

Precise MSSM Higgs mass prediction
combining diagrammatic and EFT calculations

Henning Bahl

Elementary Particle Physics Seminar
December 14, 2017
Universität Würzburg

Current situation:

- ▶ no direct evidence for BSM physics at LHC yet

BSM models constrained by

- ▶ direct searches
- ▶ indirect constraints → precision observables

One of the most common BSM models: MSSM

- ▶ Higgs sector of MSSM corresponds to a THDM type II
- ▶ Two Higgs doublets results in five physical Higgs states:
 h, H, A, H^\pm
- ▶ A general THDM type II has 9 free parameters
→ SUSY reduces these to 2 (M_A and $\tan \beta = v_2/v_1$)

Special feature of MSSM

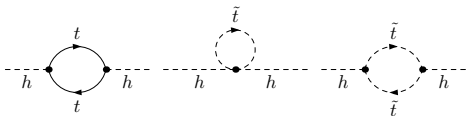
Mass of lightest \mathcal{CP} -even Higgs M_h is calculable in terms of model parameters \Rightarrow can be used as a precision observable

- ▶ at tree-level $M_h^2 \simeq M_Z^2 \cos(2\beta)^2 \leq M_Z^2$
- ▶ M_h is however heavily affected by loop corrections (up to $\sim 100\%$)

To fully profit from experimental precision, higher order calculations are needed. Two standard approaches:

- ▶ Fixed-order techniques
- ▶ Effective field theories

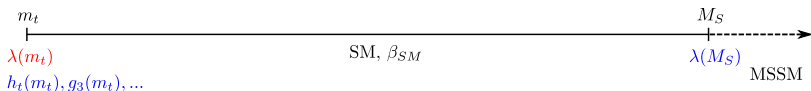
Fixed-order techniques



- ▶ diagrammatic approach
status: $\mathcal{O}(\text{full 1L}, \alpha_s(\alpha_b + \alpha_t), (\alpha_b + \alpha_t)^2)$
- ▶ effective potential approach
status: same + partial three-loop results

→ precise for low SUSY scales,
but for high scales large logarithms appear, $\ln(M_{\text{SUSY}}/M_t)$,
spoil convergence of perturbative expansion

EFT calculation



- ▶ integrate out all SUSY particles \rightarrow SM as EFT
- ▶ Higgs self-coupling fixed at matching scale

$$\lambda(M_{\text{SUSY}}) = \frac{1}{4}(g^2 + g_y^2) + \dots$$
- ▶ status: full LL+NLL, $\mathcal{O}(\alpha_s, \alpha_t, \alpha_b)$ NNLL

\rightarrow precise for high SUSY scales (logs resummed),
 but for low scales $\mathcal{O}(M_t/M_{\text{SUSY}})$ terms are important

How to deal with intermediary SUSY scales?

For sparticles in the LHC range, both logs and suppressed terms might be relevant. We could try to improve

- ▶ fixed-order calculation → need to calculate more three- and two-loop corrections,
- ▶ EFT calculation → need to include higher-dimensional operators into calculation.

or ...



Hybrid approach

Combine both approaches to get precise results for both regimes

Such an approach is implemented e.g. in **FeynHiggs**

[HB, T. Hahn, S. Heinemeyer, W. Hollik, S. Paßehr, H. Rzehak, G. Weiglein]

Procedure in FeynHiggs

1. Calculation of diagrammatic fixed-order self-energies $\hat{\Sigma}_{hh}$
2. Calculation of EFT prediction $2\lambda(M_t)v^2$
3. Add non-logarithmic terms contained in fixed-order result and the logarithms contained in EFT result

$$\hat{\Sigma}_{hh}(m_h^2) \longrightarrow [\hat{\Sigma}_{hh}(m_h^2)]_{\text{nolog}} - [2v^2\lambda(M_t)]_{\text{log}}$$

In practice, this is achieved by using subtraction terms.

Additional complication:

FH by default uses OS scheme, for EFT calculation however $\overline{\text{DR}}$ parameters needed (i.e. $X_t^{\overline{\text{DR}}}$)

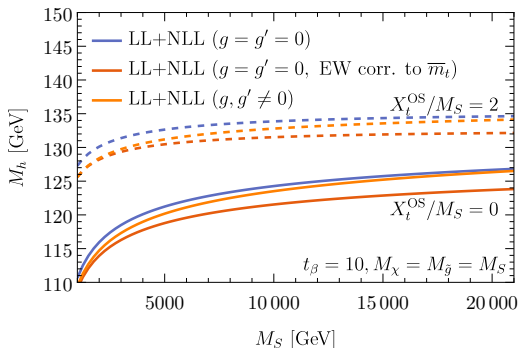
→ 1L log only conversion of X_t sufficient

Development history (and talk outline)

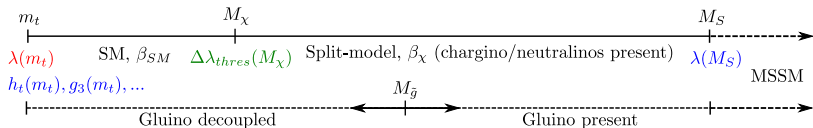
- ▶ First implementation:
 $\mathcal{O}(\alpha_s, \alpha_t)$ LL and NLL resummation [Hahn et. al. (2013)]
- ▶ Improvement of EFT calculation:
 full LL and NLL and $\mathcal{O}(\alpha_s, \alpha_t)$ NNLL resummation,
 gaugino thresholds [HB & W. Hollik (2016)]
- ▶ Comparison to pure EFT calculations:
 handling of $\overline{\text{DR}}$ input, improved pole mass determination
 [HB, S. Heinemeyer, W. Hollik, G. Weiglein (2017)]
- ▶ More complicated mass hierarchies:
 THDM as low-energy EFT [HB & W. Hollik (in preparation)]

