A lower bound for BR($\mu \to e\gamma$) given BR($\tau \to e\gamma$) BR($\tau \to \mu\gamma$)

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Abstract

Neutrino oscillations obviously lead to lepton family number violation. In the case of $l_i \to l_j\gamma$, on which we focus in this work, this contribution is small. But if the theory is supersymmetric and the seesaw mechanism is responsible for generating the neutrino masses, there are additional contributions exceeding the previous ones by orders of magnitude. Esp. the BR($\mu \to e\gamma$) has been measured well and is already constraining this scenario, but it is of course possible that just this transition is suppressed somehow. So the goal of this work is to figure out a lower bound of the BR($\mu \to e\gamma$) given BR($\tau \to \mu\gamma$) and BR($\tau \to e\gamma$) in complete generality supposing only that no miraculous cancellations take place. Thus one can make predictions for the other BR if two BRs have been measured and we get a tool to test the scenario.

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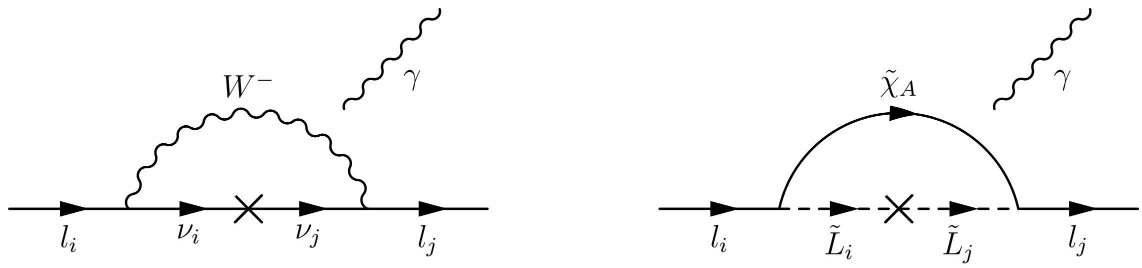

Figure 1: Feynman diagrams contributing to $l_i \to l_j \gamma$. The photon-line can be inserted at each charged line. **(a)** The transition on the left hand side originates from the neutrino-mixing. **(b)** In the right graph $\tilde{\chi}_A$ denotes the neutralinos and charginos and $\tilde{L}_i$ the lefthanded sleptons.

1 Introduction

While in the pure Standard Model the lepton family is (aside from tiny anomaly effects) a conserved quantity, the measured neutrino oscillations already show that it is not (look at fig. 1(a)). If we calculate the BR($l_i \to l_j \gamma$)in an effective low energy theory adding only a left handed neutrino mass term to the Lagrangian of the Standard Model, we get a violation suppressed by $\left(\frac{\text{neutrino mass}}{\text{cut-off mass}}\right)^2$, which yields *e.g.* to BR($\mu \to e\gamma$)$<$ 10^{-50}. This is still far from the present experimental bounds.

On the other hand it is clear that this effective theory has to be extended for higher energies, introducing new degrees of freedom and potentially new sources of lepton flavour violation. Thus there is effort in experiments improving the bounds in this sector. The improvements of the experiments are collected in fig. 2.

One of the currently most favored extensions to include the neutrino masses to the Standard Model is the so called seesaw mechanism, because it not only provides neutrino masses and mixings, but is also able to explain the smallness of the masses. Additionally it is a very natural expansion, since the Lagrangian contains all possible renormalizable terms with the new degrees of freedom of the right handed neutrinos, *i.e.* the kinetic term, the Yukawa coupling as well as a majorana mass term.

Unfortunately the so called hierarchy problem, *i.e.* the dependence of the Higgs mass on the mass of the heaviest particle in the theory, becomes acute: The seesaw model just predicts such a particle, namely the righthanded neutrino, what makes it difficult to explain, why the Higgs mass is so small. Another problem to the Standard Model is coming from astronomy: There has to be some dark matter, which does not originate from any of the known particles. Both flaws are solved in the SUSY, and SUSY furthermore predicts the unification of gauge couplings and seems to make the inclusion of gravity into the theory more feasible. All in all there are good reasons to assume that SUSY exists. The most conservative assumption for a SUSY model is the MSSM, which does not allow for neutrino masses. Fortunately it doesn't spoil any of the benefits of SUSY to combine it with the seesaw, so the theory MSSM + seesaw is viable and it will be the framework of this work.

