Resummation of logarithmic contributions in MSSM Higgs mass calculations

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31.4.2015

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   - Conclusion
   - Outlook

- mass of the lightest Higgs can be calculated in the MSSM
- terms up to 3-loop level are known
- calculation yields terms $\propto \ln\left(\frac{M_S^2}{m_t^2}\right)$
- for large sparticles masses $M_S \gg m_t$ logarithms get large

Higher order logarithms are relevant

$\Rightarrow$ **resummation of logarithms needed**

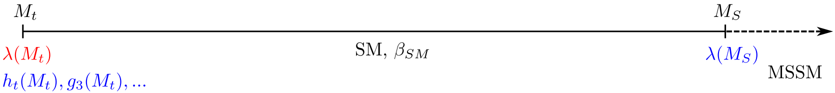
Higgs mass

$M_h^2 = 2\lambda(Q = m_t)v^2$, how to get $\lambda(m_t)$?

Idea: Effective field theory

M_S mass scale of SUSY-particles, above $\rightarrow$ MSSM, below $\rightarrow$ SM

- λ fixed in MSSM: $\lambda(M_S) = \frac{1}{4}(g^2 + g'^2)c_{2\beta}^2$ at tree-level



$\Rightarrow$ **use SM-RGEs to run λ down:**

$$\lambda(M_S) \overset{\beta_{\text{SM}}}{\rightarrow} \lambda(m_t)$$

Notation: $t = \ln Q^2, k = 1/(16\pi^2), h_t = m_t/v$

$$\frac{d\lambda}{dt} = 6k\left(\lambda^2 + \lambda h_t^2 - h_t^4\right)$$

Solve iteratively:

$$\lambda(m_t) \approx \lambda(M_S) + \int_{M_S}^{m_t} \frac{d\lambda}{dt} dt \approx$$

$$\approx \lambda(M_S) - 6k\left(\lambda^2(M_s) + \lambda(M_s) h_t^2(m_t) - h_t^4(m_t)\right) \ln\left(\frac{M_S^2}{m_t^2}\right) \approx$$

$$\approx \lambda_{tree} + 6k h_t^4(m_t) \ln\left(\frac{M_S^2}{m_t^2}\right)$$

Solve **numerically**:

$\Rightarrow$ **Resummation of large logarithms to all orders**

(sub)leading logarithms

Use 1-loop RGEs → Resummation of leading logarithms ($k^n L^n$)
Use 2-loop RGEs → Also subleading logarithms ($k^n L^{n-1}$)

Additional complication:

Threshold corrections

$$\lambda(M_S) = \frac{1}{4}(g^2 + g'^2)c_{2\beta}^2 + \Delta\lambda_{thres}$$

- originate from integrating out heavy sparticles
- n-loop threshold corrections result in sub^n-leading logarithms
- $\Delta\lambda_{thres} = 6kh_t^4\left(\hat{X}_t^2 - \frac{1}{12}\hat{X}_t^4\right)$
  $(X_t = A_t - \mu/t_\beta, \hat{X}_t = X_t/M_s)$

Double counting has to be avoided:

$\Rightarrow$ Subtract logarithms from the diagrammatic result

RGEs derived in $\overline{\text{MS}}$, diagrammatic calculation in OS:

$\Rightarrow$ Conversion $\overline{\text{MS}} \leftrightarrow$ OS is mandatory: $A^{\text{OS}} = A^{\overline{\text{MS}}} + \delta A^{\text{OS}}_{\text{fin.}}$

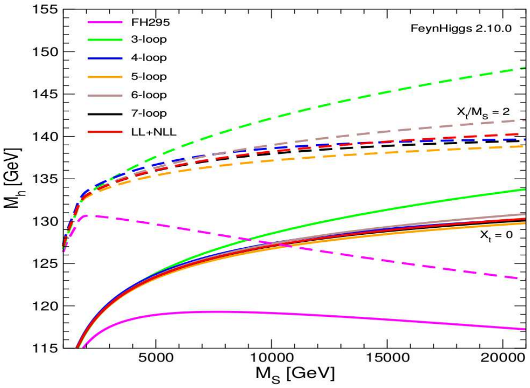
$$\Downarrow$$

$$M_h^2 = (M_h^2)^{\text{FD}} + \Delta M_h^2$$
$$\Delta M_h^2 = (\Delta M_h^2)^{\text{RGE}}(X_t^{\overline{\text{MS}}}) - (\Delta M_h^2)^{\text{FD,Logs}}(X_t^{OS})$$

