Reconciling EFT and hybrid calculations

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in collaboration with
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- EFT calculations allow to resum large logarithms
  $\rightarrow$ should be accurate for high SUSY scale M_{Susy}
- miss however terms $\propto v/M_{\text{Susy}}$
- diagrammatic calculation expected to be more accurate for low M_{Susy} ($\lesssim$ few TeV)

Goal

Combine both approaches to get precise results for both regimes.

If not stated otherwise all plots with parameters

$$\tan\beta = 10, \;\; M_{\text{soft}} = \mu = M_A \equiv M_{\text{Susy}}, \;\; A_{b,c,s,e,\mu,\tau} = 0$$

Current status

`FeynHiggs` resummation procedures at the very similar level of accuracy as pure EFT calculations

$$\downarrow$$

expected to see correspondence for high scales, but so far still large discrepancies could be observed

Discussions mainly about

- $\overline{\text{DR}} \leftrightarrow$ OS conversion
- terms induced by momentum dependence of Higgs self-energy

`FeynHiggs` uses mixed OS/$\overline{\text{DR}}$ scheme
$\rightarrow$ to use $\overline{\text{DR}}$ input parameters conversion necessary

Procedure so far

- $m_{\tilde{t}_{1,2}}^{\overline{\text{DR}}}, X_t^{\overline{\text{DR}}}, X_b^{\overline{\text{DR}}} \overset{\mathcal{O}(\alpha_s,\alpha_t,\alpha_b)}{\longrightarrow} M_{\tilde{t}_{1,2}}, X_t^{\text{OS}}, X_b^{\text{OS}}$
- Forget about $m_{\tilde{t}_{1,2}}^{\overline{\text{DR}}}, X_t^{\overline{\text{DR}}}, X_b^{\overline{\text{DR}}}$, use $M_{\tilde{t}_{1,2}}, X_t^{\text{OS}}, X_b^{\text{OS}}$ as 'new' input parameters
- No conversion of $\mu, M_A, M_{\tilde{b}_{1,2}}, ...$

Two problems with this approach

1. Conversion induces terms beyond 2L level
2. X_t, entering in resummation procedure, is calculated by

$$X_t^{\overline{\text{DR}},\text{EFT}} = X_t^{\text{OS}} \left[1 + \left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi} \left(1 - \hat{X}_t^2 \right) \right) \ln \left(\frac{M_S^2}{m_t^2} \right) \right].$$

$\Rightarrow X_t^{\overline{\text{DR}},\text{EFT}} \neq X_t^{\overline{\text{DR}}}$

How big are these effects?

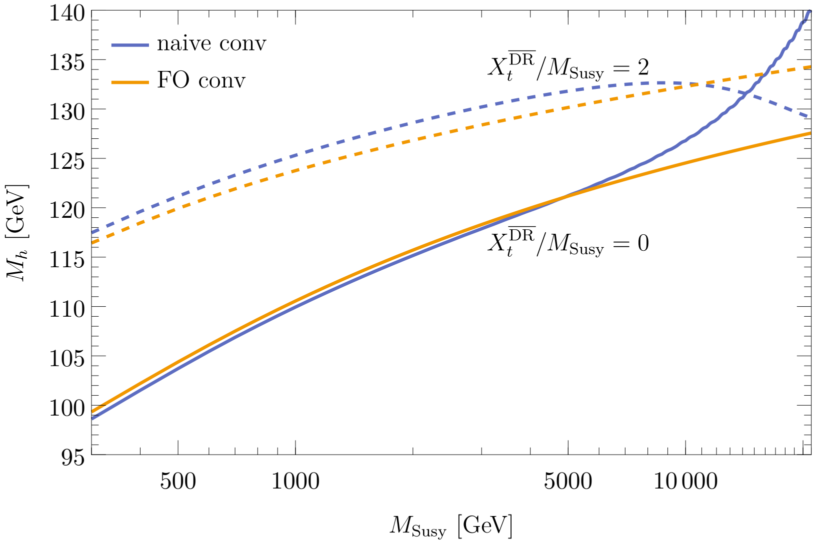
Change renormalization of X_t, $M_{\tilde{t}_{1,2}}$ from OS to $\overline{\text{DR}}$ scheme and compare.

