Determination of the Higgs propagator poles

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MSSM Higgs sector

- In the MSSM, we have at 5 physical Higgs bosons.
- tree-level mass eigenstates
  (obtained via diagonalization of mass matrices):
  - $\mathcal{CP}$-even h and H
  - $\mathcal{CP}$-odd A
  - charged $H^{\pm}$
- loop corrections lead to mixing between h and H and A in case of $\mathcal{CP}$-violation (and Goldstone boson G^0)

How to we determine the Higgs pole masses?

1. Calculate Higgs self-energies
2. Construct inverse Higgs propagator matrix
3. Find poles of inverse propagator matrix

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   $\rightarrow$ most work intensive

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1. Calculate Higgs self-energies
   $\rightarrow$ most work intensive

2. Construct inverse Higgs propagator matrix
   $\rightarrow$ trivial

3. Find poles of inverse propagator matrix
   $\rightarrow$ straightforward??

1. Calculate Higgs self-energies

Hybrid approach of `FeynHiggs`:

$$\hat{\Sigma}_{ij}(p^2) = \hat{\Sigma}^{(1)}_{ij}(p^2) + \hat{\Sigma}^{(2)}_{ij}(0)\big|_{\text{g=g'=0}} + \text{higher-order logs}$$

- 1L and 2L self-energies obtained in diagrammatic fixed-order approach
- approximation of vanishing electroweak gauge couplings and external momentum @ 2L

  ($p^2 \neq 0$ can be included for QCD corrections)
- large logarithms resummed in EFT approach

  (full LL+NLL, $\mathcal{O}(\alpha_s, \alpha_t)$ NNLL)

2. Construct inverse Higgs propagator matrix

$$i\Delta_{hH}^{-1}(p^2) = \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2) \end{pmatrix}$$

General remarks:

- Discussion here restricted to 2×2 mixing between $\mathcal{CP}$ even states h and H
  (but also applies for 3×3 mixing)
- Pole masses labelled by $M_{h_1} \leq M_{h_2} (\leq M_{h_3})$
- $M_h \to h$-like state, $M_H \to H$-like state

3. Find poles of inverse propagator matrix

Have to solve

$$\det\left(\Delta_{hH}^{-1}(p^2)\right) = 0$$

How to solve this equation?

1. Numerical determination
2. Fixed-order determination
3. Numerical determination with finite field renormalization

Numerical pole determination

- Conceptionally very easy
- "Just" have to employ numerical algorithm

Solutions:

$$M_{h_1}^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(0)\Big|_{\text{g=g'=0}} + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2) + \frac{\left(\hat{\Sigma}_{hH}^{(1)}(m_h^2)\right)^2}{m_h^2 - m_H^2} + \ldots$$

$M_{h_2}^2$ determined by same equation with $(h \leftrightarrow H)$

Problems of numerical pole determination

For $M_{\text{SUSY}} \gg M_t$, we have

$$\hat{\Sigma}^{(1)} = \hat{\Sigma}^{(1),\text{SM}} + \hat{\Sigma}^{(1),\text{nonSM}}$$

Comparison between EFT and hybrid approach showed

- $\hat{\Sigma}_{hh}^{(1),\text{nonSM}\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2)$ is cancelled by parts of subloop renormalization contained in $\hat{\Sigma}_{hh}^{(2)}(0)\big|_{\text{g=g'=0}}$
- cancellation incomplete, since terms are included at different orders of accuracy
- similar incomplete cancellation at higher orders

$\rightarrow$ easy to solve in decoupling limit, but what's for low M_A?

