Heavy Quark Effective Theory

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Introduction and motivation

Although the Standard Model satisfactorily describes most of the observable phenomena in elementary particles physics, several questions, important for our understanding of the Universe, remain unanswered. Among them the most striking unsolved problem in high energy physics is the phenomenon of generations.

The gauge forces in the Standard Model do not discriminate fermions relating to different generations. Corresponding particles (quarks and gluons) belonging to the three generations are identical in almost all respects. And with it we do not know the origin of generations; what does stipulate their number (three)? How can we explain the present structure of hierarchies of fermions' masses and mixing angles? We may expect that if we could find solution of this problem, we shall have a chance to understand hitherto unknown fundamental laws of the Universe and make new great discoveries.

Another crucial problem is the origin of baryogenesis. Standard Model satisfies the necessary conditions for baryogenesis, which consist in the so-called Sakharov criteria. Baryon-number violating process are unsuppressed at high temperature, CP-violating interactions are present due to complex couplings in the quark sector, and non-equilibrium processes can occur during phase transitions driven by the expansion of the Universe. However, Standard Model can not explain on quantitative level the observed matter-antimatter asymmetry. And so it is obvious that we need new CP-violating phases or new mechanisms of CP-violation.

There are several more nontrivial questions in the theory of elementary particles concerned with flavor physics. But the problems mentioned above are enough to underscore that we must carry out the experimental measurements and theoretical predictions of the parameters of the Standard Model, such as the elements of the Cabibbo-Kobayashi-Maskawa matrix, as accurately as possible, since comparing precise theoretical and experimental results gives us a chance to find a hint to the physics beyond the Standard Model.

However, the computational complexity of the SM, in particular QCD, is such that we can not do calculations with sufficient accuracy. A possible way out is to give a formulation of an effective theory which approximates QCD under certain conditions, and possesses all necessary phenomenological properties, allowing to perform simple calculations. One of such effective theories is the Heavy Quark Effective Theory, which corresponds to QCD in the limit of infinite (or very large) heavy quark mass.

Structure of HQET

The main idea of the heavy quarks effective theory is as follows. Let us consider a heavy hadron, which contains a charm or a bottom quark, interacting with light constituents by the exchange of soft gluons. For such system we have an energy scale set by the heavy quark mass m_Q. The soft gluons and the spectator quark have energies, which are represented by the other scale, namely Λ_{QCD}. Hence, the dynamics of such a heavy quark system can be solved as a perturbation in $\frac{\Lambda_{QCD}}{m_Q}$<<1. Hence, as a first approximation we can suppose, that the heavy quark moves with the hadron's velocity (so-called infinite mass limit) and there are no dynamical degrees of freedom. With this assumption, description of the heavy quark system becomes significantly simpler, and deviations of the behavior of real system from the ideal limiting case could be taken into account by the introduction of correction terms, inversely proportional to the powers of the heavy quark mass, in expressions of the physical observables.

Let us give a brief description of the HQET structure and it's applications to the calculation of physical processes [1]. As we are not interested in the study of the physics of the processes that occur at energies above m_Q, we may choose a cutoff $\Lambda<m_Q$ and separate fields of the theory into two terms, corresponding to the Fourier modes with high frequency $\omega>\Lambda$ (φ_H) and low frequency $\omega<\Lambda$ (φ_L):

$$\varphi=\varphi_L+\varphi_H\,.$$

By the construction of this theory, all low-energy physical processes are described in φ_L-fields terms. So, we can use the standard apparatus of quantum field theory and obtain all necessary information from correlation functions of low-frequency fields:

$$\left\langle 0\left|T\left\{\varphi(x_1)\ldots\varphi(x_n)\right\}\right|0\right\rangle=\frac{1}{Z[0]}\left(-i\frac{\delta}{\delta J_L(x_1)}\right)\ldots\left(-i\frac{\delta}{\delta J_L(x_n)}\right)Z\left[J_L\right]\Big|_{J_L=0},$$

where

$$Z\left[J_L\right]=\int D\varphi_L D\varphi_H\, e^{iS(\varphi_L,\varphi_H)+i\int d^Dx J_L(x)\varphi_L(x)}$$

is the generating functional of the theory. Here $S\left(\varphi_L,\varphi_H\right)=\int d^D x\, L(x)$ is the action, D is the dimension of the space-time, and J_L are the sources of light fields.