Inclusion of electroweak contributions

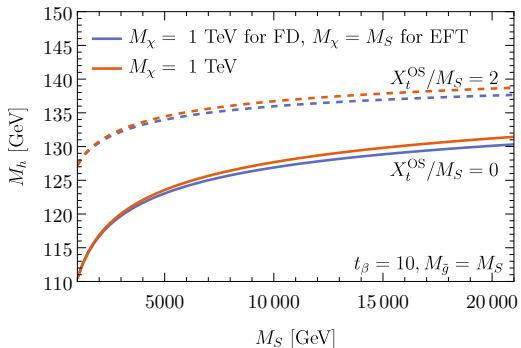
- ▶ included at the LL+NLL level
(full SM 2L RGEs, full 1L thresholds)
- ▶ include electroweak 1L corrections to SM \overline{MS} top mass, used in the diagrammatic calculation



Separate gaugino thresholds

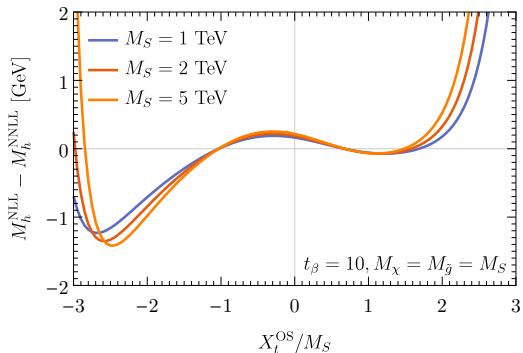


- Separate threshold for EWinos (neutralinos/charginos) and gluino



Inclusion of NNLL resummation

- ▶ 2L threshold for λ , 3L RGEs



This work brought EFT calculation in `FeynHiggs` to same level of accuracy as pure EFT calculations

Next step: Comparison to pure EFT calculations

⇒ expected to see agreement with EFT codes for high scales,
but at this time large discrepancies could be observed

Two main origins found

- ▶ $\overline{\text{DR}} \leftrightarrow \text{OS}$ conversion
- ▶ determination of Higgs propagator pole

We focused on single scale scenario:

$$\tan \beta = 10, \quad M_{\text{soft}} = \mu = M_A \equiv M_{\text{Susy}}, \quad A_{b,c,s,e,\mu,\tau} = 0$$

FeynHiggs mixed OS/ $\overline{\text{DR}}$ scheme \leftrightarrow EFT codes typically $\overline{\text{DR}}$

\rightarrow for comparison parameter conversion necessary

Especially relevant: stop mixing parameter X_t
(large impact on Higgs mass, large logarithms in conversion)

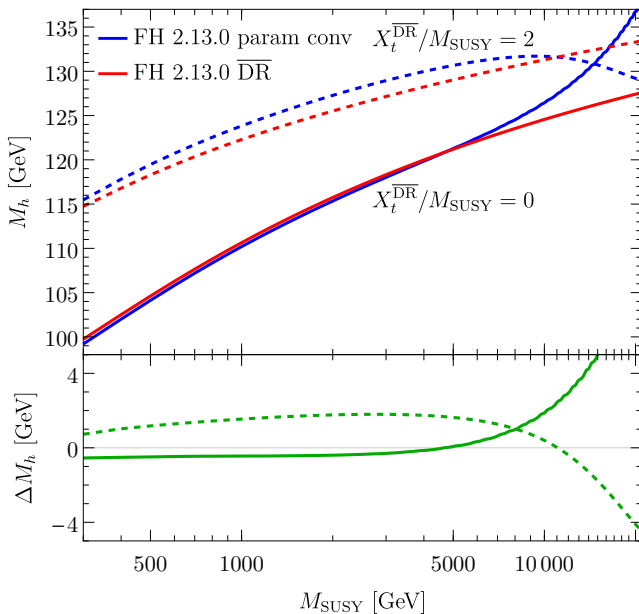
Procedure at this time

- ▶ $X_t^{\overline{\text{DR}}} \xrightarrow{\mathcal{O}(\alpha_s, \alpha_t, \alpha_b)} X_t^{\text{OS}}$
- ▶ Forget about $X_t^{\overline{\text{DR}}}$, use X_t^{OS} as 'new' input parameter

Problem: result contains resummed logarithms

\rightarrow conversion induces additional logarithms not present in a genuine $\overline{\text{DR}}$ calculation

→ solution: optional $\overline{\text{DR}}$ renormalization of fixed-order result



How is the pole mass determined?

