Update on large log resummation in `FeynHiggs`

Henning Bahl

Max-Planck-Institut für Physik, München

Higgs Days
20.9.2017, Santander

- EFT calculations allow to resum large logarithms
  $\rightarrow$ should be accurate for high SUSY scale M_{Susy}
- miss however terms $\propto v/M_{\text{Susy}}$
- diagrammatic calculation expected to be more accurate for low M_{Susy} ($\lesssim$ few TeV)

Goal

Combine both approaches to get precise results for both regimes.

Procedure in `FeynHiggs`

1. calculate fixed-order corrections
2. subtract logarithms already contained in fixed-order result
3. resum logarithms using EFT approach
4. add resummed logarithms to fixed-order result

Current status

- fixed-order → full 1L + $\mathcal{O}\left(\alpha_s(\alpha_t+\alpha_b),(\alpha_t+\alpha_b)^2\right)$
- EFT → full LL+NLL, $\mathcal{O}(\alpha_s\alpha_t,\alpha_t^2)$ NNLL,
  intermediary EWino threshold

FeynHiggs 2.14.0

implements changes discussed in [HB Heinemeyer Hollik Weigglein 1706.00346]

- optional $\overline{\text{DR}}$ renormalization of stop sector
- improved calculation of pole masses/Z factors
- small improvements of resummation routines
  - now $v_{\overline{\text{MS}}}$ is used
  - improved 2L subtraction term for `runningMT = 1` ($\overline{\text{MS}}$ top mass)

Optional $\overline{\text{DR}}$ renormalization of stop sector

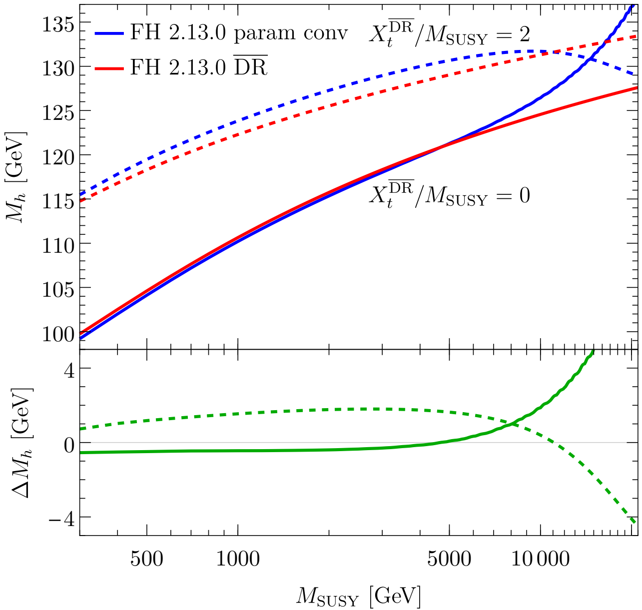
So far

- FH uses OS scheme for renormalization of stop sector
- 1L parameter conversion in case of $\overline{\text{DR}}$ input parameters

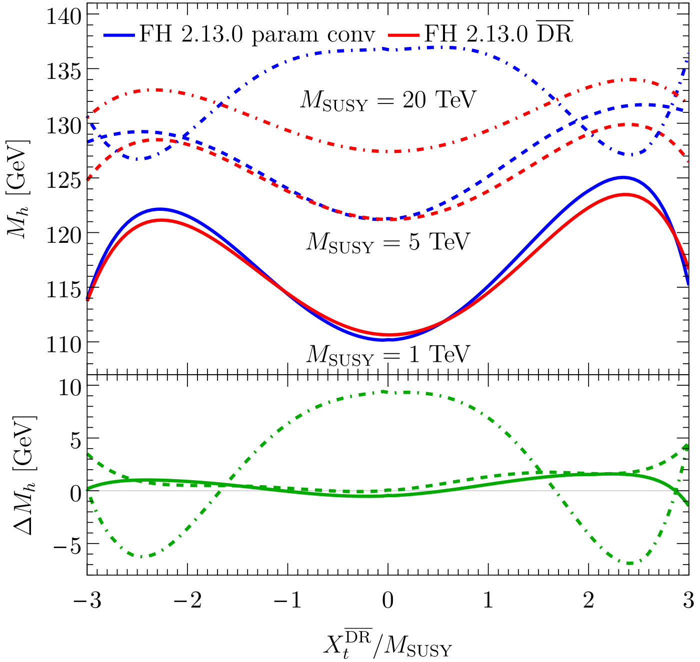
↯ conversion not adequate for result containing resummed logs

Therefore

- optional $\overline{\text{DR}}$ renormalization of stop sector
- automatically active if parameter Qt $\neq 0$
- for sbottom sector still a parameter conversion is used



FH 2.13.0 param conv
FH 2.13.0 $\overline{\mathrm{DR}}$
$X_t^{\overline{\mathrm{DR}}}/M_{\mathrm{SUSY}} = 2$
$X_t^{\overline{\mathrm{DR}}}/M_{\mathrm{SUSY}} = 0$
M_h [GeV]
135
130
125
120
115
110
105
100
ΔM_h [GeV]
4
0
−4
500
1000
5000
10 000
M_{SUSY} [GeV]


FH 2.13.0 param conv
FH 2.13.0 $\overline{\text{DR}}$
$M_{\text{SUSY}} = 20$ TeV
$M_{\text{SUSY}} = 5$ TeV
$M_{\text{SUSY}} = 1$ TeV
M_h [GeV]
ΔM_h [GeV]
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}}$

Improved calculation of pole masses/Z factors I

For $M_A \gg M_Z$, we have to solve $p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) = 0$

$$\Rightarrow M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(m_h^2) + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2) + ...$$

- non-SM contributions to $\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2)$ are cancelled by subloop-renormalization in $\hat{\Sigma}_{hh}^{(2)}(m_h^2) \to$ vev-CT
- holds generally at 2L (probably also at higher orders)
- but FH includes $\hat{\Sigma}_{hh}^{(2)}$ only for vanishing electroweak couplings $\to$ incomplete cancellation

Solution easy for $M_A \gg M_Z$, but what to do for $M_A \sim M_Z$?

