A first attempt to compare the results of Monte Carlo generators for the Drell-Yan process to the NLL QCD predictions

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Abstract

Theoretical predictions for the functional form of the cross-section of Z boson production in the Drell-Yan process show that the result can be expressed in a simple analytic form. The fourier transform of this theoretical result is particularly concise form and is simply the product of a Sudakov factor and parton distribution functions. In this project PYTHIA was used to extract Monte Carlo predictions for the cross-section of the $p + \overline{p} \to Z^0 + X$ process and a comparison with the theoretical calculations was undertaken using the 1-loop expression for the strong coupling constant. The two were found to approximately agree on the shape, with the absolute magnitudes for the cross-section differing by a factor of 5. More meaningful results could not be reached due to time constraints and therefore a list of ideas for future work is presented.

1 Introduction

In 1970 Sidney D. Drell and Tung-Mow Yan studied the annihiliation of a quark and an antiquark into a virtual photon which then proceeds to creates a real lepton-antilepton pair [1]. The original study was within the framework of the parton model, treating the partons within the proton as free particles. While partons have in the meantime been replaced with quarks and gluons (although the term *parton* is now used to refer to both), the process is still called the *Drell-Yan process*. Strictly speaking the Drell-Yan process refers to the creation of a virtual photon, in the present context we will also use it to refer to the process whereby the incoming quark-antiquark pair annihiliate to create a Z^0 boson.

The Drell-Yan process is interesting for several reason. To begin with, its predictions lent credence to the parton model which at the time was not universally accepted. Additionally, the fact that on-shell photons are massless implies that no resonances would be observed in the spectrum of the resultant pair's rest mass. The observation of such observations led to the discovery of the Z boson at the SPS collider in 1983 [2] through the study of the process

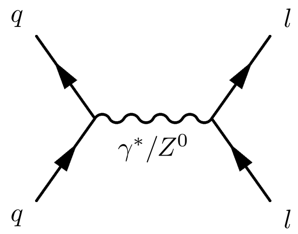

Figure 1: Feynman diagram of the Drell-Yan process.

$$p + \overline{p} \rightarrow Z^0 + X$$

The Z boson then proceeds to decay into a pair of leptons. The spectrum of the rest mass of the resultant dilepton pair was indeed sharply peaked at $M = (95.2 \pm 2.5) GeV/c^2$, vindicating the predictions of the Glashow-Salam-Weinberg theory of electroweak interactions.

Apart from its historical interest, the Drell-Yan process remains of interest because the details of the interaction depend in a direct manner on the parton distribution functions (PDFs). Particles like the proton are not fundamental; in this case, the proton consists of an (uud) quark arrangement. These are referred to as the *valence* quarks to distinguish them from the *sea* quarks that occur in $q\overline{q}$ pairs and, along with the gluons, account for most of the proton's rest mass. PDFs describe the probability for finding a quark with a given dynamical state within the nucleus. Difficulties in the application of QCD to bound states forbid the direct theoretical evaluation of PDFs. Predicting any process whose description requires the quark model will necessarily require knowledge of PDFs. Therefore, to allow any progress to be made, PDFs must be inferred from experiments.

Just as the Drell-Yan process led to the discovery of the Z^0 boson, it remains to be seen whether it will in the future lead to the discovery of heavier neutral gauge bosons. These Z' bosons are not part of the Standard Model and have never been observed experimentally[1]. Nevertheless they are an important aspect of several more ambitious theories (e.g. string theories in which the Stückelberg term arises, Kaluza-Klein theories etc.).

Finally, the important aspect of the Drell-Yan process is its prevalence in LHC physics. In the design and optimisation of the experiments, Monte-Carlo event generators such as PYTHIA are used to simulate the underlying physics. One must therefore establish that these generators faithfully reproduce the current theoretical knowledge of the underlying QCD processes. Determination of the properties of the Z boson in the Drell-Yan process is important because the calibration of LHC experiments will be done partly by Z boson measurements [5]. At the LHC parameters one can expect production rates of a few

[1] In January 2007 the CDF collaboration announced [3] a possible hint of a Z' at 240 MeV. This by no means constituted a discovery - these can only be claimed with 5σ departures from the expected distribution while the CDF result only exhibited a 3.8 σ departure. The D0 collaboration followed up with a larger data set and a preliminary analysis [4] shows that that there appears to be no disagreement with the standard model. So it's not that people haven't been looking!

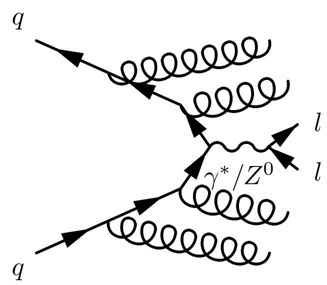

Figure 2: Initial state gluon radiation in the Drell-Yan process. In reality there will also be photon emission and similar QED/QCD radiation in the final state. The theory however only takes into account the initial state QCD radiation.