EFT calculation

$$\begin{aligned}
 p^2 - 2\lambda(M_t)v^2 + \hat{\Sigma}_{hh}^{\text{SM}}(p^2) &= 0 \\
 \rightarrow (M_h^2)_{\text{EFT}} &= 2\lambda(M_t)v^2 - \hat{\Sigma}_{hh}^{\text{SM}}(m_h^2) \\
 &\quad - \hat{\Sigma}_{hh}^{\text{SM}'}(m_h^2) \left[2\lambda(M_t)v^2 - \hat{\Sigma}_{hh}^{\text{SM}}(m_h^2) - m_h^2 \right] + \dots
 \end{aligned}$$

Hybrid calculation

In limit $M_A \gg M_Z$ Higgs pole mass is determined by

$$\begin{aligned}
 p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{MSSM}}(p^2) - [2v^2\lambda(M_t)]_{\log} - [\hat{\Sigma}_{hh}(m_h^2)]_{\log} &= 0 \\
 \rightarrow (M_h^2)_{\text{FH}} &= m_h^2 + [2v^2\lambda(M_t)]_{\log} - [\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)]_{\text{nolog}} \\
 &\quad - \hat{\Sigma}_{hh}^{\text{MSSM}'}(m_h^2) \left([2v^2\lambda(M_t)]_{\log} - [\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2)]_{\text{nolog}} \right) \\
 &\quad + \dots
 \end{aligned}$$

Comparison of logarithmic terms

In decoupling limit, we can split up MSSM self-energy

$$\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2) = \hat{\Sigma}_{hh}^{\text{SM}}(m_h^2) + \hat{\Sigma}_{hh}^{\text{nonSM}}(m_h^2).$$

We straightforwardly obtain

$$\begin{aligned} \Delta^{\log} &\equiv (M_h^2)_{\text{FH}}^{\log} - (M_h^2)_{\text{EFT}}^{\log} \\ &= \left[\hat{\Sigma}_{hh}^{\text{nonSM}'}(m_h^2) \right]_{\log} \left[\hat{\Sigma}_{hh}^{\text{MSSM}}(m_h^2) \right]_{\text{nolog}} \\ &\quad - \hat{\Sigma}_{hh}^{\text{nonSM}'}(m_h^2) \left[2v^2 \lambda(M_t) \right]_{\log} + \dots \end{aligned}$$

Very similar for non-logarithmic terms.

Observation

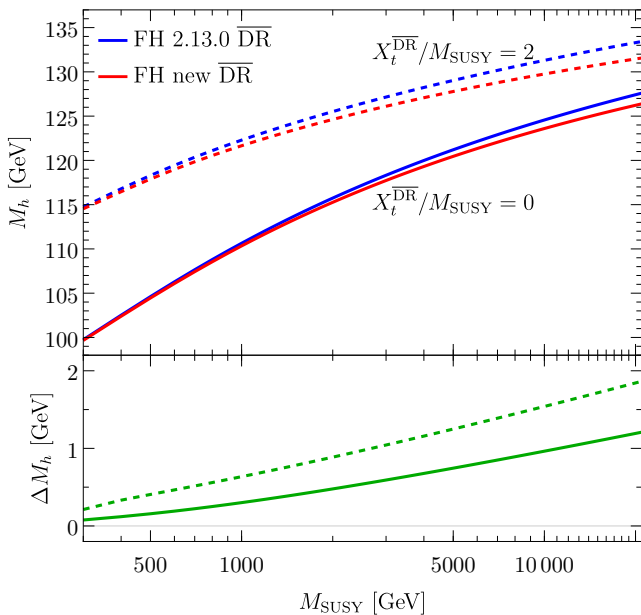
vev counterterm appearing in 2L subloop-renormalization
cancels 2L terms in Δ^{\log}

$$\frac{(\delta v^2)^{\text{MSSM}}}{v^2} = \frac{(\delta v^2)^{\text{SM}}}{v^2} - \hat{\Sigma}_{hh}^{\text{nonSM},(1)'}(m_h^2) + \mathcal{O}(v/M_{\text{SUSY}})$$

- ▶ Argument holds for all 2L contributions
- ▶ Full 2L calculation however not available
→ induced terms of e.g. $\mathcal{O}(\alpha_t \alpha)$ are not compensated
- ▶ Likely also holds for higher loop orders

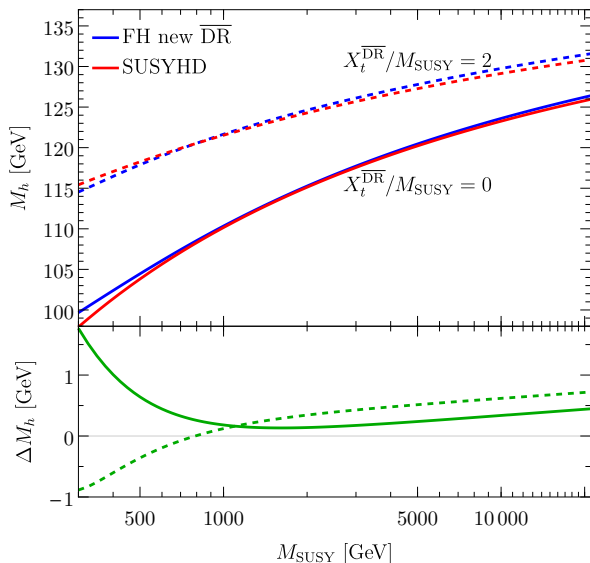


adapted determination of Higgs propagator pole to avoid these
terms (truncate expansion around tree-level mass)



