposition of the mean square fluctations $\left\langle\delta\rho^2\right\rangle$ on a logarithmic momentum scale:

$$\left\langle\delta\rho^2\left(t,\vec{x}\right)\right\rangle = \int d\ln\frac{k}{H}\;\delta\rho_k^2 \tag{41}$$

At the onset of galaxy formation, when the energy in cold matter has come to dominate, $\delta\rho_k$ can be estimated by standard methods [3]:

$$\frac{\delta\rho_k}{\rho_0} = \frac{16\sqrt{6\pi}}{5M_p^3}\left.\frac{V^{3/2}}{V_{,\phi}}\right|_{k=aH} \tag{42}$$

In hybrid inflation for $\sigma = 0$:

$$\frac{\delta\rho_k}{\rho_0} = \frac{16\sqrt{6\pi}}{5M_p^3}\left.\frac{\left(\frac{1}{2}m^2\phi^2 + \frac{M^4}{4\lambda}\right)^{3/2}}{m^2\phi}\right|_{k=aH} \tag{43}$$

As mentioned at the end of Sec. II B the galactic scales cross the Hubble horizon during vacuum domination:

$$\frac{\delta\rho_k}{\rho_0} \simeq \frac{2\sqrt{6\pi}}{5M_p^3}\frac{M^6}{\lambda\sqrt{\lambda}m^2\phi_c}\left.\frac{\phi_c}{\phi}\right|_{k=aH} \tag{44}$$

With $\phi/\phi_c = e^{rN} = \left(a_c/a\right)^r \to \left(k_c/k\right)^r$ we finally obtain:

$$\frac{\delta\rho_k}{\rho_0} \simeq \frac{2\sqrt{6\pi}}{5M_p^3}\frac{gM^5}{\lambda\sqrt{\lambda}m^2}\left(\frac{k}{k_c}\right)^r \tag{45}$$

As a very usual feature of the hybrid inflation model we note its blue spectrum. The scalar spectral index n_s always is larger than 1:

$$n_s - 1 = \frac{d\ln\delta\rho_k^2}{d\ln k} = 2r = \frac{2m^2}{3H_c^2} = \frac{\lambda m^2M_p^2}{\pi M^4} \tag{46}$$

Since $m^2 \ll H^2$, cf. Eq. (8), the deviation from 1 is vanishingly small. But still, observations clearly indicate $n_s < 1$. The ΛCDM fit of the WMAP7 data yields:

$$n_s = 0.963 \pm 0.014 \tag{47}$$

Moreover, notice that for fluctuations with very small k, which left the Hubble horizon while the inflaton contribution to the potential energy density still dominated over the vacuum term, the spectrum begins growing again.

Neglecting the k-dependence in Eq. (45) and using

Eq. (8) we may write:

$$\frac{1}{H_c}\cdot\frac{2\sqrt{6}\pi}{5M_p^3}\frac{gM^5}{\lambda\sqrt{\lambda}m^2} \ll \frac{1}{m}\cdot\frac{\delta\rho_k}{\rho_0} \tag{48}$$

$$\Leftrightarrow \quad \sqrt{\frac{3\lambda}{2\pi}}\frac{M_p}{M^2}\frac{2\sqrt{6}\pi}{5M_p^3}\frac{gM^5}{\lambda\sqrt{\lambda}m} \ll \frac{\delta\rho_k}{\rho_0} \tag{49}$$

$$\Leftrightarrow \quad M^3 \ll \frac{\delta\rho_k}{\rho_0}\cdot\frac{5\lambda}{6g}mM_p^2 \tag{50}$$

The typical scale of the amplitude of the power spectrum has been measured by the COBE satellite, $\delta\rho_k/\rho_0 \sim 5\times 10^{-5}$. In order to generate density perturbations of the right size the parameters of the hybrid inflation model thus have to satisfy:

$$\boxed{M^3 \ll 5\times 10^{-5}\cdot\frac{\lambda}{g}mM_p^2} \tag{51}$$

This means that for a wide range of values of the coupling constants λ and g the waterfall conditions (26) and (39) are automatically satisfied merely by virtue of the assumption $m^2 \ll H^2$ and the requirement $\delta\rho_k/\rho_0 \sim 5\times 10^{-5}$. In cosmologically realistic scenarios hybrid inflation hence tends to end in a waterfall regime.