also set $X_t^{\overline{\text{DR}},\text{EFT}} = X_t^{\overline{\text{DR}}}$

$$\Downarrow$$

both problems solved by construction

- practical implementation: reparametrization of final result (fixed-order conversion)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FO conv
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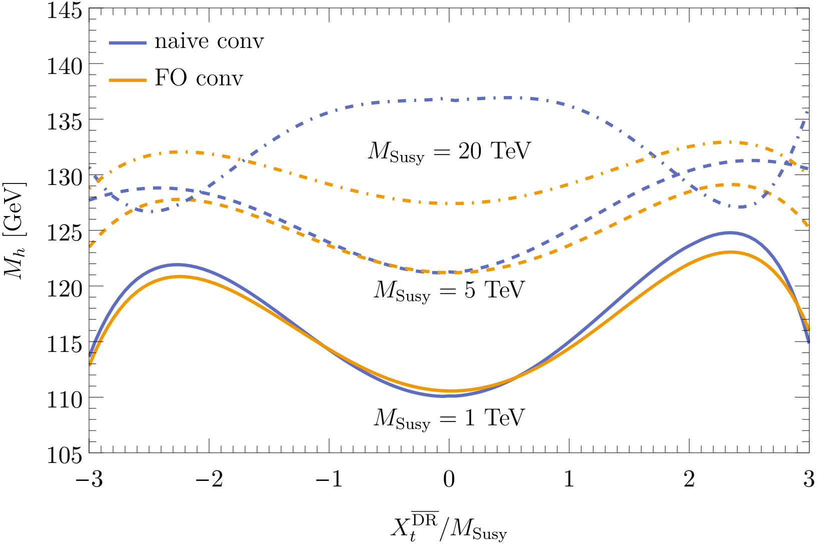
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145
140
135
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125
120
115
110
105
−3
−2
−1
0
1
2
3

Diagrammatic calculation

In limit $M_A \gg M_Z$ Higgs pole mass is determined by

$$(M_h^2)_{\text{FD}} = m_h^2 - \hat{\Sigma}_{hh}^{\text{MSSM}}(M_h^2) = \\ = m_h^2 - \hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2) + \hat{\Sigma}_{hh}^{\text{MSSM}\prime}(m_h^2)\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2) + \ldots$$

EFT calculation

Calculate $\lambda(M_t)$ by RGE running. Extract pole mass out of $\lambda(M_t)$ via

$$
\begin{aligned}
&(M_h^2)_{\text{EFT}} = v^2\lambda_{\text{OS}} = v^2\lambda(M_t) - v^2\delta\lambda = \text{ (finite parts only)} \\
&= v^2\lambda(M_t) - \frac{\delta T}{v} - \delta M_h^2 + \lambda\delta v^2 + ... = \\
&= v^2_{\overline{\text{MS}}}\lambda(M_t) + \frac{\tilde{T}^{\text{SM}}}{v} - \tilde{\Sigma}_{hh}^{\text{SM}}(M_h^2) + ... = \\
&= v^2_{\overline{\text{MS}}}\lambda(M_t) + \frac{\tilde{T}^{\text{SM}}}{v} - \tilde{\Sigma}_{hh}^{\text{SM}}(m_h^2) + \tilde{\Sigma}_{hh}^{\text{SM}\prime}(m_h^2)\Big(...\Big) + ...
\end{aligned}
$$

Hybrid approach in FeynHiggs

Calculate $\lambda(M_t)$ by RGE running. Extract pole mass out of $\lambda(M_t)$ via

$$
\begin{aligned}
(M_h^2)_{\text{FH}} =& \\
=& m_h^2 - \underbrace{\hat{\Sigma}_{hh}^{\text{MSSM}}(M_h^2)}_{\text{FO result}} + \underbrace{\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}}}_{\text{EFT result}} + \underbrace{\left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{logs}}}_{\text{subtraction term}} = \\
=& m_h^2 + \left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} - \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{nolog}} \\
& - \hat{\Sigma}_{hh}^{\text{MSSM}\prime}(m_h^2)\left(\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} - \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)\right]_{\text{nolog}}\right) + \ldots
\end{aligned}
$$