The high-frequency fields may be integrated out in the functional integral of the system:

$$Z\left[J_L\right]\equiv\int D\varphi_L\, e^{iS_\Lambda(\varphi_L)+i\int d^Dx J_L(x)\varphi_L(x)},$$

where

$$e^{iS_\Lambda(\varphi_L)}=\int D\varphi_H\, e^{iS(\varphi_L,\varphi_H)}$$

is called the "Wilsonian effective action".

We must note here, that although we do not consider heavy particles in our theory, pair creation of heavy quarks on the virtual level could not be excluded. So, the effective action of the theory becomes non-local on scales $\Delta x^{\mu} \sim \frac{1}{\Lambda}$, since the description of such virtual processes in terms of Feynman diagrams of effective theory is impossible (we do not have the corresponding analytical expressions for heavy-particles propagators). But we can expand this non-local action functional in terms of local operators composed of light fields. This technique is called Operator Product Expansion, and convergence of the series is provided by the small parameter $\frac{E}{\Lambda}$.

The result can be expressed in form:

$$S_{\Lambda}\left(\varphi_L\right)=\int d^D x\, L_{\Lambda}^{eff}(x),$$

where

$$L_{\Lambda}^{eff}=\sum_i g_i\, Q_i\left(\varphi_L(x)\right)\cdot$$

The effective Lagrangian presents infinite sum over local operators Q_i, multiplied by coupling constants g_i, which are called the Wilson coefficients. It is quite difficult to work with an infinite number of operators, and so we need to simplify this object. The trick of "naive dimensional analysis" can help us. Denote by $[g_i]=-\gamma_i$ the mass dimension of the effective coupling constants. And it can be written as:

$$g_i=C_i\, M^{-\gamma_i},$$

where C_i are dimensionless coefficients. Because there is only one fundamental scale in the HQET, we can expect that $C_i=O(1)$. This assumption is named the hypothesis of naturalness. Let us assume for simplicity that observables are dimensionless. In this case, the quantitative contribution of each operator in OPE is expected to scale as

$$C_i\left(\frac{E}{M}\right)^{\gamma_i}=\begin{cases} O(1),\ if\ \gamma_i=0, \\ <<1,\ \ if\ \gamma_i>0, \\ >>1,\ if\ \ \gamma_i<0\cdot \end{cases}$$

It is clear that only few operators whose couplings have $\gamma_i \leq 0$ are important to study physical processes, and the infinite series becomes a short sum.

Besides the general discussion, it is instructive to give the explicit expression of the Lagrangian of HQET [2]. Denote the quark spinor field as Q(x), and define the large and small component fields h_v and H_v by

$$h_v(x)=e^{im_Q v\cdot x}P_+\,Q(x),$$

$$H_v(x)=e^{im_Q v\cdot x}P_-\,Q(x)\cdot$$

So

$$Q(x)=e^{-im_Q v\cdot x}\left[h_v(x)+H_v(x)\right]\cdot$$

The effective Lagrangian is defined in these terms as:

$$L_{eff}=\overline{h}_v\, iv\cdot Dh_v+\overline{h}_v\, i\hat{D}_{\perp}\frac{1}{iv\cdot D+2m_Q-i\varepsilon}\, i\hat{D}_{\perp}\, h_v,$$

where $\hat{D}_{\perp} = \left(D^{\mu} - v^{\mu}\, v \cdot D\right)\gamma_{\mu}$ is orthogonal to heavy quark velocity.