Hz, allowing the Z bosons to be used as luminosity monitors [5]. Additionally, knowledge of the dileptron spectrum will allow a more accurate determination of the W/Z masses. Precise estimation of the background signal also relies on detailed knowledge of the Drell-Yan dynamics. The purpose of this study is to establish whether such an agreement of theory and Monte-Carlo generators exists.

Assume that two protons collide in the CM frame frame. Let Q denote the 4-momentum transfer between them and Q_T be the component of Q perpendicular to the direction of motion of the pair. As this is an s-channel process it is clear that Q and Q_T are also the corresponding dynamical variables of the intermediate gauge boson. By x we denote the *Bjorken-x* of a particle, given by the equation

$$x = \frac{Q^2}{2p \cdot q}. \tag{1}$$

In reality, the physics underlying the Drell-Yan process is rarely as simple as described above. The quarks are allowed to radiate gluons. This initial-state radiation can be calculated in the high Q_T regime. In this regime the strong force coupling constant $\alpha_s(Q^2)$ is small enough to allow the application of perturbation theory and so, in principle, the result of the interaction can be computed to any desired level of accuracy.

Difficulties in calculating the gluon radiation are encountered in the low Q_T regime where perturbation theory fails. Fortunately, a technique known as *resummation* has been devised which allows theoretical predictions in the low Q_T region to be made. The results will be described in detail in the next section.

Direct comparison with experimental data is not easy. This is because the effects of gluon radiation will be overshadowed by effects that cannot be simply turned off in real life. In practice when a $q\bar{q}$ pair interacts there will be QED radiation in both the initial and final state. Hadronisation [2]of the resultant quarks will also affect the cross-section as will the effects of the interactions between the resultant particles. Furthermore the annihiliation into a Z^0 boson

[2]The arrangement of gluons and quarks into hadrons.

competes with the real Drell-Yan process (i.e. the virtual photon) and the exchange of virtual gluons.

The solution to the difficulties presented above is the simulation of the process via Monte Carlo methods. Monte Carlo methods in particle physics rely on randomly creating events (i.e. sets of particle 4-vectors) and using them to evaluate the integrals which along with the matrix elements, determine the cross section. Using a sufficiently large number of events the underlying process can be accurately modelled. The advantage of this approach is that one is, contrary to conventional experiments, free to set any parameters to any value [3] and, most importantly, isolate the processes of interest. MC methods also sidestep the formidable computational complexity of numerically evaluating these high-dimensional integrals.

2 Overview of theoretical predictions

Progress on the problem of the Drell-Yan cross-section was stimulated by the QCD factorisation theorem which allowed [6] the first predictions about the cross-section [7]. These predictions worked sufficiently well in the $Q_T \sim Q$ regime but proved to be problematic when $Q_T < Q$.

The analytic result for the cross-section in limit where $Q_T \ll Q$ can be written as [6]

$$
\begin{aligned}
\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q_T^2\mathrm{d}y} = \frac{4\pi^3\alpha}{3s}\frac{1}{(2\pi)^2} & \int \mathrm{d}^2\boldsymbol{b}\, e^{i\boldsymbol{Q_T}\cdot\boldsymbol{b}} \\
& \times \sum_{a,b}\int_{x_A}^{1}\frac{\mathrm{d}\eta_a}{\eta_a} f_a(\eta_a,(\frac{C}{b}))^2 \int_{x_B}^{1}\frac{\mathrm{d}\eta_b}{\eta_b} f_b\big(\eta_b,\frac{C^2}{b^2}\big) \\
& \times \exp\Big\{ - C_F \int_{C^2/b^2}^{M_z^2} \frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s(k^2)}{\pi}\big[\ln\frac{{M_z}^2}{k^2} - 1.5\big]\Big\} \\
& \times \sum_{c,d} H^0_{c,d} C_{a,c}\Big(\frac{x_A}{\eta_a},\alpha_s\big(C^2/b^2\big)\Big) C_{b,d}\Big(\frac{x_B}{\eta_b},\alpha_s\big(C^2/b^2\big)\Big). \qquad (2)
\end{aligned}
$$

In the above equation y is the rapidity of the Z^0 boson, b is a fourier parameter, $f_{a,b}(x,Q^2)$ are parton distribution functions, $C_F = 4/3$, $C = e^{2\gamma}$ where $\gamma \simeq 0.5772$, the Euler-Mascheroni constant and $\alpha_{fs} = 1/137.036$ is the familiar fine structure constant. The sums are all over all parton species [4]. The top quark was not taken into account since its high mass implies that top production will have a negligible impact on the analysis. The variables x_A, x_B are, in the $Q_T \to 0$ limit, given by

[3]You could for instance see what the world would look like if the top quark did not exist, how supersymmetric particles fit into the big picture and what the implications of higher dimensions are. Of course most programs necessarily come with some built-in restrictions on what may or may not be changed but that is rarely an issue.

[4]Including gluons!

$$x_A = \frac{M_z}{s} e^{y}, \qquad x_B = \frac{M_z}{s} e^{-y}, \tag{3}$$

where s is the center-of-momentum energy of the system. The mass of the Z^0 boson was taken to be $M_z = 91.187$ GeV [8].