III. TACHYONIC PREHEATING

The basic form of the effective potential in hybrid inflation scenario is given by Eq. (3). As stated above, inflation in this model occurs while the ϕ field rolls slowly in the $\sigma = 0$ valley towards the global minimum at $\phi = 0$ and $|\sigma| = v$. In realistic versions of this model, the mass m and the velocity $\dot{\phi}$ are small after inflation. The fields then fall along a trajectory $\phi(t)$, $\sigma(t)$ in such a way that the initial trajectory is flat, then it rapidly falls down, and becomes flat again near the minimum. In between, the curvature of the effective potential is initially negative, and therefore, the field should experience a tachyonic instability along the way.

A. Dynamics of the Symmetry breaking

The main goal of this section is to show that the tachyonic instability leads to SSB so efficient that it completes typically within an oscillation. We will see that the quantum fluctuations of the Higgs field grow such that it produces Higgs classical waves.

At the initial stages of Spontaneous Symmetry Breaking (SSB), the Higgs modes follow the linear Eq. (19). In the symmetric phase $\sigma = 0$, i.e. when $\phi > \phi_c$, we take the quantum fluctuations of the mode functions to be the same as the one for a massless field

$$\sigma_k(t) = \frac{1}{\sqrt{2k}}\, e^{-ikt}, \qquad k \equiv |\vec{k}|\,. \tag{52}$$

At $t = 0$, one "turns on" the term $-M^2\sigma^2/2$, corresponding to a negative mass squared $-M^2$. Consequently, in this quench approximation, all modes $k < M$ grow exponentially

$$\boxed{\sigma_k(t) = \frac{1}{\sqrt{2k}}\, e^{t\sqrt{M^2-k^2}}}\,, \tag{53}$$

At latter stages, the mean value of the field stays equal to zero $\langle\sigma\rangle = 0$. The first non-vanishing quantity is the second moment of σ, i.e. $\langle\sigma^2\rangle$. This is this quantity that we use to characterize the symmetry breaking process, stating that symmetry breaking occurs when $\langle\sigma^2\rangle = v^2$.

Using the Heisenberg representation of the quatum fluctuations of the σ field is

$$\sigma(t,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\left[a_{\mathbf{k}}\,\sigma_{\mathbf{k}}(t)\,e^{-i\mathbf{kx}} + a_{\mathbf{k}}^{\dagger}\,\sigma_{\mathbf{k}}^{*}(t)e^{i\mathbf{kx}}\right]\,, \tag{54}$$

where $\mathbf{x}$ and $\mathbf{k}$ denote the position and momentum vectors, one can deduce the initial ($t = 0$) dispersion of all growing fluctuations with $k < M$

$$\langle\sigma^2\rangle \equiv \langle 0|\sigma(\mathbf{x})\sigma(\mathbf{x})|0\rangle\,, \tag{55}$$

$$= \int_0^{m^2}\frac{dk^2}{8\pi^2} = \frac{m^2}{8\pi^2}\,. \tag{56}$$

where the canonical commutation relation $[a_{\mathbf{k}}, a_{\mathbf{k}'}^{\dagger}] = \delta^{(3)}(\mathbf{k}-\mathbf{k}')$ has been used. Using the 2-point correlation function

$$\langle\sigma(\mathbf{x})\sigma(\mathbf{y})\rangle = \int\frac{dk}{k}\,\frac{\sin(k|\mathbf{x}-\mathbf{y}|)}{k|\mathbf{x}-\mathbf{y}|}\,\delta\sigma_k^2\,, \tag{57}$$

one can deduce the average initial amplitude $\delta\sigma_k$ of all fluctuations with $k < M$

$$\delta\sigma_{\mathbf{k}} = \frac{k}{2\pi}\,. \tag{58}$$

At $t > 0$, the modes with $k < M$ start to growth exponentially, and their dispersion relation becomes

$$\langle\sigma^2\rangle = \int_0^{M^2} \frac{dk^2}{8\pi^2}\, e^{2t\sqrt{M^2-k^2}}\,, \tag{59a}$$

$$= \frac{e^{2Mt}(2Mt-1)+1}{16\pi^2 t^2}\,, \tag{59b}$$

$$\boxed{\langle\sigma^2\rangle \sim \frac{M^2}{16\pi^2}\, e^{2Mt}}\,. \tag{59c}$$

This means that the average amplitude $\delta\sigma_{\mathbf{k}}$ of quantum fluctuation with momenta k initially was $\delta\sigma_{\mathbf{k}} = k/2\pi$ and then started to grow as $e^{t\sqrt{M^2-k^2}}$.