FeynHiggs 2.10

Resummation of leading/subleading logarithms $\propto \alpha_t, \alpha_s$

- weak gauge couplings are neglected ($g = g' = 0$)



Hahn et al.: arXiv:1312.4937

Extension I

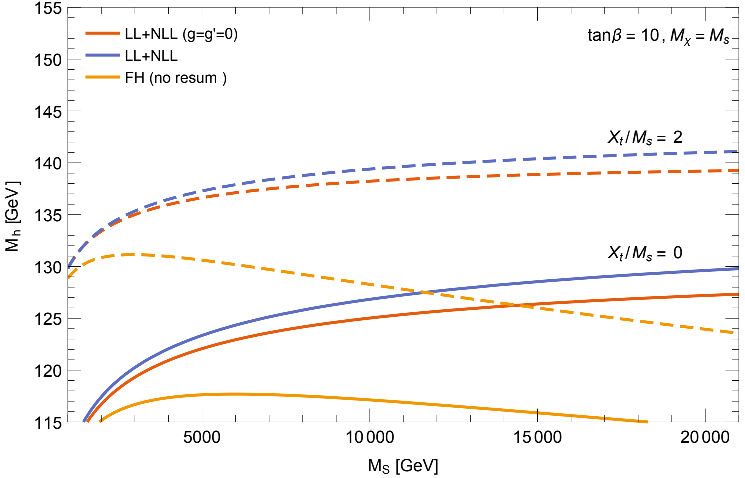
Use of full 2-loop SM-RGEs, including g, g'

- avoid double-counting of electroweak logarithms at 1-loop
- new threshold corrections (e.g. Draper et al.: arXiv:1312.5743)

$$\lambda(M_S) = \lambda_{tree} + \Delta\lambda_{\text{stop}} + \Delta\lambda_{\text{heavy Higgs}} + \Delta\lambda_{\text{chargino/neutralino}}$$

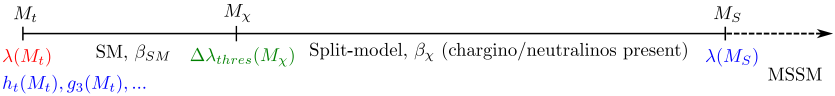
- additional terms in $\overline{\text{MS}} \leftrightarrow$ OS conversion

$$X_t^{\overline{\text{MS}}} = X_t^{\text{OS}} \left[1 + \left(\underbrace{\frac{\alpha_s}{\pi}}_{g,\tilde{g}} \underbrace{- \frac{3\alpha_t}{16\pi}\left(1 - \hat{X}_t^2\right)}_{\text{Higgs}} \underbrace{- \frac{\alpha}{96\pi}(1 - 26c_w^2)}_{\text{Z,W}^\pm} \right) \ln\left(\frac{M_S^2}{m_t^2}\right) \right]$$


LL+NLL (g=g'=0)
LL+NLL
FH (no resum)
tanβ = 10 , M_χ = M_s
X_t/M_s = 2
X_t/M_s = 0
155
150
145
140
135
130
125
120
115
5000
10 000
15 000
20 000
M_h [GeV]
M_S [GeV]

Extension II

Additional threshold $M_\chi \equiv M_1 = M_2 = \mu$ ($m_t \ll M_\chi < M_S$), above which charginos/neutralinos contribute to RGE running