Comparison to SUSYHD as exemplary EFT code

[J.P. Vega, G. Villadoro]



→ **overall very good agreement**

Remaining differences

- ▶ derivation for small scales due to suppressed terms not captured in EFT framework
- ▶ constant shift due to different parametrizations of non-logarithmic terms (i.e. top mass and vev)

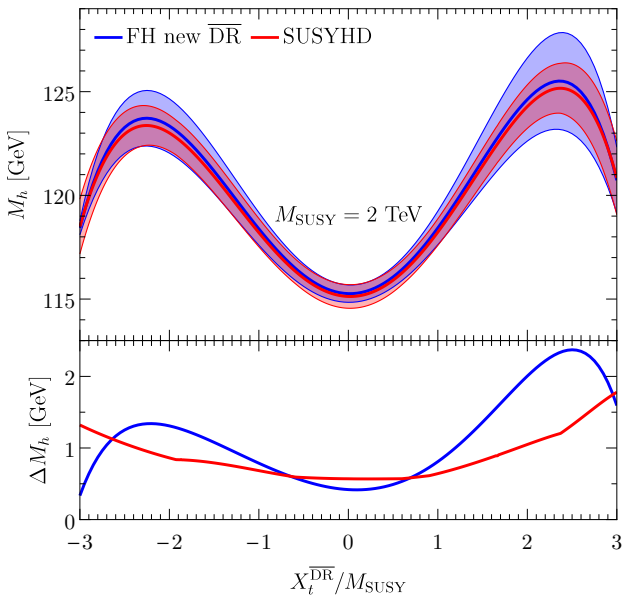
Comparison of uncertainty estimates

FeynHiggs

- ▶ variation of renormalization scale between $M_t/2$ and $2M_t$
- ▶ change of renormalization scheme; switch between OS top mass and SM $\overline{\text{MS}}$ top mass
- ▶ deactivating the resummation of bottom Yukawa coupling

SUSYHD

- ▶ variation of matching scale between $M_{\text{SUSY}}/2$ and $2M_{\text{SUSY}}$
- ▶ switching between NNLO and NNNLO top Yukawa coupling
- ▶ estimate of suppressed terms, $\mathcal{O}(M_t/M_{\text{SUSY}})$



What is about more complicated hierarchies?

Assumption so far

All sfermions and non-SM Higgs share common mass scale

Therefore, prediction might be unreliable e.g. if

- ▶ one stop is much lighter than the other
[Espinosa & Navarro (2001)]
- ▶ non SM Higgs are much lighter than sfermions
[Haber & Hempfling (1993), Lee & Wagner (2015)]
- ▶ ...

What is about more complicated hierarchies?

Assumption so far

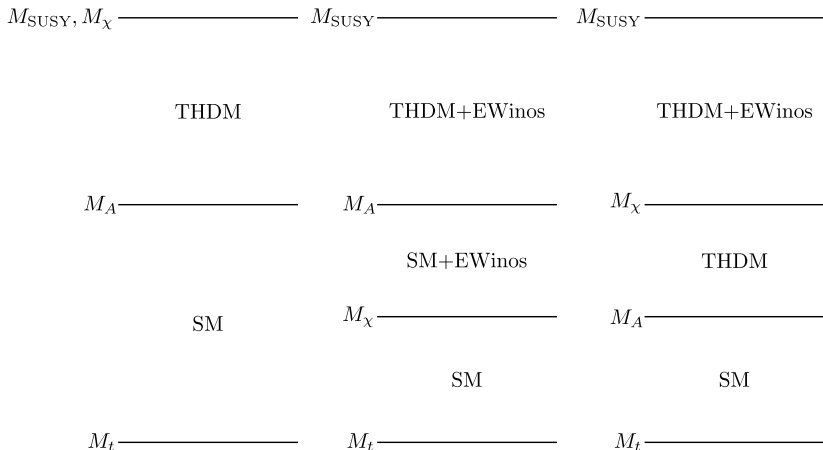
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- ▶ ...

→ Low-energy THDM is needed for correct resummation

EFTs for low M_A



$$M_\chi = M_1 = M_2 = \mu;$$

additional freely variable gluino threshold not shown

EFT calculation

- ▶ all possible hierarchies taken into account
 - THDM type III \rightarrow 12 effective couplings $(\lambda_{1..7}, h_t, h'_t)$
 - THDM type III + EWinos \rightarrow 20 effective couplings $(\lambda_{1..7}, h_t, h'_t + \text{gaugino-Higgsino-Higgs couplings})$
- ▶ full 2L running for all effective couplings (RGEs via SARAH)
- ▶ full 1L threshold corrections for all effective couplings
- ▶ $\mathcal{O}(\alpha_s \alpha_t)$ threshold corrections for λ_i 's

\rightarrow most precise EFT calculation available

Matching to fixed order calculation

- ▶ Running from M_{SUSY} to $M_A \rightarrow \Delta\hat{\Sigma}_{11}, \Delta\hat{\Sigma}_{12}, \Delta\hat{\Sigma}_{22}$, e.g.

$$\Delta\hat{\Sigma}_{11} =$$

$$= \left[M_A^2 s_\beta^2 + v^2 \left(3\lambda_1 c_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5) s_\beta^2 + 6\lambda_6 s_\beta c_\beta \right) \right]_{Q=M_A}$$