To leading order we can use the following expressions for the functuibs $C_{a,b}$:

$$C_{a,b}(\frac{x_A}{\eta_a}) = \delta_{ab}\delta(1 - \frac{x_A}{\eta_a}).$$

For the H_{ab} functions we can use

$$H_{ab} = \delta_{ab}\frac{1 + [1 - 4|e_q|\sin^2\theta_w]^2}{16\sin^2\theta_w\cos^2\theta_w},$$

with e_q being the charge of the parton and θ_w being the Weinberg angle ($sin^2\theta_w = 0.2319$ [8]).

Using the above relations, formula 2 can therefore be simplified to

$$\begin{aligned}\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q_T^2\mathrm{d}y} &= \frac{1}{(2\pi)^2}\frac{4\pi^3 a_{fs}}{3s}\int \mathrm{d}^2\boldsymbol{b}\, e^{i\boldsymbol{Q_T}\cdot\boldsymbol{b}} \\ &\times \exp\Big\{ - C_F \int_{C^2/b^2}^{M_z^2} \frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s(k^2)}{\pi}\big[\ln\frac{{M_z}^2}{k^2} - 1.5\big]\Big\} \\ &\times \sum_a H^0_{a,a} f_a\big(x_A, C^2/b^2\big) f_a\big(x_B, C^2/b^2\big).\end{aligned}$$

Exploiting the cylindrical symmetry of the integrand we obtain

$$\begin{aligned}\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q_T^2\mathrm{d}y} &= \frac{4\pi^3 a_{fs}}{3s}\frac{1}{2\pi}\int \mathrm{d}b\, b J_0(pb) \\ &\times \exp\Big\{ - C_F \int_{C^2/b^2}^{M_z^2} \frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s(k^2)}{\pi}\big[\ln\frac{{M_z}^2}{k^2} - 1.5\big]\Big\} \\ &\times \sum_a H^0_{a,a} f_a\big(x_A, C^2/b^2\big) f_a\big(x_B, C^2/b^2\big),\end{aligned} \tag{4}$$

where $J_0(x)$ is the usual zero'th order Bessel function.

At this stage no more can be said without knowledge of the form of α_s. Fortunately the integral appearing in the exponential can be done analytically for the 1-loop and 2-loop approximations to the coupling constant.

For the 1-loop case we have

$$\alpha_s^{(1)}(\mu^2) = \frac{4\pi}{\beta_0 \ln\frac{\mu^2}{\Lambda^2_{QCD}}}, \qquad \beta_0 = 11 - \frac{2}{3}n_f, \tag{5}$$

where Λ_{QCD} is a scale variable for QCD determined by experiment (and depending on the PDFs used) and $n_f = 5$, the number of flavours we are considering. Using this form for α_s we obtain

$$\int \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_s^{(1)}(k^2)}{\pi} \big(\ln M_z^2/k^2 - 1.5 \big) = -\frac{4}{\beta_0} \Big\{ \ln k^2 + (1.5 - \ln M_z^2/\Lambda_{QCD}^2) \ln \big| \ln k^2/\Lambda_{QCD}^2 \big| \Big\} + (const) \tag{6}$$

Similarly, we can choose to use the 2-loop form of the coupling constant

$$\alpha_s^{(2)}(\mu^2) = \frac{4\pi}{\beta_0 \ln \mu^2/\Lambda_{QCD}^2} \left(1 - 2\frac{\beta_1}{\beta_0^2} \frac{\log \big| \log \mu^2/\Lambda_{QCD}^2 \big|}{\ln \mu^2/\Lambda_{QCD}^2} \right) \tag{7}$$

$$\beta_1 = 51 - \frac{19}{3} n_f,$$

in which case the analytic form of the integral becomes

$$\begin{aligned} \int \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_s^{(2)}(k^2)}{\pi} \big(\ln M_z^2/k^2 - 1.5 \big) = -\frac{4}{\beta_0} \Bigg\{ & \ln k^2 \\ & + (1.5 - \ln M_z^2/\Lambda_{QCD}^2) \Bigg[\ln \big| \ln k^2/\Lambda_{QCD}^2 \big| \\ & \times (1 + \frac{2\beta_1}{\beta_0^2 \ln k^2/\Lambda_{QCD}^2}) \\ & + \frac{2\beta_1}{\beta_0^2 \ln k^2/\Lambda_{QCD}^2} \Bigg] - \frac{2\beta_1}{\beta_0^2} \ln^2 \ln k^2/\Lambda_{QCD}^2 \Bigg\} \\ & + (const). \end{aligned} \tag{8}$$

As will be discussed in the next section, cuts were applied to the rapidity of the Z^0 boson so that $|y| \leq 2$. Hence

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q_T^2} = \int_{y=-2}^{y=2} \frac{\mathrm{d}^2\sigma}{\mathrm{d}Q_T^2 \mathrm{d}y} dy$$