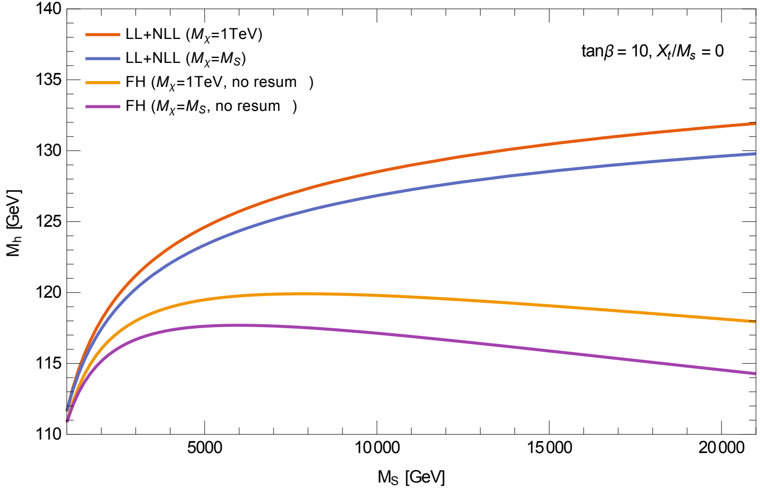
- gaugino-gaugino-Higgs couplings $\tilde{g}_{1u,1d,2u,2d}$ fixed at $Q = M_S$

  (e.g. Giudice et al. arXiv:1108.6077)

- threshold corrections at $Q = M_\chi$

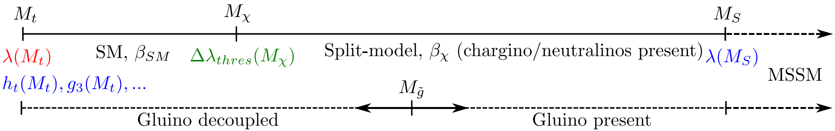
  $\lambda_{\mathrm{SM}}(M_\chi) = \lambda_\chi(M_\chi) + \Delta\lambda_{\mathrm{chargino/neutralino}}$

  $h_{t,\mathrm{SM}}(M_\chi) = h_{t,\chi}(M_\chi) + \Delta h_{t,\mathrm{chargino/neutralino}}$


140
135
130
125
120
115
110
5000
10000
15000
20000
M_h [GeV]
M_S [GeV]
LL+NLL (M_χ=1TeV)
LL+NLL (M_χ=M_S)
FH (M_χ=1TeV, no resum)
FH (M_χ=M_S, no resum)
tanβ = 10, X_t/M_s = 0

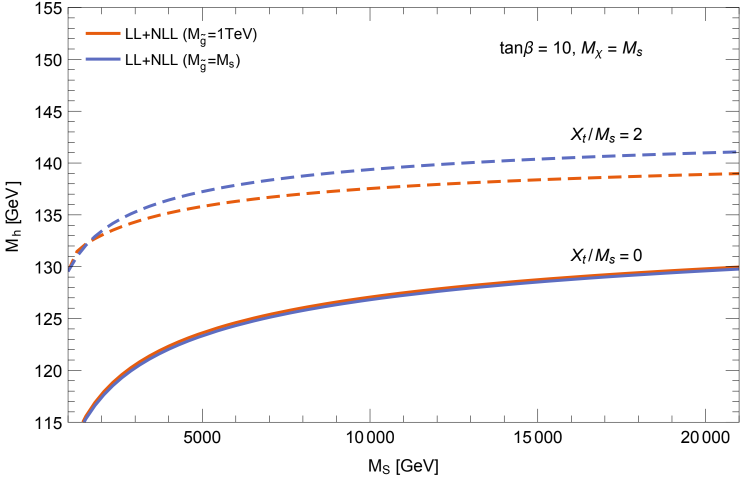
Extension III

Additional threshold $M_{\tilde{g}}$ ($m_t \ll M_{\tilde{g}} < M_S$), above which gluinos contribute to RGE running



- no additional threshold corrections at one-loop (gluino enters at two-loop level)
- only modifications of RGEs above $Q = M_{\tilde{g}}$ necessary, e.g.

$$\frac{dg_3}{dt} = \begin{cases} -\frac{7}{2} k g_3^3 & \text{for } Q < M_{\tilde{g}} \\ -\frac{5}{2} k g_3^3 & \text{for } Q > M_{\tilde{g}} \end{cases}$$


LL+NLL (M$_{\tilde{g}}$=1TeV)
LL+NLL (M$_{\tilde{g}}$=M$_s$)
tanβ = 10, M_χ = M_s
$X_t/M_s = 2$
$X_t/M_s = 0$
155
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M$_h$ [GeV]
M$_S$ [GeV]

How to explain difference between $\hat{X}_t = 0$ and $\hat{X}_t = 2$?