Comparison of logarithmic terms

$$(M_h^2)^{\text{logs}}_{\text{EFT}} = \left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} - \tilde{\Sigma}^{\text{SM}\prime}_{hh}(m_h^2)\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \ldots$$

$$\begin{aligned}(M_h^2)^{\text{logs}}_{\text{FH}} = &\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \left[\hat{\Sigma}^{\text{MSSM}\prime}_{hh}(m_h^2)\right]_{\text{logs}}\left[\hat{\Sigma}^{\text{MSSM}}_{hh}(m_h^2)\right]_{\text{nolog}} \\ &- \hat{\Sigma}^{\text{MSSM}\prime}_{hh}(m_h^2)\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \ldots\end{aligned}$$

In heavy SUSY limit $\hat{\Sigma}^{\text{MSSM}}_{hh} \simeq \hat{\Sigma}^{\text{SM}}_{hh} + \hat{\Sigma}^{\text{nonSM}}_{hh}$. Therefore

$$\begin{aligned}\Delta^{\text{logs}}_{p^2} \equiv &(M_h^2)^{\text{logs}}_{\text{FH}} - (M_h^2)^{\text{logs}}_{\text{EFT}} = \\ = &\left[\hat{\Sigma}^{\text{nonSM}\prime}_{hh}(m_h^2)\right]_{\text{logs}}\left[\hat{\Sigma}^{\text{MSSM}}_{hh}(m_h^2)\right]_{\text{nolog}} \\ &- \hat{\Sigma}^{\text{nonSM}\prime}_{hh}(m_h^2)\left[v^2_{\overline{\text{MS}}}\lambda(M_t)\right]_{\text{logs}} + \ldots\end{aligned}$$

Very similar for non-logarithmic terms.

At strict two-loop level

$$
\begin{aligned}
(M_h^2)_{\text{FD}} =& m_h^2 - \hat{\Sigma}_{hh}^{\text{MSSM},(1)}(m_h^2) - \hat{\Sigma}_{hh}^{\text{MSSM},(2)}(m_h^2) \\
&+ \left(\hat{\Sigma}_{hh}^{\text{nonSM},(1)\prime}(m_h^2) + \hat{\Sigma}_{hh}^{\text{SM},(1)\prime}(m_h^2)\right)\hat{\Sigma}_{hh}^{\text{MSSM},(1)}(m_h^2).
\end{aligned}
$$

The renormalized two-loop self-energy reads

$$
\begin{aligned}
\hat{\Sigma}_{hh}^{\text{MSSM},(2)}(0) &= \Sigma_{hh}^{\text{MSSM},(2)}(0) + \frac{\partial}{\partial v^2}\hat{\Sigma}_{hh}^{\text{MSSM},(1)}(m_h^2)\cdot(\delta v^2)^{\text{MSSM}} + \ldots = \\
&= \Sigma_{hh}^{\text{MSSM},(2)}(0) - \hat{\Sigma}_{hh}^{\text{MSSM},(1)}(m_h^2)\frac{(\delta v^2)^{\text{MSSM}}}{v^2} + \ldots
\end{aligned}
$$

In the decoupling limit, we verified by explicit calculation

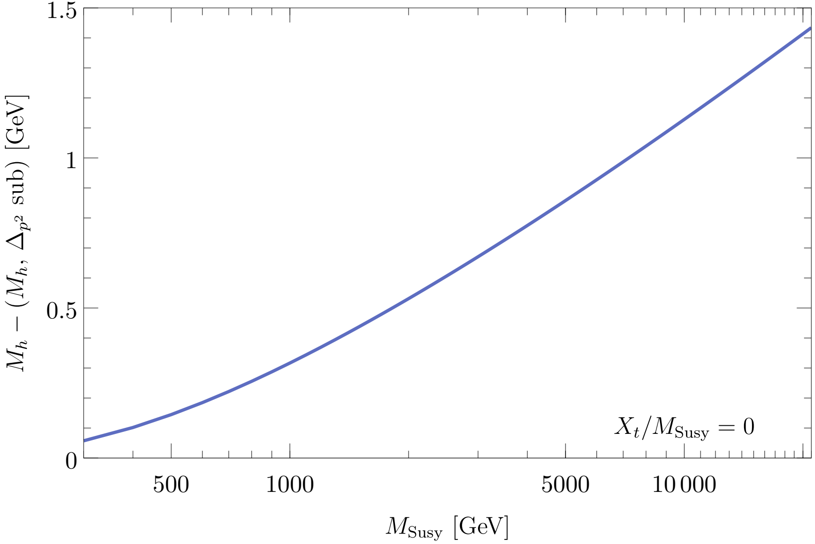
$$
\frac{(\delta v^2)^{\text{MSSM}}}{v^2} = \frac{(\delta v^2)^{\text{SM}}}{v^2} - \hat{\Sigma}_{hh}^{\text{nonSM},(1)\prime}(m_h^2)
$$