Returning to equation 2 we can immediately write down an expression for the fourier coefficients of the cross-section $\frac{\mathrm{d}\sigma}{\mathrm{d}Q_T^2}$:

$$\begin{aligned} \Sigma(b) \equiv & \int \mathrm{d}^2 \boldsymbol{Q_T} e^{-\boldsymbol{Q_T}\cdot\boldsymbol{b}} \frac{\mathrm{d}\sigma}{\mathrm{d}Q_T^2} \\ & = \exp \Bigg\{ -C_F \int_{C^2/b^2}^{M_z^2} \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_s(k^2)}{\pi} \big[\ln \frac{{M_z}^2}{k^2} - 1.5 \big] \Bigg\} \\ & \times \int_{y=-2}^{y=2} \mathrm{d}y \sum_a H_{a,a}^0 f_a\Big(x_A, C^2/b^2\Big) f_a\Big(x_B, C^2/b^2\Big). \end{aligned} \tag{9}$$

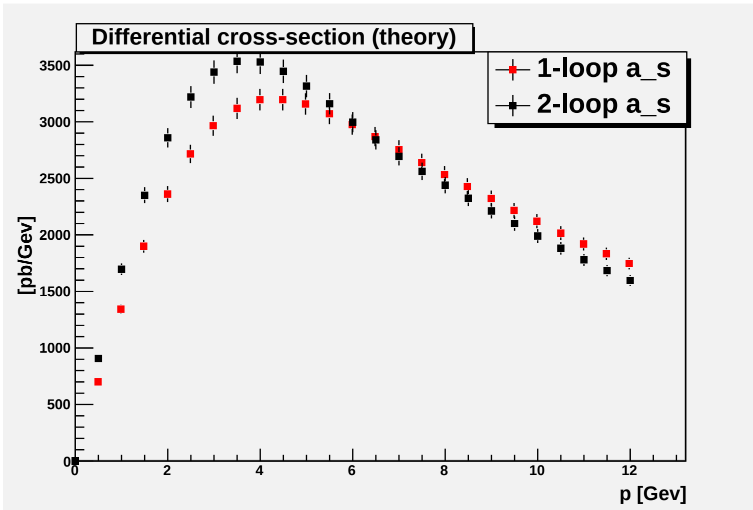

Figure 3: Results of theoretical calculation for the cross-section using the 1-loop and 2-loop approximations to the strong coupling constant.

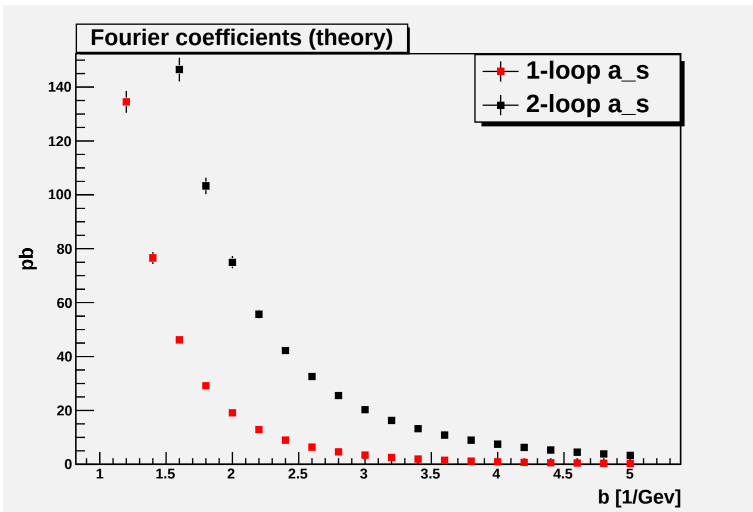

Figure 4: The fourier coefficients of the cross-section plotted against transverse momentum (eq 9).

A practical difficulty in evaluating the theoretical predictions is the divergence of the strong coupling constant when $\mu^2 \rightarrow \Lambda^2_{QCD}$. This divergence is known as the *Landau pole*. The singularity is clearly non-integrable - to avoid it the value of α_s was 'frozen' under $\mu^2 = 1$ GeV^2, so that $\alpha_s(\mu^2) = \alpha_s(1\text{GeV}^2)$, $\mu^2 < 1$ GeV^2.

Results were obtained using both the 1-loop and 2-loop approximation to the strong coupling constant and are shown in figures 3,4.

To obtain $\frac{d\sigma}{dQ_T^2}$ two integrations have to be performed: First over the fourier parameter b and then over the rapidity y. The first integration was performed using the GNU Scientific Libary (GSL) v1.12 [9] with the `gsl_integration_qagiu` routines for improper integration in the semi-infinite interval with an upper bound at infinity and a lower bound dictated by the limits on the momentum variable imposed by the PDF sets. The second integration was based on Gaussian quadrature, using ROOT's `TF1` implementation of the CERNLIB `DGAUSS` routines. Figures 3,4 include error bars that depict the error due to integration but do not take into account the uncertainty of the PDF sets.