$$\frac{dh_t}{dt} = \frac{1}{2} h_t k \left(\frac{9}{2} h_t^2 - 8 g_3^2 \right)$$

$$\frac{dg_3}{dt} = -\langle \frac{7}{2}; \frac{5}{2} \rangle k g_3^3$$

$$\Delta\lambda_{stop} = 6 k h_t^4 (M_s) \left[\left(\frac{X_t}{M_s} \right)^2 - \frac{1}{12} \left(\frac{X_t}{M_s} \right)^4 \right]$$

Scheme conversion

$\overline{\text{MS}} \leftrightarrow$ OS conversion: $X_t, M_S, M_\chi, M_{\tilde{g}}, m_t, M_W, M_Z$

- only logarithmic terms relevant
- definition of counterterms:
  - $M_s \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \to \delta M_S^2 = \frac{1}{2}\left(\frac{m_{\tilde{t}_2}}{m_{\tilde{t}_1}}\delta M_{\tilde{t}_1}^2 + \frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}}\delta M_{\tilde{t}_2}^2\right)$
  - $M_\chi \equiv M_{\tilde{\chi}_2^0} \to$ use mass counterterm of $\tilde{\chi}_2^0$
  - $M_{\tilde{g}} \to$ use mass counterterm of $\tilde{g}$
- $m_t \to$ use running mass in Feynman diagrammatic result
- M_W, M_Z $(g, g') \to$ no logarithms (effective theory is SM)

Question

How to handle t_β?

- $\underbrace{\overline{\text{DR}}}_{\text{FeynHiggs}} \leftrightarrow \underbrace{\overline{\text{MS}}}_{\text{RGEs}}$ conversion
- running of t_β:
  - FeynHiggs takes $t_\beta(\mu = m_t)$ as input, $t_\beta(M_s)$ needed
  - definition of t_β in effective model below M_s?
- so far ($\tilde{h}_t = h_t / s_\beta$):

$$\frac{1}{\tan^2\beta}\frac{d\tan^2\beta}{dt} = -3k\tilde{h}_t^2 + k^2\left[9\tilde{h}_t^4 - \left(\frac{4}{3}g'^2 + 16g_3^2\right)\tilde{h}_t^2\right]$$

- Large SUSY-scale $\rightarrow$ large logarithms $\Rightarrow$ resummation necessary
- FeynHiggs 2.10: resummation of logarithms $\propto \alpha_t, \alpha_s$
- Extension I: resummation of logarithms $\propto \alpha_{em}$
  up to ~ 3 GeV
- Extension II: intermediate chargino/neutralino threshold
  up to ~ 2 GeV
- Extension III: intermediate gluino threshold
  up to ~ 0.3 GeV ($\hat{X}_t = 0$), ~ 2 GeV ($\hat{X}_t = 2$)

- inclusion of (s)bottom contributions
- hierarchical stop spectrum $\rightarrow$ two stop thresholds
- heavy Higgs threshold
- subsubleading logarithms using 3-loop RGEs ($\lesssim 0.1$ GeV)

Standard Model RGEs (1 loop):

$$\frac{d\lambda}{dt} = k\left[6\left(\lambda^2 + \lambda h_t^2 - h_t^4\right) - \lambda\left(\frac{9}{2}g^2 + \frac{3}{2}g'^2\right) + \frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2 g'^2\right]$$

$$\frac{dh_t}{dt} = \frac{1}{2}h_t k\left(\frac{9}{2}h_t^2 - 8g_3^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right)$$

$$\frac{dg_3}{dt} = -\langle\frac{7}{2};\frac{5}{2}\rangle k g_3^3$$

$$\frac{dg'}{dt} = \frac{41}{12}g'^3$$

$$\frac{dg}{dt} = -\frac{19}{12}g^3$$

Split Model Langrangian:

$$
\begin{aligned}
\mathcal{L}_{\text{split}} \supset & \langle 0; -\frac{1}{2} M_3 \tilde{g}^a \tilde{g}^a \rangle - \frac{1}{2} M_\chi \tilde{W}^a \tilde{W}^a - \frac{1}{2} M_\chi \tilde{B}^a \tilde{B}^a - M_\chi \tilde{H}_u^T \epsilon \tilde{H}_d \\
& - \frac{1}{\sqrt{2}} H^\dagger \left(\tilde{g}_{2u} \sigma^a \tilde{W}^a + \tilde{g}_{1u} \tilde{B} \right) \tilde{H}_u \\
& - \frac{1}{\sqrt{2}} H^T \epsilon \left(-\tilde{g}_{2d} \sigma^a \tilde{W}^a + \tilde{g}_{1d} \tilde{B} \right) \tilde{H}_d \\
& + h.c.
\end{aligned}
$$