The first term here is the infinite mass limit of the QCD-Lagrangian, and the second term provides corrections that correspond to the finite mass of the heavy quark. Since there is an inverse differential operator in the second term, we can deal with the non-locality. OPE gives us:

$$L_{eff} = \overline{h}_v\, iv \cdot D h_v + \frac{1}{2m_Q}\overline{h}_v \left(iD_{\perp}\right)^2 h_v + \frac{g}{4m_Q}\overline{h}_v\, \sigma_{\alpha\beta} G^{\alpha\beta} h_v + O\left(\frac{1}{m_Q^2}\right).$$

Here $\left[iD^{\alpha}, iD^{\beta}\right] = iG^{\alpha\beta} = igT_a\, G_a^{\alpha\beta}$ is the gluon field strength tensor.

The physical meaning of the two new operators at order $\frac{1}{m_Q}$ is quite simple.

$$O_{kin} = \frac{1}{2m_Q}\overline{h}_v \left(iD_{\perp}\right)^2 h_v$$

is the gauge covariant extension of the kinetic energy, provided by the off-shell residual motion of heavy quark, and

$$O_{mag} = \frac{g}{4m_Q}\overline{h}_v\, \sigma_{\alpha\beta} G^{\alpha\beta} h_v$$

describes the interaction of the heavy quark spin with the gluon field (chromo-magnetic hyperfine interaction).

If we solve the equations of motion corresponding to the full non-local Lagrangian:

$$-iv \cdot D h_v = i\hat{D}_{\perp} H_v,$$

$$\left(iv \cdot D + 2m_Q\right) H_v = i\hat{D}_{\perp} h_v,$$

we get:

$$H_v = \frac{1}{\left(iv \cdot D + 2m_Q - i\varepsilon\right)} i\hat{D}_{\perp} h_v.$$

Finally, expansion for the full quark field is

$$Q(x) = e^{-im_Q v \cdot x}\left[1 + \frac{1}{\left(iv \cdot D + 2m_Q - i\varepsilon\right)}\right] h_v(x) = e^{-im_Q v \cdot x}\left(1 + \frac{i\hat{D}_{\perp}}{2m_Q} + \ldots\right) h_v(x).$$

Now, when we know explicit expression for heavy quark field, we can calculate matrix elements of various observables; determine the cross-sections of decays and scatterings etc.

Experimental results and perturbative calculations

Applications of the heavy quark effective theory are to be compared with the experimental measurements. The analytical expressions of decay rates can be written in terms of the operator product expansion through several coefficients, which are not determined within the framework of the HQET. So, we need to determine them from the experiment. In this aspect the B-meson rare and inclusive decays are very useful tools, because they are described by the HQET and allow one to find the values of CKM elements $\left|V_{cb}\right|, \left|V_{ub}\right|$ and the heavy quark masses.

Let us consider the explicit expression for the semileptonic B decay width through order $\frac{1}{m_Q^3}$ [3]:

$$\Gamma_{sl}(b \mapsto c) = \frac{G_F^2\, m_b^5(\mu)}{192\pi^3}\left|V_{cb}\right|^2\left(1 + A_{ew}\right)\left[z_0(r)\left[1 + A_3^{pert}(r,\mu)\right]\left(1 - \frac{\mu_\pi^2(\mu) - \mu_G^2(\mu) + \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b(\mu)}}{2m_b^2(\mu)}\right)\right.$$

$$- \left(1 + A_5^{pert}(r,\mu)\right) 2(1-r)^4 \frac{\mu_G^2(\mu) - \frac{\rho_D^3(\mu) + \rho_{LS}^3(\mu)}{m_b^3(\mu)}}{m_b^2(\mu)} + \left(1 + A_D^{pert}\right) d(r) \frac{\rho_D^3(\mu)}{m_b^3(\mu)}$$

$$+ 32\pi^2\left(1 + A_{6c}^{pert}(r)\right)\left(1 - \sqrt{r}\right)^2 \frac{H_c}{m_b^3(\mu)} + 32\pi^2 \widetilde{A}_{6c}^{pert}(r)\left(1 - \sqrt{r}\right)^2 \frac{\widetilde{H}_c}{m_b^3(\mu)}$$

$$\left. + 32\pi^2 A_{6q}^{pert}(r) \frac{F_q}{m_b^3(\mu)} + O\left(\frac{1}{m_b^4}\right)\right].$$