– subtraction terms

- ▶ Running from M_A to $M_t \rightarrow \Delta\hat{\Sigma}_{22} = \lambda(m_t)v^2/c_\alpha^2$
(as done for $M_A = M_{\text{SUSY}}$)

Matching to fixed order calculation II

Normalization of Higgs doublets

MSSM and THDM Higgs doublets have not the same normalization

- ▶ LSZ theorem yields (at the 1L level)

$$\begin{pmatrix} \Phi_1^{\text{THDM}} \\ \Phi_2^{\text{THDM}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 + \frac{1}{2}\Delta\Sigma'_{11} & \frac{1}{2}\Delta\Sigma'_{12} \\ \frac{1}{2}\Delta\Sigma'_{12} & 1 + \frac{1}{2}\Delta\Sigma'_{22} \end{pmatrix}}_{=U_{\Delta\Phi}} \begin{pmatrix} \Phi_1^{\text{MSSM}} \\ \Phi_2^{\text{MSSM}} \end{pmatrix}$$

$$\text{with } \Delta\Sigma'_{ij} = \Sigma_{ij}^{\text{MSSM}'} - \Sigma_{ij}^{\text{THDM}'}$$

$$\Rightarrow \Delta_{\text{MSSM}}^{-1}(p^2) = U_{\Delta\Phi}^T \Delta_{\text{THDM}}^{-1}(p^2) U_{\Delta\Phi}$$

- ▶ pole masses do not depend on absolute field normalization
→ not important for pure EFT calculation

Matching to fixed order calculation III

Hybrid calculation in FeynHiggs:

$$\Delta_{\text{FH}}^{-1}(p^2) =$$

$$= \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}^{\text{FO}}(p^2) + \Delta\Sigma_{hh}^{\text{logs}} & \hat{\Sigma}_{hH}^{\text{FO}}(p^2) + \Delta\Sigma_{hH}^{\text{logs}} \\ \hat{\Sigma}_{hH}^{\text{FO}}(p^2) + \Delta\Sigma_{hH}^{\text{logs}} & p^2 - m_H^2 + \hat{\Sigma}_{HH}^{\text{FO}}(p^2) + \Delta\Sigma_{HH}^{\text{logs}} \end{pmatrix}$$

with $\Delta\Sigma_{ij}^{\text{logs}} = \Sigma_{ij}^{\text{EFT}} - \Sigma_{ij}^{\text{sub}}$

“Relative” normalization important for

- ▶ correct merging of EFT result (THDM normalization) with fixed order result (MSSM renormalization)
- ▶ calculation of 1L and 2L subtraction terms

Matching to fixed order calculation IV

How to implement different normalization?

→ finite field normalization in fixed-order calculation

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 + \frac{1}{2}\delta^{(1)}Z_{11} + \frac{1}{2}\Delta^{(2)}Z_{11} & \frac{1}{2}\delta^{(1)}Z_{12} + \frac{1}{2}\Delta^{(2)}Z_{12} \\ \frac{1}{2}\delta^{(1)}Z_{12} + \frac{1}{2}\Delta^{(2)}Z_{12} & 1 + \frac{1}{2}\delta^{(1)}Z_{22} + \frac{1}{2}\Delta^{(2)}Z_{22} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

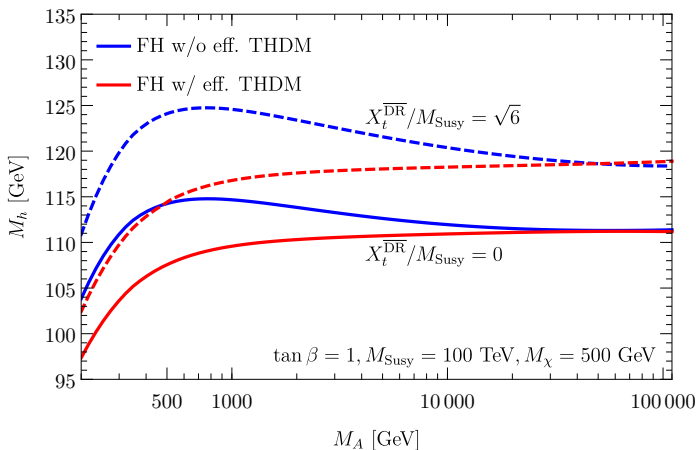
$$\Delta Z_{ij} = \delta^{(2)}Z_{ij} - \frac{1}{4} \left(\delta^{(1)}Z_{ij} \right)^2$$

- ▶ choose $\delta^{(1)}Z_{ij}|_{\text{fin}} = \Delta\Sigma'_{ij}$
- ▶ $\delta^{(2)}Z_{ij}$ drops out completely
 - 2L relation between Φ^{MSSM} and Φ^{THDM} not needed

