

Resummation of logarithmic contributions in MSSM Higgs-boson mass calculations

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1 Introduction

- Problem
- General idea
- Combination with Feynman diagrammatic result
- Current status

2 Improvements

- Electroweak contributions
- Chargino/Neutralino threshold
- Gluino threshold
- Scheme conversion and t_β

3 Conclusion and Outlook

- Conclusion
- Outlook

- ▶ mass of the lightest Higgs can be calculated in the MSSM
- ▶ terms up to the 3-loop level are known
- ▶ calculation yields terms $\propto \ln\left(\frac{M_S^2}{m_t^2}\right)$ (M_S is stop mass scale)
- ▶ for large sparticle masses ($M_S \gg m_t$) logarithms get large

Higher order logarithms are relevant.

\Rightarrow resummation of logarithms needed

Higgs mass

$$M_h^2 = 2\lambda(Q = m_t)v^2, \text{ how to get } \lambda(m_t)?$$

Idea: Effective field theory

M_S mass scale of SUSY-particles, above \rightarrow MSSM, below \rightarrow SM

- ▶ λ fixed in MSSM: $\lambda(M_S) = \frac{1}{4}(g^2 + g'^2)c_{2\beta}^2$ at tree-level

\Rightarrow use **SM RGEs** to run λ down:

$$\lambda(M_S) \xrightarrow{\beta^{\text{SM}}} \lambda(m_t)$$

Notation: $t = \ln Q^2$, $k = 1/(16\pi^2)$, $h_t = m_t/v$

$$\frac{d\lambda}{dt} = 6k \left(\lambda^2 + \lambda h_t^2 - h_t^4 \right)$$

Solve iteratively:

$$\begin{aligned} \lambda(m_t) &\approx \lambda(M_S) + \int_{M_S}^{m_t} \frac{d\lambda}{dt} dt \approx \\ &\approx \lambda(M_S) - 6k \left(\lambda^2(M_S) + \lambda(M_S) h_t^2(m_t) - h_t^4(m_t) \right) \ln \left(\frac{M_S^2}{m_t^2} \right) \approx \\ &\approx \lambda_{tree} + 6k h_t^4(m_t) \ln \left(\frac{M_S^2}{m_t^2} \right) \end{aligned}$$

Solve **numerically**:

\Rightarrow **Resummation of large logarithms to all orders**

(sub)leading logarithms

Use 1-loop RGEs \rightarrow Resummation of leading logarithms ($k^n L^n$)

Use 2-loop RGEs \rightarrow Also next-to-leading logarithms ($k^n L^{n-1}$)

Additional complication:

Threshold corrections

$$\lambda(M_S) = \frac{1}{4}(g^2 + g'^2)c_{2\beta}^2 + \Delta\lambda_{thres}$$

- ▶ originate from integrating out heavy sparticles
- ▶ n -loop threshold corrections result in (next-to) n -leading logarithms
- ▶ e.g. $\Delta\lambda_{stop} = 6kh_t^4 \left(\hat{X}_t^2 - \frac{1}{12}\hat{X}_t^4 \right) \quad (\hat{X}_t = X_t/M_S)$

FeynHiggs already contains full 1-loop and partial 2-loop results



Double counting has to be avoided:

- ⇒ Subtract logarithms from the diagrammatic result
- ⇒ Subtract non-logarithmic terms from the RGE result

RGEs derived in $\overline{\text{MS}}$, diagrammatic calculation in OS:

- ⇒ Conversion $\overline{\text{MS}} \leftrightarrow \text{OS}$ is mandatory: $A^{\text{OS}} = A^{\overline{\text{MS}}} + \delta A_{\text{fin.}}^{\text{OS}}$