Improved calculation of pole masses/Z factors II

Need to determine poles of inverse propagator matrix

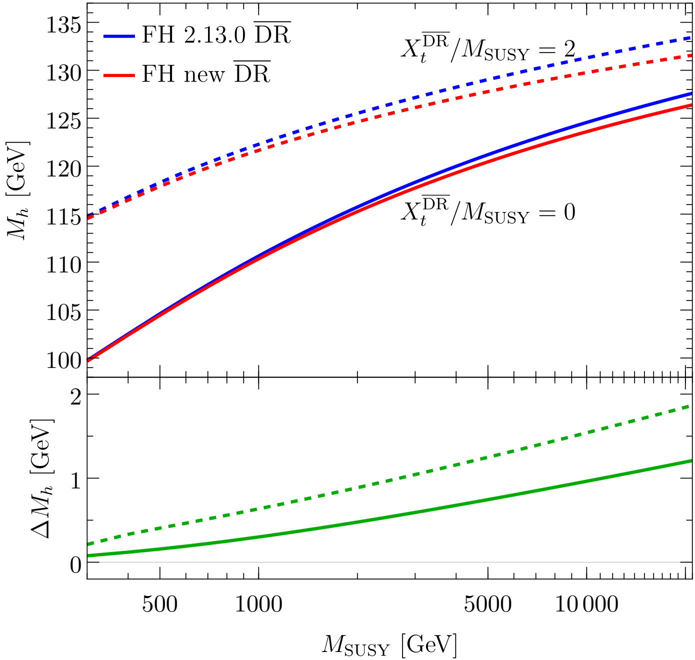
$$\Delta^{-1}(p^2) =$$
$$= \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}^{(1)}_{hh}(p^2) + \hat{\Sigma}^{(2)}_{hh}(0) & \hat{\Sigma}^{(1)}_{hH}(p^2) + \hat{\Sigma}^{(2)}_{hH}(0) \\ \hat{\Sigma}^{(1)}_{hH}(p^2) + \hat{\Sigma}^{(2)}_{hH}(0) & p^2 - m_H^2 + \hat{\Sigma}^{(1)}_{HH}(p^2) + \hat{\Sigma}^{(2)}_{HH}(0) \end{pmatrix}$$

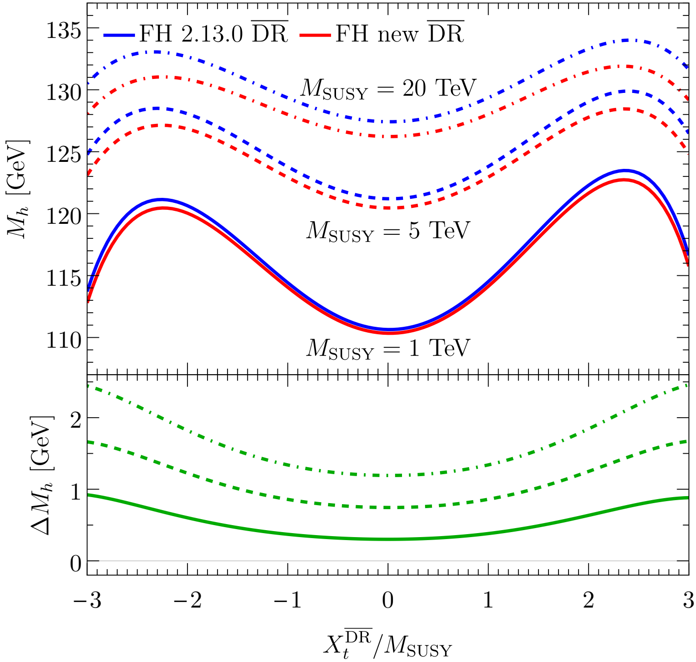
At 1L level $M_h^2 = m_h^2 - \hat{\Sigma}^{(1)}_{hh}(m_h^2) \to$ expand around 1L solution

$\Rightarrow$ determine poles of

$$\Delta^{-1}_{hh}(p^2) = p^2 - m_h^2 + \hat{\Sigma}^{(1)}_{hh}(m_h^2) + \hat{\Sigma}^{(2)}_{hh}(0) - \left[\hat{\Sigma}^{(1)\prime}_{hh}(m_h^2)\hat{\Sigma}^{(1)}_{hh}(m_h^2)\right]_{g=g_Y=0}$$
$$\Delta^{-1}_{hH}(p^2) = + \hat{\Sigma}^{(1)}_{hH}(m_h^2) + \hat{\Sigma}^{(2)}_{hH}(0) - \left[\hat{\Sigma}^{(1)\prime}_{hH}(m_h^2)\hat{\Sigma}^{(1)}_{hh}(m_h^2)\right]_{g=g_Y=0}$$
$$\Delta^{-1}_{HH}(p^2) = p^2 - m_H^2 + \hat{\Sigma}^{(1)}_{HH}(m_h^2) + \hat{\Sigma}^{(2)}_{HH}(0) - \left[\hat{\Sigma}^{(1)\prime}_{HH}(m_h^2)\hat{\Sigma}^{(1)}_{hh}(m_h^2)\right]_{g=g_Y=0}$$

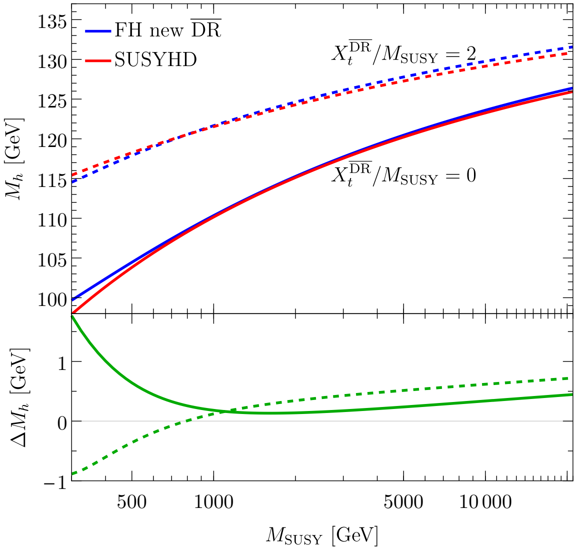
For determination of M_H expand around $M_H^2 = m_H^2 - \hat{\Sigma}^{(1)}_{HH}(m_H^2)$