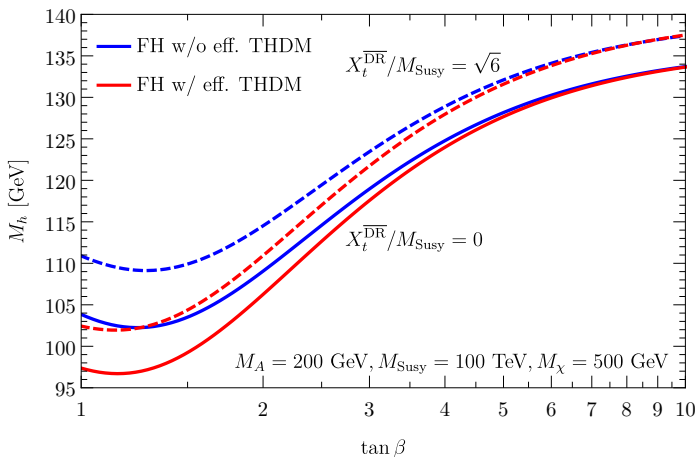
→ definition of $\tan\beta$ is changed

$$t_{\beta}^{\text{MSSM}}(M_t) \xrightarrow{\delta Z|_{\text{fin}}} t_{\beta}^{\text{THDM}}(M_A)$$

Results I: M_A scan

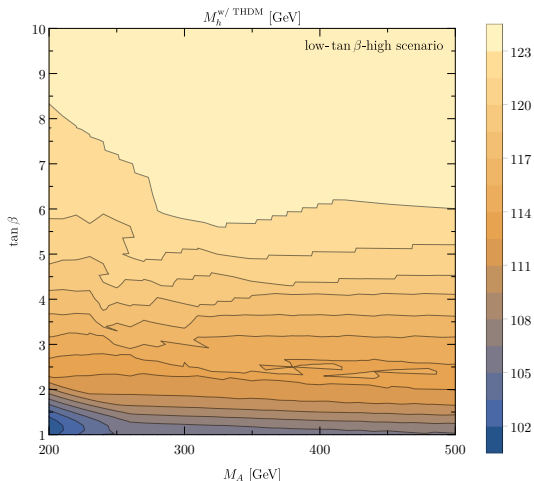


Results II: $\tan \beta$ scan

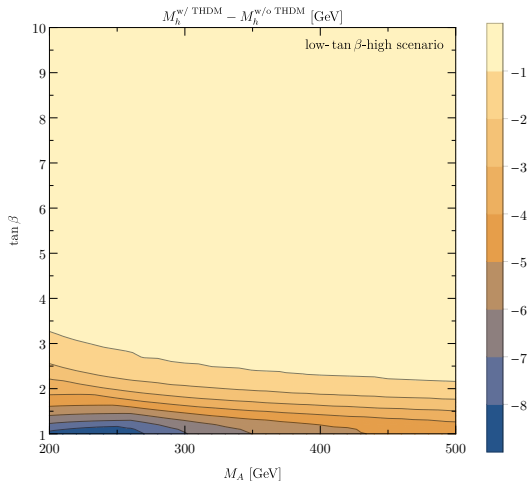


Results III: low-tan β -high scenario ($\overline{\text{DR}}$)

$\mu = 1.5$ TeV, $M_2 = 2$ TeV, $A_{b,\dots} = 2$ TeV, M_{SUSY} and X_t chosen to get $M_h = 125$ GeV



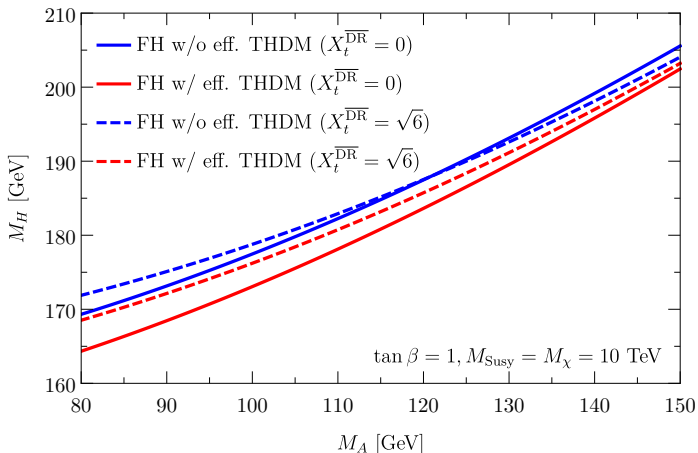
Results IV: shift in low-tan β -high scenario ($\overline{\text{DR}}$)



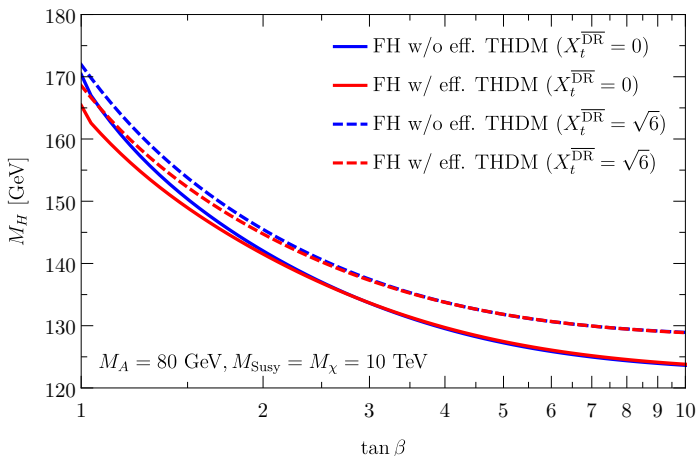
→ need to define new benchmark scenario (LHCHXSWG)

Results for M_H I

How important is the eff. THDM, when H plays role of SM Higgs?