To get a qualitative understanding, consider a single sinusoidal wave $\sigma = \Delta(t)\cos(kx)$ with $k \sim M$, and with an initial amplitude $\Delta(t) \sim M/2\pi$ in 1D. The amplitude of this wave grows until it becomes $\mathcal{O}(v) \sim M/\sqrt{\lambda}$. This leads to the division of the Universe in domains of size $\mathcal{O}(M^{-1})$, in which the field changes from $\mathcal{O}(v)$ to $\mathcal{O}(-v)$. The gradient energy density of domain walls separating areas will be $\sim k^2\sigma^2 = \mathcal{O}(M^4/\lambda)$. This is of the same order as the total initial potential energy of the field $V(0) = M^4/4\lambda$. Because the initial state contains many fluctuations with different phases growing at different rates, the resuling field is a Gaussian random field. It cannot coherently give all its gradient energy back and return to its initial state $\sigma = 0$. This is one of the reason why SSB occurs within a single oscillation of the field σ.

At $\sigma \sim v/\sqrt{3}$, the curvature of the effective potential vanishes, i.e. the Higgs mass becomes positive. In consequence, the tachyonic growth of all fluctuations with $k < M$ continues until $\sqrt{\langle\delta\sigma^2\rangle} \sim v/2$. Subsequently, all the modes oscillates.

The symmetry is broken when $\langle\sigma^2\rangle \simeq v^2$. Using Eq. (59c), one can estimate the time it takes for symmetry breaking to happen

$$\boxed{t_\star \simeq \frac{1}{2M}\,\ln\left(\frac{32\pi^2}{\lambda}\right)}\,. \tag{60}$$

The exponential growth of fluctuations can be interpreted as the growth of the occupation number, which is defined by [8]

$$n_{\mathbf{k}} + \frac{1}{2} \equiv \frac{1}{2}|\sigma^*_{\mathbf{k}}(t)\dot{\sigma}_{\mathbf{k}}(t)|\,. \tag{61}$$

Using Eq. (53), the occupation number

$$\boxed{n_{\mathbf{k}} + \frac{1}{2} = \frac{1}{2}\left|e^{t\sqrt{M^2-k^2}}\right|^2 \approx \frac{1}{2}\, e^{2Mt}\, e^{-k^2/(2k_\star^2)}}\,, \tag{62}$$

becomes exponentially large very quickly for the long wavelength modes, and drops abruptly for $k > k_\star \equiv M(2Mt)^{-1/2}$.

Example : For $\lambda = 10^{-3}$ and $v = 10^{-4}M_P$, the symmetry is broken within $Mt_\star \sim 6$ and the typical cutoff frequency is $k_\star \sim M/3$. The occupation numbers of modes with $k < k_\star$ is exponentially large

$$n_k(t_\star) \approx \frac{1}{2}e^{2Mt_\star} \simeq \frac{16\pi^2}{\lambda} \sim 2\times 10^5\,. \tag{63}$$

Importantly, one remarks that *for small λ, the fluctuations with $k < M$ have very large occupation numbers, and therefore they can be interpreted as classical waves of the field σ.*

When the fields rolls down to the minimum of its effective potential, its fluctuations scatter off each other as classical waves due to the $\lambda\sigma^4$ interaction. It is difficult to study this process analitycally, but fortunately, it can be studied using lattice simulations. In Fig. 3 is plotted the result of the evolution of the Higgs vev, together with the approximate expression

$$\sigma(t) \equiv \langle\sigma^2(t)\rangle^{1/2} = \frac{v}{2}\left(1 + \tanh\frac{M(t-t_\star)}{2}\right)\,. \tag{64}$$

As can be seen, the vacuum expectation value never comes close to the initial point $\sigma = 0$ again, which implies that symmetry becomes broken within a single oscillation of the distribution of the field σ. After SSB, space becomes divided into domains with $\sigma = \pm v$. The initial size of each domain is $\mathcal{O}(M^{-1})$, and inside each domain, the deviation from $\sigma = |v|$ is much smaller than v. Gradually, the size of each domain grows and the domain wall structure becomes more and more stable.