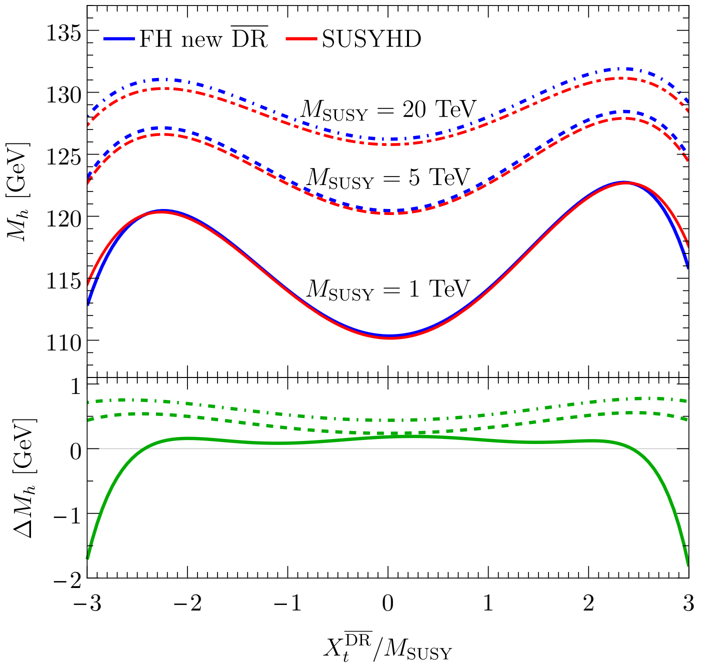

FH 2.13.0 $\overline{\text{DR}}$
FH new $\overline{\text{DR}}$
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 2$
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 0$
M_h [GeV]
135
130
125
120
115
110
105
100
ΔM_h [GeV]
2
1
0
500
1000
5000
10 000
M_{SUSY} [GeV]


FH 2.13.0 $\overline{\text{DR}}$
FH new $\overline{\text{DR}}$
$M_{\text{SUSY}} = 20$ TeV
$M_{\text{SUSY}} = 5$ TeV
$M_{\text{SUSY}} = 1$ TeV
M_h [GeV]
ΔM_h [GeV]
135
130
125
120
115
110
2
1
0
−3
−2
−1
0
1
2
3
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}}$

Comparison to SUSYHD for single-scale scenario

$$\tan\beta = 10,$$
$$M_{\text{soft}} = M_{\text{Susy}},$$
$$\mu = M_A = M_{\text{Susy}},$$
$$A_{b,c,s,e,\mu,\tau} = 0$$



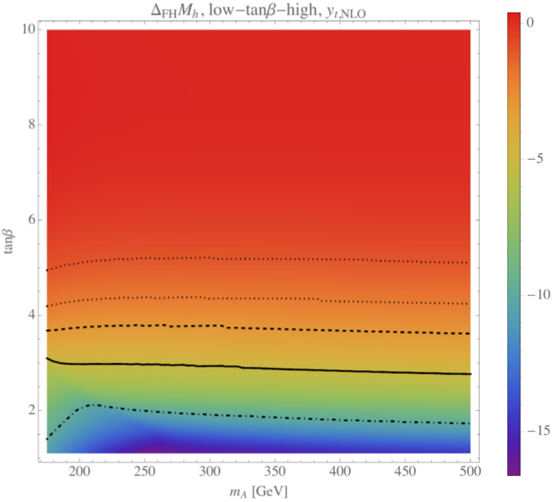

FH new $\overline{\text{DR}}$
SUSYHD
$M_{\text{SUSY}} = 20$ TeV
$M_{\text{SUSY}} = 5$ TeV
$M_{\text{SUSY}} = 1$ TeV
M_h [GeV]
ΔM_h [GeV]
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}}$
135
130
125
120
115
110
1
0
−1
−2
−3
−2
−1
0
1
2
3

Low M_A: Current status

Resummation routines built into FH assume $M_A = M_{\text{SUSY}}$
$\rightarrow$ what if $M_{\text{SUSY}} \gg M_t$ but $M_A \sim M_t$?

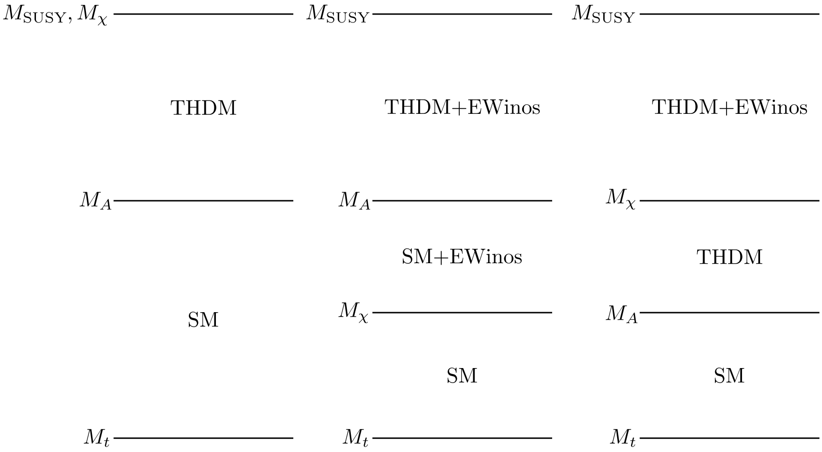
- Need to consider effective THDM for correct resummation
- Haber & Hempfling (1993), Lee & Wagner (2015), ...