Results for M_H II



→ negligible in these scenarios

Conclusion

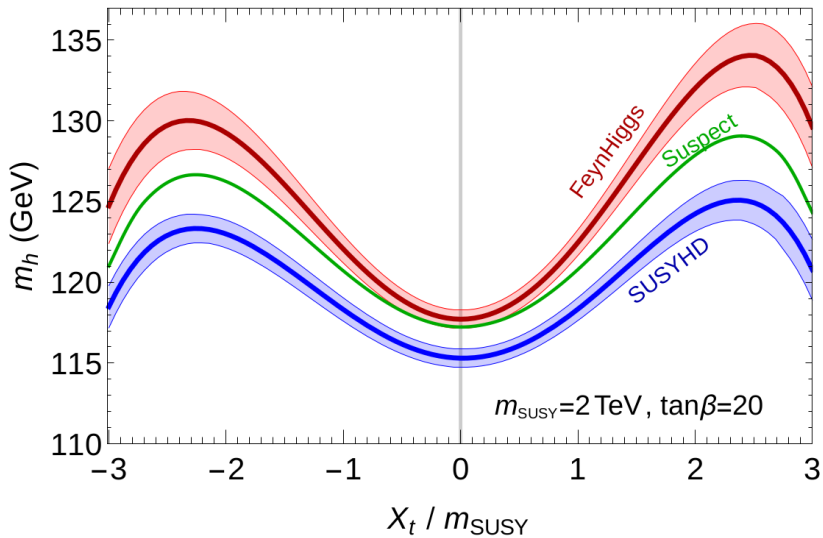
- ▶ SM-like Higgs mass is an important constraint on MSSM parameter space
- ▶ To gain precise prediction for all SUSY scales, we combined
 - state-of-the-art fixed-order calculation
 - state-of-the-art EFT calculation
- ▶ Optional $\overline{\text{DR}}$ renorm. and improved pole determination
 - excellent agreement of `FeynHiggs` with pure EFT codes found for high scales
- ▶ For low M_A , implemented effective THDM as EFT
 - Large numerical effects

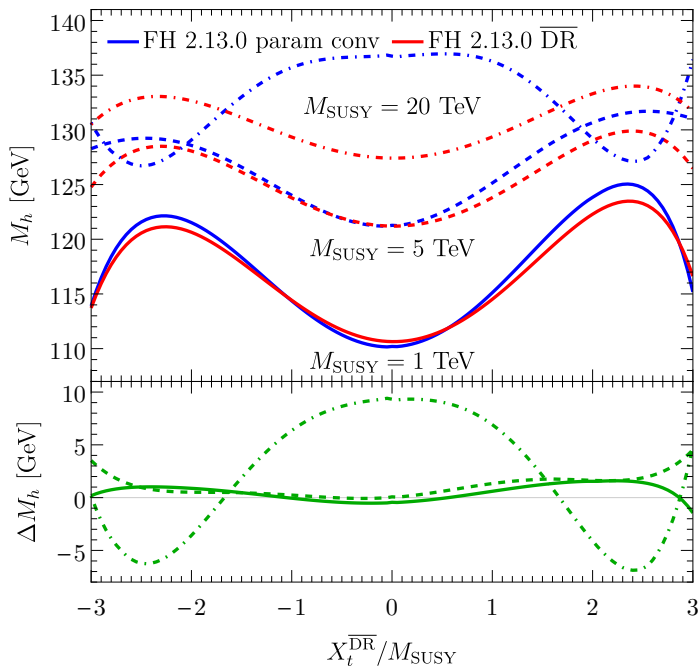


For multi-scale scenarios, proper EFT treatment is essential

Outlook

- ▶ For each hierarchy the same steps are always repeated:
 - define EFTs
 - calculate RGEs and threshold corrections
 - merge with diagrammatic calculation
(calculate subtraction terms)
- automatizing these steps would allow for a precise prediction for arbitrary hierarchies
- ▶ application to other observable and models





Need to determine poles of inverse propagator matrix

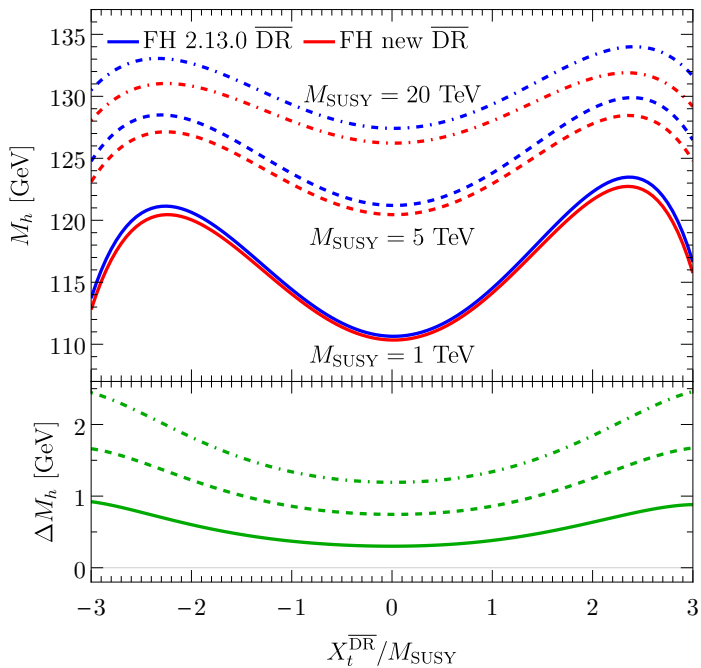
$$\Delta^{-1}(p^2) = \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}^{(1)}(p^2) + \hat{\Sigma}_{hh}^{(2)}(0) & \hat{\Sigma}_{hH}^{(1)}(p^2) + \hat{\Sigma}_{hH}^{(2)}(0) \\ \hat{\Sigma}_{hH}^{(1)}(p^2) + \hat{\Sigma}_{hH}^{(2)}(0) & p^2 - m_H^2 + \hat{\Sigma}_{HH}^{(1)}(p^2) + \hat{\Sigma}_{HH}^{(2)}(0) \end{pmatrix}$$