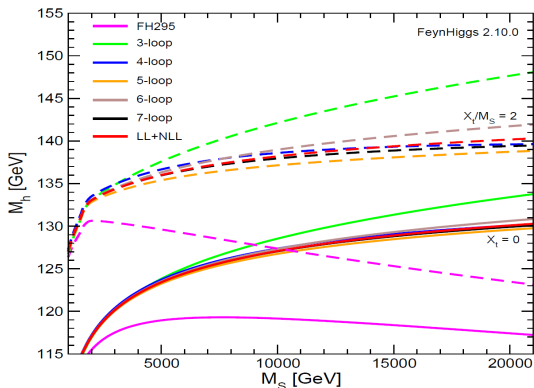


$$M_h^2 = (M_h^2)^{\text{FD}} + (\Delta M_h^2)^{\text{RGE}}(X_t^{\overline{\text{MS}}}) \\
 - (\Delta M_h^2)^{\text{RGE,non-log}}(X_t^{\text{OS}}) - (\Delta M_h^2)^{\text{FD,Logs}}(X_t^{\text{OS}})$$

FeynHiggs 2.10

Resummation of leading/next-to-leading logarithms $\propto \alpha_t, \alpha_s$

- ▶ weak gauge couplings are neglected ($g = g' = 0$)



Extension I

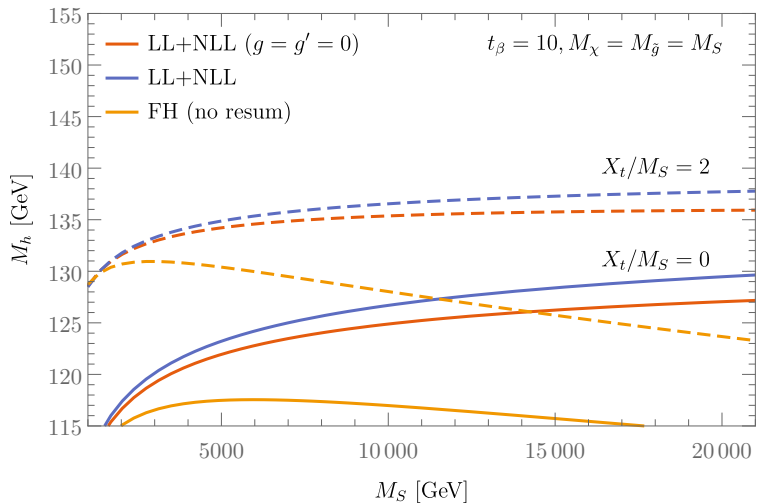
Use of full 2-loop SM RGEs including g, g' with all sparticles at the scale M_S

- ▶ avoid double-counting of electroweak logarithms at 1-loop
- ▶ new threshold corrections (e.g. Bagnaschi et al.: arXiv:1407.4081)

$$\lambda(M_S) = \lambda_{\text{tree}} + \Delta\lambda_{\text{stop}} + \Delta\lambda_{\text{heavy Higgs}} + \Delta\lambda_{\text{chargino/neutralino}} + \Delta\lambda_{\overline{\text{DR}} \rightarrow \overline{\text{MS}}}$$

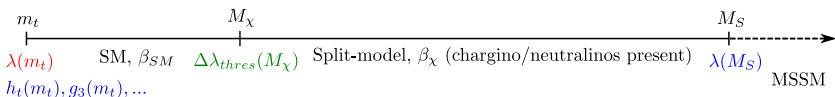
- ▶ additional terms in $\overline{\text{MS}} \leftrightarrow \text{OS}$ conversion

$$X_t^{\overline{\text{MS}}} = X_t^{\text{OS}} \left[1 + \underbrace{\left(\frac{\alpha_s}{\pi} \right)}_{g, \tilde{g}} - \underbrace{\frac{3\alpha_t}{16\pi} (1 - \hat{X}_t^2)}_{\text{Higgs}} - \underbrace{\frac{\alpha}{96\pi} (1 - 26c_w^2)}_{Z, W^\pm} \right] \ln \left(\frac{M_S^2}{m_t^2} \right)$$



Extension II

Additional threshold $M_\chi \equiv M_1 = M_2 = \mu$ ($m_t \ll M_\chi < M_S$),
 above which charginos/neutralinos contribute to RGE running