B. Particle production

We now turn to the study of the production of scalars χ and fermions ψ, coupled to the Higgs via the usual

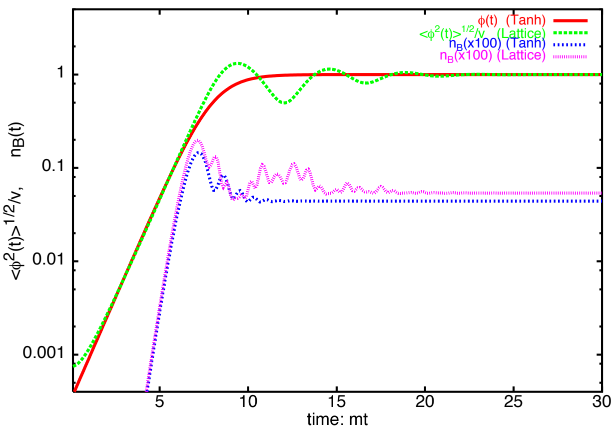

FIG. 3: Time evolution of the vacuum expectation value $\langle\sigma^2(t)\rangle^{1/2}/v$ (here ϕ) compared with the approximate solution Eq. (64) with $Mt_\star = 8$. Figure taken from Ref. [7].

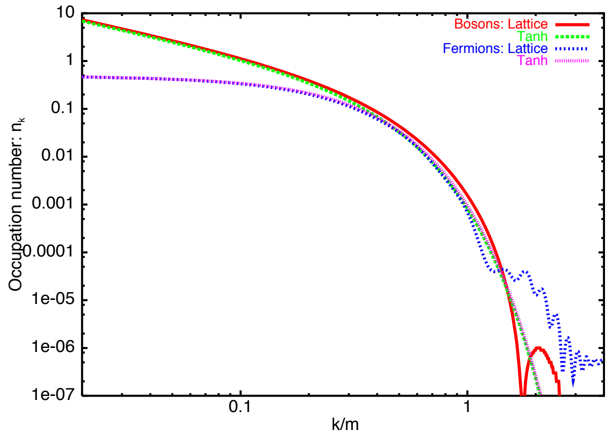

FIG. 4: Spectrum of occupation number for bosons and fermions in the true vacua, in both lattice and analytical approximation. Here the parameters are fixed to $\lambda = 0.01$, $g = 0.5$ and $h = 0.5$ Figure taken from Ref. [7].

following interactions

$$\mathcal{L}_{\rm int} = \frac{1}{2} g^2\sigma^2\chi^2 + h\sigma\bar{\psi}\psi\,. \tag{65}$$

Since the backreactions of the produced χ and ψ modes on the evolution of the Higgs expectation value, is negligible, we can solve first the process of symmetry breaking (see below), and then take the resulting evolution of the Higgs as a background fields that induces particle production.

The mode equations in terms of rescaled fields

$$X_k(t) = a^{3/2}\chi_k\,, \qquad \Psi(t) = a^{3/2}\psi\,, \tag{66}$$

are

$$\partial_t^2 X_k + \left(k^2 + m_B^2(t)a^2(t)\right) X_k = 0\,, \tag{67a}$$

$$\left(i\gamma^\mu\partial_\mu - m_F(t)a(t)\right)\Psi = 0\,. \tag{67b}$$

As we have seen previously, the quench approximation, i.e. the sudden appearance of the mass term, is valid in hybrid inflation models fulfilling the waterfall conditions. In that case, the rate of expansion is typically much smaller than the masses involved, so that we take it constant here ($a = 1$) during SSB. The change of vacuum induces a sudden change in the masses of bosons and fermions through the Higgs mechanism

$$m_B^2(t) = g^2\langle\sigma^2\rangle\,, \quad m_F^2(t) = h\langle\sigma^2\rangle\,. \tag{68}$$

It will be responsible for the non-adiabatic production of particles which can be studied using the formalism of quantum fields in strong background.