Here $z_0(r)$ is the tree-level phase factor and $r = \frac{m_c^2(\mu)}{m_b^2(\mu)}$:

$$z_0(r) = 1 - 8r + 8r^3 - r^4 - 12r^2 \ln r,$$

and the expression for $d(r)$ is

$$d(r) = 8\ln r + \frac{34}{3} - \frac{32}{3}r - 8r^2 + \frac{32}{3}r^3 - \frac{10}{3}r^3 - \frac{10}{3}r^4 \cong -18.3 z_0 \text{ at } \sqrt{r} = 0.25$$

The electroweak correction A_{ew} that corresponds to the ultraviolet renormalization of the Fermi interaction is known:

$$1 + A_{ew} \cong \left(1 + \frac{\alpha}{\pi}\ln\frac{M_Z}{m_b}\right)^2 \cong 1.014\ .$$

The quantities $A_{\dots}^{pert}$ correspond to the perturbative corrections. We can account for them by carrying out calculations within the framework of the perturbative QCD. We will not give here the corresponding computations. The quantities $\mu_\pi^2, \mu_G^2, \rho_D^3$ and ρ_{LS}^3 denote the expectation values of the kinetic, chromo-magnetic, Darwin and spin-orbit operators respectively. An auxiliary scale μ is introduced to demark the border between the long- and short-distance dynamics in the OPE. Usually it is taken as $\mu \cong 1GeV$.

The leading non-perturbative corrections arise in order $\frac{1}{m_Q^2}$ and are controlled by the matrix elements $\mu_\pi^2(\mu)$ and $\mu_G^2(\mu)$ of the kinetic and the chromomagnetic dimension-five operators, respectively

$$\mu_\pi^2(\mu) \equiv \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu, \qquad \mu_G^2(\mu) \equiv \frac{1}{2M_B}\left\langle B\left|\bar{b}\,\frac{i}{2}\sigma_{jk}\,G^{jk}\, b\right|B\right\rangle_\mu$$

The Darwin and the spin-orbital LS terms $\rho_D^3(\mu)$ and $\rho_{LS}^3(\mu)$ appear from the dimension-six operators:

$$\rho_D^3(\mu) \equiv \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(-\frac{1}{2}\vec{D}\cdot\vec{E}\right)b\right|B\right\rangle_\mu, \qquad \rho_{LS}^3(\mu) \equiv \left\langle B\left|\bar{b}\left(\vec{\sigma}\cdot\vec{E}\times i\vec{D}\right)b\right|B\right\rangle_\mu.$$

The term proportional F_q denotes the effect of generic SU(3)-singlet four-quark operators, other than Darwin operator, of the form $\bar{b}\,\Gamma b\,\bar{q}\,\Gamma q$ with the sum over $q = u,d,s$ and Γ including both color and Lorentz matrices. But their Wilson coefficients are $O(\alpha_s)$, and we neglect these contributions.

In addition, it is worth to pay attention to the H_c term. One describes a possible effect of the tree-level expectation values of the four-quark operators with the charm field. Its analytical expression is:

$$H_c = \frac{1}{2M_B}\left\langle B\left|\bar{b}\,\gamma_\nu\left(1-\gamma_5\right)c\,\bar{c}\,\gamma^\rho\left(1-\gamma_5\right)b\right|B\right\rangle_\mu\left(-\delta^\nu_\rho + v^\nu v_\rho\right), \qquad v_\nu = \frac{P^B_\nu}{M_B}.$$

The effect is quite small due to the sizeable charm quark mass. But it could not be negligible totally, so, one should take it into account. Knowledge of these non-perturbative matrix elements allows one to determine the mass of the heavy quark [4]:

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu^2_\pi\left(H_Q\right) - \mu^2_G\left(H_Q\right)}{2m_Q} + O\left(\frac{1}{m_Q^2}\right),$$

Here $\bar{\Lambda}$ is the residual energy difference between M_{H_Q} and m_Q surviving in the infinite heavy quark mass limit.