Observation

2L subloop renormalization cancels 2L term induced by momentum dependence of non SM contributions to Higgs self-energy

- Argument holds for all 2L contributions
- Full 2L calculation however not availabe
  $\rightarrow$ induced terms of e.g. $\mathcal{O}(\alpha_t \alpha)$ are not compensated
- Might also holds for higher loop orders

Explicit derivation of terms induced by momentum dependence allows to investigate their numerical significance.


1.5
1
0.5
0
$M_h - (M_h,\, \Delta_{p^2}\ \text{sub})$ [GeV]
500
1000
5000
10 000
M_{Susy} [GeV]
$X_t/M_{\text{Susy}} = 0$

Differences to SUSYHD

- different renormalization schemes
- different extraction of pole mass
- small differences in EFT calculations

- different renormalization of $\tan\beta$

- $\mathcal{O}(v/M_{\text{Susy}})$ terms
- non-logarithmic terms
- ...?

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  SUSYHD by default uses NNNLO for $y_t(M_t)$ → deactivated for all comparison plots
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  → negligible for $\tan\beta = 10$ ✓
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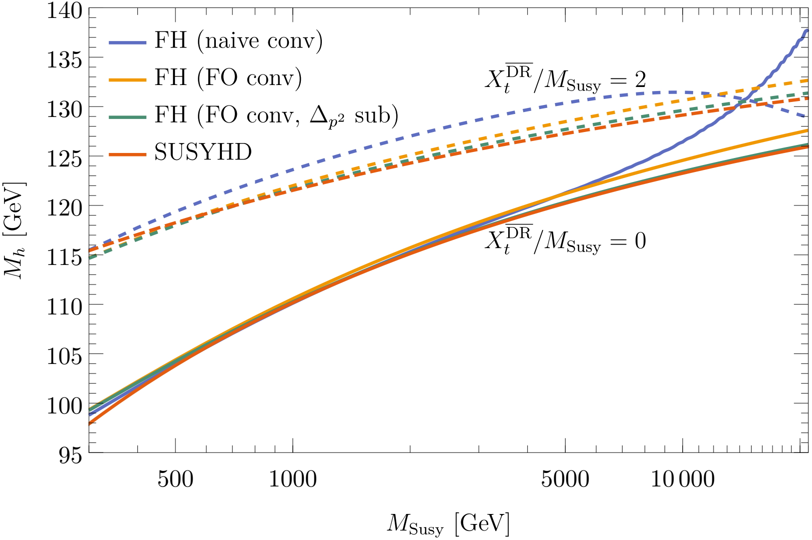
FH (naive conv)
FH (FO conv)
FH (FO conv, Δ_{p^2} sub)
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M_h [GeV]
M_{Susy} [GeV]
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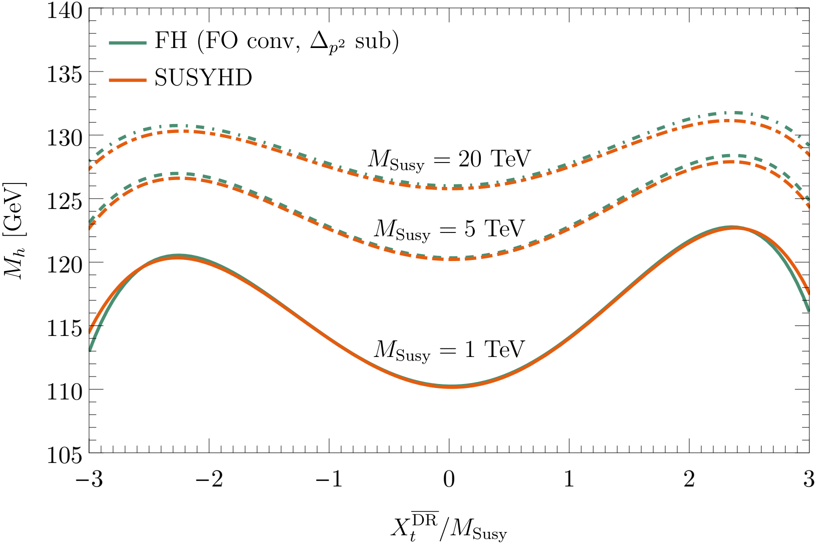
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$\longrightarrow$ **overall very good agreement**