First proposed solution: Fixed-order determination

Determination of h-like state:

1. Expand $i\Delta^{-1}_{hH}$ around 1L solution $\left(M_h^{(1)}\right)^2 = m_h^2 - \hat{\Sigma}^{(1)}_{hh}(m_h^2)$
2. Get eigenvalues of expanded matrix

$$\left(i\Delta^{-1,h-\text{exp}}_{hH}(p^2)\right)_{jk} = (p^2 - m_j^2)\delta_{jk} + \hat{\Sigma}^{(1)}_{jk}(m_h^2) + \hat{\Sigma}^{(2)}_{jk}(0) + \Delta^{\text{logs}}_{jk} - \left[\hat{\Sigma}^{(1)\prime}_{jk}(m_h^2)\hat{\Sigma}^{(1)}_{hh}(m_h^2)\right]_{\text{g=g'=0}},$$

3. Pick h-like eigenvalue corresponding to $M_h^2 = m_h^2 - \hat{\Sigma}^{(1)}_{hh}(m_h^2) + \ldots$
   other eigenvalue would be $m_H^2 - \hat{\Sigma}^{(1)}_{HH}(m_h^2) + \ldots$

Determination of H-like state analogously, just have to $h \leftrightarrow H$

Assement of the numerical pole determination

- brought hybrid and EFT approach to much better agreement
- faster since no numerical pole search is required

$\rightarrow$ everything fine?

But: MH125 scenario

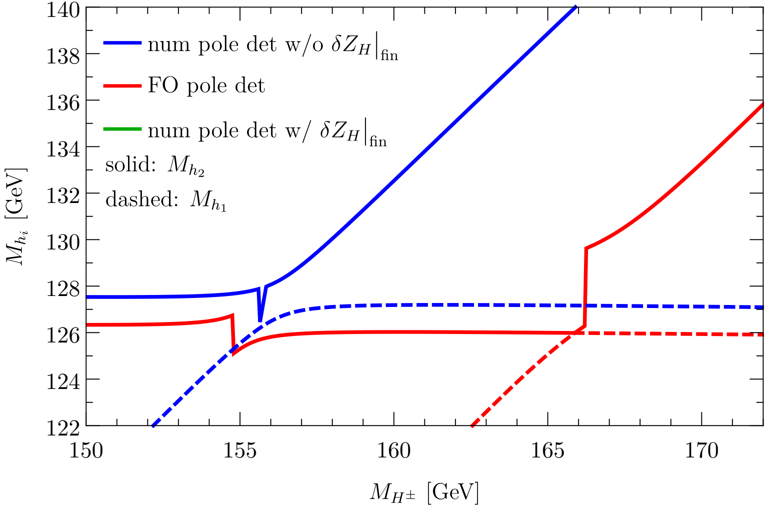
- new benchmark scenario under development in the LHCHXSWG
- parameters:

$$M_{\text{SUSY}} = 2\ \text{TeV},\ M_{\tilde{Q}_3} = M_{\tilde{U}_3} = 700\ \text{GeV},$$
$$\mu = 6\ \text{TeV},\ M_1 = 675\ \text{GeV},\ M_2 = 1\ \text{TeV},\ M_3 = 2.5\ \text{TeV},$$
$$A_t = 450\ \text{GeV},\ A_{b,c,s,u,d} = 0.$$

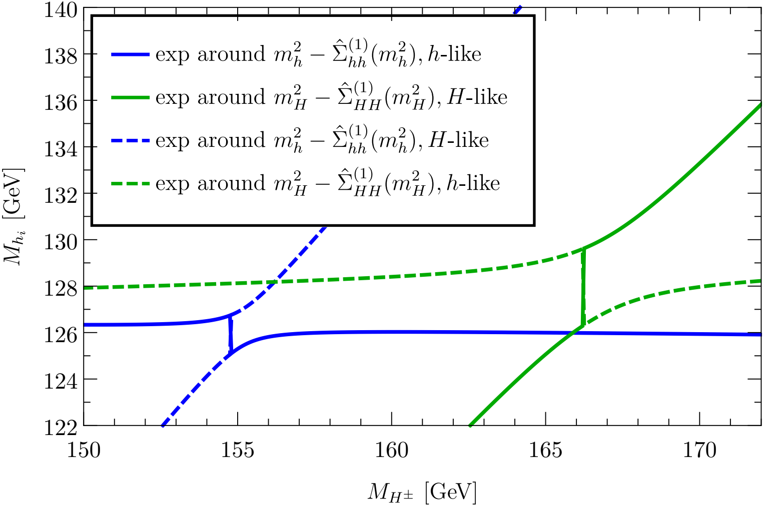
- scan over $M_{H^\pm}$ and $\tan\beta$

M_{h_2} is suppossed to play role of SM-like Higgs boson

But: MH125 scenario



What's going on?