Low-tanb-high scenario



$\mu = 1.5$ TeV, $M_2 = 2$ TeV, $A_{b,..} = 2$ TeV, $M_{\rm SUSY}$ and X_t chosen to get $M_h = 125$ GeV

EFTs for low M_A



$$M_\chi = M_1 = M_2 = \mu$$

EFT calculation

- all possible hierarchies taken into account
  - THDM type III → 12 effective couplings ($\lambda_{1..7}, h_t, h'_t$)
  - THDM type III + EWinos → 20 effective couplings ($\lambda_{1..7}, h_t, h'_t$ + gaugino-Higgsino-Higgs couplings)
- full 2L running for all effective couplings (RGEs via SARAH)
- full 1L threshold corrections for all effective couplings
- $\mathcal{O}(\alpha_s\alpha_t)$ threshold corrections for λ_i's

Matching to fixed order calculation

- Running from M_{SUSY} to $M_A \to \Delta\hat{\Sigma}_{11}, \Delta\hat{\Sigma}_{12}, \Delta\hat{\Sigma}_{22}$, e.g.

  $$\Delta\hat{\Sigma}_{11} = \\ = \left[M_A^2 s_\beta^2 + v^2 \left(3\lambda_1 c_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5) s_\beta^2 + 6\lambda_6 s_\beta c_\beta\right)\right]_{Q=M_A} \\ - \text{subtraction terms}$$

- Running from M_A to $M_t \to \Delta\hat{\Sigma}_{22} = \lambda(m_t) v^2 / c_\alpha^2$ (as done for $M_A = M_{\text{SUSY}}$)

Matching to fixed order calculation II

Normalization of Higgs doublets

MSSM and THDM Higgs doublets have not the same normalization

- LSZ theorem yields (at the 1L level)

$$\begin{pmatrix} \Phi_1^{\text{THDM}} \\ \Phi_2^{\text{THDM}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 + \frac{1}{2}\Delta\Sigma'_{11} & \frac{1}{2}\Delta\Sigma'_{12} \\ \frac{1}{2}\Delta\Sigma'_{12} & 1 + \frac{1}{2}\Delta\Sigma'_{22} \end{pmatrix}}_{=U_{\Delta\Phi}} \begin{pmatrix} \Phi_1^{\text{MSSM}} \\ \Phi_2^{\text{MSSM}} \end{pmatrix}$$

with $\Delta\Sigma'_{ij} = \Sigma_{ij}^{\text{MSSM}} - \Sigma_{ij}^{\text{THDM}}$

$$\Rightarrow \Delta^{-1}_{\text{MSSM}}(p^2) = U^T_{\Delta\Phi}\Delta^{-1}_{\text{THDM}}(p^2)U_{\Delta\Phi}$$

- pole masses do not depend on absolute field normalization $\rightarrow$ not important for pure EFT calculation

Matching to fixed order calculation III

Hybrid calculation in `FeynHiggs`:

$$\Delta_{\texttt{FH}}^{-1}(p^2) =$$
$$= \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{FO}}(p^2) + \Delta\Sigma_{hh}^{\text{logs}} & \hat{\Sigma}_{hH}^{\text{FO}}(p^2) + \Delta\Sigma_{hH}^{\text{logs}} \\ \hat{\Sigma}_{hH}^{\text{FO}}(p^2) + \Delta\Sigma_{hH}^{\text{logs}} & p^2 - m_H^2 + \hat{\Sigma}_{HH}^{\text{FO}}(p^2) + \Delta\Sigma_{HH}^{\text{logs}} \end{pmatrix}$$

with $\Delta\Sigma_{ij}^{\text{logs}} = \Sigma_{ij}^{\text{EFT}} - \Sigma_{ij}^{\text{sub}}$

"Relative" normalization important for

- correct merging of EFT result (THDM normalization) with fixed order result (MSSM renormalization)
- calculation of 1L and 2L subtraction terms

Matching to fixed order calculation IV

How to implement different normalization?
→ finite field normalization in fixed-order calculation

$$\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} \to \begin{pmatrix} 1+\frac{1}{2}\delta^{(1)}Z_{11}+\frac{1}{2}\Delta^{(2)}Z_{11} & \frac{1}{2}\delta^{(1)}Z_{12}+\frac{1}{2}\Delta^{(2)}Z_{12} \\ \frac{1}{2}\delta^{(1)}Z_{12}+\frac{1}{2}\Delta^{(2)}Z_{12} & 1+\frac{1}{2}\delta^{(1)}Z_{22}+\frac{1}{2}\Delta^{(2)}Z_{22}\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix},$$

$$\Delta Z_{ij} = \delta^{(2)}Z_{ij} - \frac{1}{4}\left(\delta^{(1)}Z_{ij}\right)^2$$

- choose $\delta^{(1)}Z_{ij}\big|_{\text{fin}} = \Delta\Sigma_{ij}$
- $\delta^{(2)}Z_{ij}$ drops out completely
  → 2L relation between Φ^{MSSM} and Φ^{THDM} not needed

Affect on $\tan\beta$

$$\delta^{(1)} t_\beta = \tfrac{1}{2} t_\beta \left(\delta Z_{22}^{(1)} - \delta Z_{11}^{(1)}\right) + \tfrac{1}{2}\left(1 - t_\beta^2\right)\delta Z_{12}^{(1)}$$

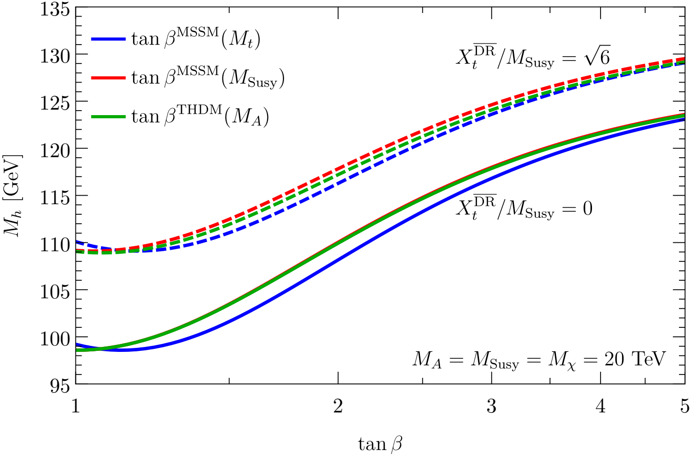
- finite field normalization changes definition of t_β
- renormalization scale of fixed-order calculation by default chosen to be M_t
- scale of THDM $\to M_A$

$\Rightarrow t_\beta^{\text{MSSM}}(M_t) \xrightarrow{\delta Z\big|_{\text{fin}}} t_\beta^{\text{THDM}}(M_A)$ in fixed-order calculation