To be honest there is a further assumption underlying this work. If SUSY was broken in an arbitrary way, this would imply flavour mixing and CP violation of an

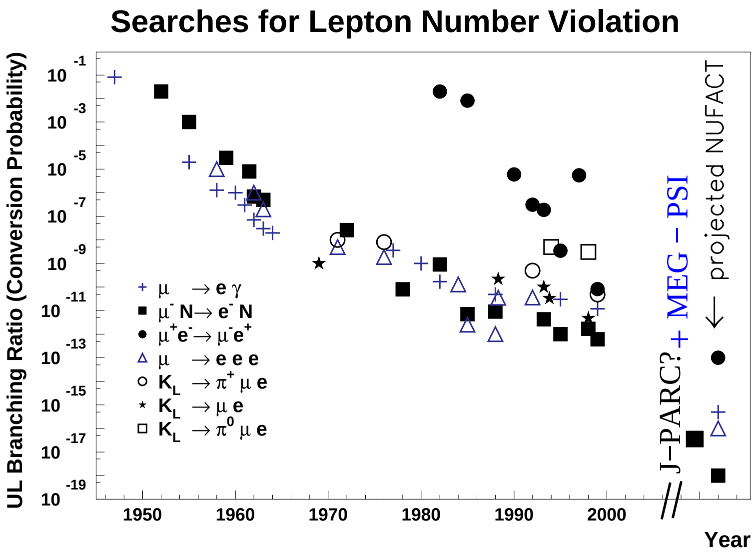

Figure 2: Upper bounds for lepton family violation [1]

BR	bound
$\mu \to e\gamma$	$1.2\,10^{-11}$
$\tau \to \mu\gamma$	$6.8\,10^{-8}$
$\tau \to e\gamma$	$1.1\,10^{-7}$

Figure 3: cf. [3]

order already seriously restricted by experiment. So it is expected that the SUSY breaking mechanism is flavour-blind, *i.e.* all of the squark and slepton squared mass matrices are proportional to the unit matrix, the trilinear couplings are proportional to the corresponding Yukawa couplings and all of the soft breaking terms are real in a certain basis (see *e.g.*[2]). Of course all of these restrictions hold only at the mass scale M_X, where SUSY breaking is imposed, and are altered by the renormalization group equations (RG). In the next chapter we will clarify all this in a more formal way.

Now we are ready to jump into the topic. The main idea is the following:

Because of SUSY there are also the partners of the leptons called sleptons. They can induce flavour violation via fig. 1(b) analogue to 1(a). On the one hand the neutrino mixing amplitude is proportional to $\mathbf{Y}_\nu{}^T \mathcal{M}^{-1} \mathbf{Y}_\nu$, where $\mathbf{Y}_\nu$ is the the neutrino Yukawa coupling and $\mathcal{M}$ is the majorana mass matrix, which can be chosen diagonal (cf. eq. (6)). Because we know that the mixing takes place between all families there are at least two different off-diagonal non-vanishing entries in $\mathbf{Y}_\nu{}^T \mathcal{M}^{-1} \mathbf{Y}_\nu$. The slepton mixing instead is proportional to $\mathbf{m}^2_\mathbf{L}$, which is constrained to be diagonal at M_X, but gets off-diagonal terms due to the RGs (cf. chapter 3) (t denotes the logarithm of different mass scales):

$$\frac{d(\mathbf{m}^2_\mathbf{L})_{ij}}{dt} \propto \frac{1}{16\pi^2} \left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu\right)_{ij} \qquad i \neq j \tag{1}$$

Without miraculous cancellations we expect that also at least two entries of $\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu$ are non vanishing.