Solid lines: "right" solutions; dashed lines: "wrong" solutions

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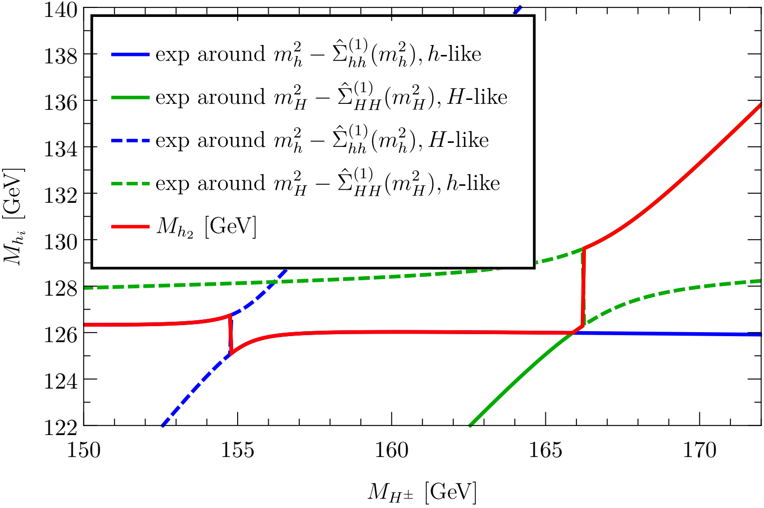


Solid lines: "right" solutions; dashed lines: "wrong" solutions

Assement of the fixed-order pole determination

- Algorithm works as wanted
- 2L truncation, however, introduces “unphysical” jumps
- Could be associated with high theoretical uncertainty

→ Still unsatisfying, can we find better method?

What is the origin of the observed cancellation?

- uncancelled terms originate from p^2 dependence of **non-SM** contributions to 1L self-energies

$$\begin{aligned}\hat{\Sigma}(p^2) &= \Sigma(p^2) + \delta Z\,(p^2 - m^2) - \delta m^2 = \\ &= \Sigma^{\text{nonSM}}(p^2) + \Sigma^{\text{SM}}(p^2) \\ &\quad + \delta Z\,(p^2 - m^2) - \delta m^2 + \mathcal{O}(v/M_{\text{SUSY}}) = \\ &= \Sigma^{\text{nonSM}}(m^2) + \left(\Sigma^{\text{nonSM}\prime}(m^2) + \delta Z\right)\,(p^2 - m^2) \\ &\quad + \Sigma^{\text{SM}}(p^2) - \delta m^2 + \mathcal{O}(v/M_{\text{SUSY}})\end{aligned}$$

- higher derivatives of $\hat{\Sigma}^{\text{nonSM}}$ suppressed by v/M_{SUSY}

$\rightarrow p^2$ dep. of “heavy” contributions $\sim$ field (re)normalization

Change of field normalization

Field (re)normalization

Should drop out if calculating physical observables order by order!

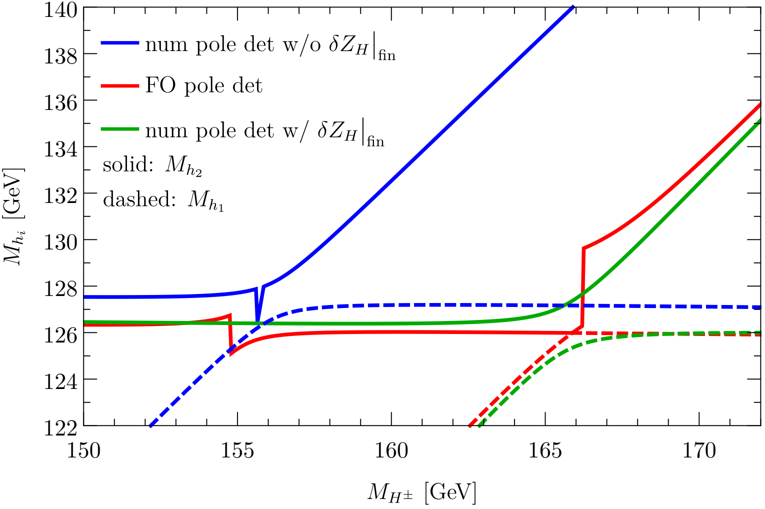
Prevent numerical det. from inducing terms $\propto \hat{\Sigma}^{\text{nonSM}\prime}(m^2)$ by:

- chosing $\delta Z = -\hat{\Sigma}^{\text{nonSM}\prime}(m^2)$, i.e.:

$$\delta^{(1)} Z_{hh} = -\hat{\Sigma}_{hh}^{\text{nonSM}\prime}(0)$$
$$\delta^{(1)} Z_{hH} = -\hat{\Sigma}_{hH}^{\text{nonSM}\prime}(0)$$
$$\vdots$$

- can be evaluated at arbitrary momentum $\ll M_{\text{SUSY}}$
- evaluating at zero convenient
  $\rightarrow$ no unphysical thresholds are introduced

MH125 scenario



Comparison to other methods

Compare in the limit $M_A \gg M_t$:

- numerical pole determination:

$$M_{h_1}^2 = \ldots + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\hat{\Sigma}_{hh}(m_h^2) + \ldots$$

- fixed-order determination:

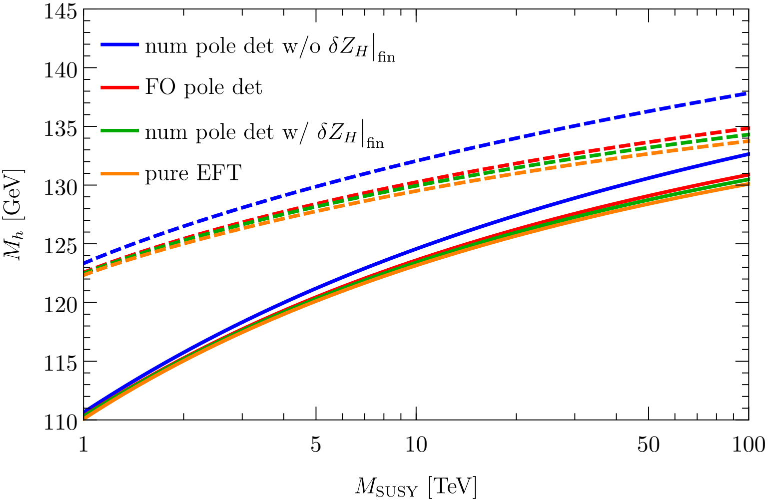
$$M_{h_1}^2 = \ldots + \left[\hat{\Sigma}_{hh}^{\mathrm{SM},(1)\prime}(m_h^2)\hat{\Sigma}_{hh}^{(1)}(m_h^2)\right]_{\mathrm{g=g'=0}} + \ldots$$

- numerical pole determination with finite field renormalization:

$$M_{h_1}^2 = \ldots + \hat{\Sigma}_{hh}^{\mathrm{SM},(1)\prime}(m_h^2)\hat{\Sigma}_{hh}(m_h^2) + \ldots$$

Implications for high-scale scenario

all SUSY particles at common scale M_{SUSY}, $\tan\beta = 10$. Solid: $X_t^{\overline{\text{DR}}} = 0$; dashed: $X_t^{\overline{\text{DR}}} = \sqrt{6}$