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- different extraction of pole mass $\rightarrow$ effect isolated ✓
- small differences in EFT calculations

- different renormalization of $\tan\beta$
  $\rightarrow$ negligible for $\tan\beta = 10$ ✓
- $\mathcal{O}(v/M_{\text{Susy}})$ terms $\rightarrow$ negligible for $M_{\text{Susy}} \gtrsim 1$ TeV ✓
- non-logarithmic terms

- ...?

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- non-logarithmic terms $\rightarrow$ different parametrization of y_t explains remaining differences
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- ...? → nothing significant

$\longrightarrow$ **overall very good agreement**

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- different extraction of pole mass $\rightarrow$ effect isolated ✓
- small differences in EFT calculations $\rightarrow$ negligible ✓

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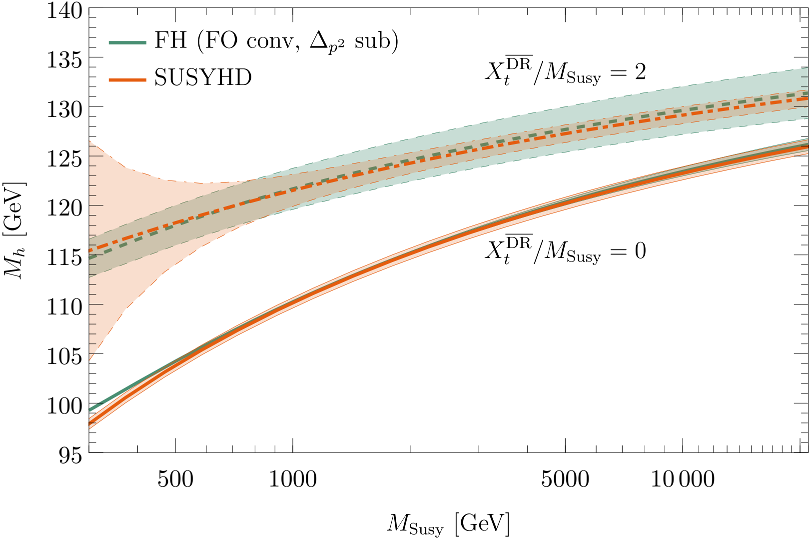
- non-logarithmic terms $\rightarrow$ different parametrization of y_t explains remaining differences
- ...? $\rightarrow$ nothing significant

$\Downarrow$

Differences between EFT and hybrid calculations completely understood?!



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Conclusion

- Naive $\overline{\text{DR}} \to$ OS conversion induces large higher order terms
- Momentum dependence of SUSY contributions to Higgs self-energy induces terms not present in pure EFT calculation
- Taking into account these effects
  $\to$ excellent agreement of `FeynHiggs` with `SUSYHD` found
- Remaining differences can largely be explained by different parametrizations of non-logarithmic terms

What's next for FeynHiggs

Next version: `FeynHiggs 2.13.0`

- Improved calculation of EWPO (2L corrections to M_W)
- Implementation of improved 1L thresholds (degenerate case, additional terms in effective EWino-Higgsino-Higgs couplings)
- Allow for `looplevel < 2` combined with `loglevel > 0`

To come later:

- Improved $\overline{\text{DR}} \to$ OS conversion (option of renormalizing X_t in the $\overline{\text{DR}}$ scheme)
- Improved handling of momentum dependence
- ... ($\to$ Peter's and Sebastian's talks)