Let us consider at some length details of the experimental measurements of the non-perturbative HQE parameters. In an experiment, the moments of an observable are defined in the general form as [5]:

$$R_n\left(E_{cut},\mu\right) = \int_{E_{cut}} \left(V-\mu\right)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}V}\,\mathrm{d}V\ ,$$

where $\frac{\mathrm{d}\Gamma}{\mathrm{d}V}$ is the spectrum in the variable V in the B rest frame, n is the order of moment, μ is the shift from the center of the distribution and E_{cut} is a lower cut on the energy of a light particle produced in the decay (lepton or photon).

Several collaborations (CLEO, DELPHI, BELLE, BABAR, CDF) carried out the corresponding experiments with the semileptonic and inclusive B-decays and determined the partial branching fraction, the second and third central and non-central moments of the lepton energy, the hadron mass (in the semileptonic $B \mapsto X_c l \nu_l$ decays), the photon energy (in the inclusive decay $B \mapsto X_s \gamma$) for the different values of the cuts on the lepton or photon energy.

Having at hand these moments, as the next step we need to process the data and extract from them values of the heavy quark masses and the necessary matrix elements. In general quark masses and coupling constants depend on the scheme. In the literature, one encounters several such schemes: the so-called 1S, PS, pole, $\overline{MS}$ and the kinetic schemes, expanding in $\frac{1}{m_c}$, not expanding in $\frac{1}{m_c}$ and using $\overline{m_c}\,(m_b)$ (see later), and not expanding in $\frac{1}{m_c}$ and using the kinetic scheme for both m_b and m_c. Data is analyzed by performing fits to the underlying theory. The fits in the various schemes are obtained by the minimization of the χ^2 function with several free parameters (these parameters present the desired values, their number is different in different schemes) [6]:

$$\chi^2 = \sum_{i,j} \left(\langle X \rangle_i^{meas} - \langle X \rangle_i^{kin} \right) cov_{ij}^{-1} \left(\langle X \rangle_j^{meas} - \langle X \rangle_j^{kin} \right).$$

Here $\langle X \rangle_j^{meas}$ are the measured moments, $\langle X \rangle_j^{kin}$ are the corresponding kinetic scheme predictions that depend on these free parameters. The covariance matrix is the sum of the experimental and theoretical error matrices. The minimum of this function in the space of the free parameters corresponds to the physical quantities of interest.

The main difference between two types of fit schemes consists in the use or not of the $\frac{1}{m_c}$ expansion. If we consider $B \mapsto X_c$ transition, two heavy quarks emerge: the bottom and the charm. One can treat the c-quark as a heavy quark. This allows one to compute the D^* meson masses as an expansion in powers of $\frac{\Lambda_{QCD}}{m_c}$. The observed D^* masses can be used to determine m_c. Since the computations are performed to $\frac{\Lambda_{QCD}^3}{m_c^3}$, this introduces errors of the fractional order $\frac{\Lambda_{QCD}^4}{m_c^4}$ in m_c, which gives the fractional errors of order $\frac{\Lambda_{QCD}^4}{m_b^2 m_c^2}$ in the inclusive B decay rates, since the charm mass effects first enter at order $\frac{m_c^2}{m_b^2}$. Free parameters in this case are $|V_{cb}|$, m_b, m_c, $\lambda_1 \equiv -\mu_\pi^2(B^*)$, $\lambda_2 \equiv -\mu_G^2(B^*)$, $\rho_{1,2}$ and τ_{1-4}. Where τ_{1-4} are the matrix elements of the time-ordered products of several operators. Their explicit expressions are quite involved and are given in Ref. [12].

An alternative approach is to avoid using the $\frac{1}{m_c}$ expansion for the charm quark, since it introduces the $\frac{\Lambda_{QCD}}{m_c}$ corrections, which are larger than the $\frac{\Lambda_{QCD}}{m_b}$ corrections. Moreover, this approach allows one to eliminate the poorly known non-local correlators [7]. With this procedure, one has in addition to seven free parameters: $|V_{cb}|$, two parameters of order the quark mass $m_{b,c}$, two parameters of order Λ_{QCD}^2: λ_1, λ_2, and two parameters of order Λ_{QCD}^3: ρ_1, ρ_2.