1. *Number densities*

At first, one can solve the mode equations Eqs. (67) within the approximation Eq. (64). For bosons, the mode functions $X_k(t)$ are solutions of the oscillator Eq. (67a) with time-dependent frequency $\omega_k^2(t) = k^2 + m_B^2(t)$. It can be rewritten

$$X_k'' + \left(\omega_-^2(k) + \frac{\alpha^2 m_B^2}{4}(1+\tanh x)^2\right) X_k = 0 \tag{69}$$

where $x = M(t - t_\star)/2$ and $\alpha \equiv g/\sqrt{\lambda}$, while $\omega_-^2(k) = k$ and $\omega_+^2(k) = \sqrt{k^2 + \alpha^2 M^2}$ are the *in*/*out* asymptotic frequencies. Solving this equation in terms of hypergeometric functions [7], one can deduce the boson occupation number

$$n_k^B(k) = \frac{\cosh[\sqrt{4\alpha^2 - 1}] - \cosh[2\pi(\omega_+ - \omega_-)/M]}{\sinh[2\pi\omega_-/M]\sinh[2\pi\omega_+/M]]} \tag{70}$$

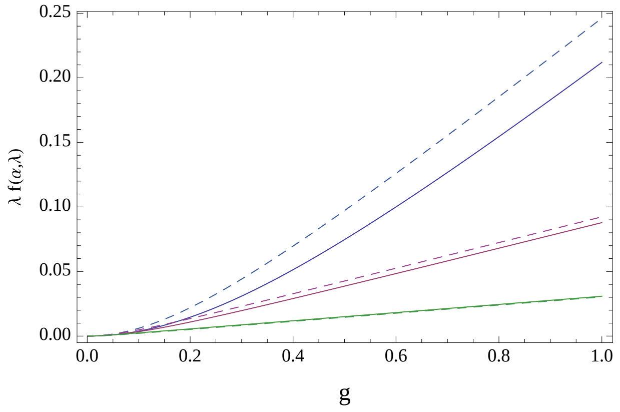

FIG. 5: Evolution of the $\lambda f(\alpha,\gamma)$ function with the coupling g (of equivalently h) for $\lambda = 0.1, 0.01, 0.001$ (from top to bottom). The case of bosons (fermions) is represented by the plain (dashed) lines.

A similar treatment can be done for the fermions, which leads to

$$n_k^F(k) = \frac{\cosh[2\pi\alpha] - \cosh[2\pi(\omega_+ - \omega_-)/M]}{2\sinh[2\pi\omega_-/M]\sinh[2\pi\omega_+/M]]} \tag{71}$$

with $\alpha \equiv h/\sqrt{\lambda}$ in this case.

More precisely, the modes equations Eqs. (67) are solved numerically, using the lattice Higgs vev evolution. The two options are illustrated in Fig. 4 where one can see that they agree very well, except at large momentum, where parametric resonance effects are responsible for the excitation of the low wavelength modes (remember : the first band in the instability chart in the parametric resonance case is sitting around the value $k \sim M$).

2. Energy densities

The ratio of energy densities of particles created to the initial false vacuum energy density $\rho_0 \equiv M^4/4\lambda$ is given by

$$\frac{\rho_X}{\rho_0} = \frac{2g_s\lambda}{\pi^2}\int d\kappa\,\kappa^2\, n_k^X(\alpha)\,\omega_+(\kappa,\alpha) \tag{72}$$

where $\kappa \equiv k/M$. Fitting functions to the final energy densities are provided by

$$\boxed{\frac{\rho_B}{\rho_0} \simeq 2\times 10^{-3}\, g_s\,\lambda\, f(\alpha, 1.3)}\,, \tag{73}$$

$$\boxed{\frac{\rho_F}{\rho_0} \simeq 1.5\times 10^{-3}\, g_s\,\lambda\, f(\alpha, 0.8)}\,, \tag{74}$$

where $f(\alpha,\gamma) \equiv \sqrt{\alpha^2+\gamma^2} - \gamma$ and $\alpha = g/\sqrt{\lambda}(h/\sqrt{\lambda})$ for bosons (fermions). The production of bosons and fermions from symmetry breaking is thus more efficient when the Higgs mass and thus λ is large, as is also clear from Fig. 5. Unless the couplings are unnaturally large, the fractional energy density in bosons and fermions is always small. As a consequence, no backreaction on the evolution of the Higgs is thus expected, justifying *a posteriori* our initial assumption of dividing the problem into two stages. Importantly, this also implies that at the end of tachyonic preheating the Universe energy density is dominated be the non-relativistic classical waves of the Higgs bosons. One would thus have to wait for the subsequent decays for the Universe to be reheated. The reheating temperature is then given by the usual expression

$$T_R \simeq 0.2(200/g_\star)^{1/4}\sqrt{\Gamma_\sigma M_P}\,. \tag{75}$$

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