3 Monte Carlo results

PYTHIA [10] version 6.420 was used as the Monte Carlo event generator. The Drell-Yan process was simulated at a center of momentum energy of 14 TeV for a pp system. A restriction $90.0 < M_z < 91.4$ GeV was placed in an effort to produce 'on-shell' bosons and to circumvent the possibility of ill-defined cuts in the phase space region (discussed in the next paragraph). Via use of manual process selection (`MSEL`=0) only the $f + \overline{f} \rightarrow \gamma/Z$ process was allowed (`MSUB(1)=1`). Fragmentation and decay were turned off (`MSTJ(1)=0`, `MSTP(111)=0`), as was the final state QCD/QED radiation (`MSTP(71)=0`), multiple interactions (`MSTP(81)=0`). Initial state radiation (`MSTP(61)=1`) was restricted to QCD branchings of quarks and gluons (`MSTJ(41)=0`) in an effort to isolate the gluon emission that we are trying to model.

PYTHIA 's default cuts were used and a post-event selection routine was used to cut the rapidity of the Z boson as $|y| \leq 2$. It should be noted that PYTHIA applies a multitude of cuts by default. The only relevant one is the non-removable — yet configurable — cut on the pseudorapidity $|\eta| < 40$. The pseudorapidity is related to the true rapidity by the formula

$$y = \frac{1}{2} \ln \frac{\sqrt{p_T^2 \cosh^2 \eta + M_z^2} + p_T \sinh \eta}{\sqrt{p_T^2 \cosh^2 \eta + M_z^2} - p_T \sinh \eta} \tag{10}$$

This is a '1-1' mapping $y \leftrightarrow \eta$ if the mass of the Z boson is fixed. Our sharp restriction on both the mass of the Z boson and its rapidity render PYTHIA 's cut irrelevant and simplify the event selection process.

It should be noted that no cut was placed on the transverse momentum of the Z boson. This was done deliberately as the evaluation of the fourier coefficients will require knowledge of the cross-section in the high p_T region; cutting off the integration at a value of p_T where $\frac{\mathrm{d}\sigma}{dp_T^2}$ is not negligible may easily lead to numerical instabilities of the Bessel function that arises during the calculation of the fourier coefficients. Additionally, to this end the results were recorded in a histogram with variable binning that allowed the fine details of the very low p_T region to be discerned while still extending in the high p_t region.

A histogram of the results for the raw count of events as a function of the transverse momentum of the boson is shown in figures 5,6. To obtain the cross-section the histogram is normalized to the bin widths, then to unit overall area and then scaled by $1/\sigma_{tot}$ (figure 7).

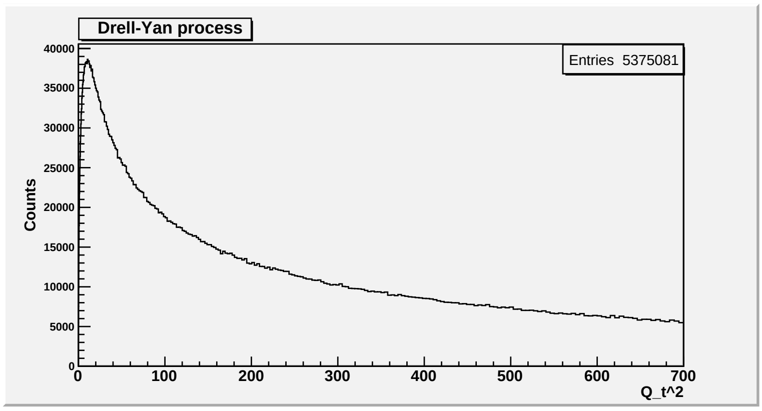

Figure 5: Results of PYTHIA simulation for s=14 TeV.

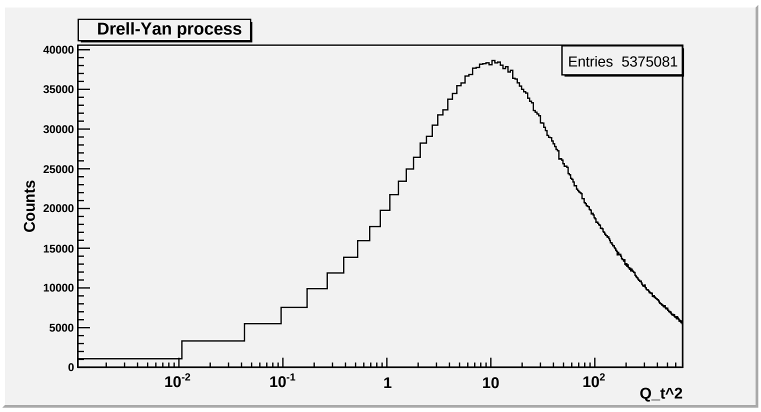

Figure 6: Same graph as above but with logarithmic x-axis. Notice the very sharp drop of the event count at low Q_T^2.