This flavour violating process is of course not suppressed by the smallness of the neutrino masses, so the experimental bounds are already interesting. As you can see in fig. 3 the bound on BR($\mu \to e\gamma$) is much more restricting than on BR($\tau \to \mu\gamma$) and BR($\tau \to e\gamma$). So the most conservative assumption to avoid these restrictions is to have

$$\left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu(M_{\mathrm{X}})\right)_{12} = \left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu(M_{\mathrm{X}})\right)_{21} \approx 0 \tag{2}$$

Of course there is no known reason for that. But if there is a reason, then at the M_{X} scale, where new physics enters.

The majorana scale M naturally should be much larger than the electroweak scale but below M_{X}. Because loops containing the righthanded neutrinos are suppressed below M, the parts of the RGs prop. to M and $\mathbf{Y}_\nu$ contribute only between M and M_{X}. In this area the following RG applies (cf. chapter 3):

$$\frac{d\left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu\right)_{ij}}{dt} \propto \left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu\right)^2_{ij} \qquad i \neq j \tag{3}$$

As you can see from the latter formula together with eq. (2) as a limiting case, we expect for every scale below M_{X} an inequality like

$$\left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu\right)_{12} > \text{const.} \left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu\right)_{13} \left(\mathbf{Y}_\nu{}^\dagger \mathbf{Y}_\nu\right)_{32} \tag{4}$$

With eq. (1) this can be written in terms of branching ratios:

$$\mathrm{BR}(\mu \to e\gamma) > \text{const.}\ \mathrm{BR}(\tau \to e\gamma)\mathrm{BR}(\tau \to \mu\gamma) \tag{5}$$

Where the constant is to be estimated in this paper.

2 Choice of the basis, MI

Before we start lets first have a look to the superpotential in the SUSY + seesaw theory:

$$W = W_{non-leptonic} - e^{c\,T}_{Ri} \mathbf{Y}_{\mathbf{e}ij} L_j \cdot H_d - \frac{1}{2}\nu^{c\,T}_{Ri} \mathcal{M}_{ij} \nu^c_{Rj} + \nu^{c\,T}_{Ri} \mathbf{Y}_{\nu ij} L_j \cdot H_u \tag{6}$$

The matrix $\mathbf{Y_e}$ is arbitrary, but can be taken diagonal with positive, real entries if one chooses the appropriate basis of e_R and L. $\mathcal{M}$ is the majorana mass matrix of order M. Because it has to be symmetric, it can also be taken for diagonal in a fixed basis of ν_R.

As a little supplement to the introduction lets first take this basis at the majorana scale and write down the effective superpotential, where the ν_R are integrated out:

$$W_{\mathit{eff}} = W_{non-leptonic} - e^{c\,T}_R \mathbf{Y_e} L \cdot H_d + \frac{1}{2}\left(\mathbf{Y}_\nu L \cdot H_u\right)^T \mathcal{M}^{-1} \left(\mathbf{Y}_\nu L \cdot H_u\right) \tag{7}$$

Because $\mathcal{M}$ will not run below the majorana scale any more (cf. chapter 3), this relation is right also at low energies.

If we use this basis instead at M_{X} the only non diagonal matrices in the flavour sector are $\mathbf{Y}_\nu$ and the corresponding trilinears $\mathbf{A}_\nu$. This will be our starting point for the running of all of the parameters.

At low energies we will make use of the so called mass insertion approximation [4]. The basis is here chosen such that the charged lepton propagators are flavour diagonal and the gauge couplings too. Then the slepton mass matrices $\mathbf{m^2}$ and the trilinear terms $\mathbf{A}$ can't be diagonal too, and the flavour mixing can be expressed in a series of insertions of these. Just in order to clear the notation a part of the SUSY soft breaking Lagrangian follows:

$$-\mathcal{L}_{\rm soft} = \left(\mathbf{m_L^2}\right)_{ij} \bar{L}_i L_j \;+\; \left(\mathbf{m_e^2}\right)_{ij} \bar{e}_{Ri} e_{Rj} \;+\; \left(\mathbf{A_e}_{ij} e^c_{Ri} H_1 L_j + \text{h.c.}\right) \;+\; \text{etc.} \tag{8}$$