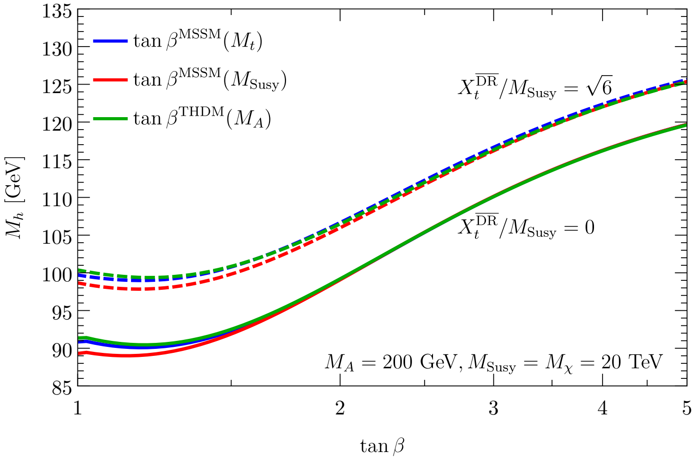
$$\downarrow$$

$$t_\beta^{\text{MSSM}}(M_t) = t_\beta^{\text{THDM}}(M_A)\cdot$$
$$\cdot \left[1 - \frac{y_t^2}{(4\pi)^2 s_\beta^2}\left(\frac{3}{2}\ln\frac{M_A^2}{M_t^2} + \frac{1}{4}(\hat{A}_t - \hat{\mu}/t_\beta)(\hat{A}_t + \hat{\mu} t_\beta)\right)\right]$$

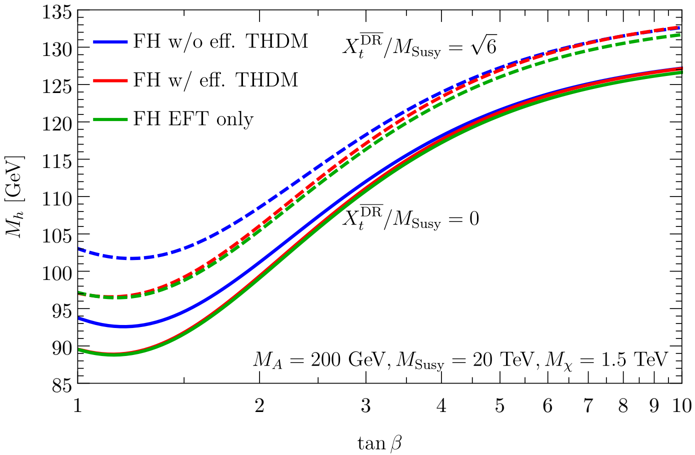
$\tan\beta$ definition ($M_A = M_{\text{SUSY}}$, fixed-order only)



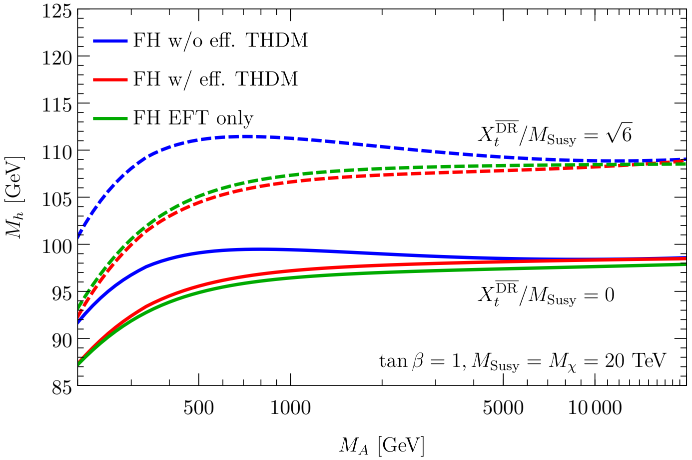
$\tan\beta$ definition ($M_A \ll M_{\text{SUSY}}$, fixed-order only)



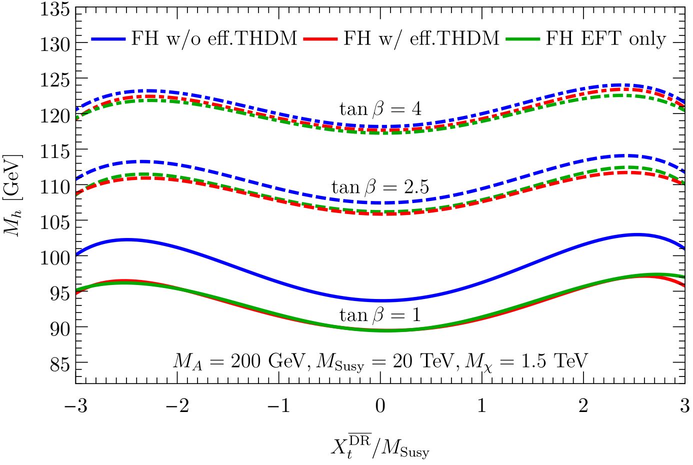
Results I: tan β scan



Results II: M_A scan



Results III: $X_t^{\overline{\mathrm{DR}}}$ scan



X_t conversion

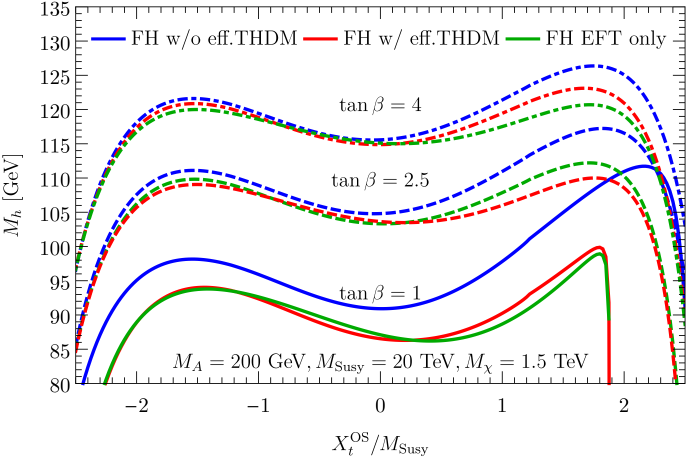
- For fixed-order calculation OS renormalization can be used
- To combine with EFT calculation conversion of X_t needed