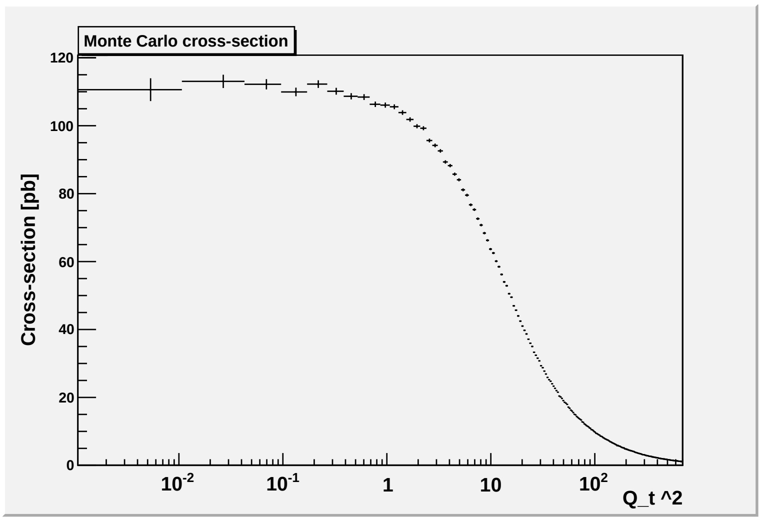

Figure 7: Monte Carlo results for the cross-section. The number of entries corresponds to the number of bins used in the event histogram of figure 5

The numerical evaluation of the fourier integrals was done again using GSL library, this using the adaptive QAG routines with a 15-point Gauss-Kronrod rule. Difficulties were encountered with round-off errors and it was these difficulties that led to the use of a 15-point, rather than a 61-point rule. In the overwhelming majority of cases the the error in the evaluation of the integral was 2 orders of magnitude smaller than the statistical uncertainty. The large error bars on some of the points are due to these round-off errors. [5]

Upon inspecting the shape of the graph one immediately sees that the fourier coefficients exhibit an oscillatory behaviour. This is in direct disagreement with the theoretical predictions. I am not sure why these oscillations arise. A plausible explanation could be that they are a result of the discontinuities in the cross-section due to the discrete nature of the histogram. Alternatively, different values of b modify the 'frequency' of the Bessel function that appears in the integrand and may result in numerical instability due to the sharp cut-off of $\frac{\mathrm{d}\sigma}{\mathrm{d}Q_T^2}$.

An attempt to cancel out these oscillations was made by binning the data for the fourier coefficients in only a few bins. The results are shown in figure 10. It is immediately obvious that there is a disagreement with the theory from

[5] I did not succeed in tracing the origin of these round-off errors. Trying non-adaptive integration routines did not alleviate the problem. It seems very odd that one cannot obtain the answer with a relative error less than even 10^{-2} in some cases. Furthermore there seems to be a gap in existing numerical software; arbitrary precision arithmetic libraries exist (a noteworthy example is the GNU Multiple Precision library - `http://gmplib.org/`). Similarly, there are several fixed-precision libraries featuring numerical integration. Nevertheless, it appears that there exists exactly one library that is capable of numerical integration with arbitrary precision - `mpmath`. It is written in Python and the project is in its infancy so no C/C++ wrappers exist. *A patch/plugin that will allow GSL to use the arbitrary precision of GMP suggests itself as a wortwhile pursuit*, although I cannot vouch for the technical feasability of the project.

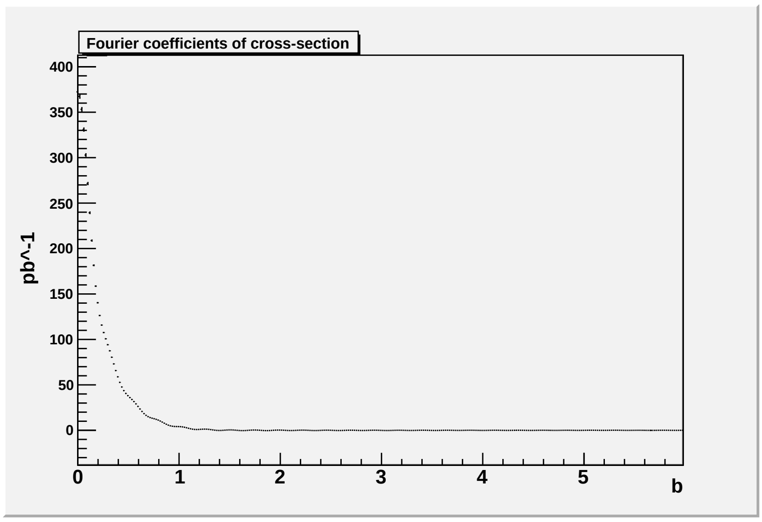

Figure 8: General behaviour of fourier coefficients as a function of the fourier parameter.