3 One loop RG-evolution

The goal is of course to have a prediction at low energies, *i.e.* at the electroweak scale and below. The one-loop RGs are *e.g.* given in the appendix of [5]. Because we are engaged in lepton flavour mixing, only the RGs, which drive off-diagonal terms are of interest. Those (namely $\mathbf{Y}_\nu$, $\mathbf{A}_\nu$) contribute only between $M_{\rm X}$ and M. In the leading-log approximation we find for the parameters of low energy:

$$\begin{aligned}
(\mathbf{Y_e})_{ij} &\simeq \frac{-1}{16\pi^2}\mathbf{Y_e}(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)_{ij}\log\frac{M_X}{M} \\
\left(\mathbf{m_L^2}\right)_{ij} &\simeq \frac{-1}{8\pi^2}(\mathbf{m_L^2} + \mathbf{m}_\nu^2 + m_{H_u}^2 + A_\nu^2)(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)_{ij}\log\frac{M_X}{M} \\
\left(\mathbf{m_e^2}\right)_{ij} &\simeq 0 \\
(\mathbf{A_e})_{ij} &\simeq \frac{-1}{16\pi^2}(A_\nu + A_e)\mathbf{Y_e}(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)_{ij}\log\frac{M_X}{M}
\end{aligned} \tag{9}$$

where $i \neq j$, $\mathbf{A}_\nu = A_\nu \mathbf{Y}_\nu$, $\mathbf{A_e} = A_e \mathbf{Y_e}$ and all parameters on the right hand sides are meant to be at the $M_{\rm X}$ scale, whereas on the left hand side they're meant to be at low energies. The off-diagonal part of $\mathbf{Y_e}$ leads to a redefinition of the basis, corresponding to additional off-diagonal terms in the soft terms. But the effect is small esp. for the case $i, j = 1, 2$, where our study is most susceptible.

$\mathbf{A_e}$ is suppressed by the lepton mass, but because a chirality flip is necessary to emit a on-shell photon anyway, it could also be relevant. Nevertheless we will not consider this case further in this work.

In principle the running of $\mathbf{A}_\nu$ is just as relevant as the running of $\mathbf{Y}_\nu$ for the mixing within the off-diagonal entries. But as the RGs have the same form and sign we will use only the RG of $\mathbf{Y}_\nu$ for the sake of simplicity.

$$\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu \;\simeq\; \left(\text{const.}\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu + \frac{-3}{8\pi^2}(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)^2\right)\log\frac{M_X}{M} \tag{10}$$

4 BR($\mu \to e\gamma$) in the one MI approximation under the most conservative assumption

As we have already explained in the introduction, the measurement of BR($\mu \to e\gamma$) is most restricting. So in order to get this BR minimal we make the most conservative assumption for $\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu$, namely eq. (2) as a limiting case. We will see soon, that this leads to the desired small BR($\mu \to e\gamma$).

Due to the RG running a 21-term and an analogue 12-term is induced at M (cf. eq. (10)):

$$\left(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu\right)_{21} \simeq \frac{-3}{8\pi^2}(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)_{23}(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)_{31}\log\frac{M_X}{M} \tag{11}$$

Inserting this into eq. (9) and adding a factor $\frac{1}{2}$, because the differential RG for $\mathbf{m_L^2}$ is now linear in the logarithm, we get:

$$\left(\mathbf{m_L^2}\right)_{21} \simeq \frac{3}{2(8\pi^2)^2}(\mathbf{m_L^2}+\mathbf{m}_\nu^2+m_{H_u}^2+A_\nu^2)(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)_{23}(\mathbf{Y}_\nu{}^+\mathbf{Y}_\nu)_{31}\left(\log\frac{M_X}{M}\right)^2 \tag{12}$$

This is already of order $\frac{1}{(16\pi^2)^2}$ and thus comparable with a two loop calculation. We will show in chapter 5, that the two loop correction doesn't dominate. The 31- and 32 entry remain unchanged in leading order.