Split Model RGEs (1 loop):

$$
\begin{aligned}
\frac{dg'}{dt} =& \frac{15}{4} k g'^3 \\
\frac{dg}{dt} =& -\frac{7}{12} k g^3 \\
\frac{dg_3}{dt} =& -\langle \frac{7}{2}; \frac{5}{2} \rangle k g_3^3 \\
\frac{dh_t}{dt} =& \frac{1}{2} k h_t \left[-\frac{9}{4} g^2 - 8 g_3^2 - \frac{17}{12} g'^2 + \frac{9}{2} h_t^2 + \frac{1}{2} (\tilde{g}_{1d}^2 + \tilde{g}_{1u}^2 + 3\tilde{g}_{2d}^2 + 3\tilde{g}_{2u}^2) \right] \\
\frac{d\lambda}{dt} =& \frac{1}{2} k \left[-\tilde{g}_{1d}^4 - \tilde{g}_{1u}^4 + \frac{9}{4} g^4 - 5\tilde{g}_{2d}^4 - 4\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u} - 5\tilde{g}_{2u}^4 \right. \\
& -2(\tilde{g}_{1u}^2 + \tilde{g}_{2d}^2)(\tilde{g}_{1d}^2 + \tilde{g}_{2u}^2) \\
& + \frac{3}{2} g^2 g'^2 + \frac{3}{4} g'^4 - 12 h_t^4 + 2(\tilde{g}_{1d}^2 + \tilde{g}_{1u}^2 + 3\tilde{g}_{2d}^2 + 3\tilde{g}_{2u}^2)\lambda \\
& \left. -9 \left(g^2 + \frac{1}{3} g'^2 \right) \lambda + 12 h_t^2 \lambda + 12 \lambda^2 \right]
\end{aligned}
$$

Split Model RGEs (1 loop), cont.:

$$\frac{d\tilde{g}_{1u}}{dt} = \frac{1}{2}k\left[3\tilde{g}_{1d}\tilde{g}_{2d}\tilde{g}_{2u} + \tilde{g}_{1u}\left(\frac{5}{4}\tilde{g}_{1u}^2 + 2\tilde{g}_{1d}^2 + \frac{9}{4}\tilde{g}_{2u}^2 + \frac{3}{2}\tilde{g}_{2d}^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right]$$

$$\frac{d\tilde{g}_{1d}}{dt} = \frac{1}{2}k\left[3\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u} + \tilde{g}_{1d}\left(\frac{5}{4}\tilde{g}_{1d}^2 + 2\tilde{g}_{1u}^2 + \frac{9}{4}\tilde{g}_{2d}^2 + \frac{3}{2}\tilde{g}_{2u}^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right]$$

$$\frac{d\tilde{g}_{2u}}{dt} = \frac{1}{2}k\left[\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d} + \tilde{g}_{2u}\left(\frac{1}{2}\tilde{g}_{1d}^2 + \frac{3}{4}\tilde{g}_{1u}^2 + \tilde{g}_{2d}^2 + \frac{11}{4}\tilde{g}_{2u}^2 - \frac{33}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right]$$

$$\frac{d\tilde{g}_{2d}}{dt} = \frac{1}{2}k\left[\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2u} + \tilde{g}_{2d}\left(\frac{1}{2}\tilde{g}_{1u}^2 + \frac{3}{4}\tilde{g}_{1d}^2 + \tilde{g}_{2u}^2 + \frac{11}{4}\tilde{g}_{2d}^2 - \frac{33}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2\right)\right]$$

$$\lambda(M_s) = \underbrace{\frac{1}{4}\cos^2(2\beta)\left(g^2 + g'^2\right)}_{\text{tree level}}$$