The OS vev-counterterm is given by

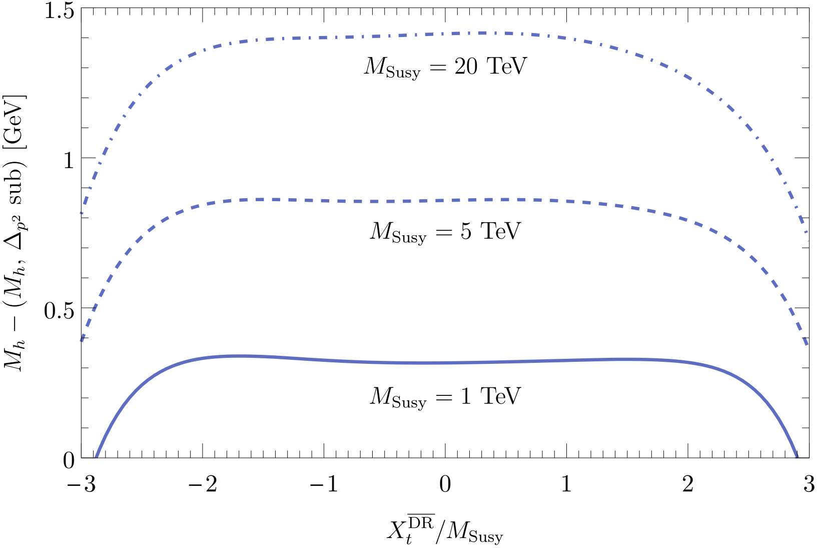
$$\delta v^2 = v^2 \left[\frac{\delta M_W^2}{M_W^2} + \frac{c_w^2}{s_w^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) - \frac{\delta e^2}{e^2} \right] \overset{\mathcal{O}(\alpha_s,\alpha_t)}{=}$$
$$= v^2 \left(-\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + \text{ SM corrections} \right).$$

The Higgs pole mass is calculated via

$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(0) + \Sigma_{hh}^{(1)\prime}(m_h^2)\Sigma_{hh}^{(1)}(m_h^2) + ...$$

The renormalized two-loop self-energy reads

$$\hat{\Sigma}_{hh}^{(2)}(0) = \Sigma_{hh}^{(2)}(0) + \frac{\partial}{\partial v^2}\hat{\Sigma}_{hh}^{(1)}(m_h^2) \cdot \delta v^2 + ... =$$
$$= \Sigma_{hh}^{(2)}(0) - \hat{\Sigma}_{hh}^{(1)}(m_h^2)\frac{\delta v^2}{v^2} + ... =$$
$$= \Sigma_{hh}^{(2)}(0) + \hat{\Sigma}_{hh}^{(1)}(m_h^2)\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + ...$$



$M_{\mathrm{Susy}} = 20$ TeV
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0.5
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1.5
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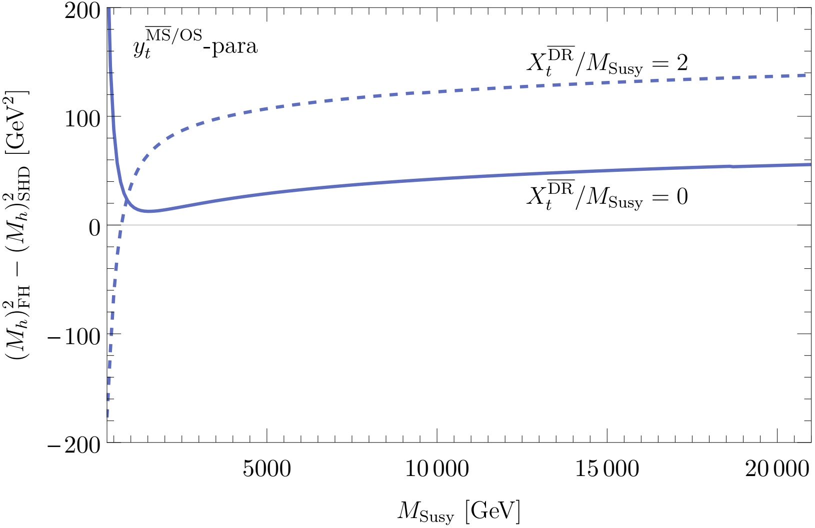
$X_t^{\overline{\mathrm{DR}}}/M_{\mathrm{Susy}}$