In the MI language the BR($l_i \to l_j\gamma$) is a result of a transition like in fig. 1(b). It is given by the approximate expression

$$\mathrm{BR}(l_i \to l_j\gamma) \sim \frac{\alpha^3}{G_F^2}\frac{|\mathbf{m}^2_{\mathbf{L}ij}|^2}{m_S^8}\tan^2\beta\ \mathrm{BR}(l_i \to l_j\nu_i\bar{\nu}_j) \tag{13}$$

where α is the electromagnetic fine-structure constant, G_F is the Fermi coupling constant, m_S is a typical SUSY scale and $\tan\beta$ is the ratio of the VEVs of H_u and H_d. This formula as well as the longish exact one are referred to in the appendix.

Putting together eqs. (9), (12) and the latter one, we can substitute the $\mathbf{Y}_\nu$. Assuming that all soft masses are about m_S and remembering that we had considered a limiting case, the desired result is:

$$\mathrm{BR}(\mu \to e\gamma) \gtrsim \frac{9}{64}\frac{m_S^4 G_F^2}{\alpha^3\tan^2\beta}\frac{\mathrm{BR}(\tau \to \mu\gamma)\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\tau\to\mu\nu_\tau\bar{\nu}_\mu)\mathrm{BR}(\tau\to e\nu_\tau\bar{\nu}_e)} \tag{14}$$

For future convenience lets call this factor $C = \frac{9}{64} = 0.14$. It should be noticed that the BR is independent of the mass scales M_X and M.

Putting in some numbers, *e.g.* $m_S = 250$ GeV and $\tan\beta = 3$ this reduces to BR($\mu \to e\gamma$) $\gtrsim 6 \cdot 10^5$ BR($\tau \to \mu\gamma$) BR($\tau \to e\gamma$). Comparing this with fig. 3 this result together with the experimental bound on BR($\mu \to e\gamma$) would forbid that both BR($\tau \to \mu\gamma$) and BR($\tau \to e\gamma$) are just at the corner.

If we want to use the correct formula (cf. the appendix) we have to insert some more sophisticated numbers, because there differences of soft masses appear in the denominator. You can look them up in fig. 4.

Trying now to solve this one MI approximation in one loop leading log approximation one gets a quadratic polynomial in $\tan\beta$. Because the factor of the quadratic term is about 2 orders of magnitude bigger than the others, we neglect them. The result is exactly of the form of eq. (13), but with an overall factor of ≈ 0.8. We are again able to easily substitute the $\mathbf{Y}_\nu$ and get $C = 0.14/0.8 = 0.18$, a slightly different result.

M_1	M_2	$\mathbf{m_L^2}$	$\mathbf{m_R^2}$	μ	any other SUSY mass	α_1	α_2
$0.4m_S$	$0.8m_S$	$1.54m_S^2$	$1.15m_S^2$	$2m_S$	m_S	$0.5\alpha_2$	$\alpha/0.23$

Figure 4: Choice of the values in the correct formulae of the one MI

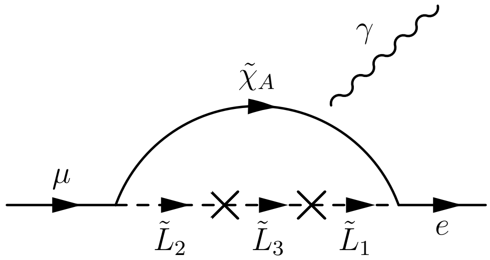

Figure 5: Transition $\mu \to e\gamma$ via two mass insertions

5 Corrections from 2 MI and two loop RGs

The $\tilde{L}_2 \to \tilde{L}_1$ vertex is in our study only emerging from the RGs. So one could ask, if a two mass insertion (see fig. 5) leads to a comparable ore even much bigger BR. The formula for the 2MI you can also find following the explanations in the appendix. Apart from the even longer expressions the procedure is the same, but it yields to:

$$\mathrm{BR}(\mu \to e\gamma) \approx 0.16 \frac{\alpha^3}{G_F^2} \frac{|\mathbf{m^2_{L}}_{23}\mathbf{m^2_{L}}_{31}|^2}{m_S^{12}} \tan^2\beta \tag{15}$$