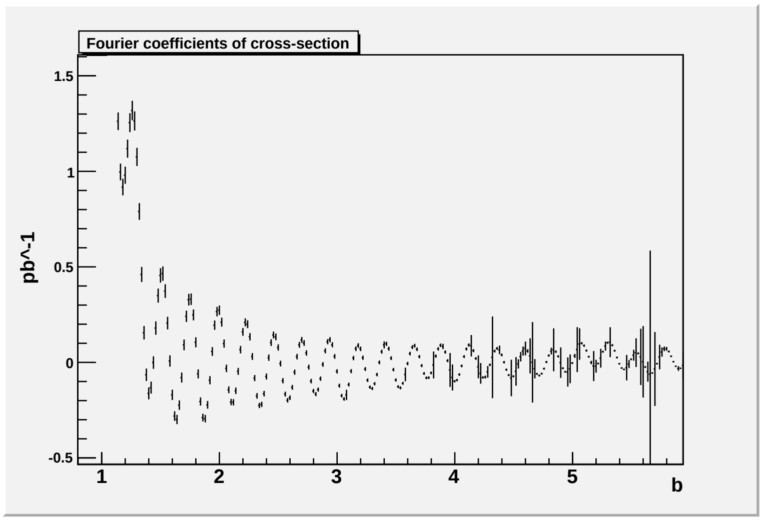

Figure 9: Zoomed-in version of figure 8. The oscillatory component is superimposed on a smoothly decreasing behaviour. As discussed in the main text, the unusually large error bars occur because of round-off error in the GSL gsl_integration_qag routines.

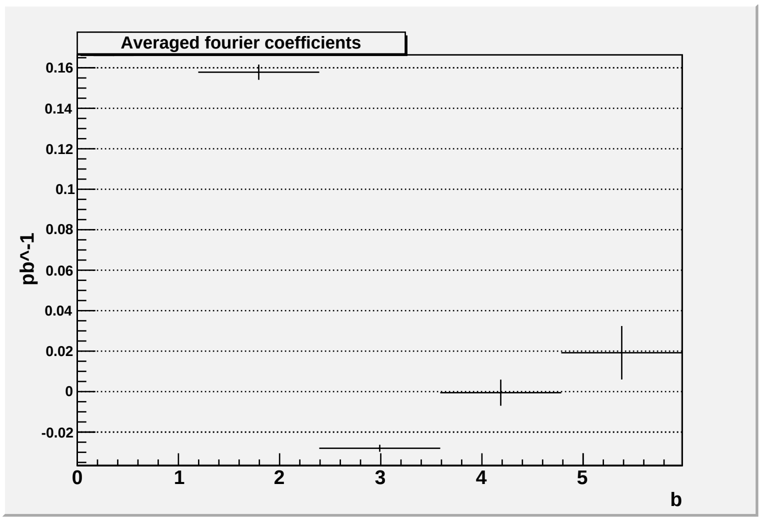

Figure 10: Histogram of fourier coefficients. This is essentially an averaging procedure. Contrary to what theory predicts it is clear that even after this there is still a bin with a negative value. This can occur due to an unlucky choice of bins (i.e. it could be that the specific bin's lower and upper edges happen to coincide with the low part of the oscillation). A different binning was tried but without success. At any rate, this averaging procedure, without a systematic and objective way of choosing the binning, is a dishonest practice. One can, by arranging for a suitable binning, sacrifice the height of a bin that is already positive to make a slightly negative bin positive as well. Playing around with the binning is therefore not a solution to the problem.

the fact that there appears to be a bin with a negative value for the fourier coefficient. This time the better agreement is expected in the high b region.

4 Agreement?

In figure 11 the ratio of the PYTHIA results for the cross-section and the theoretical predictions using the 1-loop cross-section is plotted. There is a pleasing order-of-magnitude agreement but there appears to be a discrepancy by a factor of roughly 0.15 [6]. This factor is not constant although this is to be expected; the full theoretical prediction includes another term that dominates at high p_T and has been ignored in this study. Furthermore discrepancies in the overall scale are a common situation in Monte-Carlo simulations. Without a more careful investigation it is therefore impossible to determine whether the analysis itself is somehow flawed or whether this is a genuine discrepancy. Presumably

[6] The exact value for the first bin is 0.157 ± 0.005. This is suspicously close to $1/2\pi = 0.15915$ and it makes one wonder whether a magnificent agreement is not obscured by a careless omission of a factor of 2π somewhere in the calculation. I have looked and have not found such a mistake. Then again if I have indeed such a mistake in the first place my error-catching abilities are also to be doubted!