$\longrightarrow$ nearly constant difference for high scales

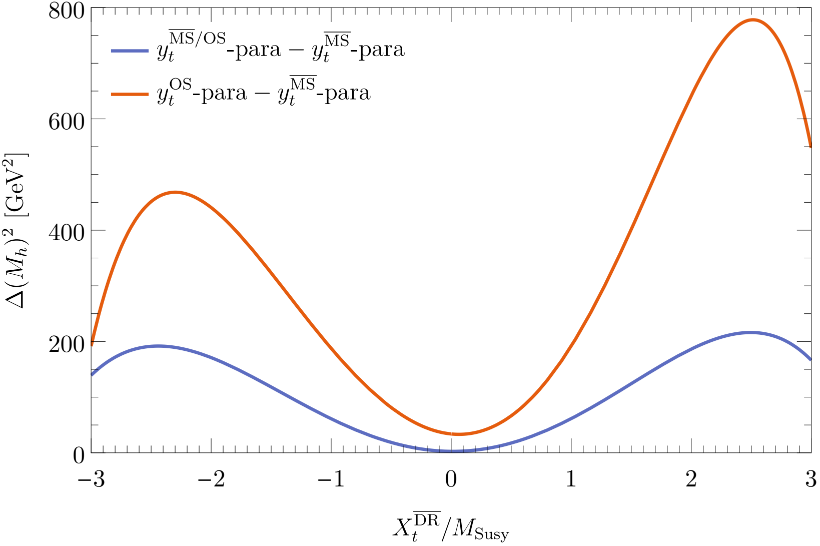
Origin

Different parametrization of non-logarithmic terms

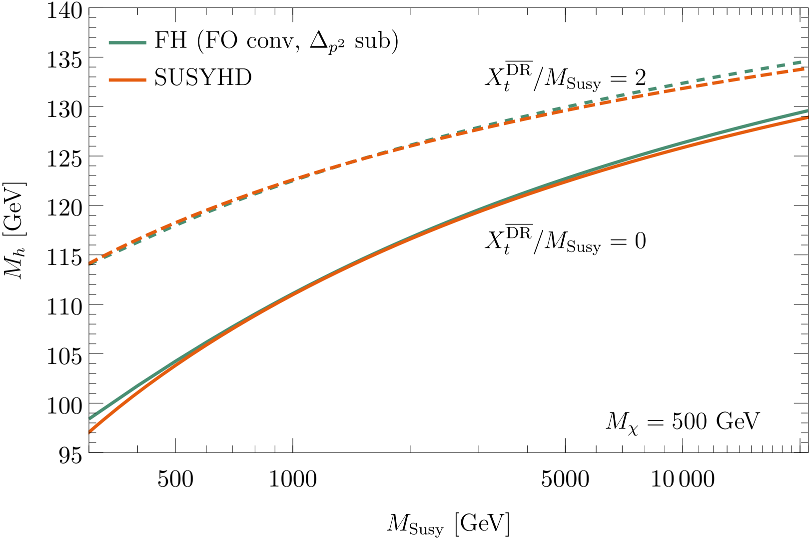
Three ways to parametrize top Yukawa coupling in FO result

- $M_t/v \rightarrow$ `FeynHiggs` with `runningMT = 0`
- $\overline{m}_t/v \rightarrow$ `FeynHiggs` with `runningMT = 1`
- $y_t^{\overline{\mathrm{MS}}} = \overline{m}_t/v_{\overline{\mathrm{MS}}} \rightarrow$ `SUSYHD`

Equivalent at 2L order, but induces differences at higher order



→ explains constant difference almost completely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Uncertainty estimate of SUSYHD

1. EFT uncertainty
    - $\mathcal{O}(v/M_S)$ terms
    - estimated by $v/M_S \cdot$(1L correction)
2. SM uncertainty:
    - higher order corrections to pole mass extraction
    - estimated by (de)activating higher order corrections to y_t and $\delta\lambda$
3. SUSY uncertainty:
    - higher order threshold corrections
    - estimated by variation of matching scale $1/2 < Q/M_S < 2$

Uncertainty estimate of `FeynHiggs`

1. Scale variation:
   - variation of renormalization scale between $1/2M_t$ and $2M_t$
2. Renormalization scheme dependence:
   - switching between OS top mass and $\overline{\text{MS}}$ top mass
3. $\tan\beta$ enhanced correction
   - (de)activating resummation of bottom Yukawa coupling