Also, the fit schemes differ due to the definition of the quark mass. We can define the quark mass through the pole of the corresponding full QCD-propagator, through relation with the meson masses, and we can use the formalism of the renormalization group to normalize the quark mass at a suitable scale. Each of these schemes has its own virtues and shortcomings. But this question deserves special consideration and we will not take up this issue in this paper (see Ref. [7-9]).

All collaborations obtained similar aggregate results, and here we produce, as an example, the results of the BELLE collaboration in the kinetic scheme [8]:

$$|V_{cb}| = (41.93 \pm 0.65 \pm 0.07 \pm 0.63) \cdot 10^{-3}$$
$$B_{cl\nu} = (10.590 \pm 0.164 \pm 0.006)\,\%$$
$$m_b = (4.564 \pm 0.076 \pm 0.003)\,GeV$$
$$m_c = (1.105 \pm 0.116 + 0.005)\,GeV$$
$$\mu_\pi^2 = (0.557 \pm 0.091 \pm 0.013)\,GeV^2$$
$$\mu_G^2 = (0.358 \pm 0.060 \pm 0.003)\,GeV^2$$
$$\tilde{\rho}_D^3 = (0.162 \pm 0.053 \pm 0.008)\,GeV^3$$
$$\tilde{\rho}_{LS}^3 = (-0.174 \pm 0.098 \pm 0.003)\,GeV^3$$

The corresponding graphical example of the analysis by the BELLE collaboration is presented in Fig.1 (Ref. [3]).

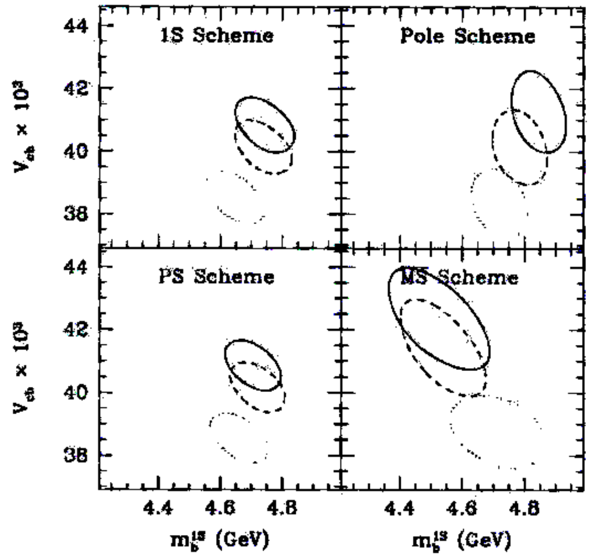

FIG. 1: The 1σ error ellipse in the m_b^{1S} vs. $|V_{cb}|$ plane, using different mass schemes for the fit. For each scheme we show the contours obtained at tree level (dotted red curves), at order ϵ (dashed blue curves), and at order ϵ_{BLM}^2 (solid black curves).

Lattice calculations

Another way to determine the coefficients in the heavy quark expansion is their numerical calculations within the technique of the lattice QCD. It is an important tool for the analysis of the low-energy regime of the QCD, where the coupling constant is big and we can not use the perturbation approach. Because the limitation of computational resources exists, we need to resort to several approximations, such as finite lattice spacing, finite total spatial size of lattice, not very small quark mass, etc.

One of the most useful approximations is the quenched approximation, in which the vacuum polarization by gluon and the quark-antiquark pair is not considered. It reduces the computational complexity by several orders of magnitude, but with it one also introduces sources of uncontrolled uncertainties. The lattice counterpart of the system, that contains the heavy quark, is described by the corresponding discrete action, a particular form of which is [4]

$$S_{LQCD} = \sum_{x,y} Q^+(x)\left(\delta_{x,y} - K_Q(x,y)\right) Q(y).$$

The kernel that describes the time evolution of heavy quark is given by

$$K_Q(x,y) \equiv \left(1-\frac{aH_0}{2n}\right)^n_{t+1}\left(1-\frac{a\delta H}{2}\right)_{t+1}\delta_4^{(-)}U_4^+(t)\left(1-\frac{a\delta H}{2}\right)_t\left(1-\frac{aH_0}{2n}\right)^n_t,$$