Or in other words, using again eqs. (9), (12) we can again bring it in the form of eq. (13):

$$\mathrm{BR}(\mu \to e\gamma) \approx 0.16 \frac{4(\mathbf{m_L^2} + \mathbf{m_\nu^2} + m_{H_u}^2 + A_\nu^2)^2}{9m_S^4} \frac{\alpha^3}{G_F^2} \frac{|\mathbf{m^2_{L}}_{21}|^2}{m_S^8} \tan^2\beta \tag{16}$$

Inserting that all masses are about m_S and using the 1 MI approximation for the other BRs we get $C_{2\mathrm{MI}} \approx 0.14 \cdot 0.16 \cdot \frac{64}{9}/0.8^2 = 0.25$. This is of the same order than in the 1 MI case.

The last thing we should check, is if the effect due to the second order RG of $(\mathbf{m_L^2})_{21}$ is comparable to the first order RG inserted the first order RG of $\mathbf{Y}_\nu$. Rules to get the second order RGs for our SUSY + seesaw case can be found in [7]. Here we give only the part driving a nonzero 21 entry:

$$\begin{aligned} \frac{d(\mathbf{m_L^2})}{dt} &= 1.\text{order} + \frac{1}{(16\pi^2)^2} \times \\ &\Big[-({\mathbf{Y}_\nu}^+\mathbf{Y}_\nu)^2(8\mathbf{m_L^2} + 8\mathbf{m_\nu^2} + 8m_{H_u}^2 + 16A_\nu^2) + \\ &+\text{terms} \propto \text{g's} + \text{something} \cdot {\mathbf{Y}_\nu}^+\mathbf{Y}_\nu + \text{diagonal terms}\Big] \end{aligned} \tag{17}$$

This can be compared more easily with eq. (12) in the form

$$\begin{aligned} \left(\mathbf{m_L^2}\right)_{21,2.order} &= \frac{4}{2(8\pi^2)^2}(\mathbf{m_L^2} + \mathbf{m_\nu^2} + m_{H_u}^2 + 2A_\nu^2)({\mathbf{Y}_\nu}^+\mathbf{Y}_\nu)_{23}({\mathbf{Y}_\nu}^+\mathbf{Y}_\nu)_{31} \log\frac{M_X}{M} \\ &\approx \frac{5}{3}\left(\log\frac{M_\mathrm{X}}{M}\right)^{-1}\left(\mathbf{m_L^2}\right)_{21,1.order} \end{aligned} \tag{18}$$

what is also a remarkable result and cannot be neglected if M is not much smaller than M_{X}.

Appendix

The $\mathrm{BR}(l_i \to l_j\gamma)$ has been studied in the MI approximation in generality in [6]. In order to clarify our calculation, we will point on the used formulas:

- In chapter 5 of [6] there is our eq. (13).
- There is also the correct formula

$$\mathrm{BR}(l_i \to l_j\gamma) = \frac{48\pi^3\alpha}{G_F^2}|A_L^{ij}|^2\mathrm{BR}(l_i \to l_j\nu_i\bar{\nu}_j) \tag{19}$$

  We consider only A_L, because the flavour violation inside the loop takes in our considerations only place between lefthanded sfermions and so the chirality flip has to be in the external legs. Thus it has to be at the incoming external leg, because otherwise the amplitude would be smaller by the ratio of the masses.

- To get the 1MI we inserted in the formula above $\left(A_L^{ij}\right)_{SU(2)}$ and $\left(A_L^{ij}\right)_{U(1)}$ from chapter 6.1, where Δ_{LL} corresponds to $\mathbf{m_L^2}$.
- For the 2MI instead the formulas are given in chapter 6.4. Here the relevant ones are $(A_{L2}^{21})_{SU(2)}$ and $(A_{L2}^{21})_{U(1)}$

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