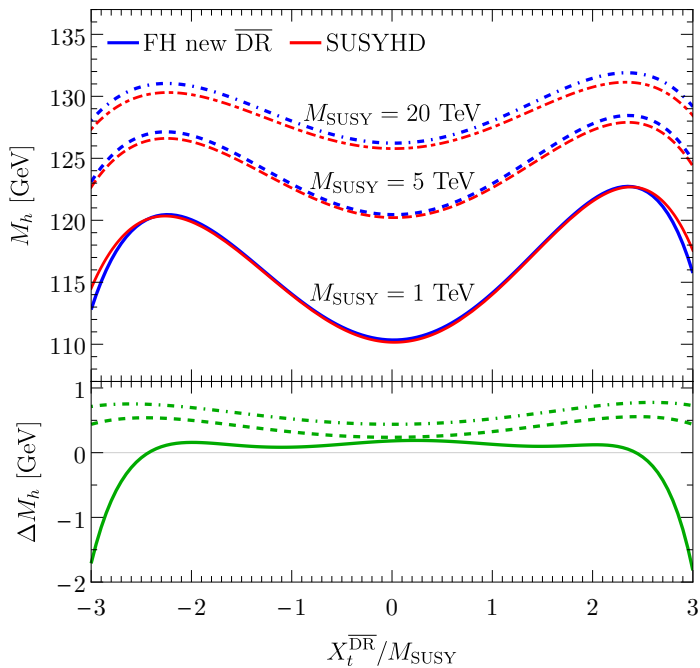
At 1L level $M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) \rightarrow$ expand around 1L solution

\Rightarrow determine poles of

$$\begin{aligned} \Delta_{hh}^{-1}(p^2) &= p^2 - m_h^2 + \hat{\Sigma}_{hh}^{(1)}(m_h^2) + \hat{\Sigma}_{hh}^{(2)}(0) - \left[\hat{\Sigma}_{hh}^{(1)'}(m_h^2) \hat{\Sigma}_{hh}^{(1)}(m_h^2) \right]_{g=g_Y=0} \\ \Delta_{hH}^{-1}(p^2) &= + \hat{\Sigma}_{hH}^{(1)}(m_h^2) + \hat{\Sigma}_{hH}^{(2)}(0) - \left[\hat{\Sigma}_{hH}^{(1)'}(m_h^2) \hat{\Sigma}_{hh}^{(1)}(m_h^2) \right]_{g=g_Y=0} \\ \Delta_{HH}^{-1}(p^2) &= p^2 - m_H^2 + \hat{\Sigma}_{HH}^{(1)}(m_h^2) + \hat{\Sigma}_{HH}^{(2)}(0) - \left[\hat{\Sigma}_{HH}^{(1)'}(m_h^2) \hat{\Sigma}_{hh}^{(1)}(m_h^2) \right]_{g=g_Y=0} \end{aligned}$$

For determination of M_H expand around $M_H^2 = m_H^2 - \hat{\Sigma}_{HH}^{(1)}(m_H^2)$





Effective Lagrangians

$$\mathcal{L}_{\text{THDM}} = \dots - V_{\text{THDM}}(H_u, H_d) - h_t \epsilon_{ij} \bar{t}_R Q_L^i H_u^j - h'_t \bar{t}_R Q_L H_d$$

→ 12 effective couplings ($\lambda_{1..7}, h_t, h'_t$)

$$\begin{aligned} \mathcal{L}_{\text{THDM+EWinos}} = & \dots - \frac{1}{\sqrt{2}} H_u^\dagger \left(\hat{g}_{2uu} \sigma^a \tilde{W}^a + \hat{g}_{1uu} \tilde{B} \right) \tilde{\mathcal{H}}_u \\ & - \frac{1}{\sqrt{2}} H_d^\dagger \left(\hat{g}_{2dd} \sigma^a \tilde{W}^a - \hat{g}_{1dd} \tilde{B} \right) \tilde{\mathcal{H}}_d \\ & - \frac{1}{\sqrt{2}} (-i H_d^T \sigma_2) \left(\hat{g}_{2du} \sigma^a \tilde{W}^a + \hat{g}_{1du} \tilde{B} \right) \tilde{\mathcal{H}}_u \\ & - \frac{1}{\sqrt{2}} (-i H_u^T \sigma_2) \left(\hat{g}_{2ud} \sigma^a \tilde{W}^a - \hat{g}_{1ud} \tilde{B} \right) \tilde{\mathcal{H}}_d \\ & + h.c. - V_{\text{THDM}}(H_u, H_d), \end{aligned}$$

→ 20 effective couplings