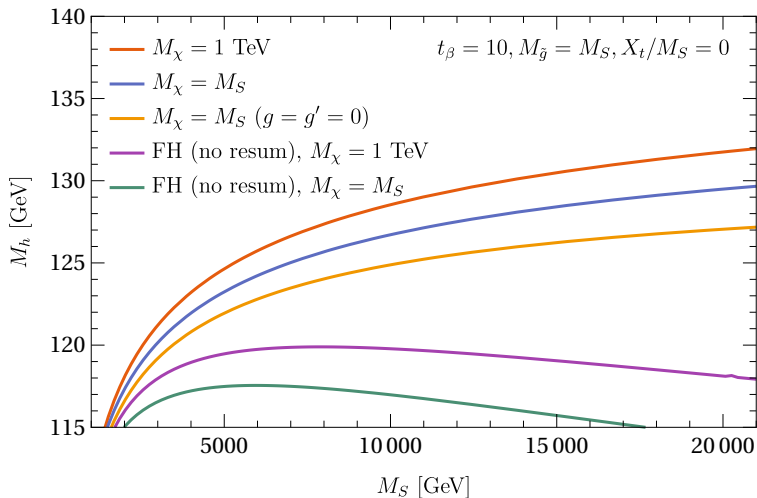
- ▶ gaugino-gaugino-Higgs couplings $\tilde{g}_{1u,1d,2u,2d}$ fixed at $Q = M_S$

(e.g. Giudice et al. arXiv:1108.6077)

- ▶ threshold corrections at $Q = M_\chi$

$$\lambda_{\text{SM}}(M_\chi) = \lambda_\chi(M_\chi) + \Delta\lambda_{\text{chargino/neutralino}}$$

$$h_{t,\text{SM}}(M_\chi) = h_{t,\chi}(M_\chi) + \Delta h_{t,\text{chargino/neutralino}}$$

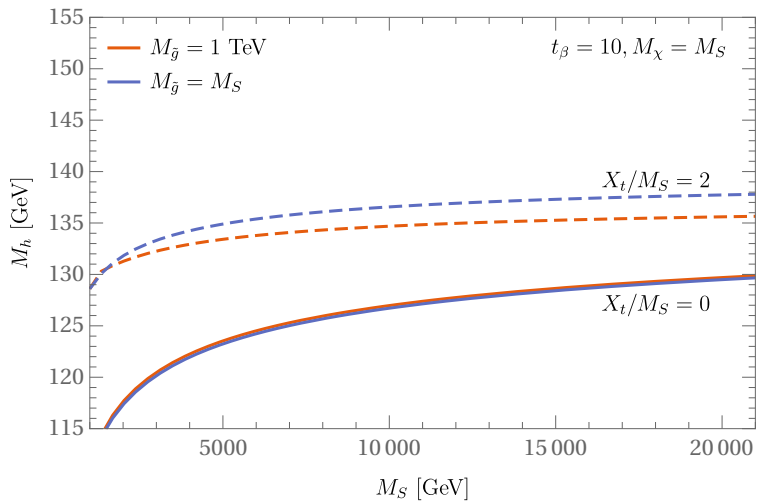


Extension III

Additional threshold $M_{\tilde{g}}$ ($m_t \ll M_{\tilde{g}} < M_S$), above which gluinos contribute to RGE running

- ▶ no additional threshold corrections at the one-loop level (gluino enters at the two-loop level)
- ▶ only modifications of RGEs above $Q = M_{\tilde{g}}$ necessary, e.g.

$$\frac{dg_3}{dt} = \begin{cases} -\frac{7}{2}kg_3^3 & \text{for } Q < M_{\tilde{g}} \\ -\frac{5}{2}kg_3^3 & \text{for } Q > M_{\tilde{g}} \end{cases}$$



Scheme conversion

$\overline{\text{MS}} \leftrightarrow \text{OS}$ conversion: $X_t, M_S, M_\chi, M_{\tilde{g}}, m_t, M_W, M_Z$