For low M_A extra log appear in 1L conversion:

$$\widetilde{X}_t(M_{\text{SUSY}}) = X_t^{\text{OS}}\left\{1 + \left[\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}(1-\hat{X}_t^2)\right]\ln\frac{M_{\text{SUSY}}^2}{M_t^2} - \frac{3}{16\pi}\frac{\alpha_t}{t_\beta^2}(1-\hat{Y}_t^2)\ln\frac{M_{\text{SUSY}}^2}{M_A^2}\right\}$$

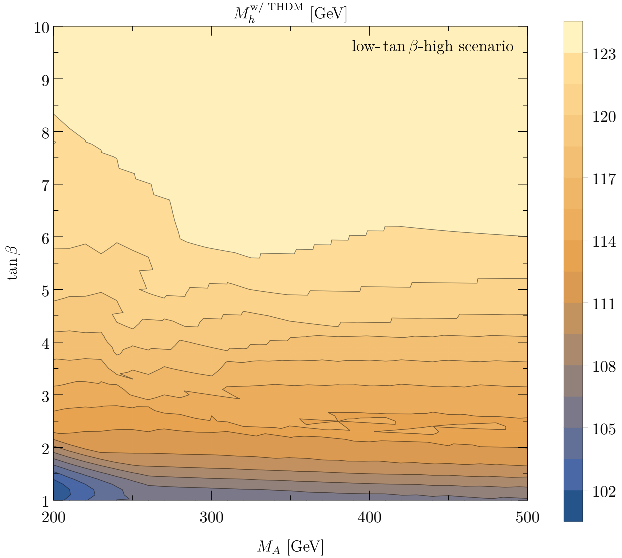
$\hat{X}_t = \hat{A}_t - \hat{\mu}/t_\beta, \;\; \hat{Y}_t = \hat{A}_t + \hat{\mu}t_\beta$

Results IV: X_t^{OS} scan

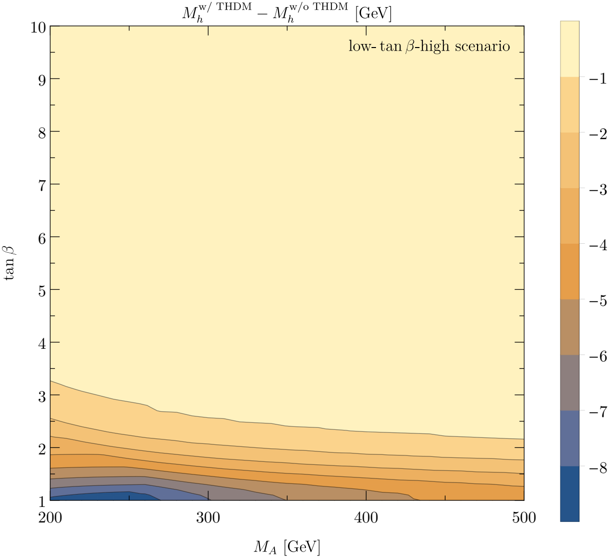


→ 1L conversion not reliable for low M_A, better use $\overline{\mathrm{DR}}$ scheme

Results V: low-tanb-high scenario ($\overline{\text{DR}}$)



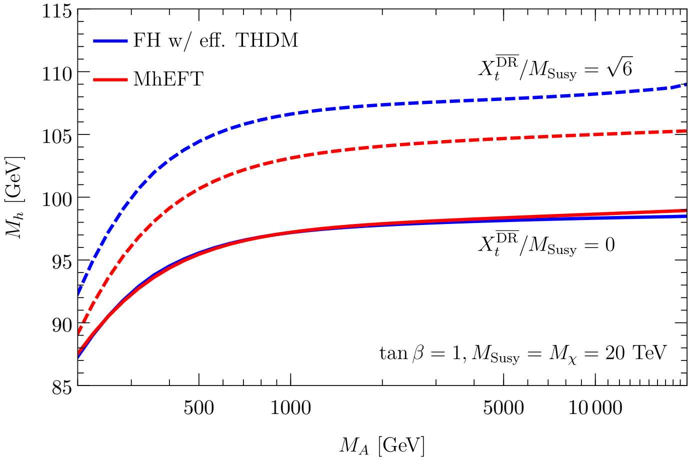
Results VI: shift in low-tanb-high scenario ($\overline{\text{DR}}$)



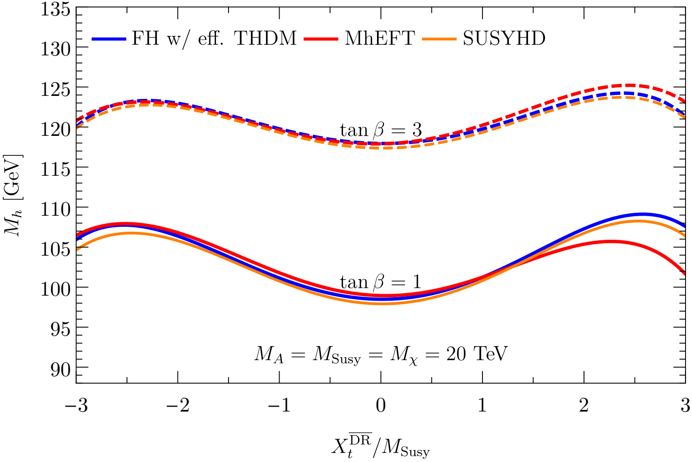
Differences observed in Lee & Wagner?