$$\underbrace{+6h_t^2 k\left\{\left[h_t^2 + \frac{1}{8}\left(g^2 + g'^2\right)c_{2\beta}\right]\left(\frac{X_t^2}{M_s^2}\right) - \frac{1}{12}h_t^2\left(\frac{X_t^4}{M_s^4}\right)\right\}}_{\text{stop-threshold corr.}}$$

$$\underbrace{-k\left[\left(\frac{3}{4} - \frac{1}{6}c_{2\beta}^2\right)g^4 + \frac{1}{2}g^2 g'^2 + \frac{1}{4}g'^4\right]}_{\text{tree-level term } \overline{\text{DR}} \to \overline{\text{MS}}}$$

$$\underbrace{-\frac{1}{16}k(g'^2 + g^2)^2 s_{4\beta}^2}_{\text{heavy Higgs threshold corr.}}$$

$$\tilde{g}_{1u}(M_s) = g s_\beta \left\{ 1 + k \left[\frac{3}{16} g^2 \left(-2 + 7 c_\beta^2 \right) + \frac{1}{20} g'^2 \left(-44 + 7 c_\beta^2 \right) + \frac{9}{4 s_\beta^2} h_t^2 \right] \right\}$$

$$\tilde{g}_{1d}(M_s) = g c_\beta \left\{ 1 + k \left[\frac{3}{16} g^2 \left(-2 + 7 s_\beta^2 \right) + \frac{1}{20} g'^2 \left(-44 + 7 s_\beta^2 \right) \right] \right\}$$

$$\tilde{g}_{2u}(M_s) = g' s_\beta \left\{ 1 + k \left[-g^2 \left(\frac{2}{3} + \frac{11}{16} c_\beta^2 \right) + \frac{1}{20} g'^2 \left(-2 + 7 c_\beta^2 \right) + \frac{9}{4 s_\beta^2} h_t^2 \right] \right\}$$

$$\tilde{g}_{2u}(M_s) = g' c_\beta \left\{ 1 + k \left[-g^2 \left(\frac{2}{3} + \frac{11}{16} s_\beta^2 \right) + \frac{1}{20} g'^2 \left(-2 + 7 s_\beta^2 \right) \right] \right\}$$

$$
\begin{aligned}
\lambda_{\mathsf{SM}}(M_\chi) = {} & \lambda_\chi(M_\chi) \\
& + k\left\{-\frac{7}{12}(\tilde{g}_{1d}^4+\tilde{g}_{1d}^4)-\frac{9}{4}(\tilde{g}_{2d}^4+\tilde{g}_{1u}^4)-\frac{3}{2}\tilde{g}_{1d}^2 g_{1u}^2-\frac{7}{2}g_{2d}^2 g_{2u}^2\right. \\
& -\frac{8}{3}\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u}-\frac{7}{6}(\tilde{g}_{1d}^2\tilde{g}_{2d}^2+\tilde{g}_{1u}^2\tilde{g}_{2u}^2)-\frac{1}{6}(\tilde{g}_{1d}^2\tilde{g}_{2u}^2+\tilde{g}_{1u}^2\tilde{g}_{2d}^2) \\
& -\frac{4}{3}(\tilde{g}_{1d}\tilde{g}_{2u}^2+\tilde{g}_{1u}\tilde{g}_{2d})(\tilde{g}_{1d}\tilde{g}_{2d}+\tilde{g}_{1u}\tilde{g}_{2u}) \\
& +\frac{2}{3}\tilde{g}_{1d}\tilde{g}_{1u}(\lambda_\chi-2\tilde{g}_{1d}^2-2\tilde{g}_{1u}^2)+2\tilde{g}_{2d}\tilde{g}_{2u}(\lambda_\chi-2\tilde{g}_{2d}^2-2\tilde{g}_{2u}^2) \\
& \left.+\frac{1}{3}\lambda_\chi(\tilde{g}_{1d}^2+\tilde{g}_{1u}^2)+\lambda_\chi(\tilde{g}_{2d}^2+\tilde{g}_{2u}^2)\right\}
\end{aligned}
$$