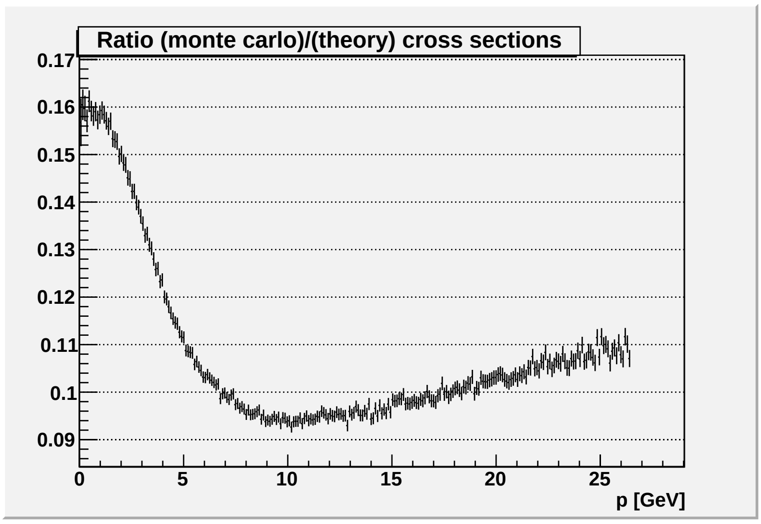

Figure 11: Ratio of Monte Carlo results over the theoretical predictions for the cross-section. The non-constancy is to be expected since the theoretical prediction omits the terms that dominate the high p_T region. Note that the theory *over*estimates the cross-section. The error bars result from combining the statistical uncertainty of the Monte-Carlor results and the numerical error in the evaluation of the theoretical predictions. It is worth noting that the two are of comparable magnitude.

a comparison with experimental data will settle the question. Suggestions for resolving the disagreement will be discussed in detail in the next section.

A comparison with the 2-loop constant was not pursued. It is clear however that doing so would only result in a larger discrepancy between the Monte-Carlo methods and theory: The 1-loop prediction *overestimates* the result [7] and the 2-loop formula for α_s a larger cross-section is predicted in the low (figure 3).

The aim of the project was to compare the fourier coefficients of the two cross-sections. This has not been done. A bug that was only spotted 2 days before the end of the summer student programme invalidate the majority of my results.

5 Conclusion and further progress

Hopefully by now it is clear from the above that this is a report on a work in progress rather than a finished project. Most of my results were obtained only during the penultimate week of the summer student programme since a misprint in the theoretical predictions delayed my work by a few weeks. This caused my to spend 3 weeks wondering why the theoretical prediction was in shocking disagreement with the Monte Carlo results. At any rate this — essentially preliminary — analysis shows a rough agreement between the results of

[7] Or the Monte-Carlo calculations *underestimate* the correct value!

Pythia and theory. The very sharp decrease of the cross-section at low Q_T was demonstrated. An order-of-magnitude agreement for differential cross-section was demonstrated and the decreasing From the data at hand a definite conclusion cannot be formed and more analysis is needed. I will conclude the report with a list of the areas on which I would focus, given enough time:

- *More accurate error analysis:* A rigorous error analysis is required, particularly for the theoretical predictions. Errors are introduced at virtually every stage involving an integration. For instance, there is an uncertainty in the evaluation of equation 4 that arises due to the uncertainty in the integration and the uncertainty in the PDF sets (among other factors). We then integrate over the rapidity in equation 2; both the uncertainty of the new integration and the propagation of the previous error should be properly taken into account. The same holds for the averaging procedure that is then carried out to produce a histogram from the theoretical prediction. Through this analysis the error was taken to be relative error in the last integration performed - the crudeness of this approach is compensated by the fact that the actual accuracy of the `GSL` integration routines usually exceeds by an order of magnitude the specified desired value.

- *Origin of oscillations:* Without an understanding of the origin of the oscillations it is not possible to make a sensible comparison; the theory predicts fourier coefficients that monotonically decrease with the fourier parameter and even if the average values agree, the disagreement in the detailed behaviour cannot be overlooked.

- *Discrepancy in cross-section:* It should be investigated whether the discrepancy is not caused by a careless omission of a constant/ conversion factor on my part. Care was obviously taken to avoid such a situation but without knowing why the absolute magnitudes differ by so much I cannot be absolutely confident that I have not made a mistake. A more useful comparison might be to forgo the overall normalisation altogether and compare only the shapes of the curves. Another interesting approach would be to compare directly to experimental data or to at least compare the Monte Carlo predictions to experiment.

- *Two-loop constant:* The results of the Monte Carlo simulation should also be compared to the 2-loop correction to the strong coupling constant. This was not carried out here due to time constraints (two-loop calculations are significantly more computationally intensive). It is also curious that the discrepancies between the 1-loop and 2-loop predictions in figure 4 are so large. Unfortunately it appears that the difference in the behaviour works in the opposite way than what we would expect, making the factor of 5 of the previous paragraph even larger.

- *Different event generator:* The current analysis used Pythia exclusively. It would be interesting to see the same analysis performed against the results of a different event simulator. This would also provide valuable information on the difference in the absolute magnitudes of the results.

- *Different Landau cuts:* The current value selected for the cut at the Landau pole was entirely arbitrary. Although the differences by selecting a different value for the cut should be very small it would be reassuring to verify this.

Concluding this report I would like to thank Hannes Jung for taking the time to carefully listen to my problems whenever an issue arised and for providing very insightful suggestions at all stages of the project. I would also like to warmly thank Zoltan Nagy for being a valuable source of information — as supervisors usually are — but also a very encouraging and inspirational presence — something not all supervisors succeed in.

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