- ▶ only logarithmic terms relevant
- ▶ definition of counterterms:
 - $M_S \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \rightarrow \delta M_S^2 = \frac{1}{2} \left(\frac{m_{\tilde{t}_2}}{m_{\tilde{t}_1}} \delta M_{\tilde{t}_1}^2 + \frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}} \delta M_{\tilde{t}_2}^2 \right)$
 - $M_\chi \equiv M_{\tilde{\chi}_2^0} \rightarrow$ use mass counterterm of $\tilde{\chi}_2^0$
 - $M_{\tilde{g}} \rightarrow$ use mass counterterm of \tilde{g}
- ▶ $m_t \rightarrow$ use running mass in Feynman diagrammatic result
- ▶ $M_W, M_Z (g, g') \rightarrow$ no logarithms (effective theory is SM)

Question

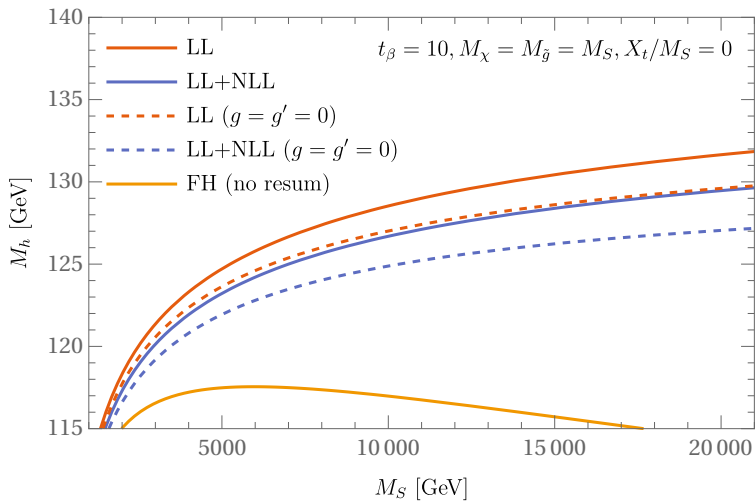
How to handle t_β ?

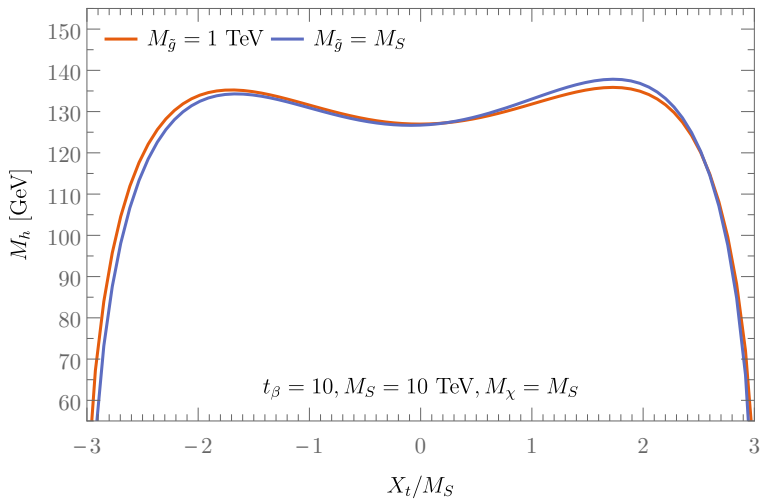
- ▶ $\underbrace{\overline{\text{DR}}}_{\text{FeynHiggs}} \leftrightarrow \underbrace{\overline{\text{MS}}}_{\text{RGEs}}$ conversion
- ▶ running of t_β :
 - FeynHiggs takes $t_\beta(\mu = m_t)$ as input, $t_\beta(M_S)$ needed
 - definition of t_β in effective model below M_S ?
- ▶ so far ($\tilde{h}_t = h_t/s_\beta$):

$$\frac{1}{t_\beta^2} \frac{dt_\beta^2}{dt} = -3k\tilde{h}_t^2$$

- ▶ Large SUSY-scale \rightarrow large logarithms \Rightarrow resummation necessary
- ▶ FeynHiggs 2.10: resummation of logarithms $\propto \alpha_t, \alpha_s$
- ▶ Extension I: resummation of logarithms $\propto \alpha_{em}$
up to ~ 3 GeV
- ▶ Extension II: intermediate chargino/neutralino threshold
up to ~ 2 GeV
- ▶ Extension III: intermediate gluino threshold
up to ~ 0.3 GeV ($\hat{X}_t = 0$), ~ 2 GeV ($\hat{X}_t = 2$)
(mainly due to two-loop non-logarithmic terms)