- In FH 2.10.2 log resummation was not very advanced (no EW contributions, no NNLL, ...).
- Resummation assumed $M_A = M_{\text{Susy}}$
- Lee & Wagner used OS parameters as input, but set $M_A = M_{\text{SUSY}}$ in conversion

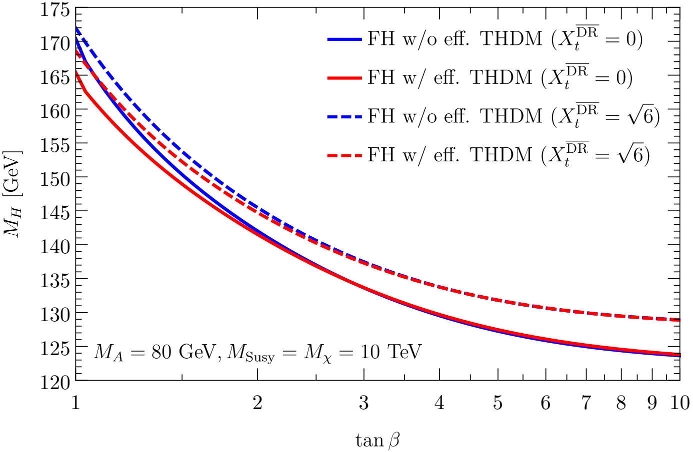
Comparison with MhEFT: M_A scan



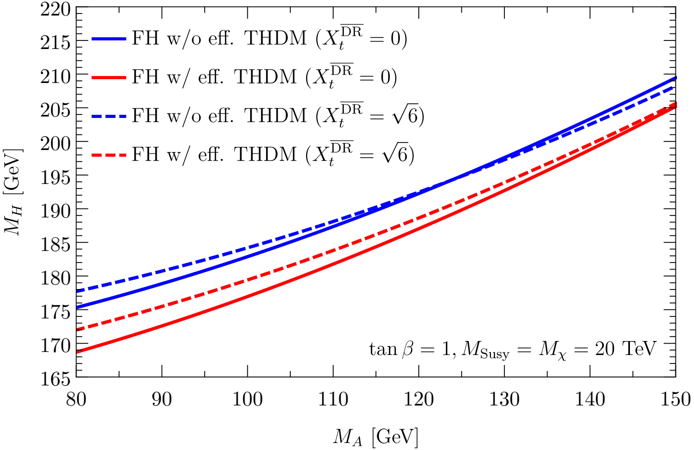
Comparison with MhEFT and SUSYHD: $X_t^{\overline{\text{DR}}}$ scan



Results for M_H: $\tan\beta$ scan



Results for M_H: M_A scan



Conclusion

Next version: `FeynHiggs 2.14.0`

- optional $\overline{\text{DR}}$ renormalization of stop sector
- improved calculation of pole masses/Z factors
- small improvements of resummation routines

Single-scale SUSY:

- good agreement between various codes
- time to look at scenarios with more mass scales

Low M_A scenario:

- upcoming extension of `FH` with effective THDM
- important to take different normalizations of Higgs doublets into account
- eff. THDM only relevant for very low $\tan\beta$
- time to update low-tanb-high scenario

The OS vev-counterterm is given by

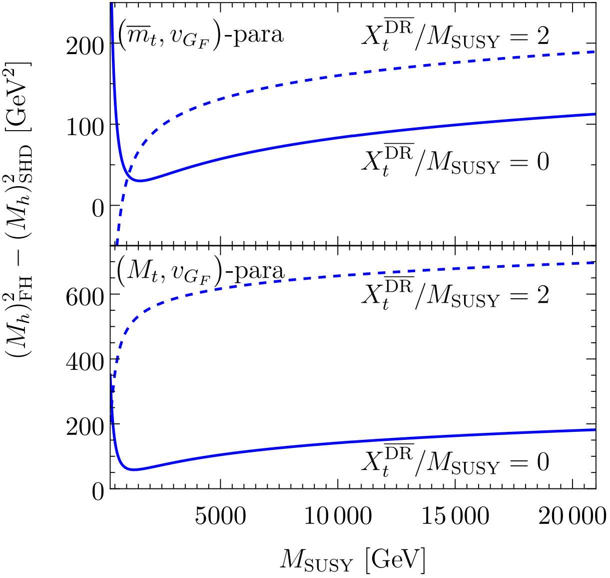
$$\delta v^2 = v^2 \left[\frac{\delta M_W^2}{M_W^2} + \frac{c_w^2}{s_w^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) - \frac{\delta e^2}{e^2} \right] \overset{\mathcal{O}(\alpha_s,\alpha_t)}{=}$$
$$= v^2 \left(-\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + \text{ SM corrections} \right).$$

The Higgs pole mass is calculated via

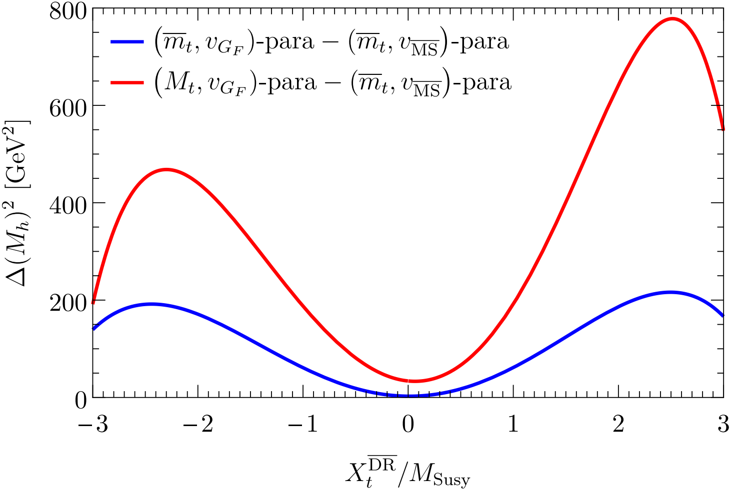
$$M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(0) + \Sigma_{hh}^{(1)\prime}(m_h^2)\Sigma_{hh}^{(1)}(m_h^2) + ...$$

The renormalized two-loop self-energy reads

$$\hat{\Sigma}_{hh}^{(2)}(0) = \Sigma_{hh}^{(2)}(0) + \frac{\partial}{\partial v^2}\hat{\Sigma}_{hh}^{(1)}(m_h^2) \cdot \delta v^2 + ... =$$
$$= \Sigma_{hh}^{(2)}(0) - \hat{\Sigma}_{hh}^{(1)}(m_h^2)\frac{\delta v^2}{v^2} + ... =$$
$$= \Sigma_{hh}^{(2)}(0) + \hat{\Sigma}_{hh}^{(1)}(m_h^2)\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2) + ...$$