where the index to label the spatial coordinate is suppressed. The operator $\delta_4^{(-)}$ is defined as $\delta_4^{(-)} \equiv \delta_{x_4-1,y_4}\delta_{\vec{x},\vec{y}}$, and

$$H_0 \equiv -\frac{\Delta^{(2)}}{2m_Q},$$

$$\delta H \equiv -c_B\frac{g}{2m_Q}\vec{\sigma}\cdot\vec{B}\cdot$$

$\Delta^{(2)}$ is a lattice covariant Laplacian

$$\Delta^{(2)}Q(x) = \sum_{i=1}^{3}\Delta_i^{(2)}Q(x) =$$

$$= \sum_{i=1}^{3}\left[U_i(x)Q\left(x+\hat{\imath}\right)+U_i^+\left(x-\hat{\imath}\right)Q\left(x-\hat{\imath}\right)-2Q(x)\right],$$

and $\vec{B}$ is the chromo-magnetic field. The parameter n in the evolution kernel is a positive integer that is necessary to stabilize unphysical momentum modes. In the limit of vanishing lattice spacing the discrete action reduces to the standard continuum action:

$$L_{QCD}^{cont} = Q^+\left[D_0+\frac{\vec{D}^2}{2M}+g\frac{\vec{\sigma}\cdot\vec{B}}{2M}\right]Q.$$

The parameters m_Q, c_B appearing in the lattice action have to be matched onto their continuum counterparts using perturbation theory. The matching of the heavy quark mass may be done through the calculation of the hadron masses. They can be obtained from study of the asymptotic behavior of the two-point function:

$$C\left(J,t\right) = \sum_{\vec{x}}\left\langle J\left(\vec{x},t\right)J^{(S)+}\left(\vec{0},0\right)\right\rangle \to e^{-E_{sim}t},$$

for sufficiently large time separation. With the lattice discrete action we can obtain the binding energy E_{sim}. The interpolating operator J is chosen such that it shares the same quantum numbers with the hadron of interest. For instead:

$$B = \overline{d}\,\gamma_4\gamma_5\,h,$$

$$B^* = \overline{d}\,\gamma_i\,h,$$

$$\Lambda_b\left(s_z=+\frac{1}{2}\right) = \varepsilon_{abc}\left(u^a\,C\gamma_5\,d^b\right)h_\uparrow^c,$$

$$\Sigma_b\left(s_z=-\frac{1}{2}\right) = -\sqrt{\frac{2}{3}}\,\varepsilon_{abc}\left(u^a\,C\frac{\gamma_1+i\gamma_2}{2}\,d^b\right)h_\uparrow^c+\frac{1}{\sqrt{3}}\varepsilon_{abc}\left(u_a\,C\gamma_3\,d^b\right)h_\downarrow^c.$$

The smeared operator $J^{(S)}$ is used at the source to enhance the overlap with the ground state. It is suitable to define this operator such that the heavy quark field is smeared according to an exponential form $e^{-a\cdot r^b}$ around the light quark field fixed at the origin. r is a distance from the origin, and the parameters a and b must be measured to specify the wave function of the interesting hadron. However, at present, the parameter c_B is not available with the one-loop level of accuracy, and we may use only the tree-level value $c_B = 1$. But final predictions for the values of the physical observables may be given in the static limit, which does not require the parameter c_B.

In one of the recent analyses the quenched approximation allows one to obtain the following results [4, 10]:

$$|V_{cb}| = (40.8 \pm 0.9) \times 10^{-3},$$
$$m_b = (4.74 \pm 0.10)\, GeV,$$
$$\overline{\Lambda} = 0.68^{+0.02}_{0.12}\, GeV,$$
$$\lambda_1 \equiv -\mu_\pi^2(B^*) = -(0.45 \pm 0.12)\, GeV^2.$$

The graphical example of the lattice data is given in Fig.2 and Fig.3 (Ref. [4]).