- ▶ inclusion of (s)bottom contributions
- ▶ hierarchical stop spectrum \rightarrow two stop thresholds
- ▶ heavy Higgs threshold
- ▶ next-to-next-to-leading logarithms using 3-loop RGEs





Standard Model RGEs (1 loop):

$$\frac{d\lambda}{dt} = k \left[6(\lambda^2 + \lambda h_t^2 - h_t^4) - \lambda \left(\frac{9}{2}g^2 + \frac{3}{2}g'^2 \right) + \frac{9}{8}g^4 + \frac{3}{8}g'^4 + \frac{3}{4}g^2g'^2 \right]$$

$$\frac{dh_t}{dt} = \frac{1}{2}h_t k \left(\frac{9}{2}h_t^2 - 8g_3^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2 \right)$$

$$\frac{dg_3}{dt} = -\left\langle \frac{7}{2}; \frac{5}{2} \right\rangle k g_3^3$$

$$\frac{dg'}{dt} = \frac{41}{12}g'^3$$

$$\frac{dg}{dt} = -\frac{19}{12}g^3$$

Split Model Langrangian:

$$\begin{aligned}\mathcal{L}_{\text{split}} = & \langle 0; -\frac{1}{2}M_3\tilde{g}^a\tilde{g}^a \rangle - \frac{1}{2}M_\chi\tilde{W}^a\tilde{W}^a - \frac{1}{2}M_\chi\tilde{B}^a\tilde{B}^a - M_\chi\tilde{H}_u^T\epsilon\tilde{H}_d \\ & - \frac{1}{\sqrt{2}}H^\dagger(\tilde{g}_{2u}\sigma^a\tilde{W}^a + \tilde{g}_{1u}\tilde{B})\tilde{H}_u \\ & - \frac{1}{\sqrt{2}}H^T\epsilon(-\tilde{g}_{2d}\sigma^a\tilde{W}^a + \tilde{g}_{1d}\tilde{B})\tilde{H}_d \\ & + h.c. + \dots\end{aligned}$$

Split Model RGEs (1 loop):

$$\frac{dg'}{dt} = \frac{15}{4} kg'^3$$

$$\frac{dg}{dt} = -\frac{7}{12} kg^3$$

$$\frac{dg_3}{dt} = -\left\langle \frac{7}{2}; \frac{5}{2} \right\rangle kg_3^3$$

$$\frac{dh_t}{dt} = \frac{1}{2} kh_t \left[-\frac{9}{4} g^2 - 8g_3^2 - \frac{17}{12} g'^2 + \frac{9}{2} h_t^2 + \frac{1}{2} (\tilde{g}_{1d}^2 + \tilde{g}_{1u}^2 + 3\tilde{g}_{2d}^2 + 3\tilde{g}_{2u}^2) \right]$$

$$\begin{aligned} \frac{d\lambda}{dt} = \frac{1}{2} k \left[-\tilde{g}_{1d}^4 - \tilde{g}_{1u}^4 + \frac{9}{4} g^4 - 5\tilde{g}_{2d}^4 - 4\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u} - 5\tilde{g}_{2u}^4 \right. \\ \left. - 2(\tilde{g}_{1u}^2 + \tilde{g}_{2d}^2)(\tilde{g}_{1d}^2 + \tilde{g}_{2u}^2) \right. \\ \left. + \frac{3}{2} g^2 g'^2 + \frac{3}{4} g'^4 - 12h_t^4 + 2(\tilde{g}_{1d}^2 + \tilde{g}_{1u}^2 + 3\tilde{g}_{2d}^2 + 3\tilde{g}_{2u}^2)\lambda \right. \\ \left. - 9 \left(g^2 + \frac{1}{3} g'^2 \right) \lambda + 12h_t^2 \lambda + 12\lambda^2 \right] \end{aligned}$$