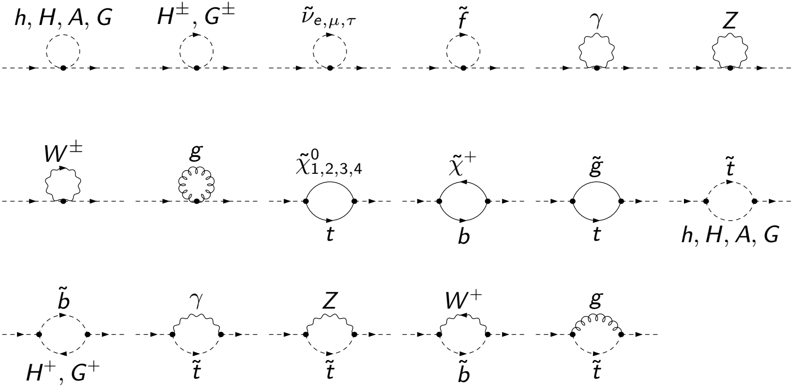
$$
\begin{aligned}
g_\chi(M_\chi) &= g_{\mathrm{SM}}(M_\chi) \\
g'_\chi(M_\chi) &= g'_{\mathrm{SM}}(M_\chi) \\
g_{3,\chi}(M_\chi) &= g_{3,\mathrm{SM}}(M_\chi) \\
h_{t,\chi}(M_\chi) &= h_{t,\mathrm{SM}}(M_\chi)\left\{1 - k\left[\frac{1}{6}\tilde{g}_{1u}\tilde{g}_{1d} + \frac{1}{12}(\tilde{g}_{1u}^2 + \tilde{g}_{1d}^2)\right.\right. \\
&\qquad\qquad\left.\left. +\ \frac{1}{2}\tilde{g}_{2u}\tilde{g}_{2d} + \frac{1}{4}(\tilde{g}_{2u}^2 + \tilde{g}_{2d}^2)\right]\right\}
\end{aligned}
$$

one-loop logarithms ($M_\chi = M_{Susy}$):

$$M_h^{\rm LL} = -\frac{\alpha}{192\pi M_W^2 s_w^2}\Big\{288 M_t^4 + 144 m_t^2 M_Z^2 c_{2\beta} + 4c_{4\beta}\left(22M_W^4 - 64M_W^2M_Z^2 + 41M_Z^4\right) - 9M_Z^4 c_{8\beta} - 56M_W^4 - 256M_W^2M_Z^2 + 101M_Z^4\Big\}\ln\left(\frac{M_s^2}{m_t^2}\right)$$

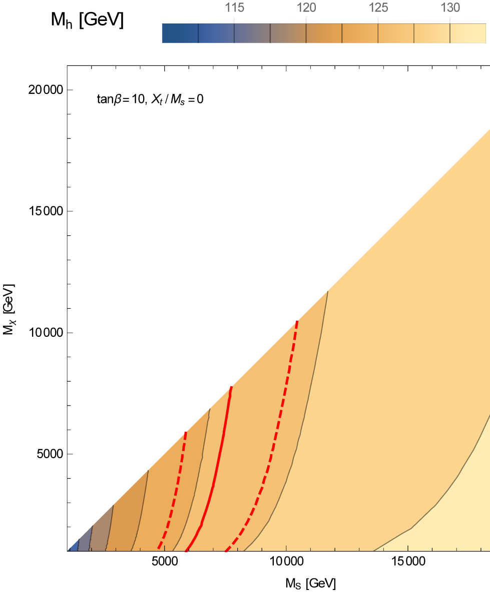
chargino/neutralino contribution:

$$\frac{\alpha M_Z^2}{24\pi c_w^4}\Big[(10c_w^2(2c_w^2-1)-1)c_{4\beta} + 2c_w^2(22c_w^2-5)+11\Big]\ln\left(\frac{M_\chi^2}{m_t^2}\right)$$



Stop self-energy diagrams

$$\delta X_t = \frac{1}{m_t}\left[(\delta M^2_{\tilde{t}_1} - \delta M^2_{\tilde{t}_2})\mathbf{U}_{\tilde{t},11}\mathbf{U}_{\tilde{t},12} + \delta M^2_{\tilde{t}_1\tilde{t}_2}(\mathbf{U}_{\tilde{t},21}\mathbf{U}_{\tilde{t},12} + \mathbf{U}_{\tilde{t},11}\mathbf{U}_{\tilde{t},22}) - X_t \delta M^2_t\right]$$



M_h [GeV]
115
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M_χ [GeV]
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