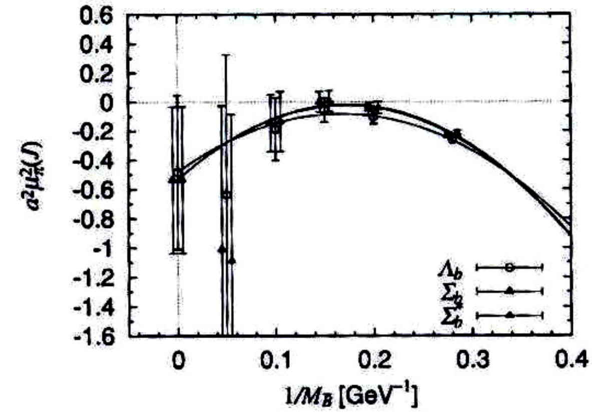

FIG. 2 : Matrix element μ_π^2 for the Λ_b, Σ_b and Σ_b^* baryons as a function of $1/M_{\bar{B}}$.

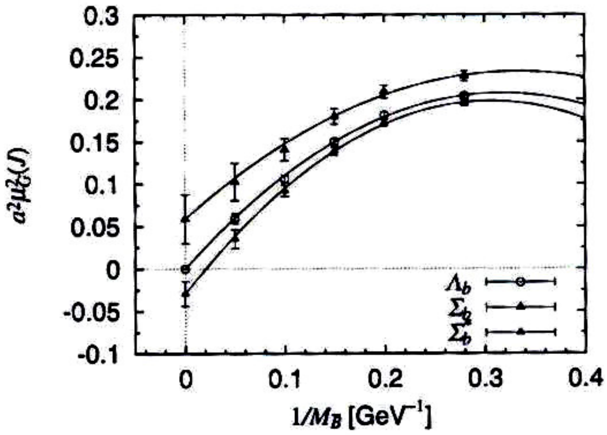

FIG. 3 : Matrix element μ_G^2 for the Λ_b, Σ_b and Σ_b^* baryons as a function of $1/M_{\bar{B}}$.

The current computational power has made it possible to carry out the simulations beyond the quenched approximation for many important quantities.

Since dynamical fermion simulations involve many inversions of the fermion matrix, it is harder to simulate the light quarks at their physical mass values. Therefore, an extrapolation in the light quark mass from feasible quark masses to the physical masses is necessary (the chiral extrapolation). These simulations are simplified by using the staggered fermions. With them there is an exact U(1) chiral symmetry and the massless limit is fixed. Of course, this method has several shortcomings. This question is considered in details in Ref. [11].

Within this approach following results were obtained [11]:

strong coupling constant $\alpha_s(M_Z) = 0.1175 \pm 0.0015$,

the strange quark mass $m_s(2\,GeV) = 78 \pm 10\,MeV$,
the bottom quark mass $\overline{m_b}\left(\overline{m_b}\right) = 4.21 \pm 0.07\,GeV$,
the form factor of semileptonic kaon decay $f_+(0) = 0.961 \pm 0.008$,
the kaon B parameter $B_K(2\,GeV) = 0.58 \pm 0.04$,
the form factors of semileptonic D meson decays:
$$f_+^{D \mapsto \pi} = 0.64 \pm 3, \qquad f_+^{D \mapsto K} = 0.73 \pm 3 \cdot$$

They are in reasonable agreement with the phenomenological values and their quantities.

Summary

In conclusion, the following should be mentioned. Use of the analytical methods of the heavy quarks effective theory allows one to obtain the several important parameters of the Standard Model such as $\left|V_{cb}\right|$, $\left|V_{ub}\right|$, m_c, m_b and some of the hadronic matrix elements. These methods have allowed to improve the precision in the knowledge of several fundamental parameters in the Nature.

At present, a number of hadronic matrix elements are determined for the moment analysis of the semileptonic and radiative B-decays. These matrix elements can eventually be calculated on the lattice; current lattice calculations are very promising but not yet precise enough.

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