Split Model RGEs (1 loop), cont.:

$$\frac{d\tilde{g}_{1u}}{dt} = \frac{1}{2}k \left[3\tilde{g}_{1d}\tilde{g}_{2d}\tilde{g}_{2u} + \tilde{g}_{1u} \left(\frac{5}{4}\tilde{g}_{1u}^2 + 2\tilde{g}_{1d}^2 + \frac{9}{4}\tilde{g}_{2u}^2 + \frac{3}{2}\tilde{g}_{2d}^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2 \right) \right]$$

$$\frac{d\tilde{g}_{1d}}{dt} = \frac{1}{2}k \left[3\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u} + \tilde{g}_{1d} \left(\frac{5}{4}\tilde{g}_{1d}^2 + 2\tilde{g}_{1u}^2 + \frac{9}{4}\tilde{g}_{2d}^2 + \frac{3}{2}\tilde{g}_{2u}^2 - \frac{9}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2 \right) \right]$$

$$\frac{d\tilde{g}_{2u}}{dt} = \frac{1}{2}k \left[\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d} + \tilde{g}_{2u} \left(\frac{1}{2}\tilde{g}_{1d}^2 + \frac{3}{4}\tilde{g}_{1u}^2 + \tilde{g}_{2d}^2 + \frac{11}{4}\tilde{g}_{2u}^2 - \frac{33}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2 \right) \right]$$

$$\frac{d\tilde{g}_{2d}}{dt} = \frac{1}{2}k \left[\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2u} + \tilde{g}_{2d} \left(\frac{1}{2}\tilde{g}_{1u}^2 + \frac{3}{4}\tilde{g}_{1d}^2 + \tilde{g}_{2u}^2 + \frac{11}{4}\tilde{g}_{2d}^2 - \frac{33}{4}g^2 - \frac{3}{4}g'^2 + 3h_t^2 \right) \right]$$

$$\begin{aligned}
\lambda(M_S) = & \underbrace{\frac{1}{4} \cos^2(2\beta) (g^2 + g'^2)}_{\text{tree level}} \\
& - k \underbrace{\left[\left(\frac{3}{4} - \frac{1}{6} c_{2\beta}^2 \right) g^4 + \frac{1}{2} g^2 g'^2 + \frac{1}{4} g'^4 \right]}_{\text{tree-level term } \overline{\text{DR}} \rightarrow \overline{\text{MS}}} \\
& + 6h_t^2 k \underbrace{\left\{ \left[h_t^2 + \frac{1}{8} (g^2 + g'^2) c_{2\beta} \right] \hat{X}_t^2 - \frac{1}{12} h_t^2 \hat{X}_t^4 \right\}}_{\text{stop-threshold corr.}} \\
& - \underbrace{\frac{1}{4} k h_t^2 (g^2 + g'^2) c_{2\beta}^2 \hat{X}_t^2}_{\text{stop-threshold corr.}} \\
& - \underbrace{\frac{3}{16} k (g'^2 + g^2)^2 s_{4\beta}^2}_{\text{heavy Higgs threshold corr.}}
\end{aligned}$$

$$\tilde{g}_{1u}(M_S) = g s_\beta \left\{ 1 + k \left[\frac{3}{16} g^2 (-2 + 7c_\beta^2) + \frac{1}{20} g'^2 (-44 + 7c_\beta^2) + \frac{9}{4s_\beta^2} h_t^2 \right] \right\}$$

$$\tilde{g}_{1d}(M_S) = g c_\beta \left\{ 1 + k \left[\frac{3}{16} g^2 (-2 + 7s_\beta^2) + \frac{1}{20} g'^2 (-44 + 7s_\beta^2) \right] \right\}$$

$$\tilde{g}_{2u}(M_S) = g' s_\beta \left\{ 1 + k \left[-g^2 \left(\frac{2}{3} + \frac{11}{16} c_\beta^2 \right) + \frac{1}{20} g'^2 (-2 + 7c_\beta^2) + \frac{9}{4s_\beta^2} h_t^2 \right] \right\}$$