$\longrightarrow$ nearly constant difference for high scales



800
600
400
200
0
−3
−2
−1
0
1
2
3
$\Delta(M_h)^2$ [GeV^2]
$X_t^{\overline{\rm DR}}/M_{\rm Susy}$
$(\overline{m}_t, v_{G_F})$-para − $(\overline{m}_t, v_{\overline{\rm MS}})$-para
(M_t, v_{G_F})-para − $(\overline{m}_t, v_{\overline{\rm MS}})$-para

Origin

Different parametrization of non-logarithmic terms

Three ways to parametrize top Yukawa coupling in FO result

- $M_t/v \rightarrow$ `FeynHiggs` with `runningMT = 0`
- $\overline{m}_t/v \rightarrow$ `FeynHiggs` with `runningMT = 1`
- $y_t^{\overline{\text{MS}}} = \overline{m}_t/v_{\overline{\text{MS}}} \rightarrow$ `SUSYHD`

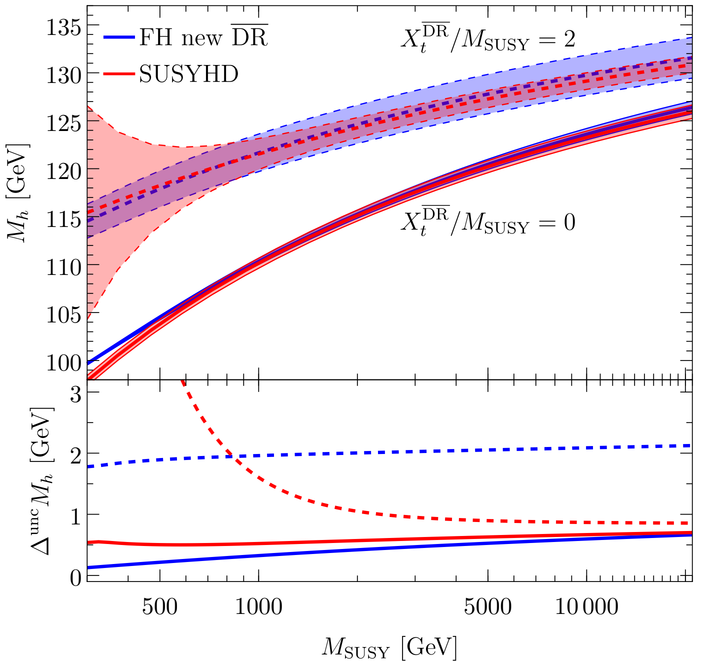
Equivalent at 2L order, but induces differences at higher order

Uncertainty estimate of SUSYHD

1. EFT uncertainty
   - $\mathcal{O}(v/M_S)$ terms
   - estimated by $v/M_S \cdot$(1L correction)
2. SM uncertainty:
   - higher order corrections to pole mass extraction
   - estimated by (de)activating higher order corrections to y_t and $\delta\lambda$
3. SUSY uncertainty:
   - higher order threshold corrections
   - estimated by variation of matching scale $1/2 < Q/M_S < 2$

Uncertainty estimate of `FeynHiggs`

1. Scale variation:
   - variation of renormalization scale between $1/2M_t$ and $2M_t$
2. Renormalization scheme dependence:
   - switching between OS top mass and $\overline{\text{MS}}$ top mass
3. $\tan\beta$ enhanced correction
   - (de)activating resummation of bottom Yukawa coupling


FH new $\overline{\text{DR}}$
SUSYHD
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 2$
$X_t^{\overline{\text{DR}}}/M_{\text{SUSY}} = 0$
M_h [GeV]
135
130
125
120
115
110
105
100
$\Delta^{\text{unc}} M_h$ [GeV]
3
2
1
0
500
1000
5000
10 000
M_{SUSY} [GeV]

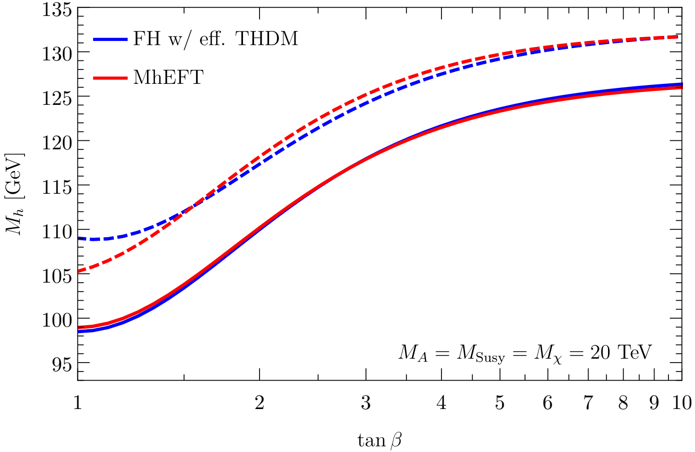
Matching to fixed order calculation V

$$\hat{\Sigma}^{(2)}_{hh}(0)\Big|_{\delta Z} = \Sigma^{(2),\text{sub}}_{hh}(0)\Big|_{\delta Z} - \frac{e}{2s_W M_W}\left(T^{(2),\text{sub}}_{h}\Big|_{\delta Z} + \frac{1}{2}s^2_\beta T^{(1)}_h \delta^{(1)} Z_{hh}\right)$$

$$\hat{\Sigma}^{(2)}_{hH}(0)\Big|_{\delta Z} = \Sigma^{(2),\text{sub}}_{hH}(0)\Big|_{\delta Z} - \frac{e}{2s_W M_W}\left(T^{(2),\text{sub}}_{H}\Big|_{\delta Z} + \frac{1}{2}s^2_\beta T^{(1)}_H \delta^{(1)} Z_{hh}\right)$$

$$\hat{\Sigma}^{(2)}_{HH}(0)\Big|_{\delta Z} = \Sigma^{(2),\text{sub}}_{HH}(0)\Big|_{\delta Z} - \Sigma^{(2),\text{sub}}_{AA}(0)\Big|_{\delta Z},$$

Comparison with MhEFT: $\tan\beta$ scan



Influence of low M_A on extraction of top Yukawa coupling