$$\tilde{g}_{2d}(M_S) = g' c_\beta \left\{ 1 + k \left[-g^2 \left(\frac{2}{3} + \frac{11}{16} s_\beta^2 \right) + \frac{1}{20} g'^2 (-2 + 7s_\beta^2) \right] \right\}$$

$$\begin{aligned}
\lambda_{\text{SM}}(M_\chi) &= \lambda_\chi(M_\chi) \\
&+ k \left\{ -\frac{7}{12}(\tilde{g}_{1d}^4 + \tilde{g}_{1u}^4) - \frac{9}{4}(\tilde{g}_{2d}^4 + \tilde{g}_{1u}^4) - \frac{3}{2}\tilde{g}_{1d}^2\tilde{g}_{1u}^2 - \frac{7}{2}\tilde{g}_{2d}^2\tilde{g}_{2u}^2 \right. \\
&- \frac{8}{3}\tilde{g}_{1d}\tilde{g}_{1u}\tilde{g}_{2d}\tilde{g}_{2u} - \frac{7}{6}(\tilde{g}_{1d}^2\tilde{g}_{2d}^2 + \tilde{g}_{1u}^2\tilde{g}_{2u}^2) - \frac{1}{6}(\tilde{g}_{1d}^2\tilde{g}_{2u}^2 + \tilde{g}_{1u}^2\tilde{g}_{2d}^2) \\
&- \frac{4}{3}(\tilde{g}_{1d}\tilde{g}_{2u}^2 + \tilde{g}_{1u}\tilde{g}_{2d}^2)(\tilde{g}_{1d}\tilde{g}_{2d} + \tilde{g}_{1u}\tilde{g}_{2u}) \\
&+ \frac{2}{3}\tilde{g}_{1d}\tilde{g}_{1u}(\lambda_\chi - 2\tilde{g}_{1d}^2 - 2\tilde{g}_{1u}^2) + 2\tilde{g}_{2d}\tilde{g}_{2u}(\lambda_\chi - 2\tilde{g}_{2d}^2 - 2\tilde{g}_{2u}^2) \\
&\left. + \frac{1}{3}\lambda_\chi(\tilde{g}_{1d}^2 + \tilde{g}_{1u}^2) + \lambda_\chi(\tilde{g}_{2d}^2 + \tilde{g}_{2u}^2) \right\}
\end{aligned}$$

$$g_{\chi}(M_{\chi}) = g_{\text{SM}}(M_{\chi})$$

$$g'_{\chi}(M_{\chi}) = g'_{\text{SM}}(M_{\chi})$$

$$g_{3,\chi}(M_{\chi}) = g_{3,\text{SM}}(M_{\chi})$$

$$h_{t,\chi}(M_{\chi}) = h_{t,\text{SM}}(M_{\chi}) \left\{ 1 - k \left[\frac{1}{6} \tilde{g}_{1u} \tilde{g}_{1d} + \frac{1}{12} (\tilde{g}_{1u}^2 + \tilde{g}_{1d}^2) \right. \right. \\ \left. \left. + \frac{1}{2} \tilde{g}_{2u} \tilde{g}_{2d} + \frac{1}{4} (\tilde{g}_{2u}^2 + \tilde{g}_{2d}^2) \right] \right\}$$

one-loop logarithms ($M_\chi = M_S$)

$$M_h^{\text{LL}} = -\frac{\alpha}{192\pi M_W^2 s_w^2} \left\{ 288M_t^4 + 144m_t^2 M_Z^2 c_{2\beta} \right. \\ \left. + 4c_{4\beta} \left(22M_W^4 - 64M_W^2 M_Z^2 + 41M_Z^4 \right) - 9M_Z^4 c_{8\beta} \right. \\ \left. - 56M_W^4 - 256M_W^2 M_Z^2 + 101M_Z^4 \right\} \ln \left(\frac{M_S^2}{m_t^2} \right)$$

chargino/neutralino contribution

$$\frac{\alpha M_Z^2}{24\pi c_w^4} \left[(10c_w^2(2c_w^2 - 1) - 1)c_{4\beta} + 2c_w^2(22c_w^2 - 5) + 11 \right] \ln \left(\frac{M_\chi^2}{m_t^2} \right)$$