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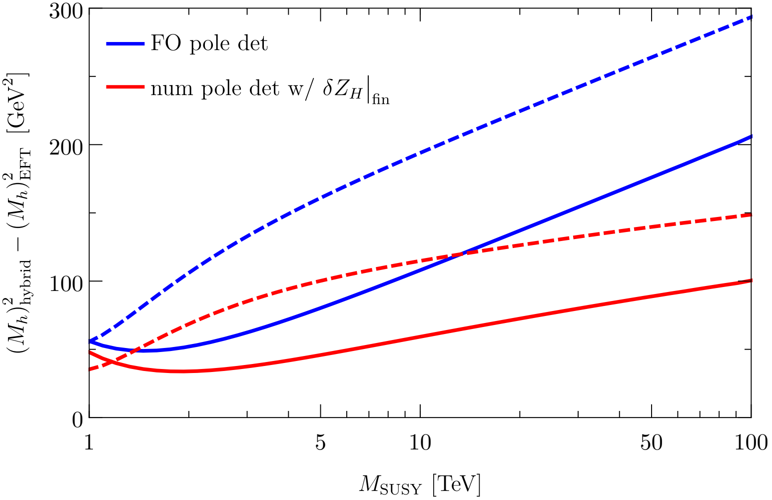


→ even better agreement with EFT

Further implications

Definition of $\tan\beta$:

- finite field renormalization affects definition of $\tan\beta$:

$$\tan\beta^{\mathrm{MSSM}}(\mu_R) \to \tan\beta^{\mathrm{THDM}}(M_t)$$

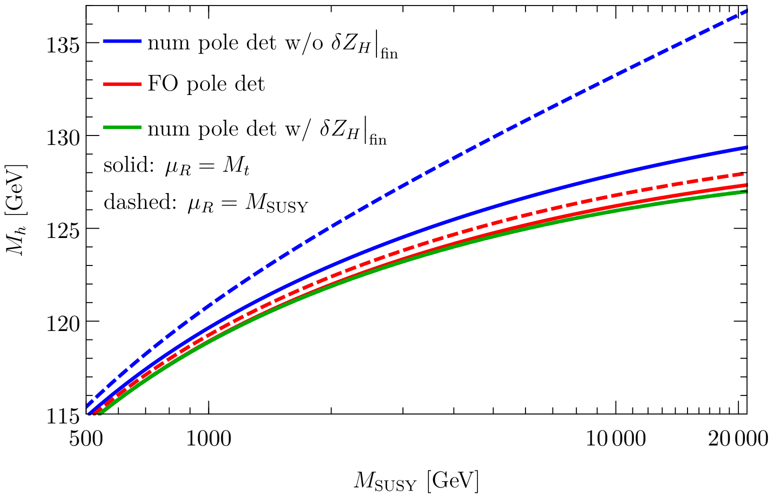
- prevent this by introducing independent finite $\tan\beta$ counterterm
- also have to introduce finite counterterms for mixing angles α, β_n, β_c

$\mathbf{Z}$ matrix connecting physical and tree-level mass states:

- definition would change from MSSM to THDM
- prevent this by using numerical pole determination without finite field renormalization for $\mathbf{Z}$ matrix

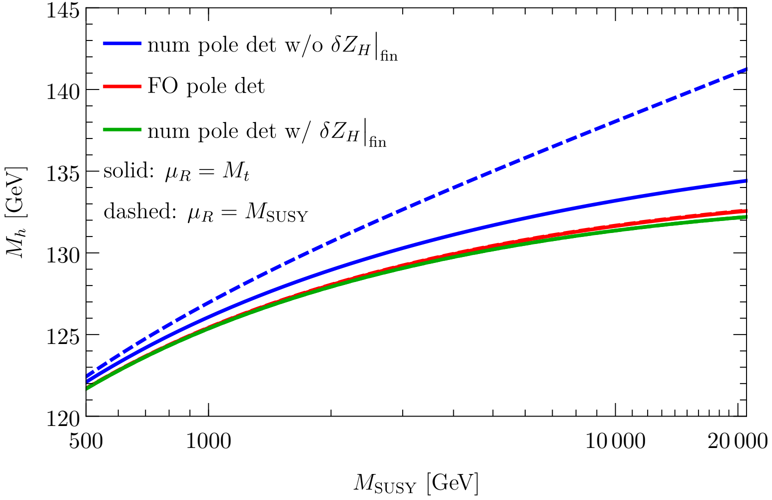
Further implications - scale variation

all SUSY particles at common scale M_{SUSY}, $\tan\beta = 4$. Solid: $X_t^{\overline{\text{DR}}} = 0$; dashed: $X_t^{\overline{\text{DR}}} = \sqrt{6}$

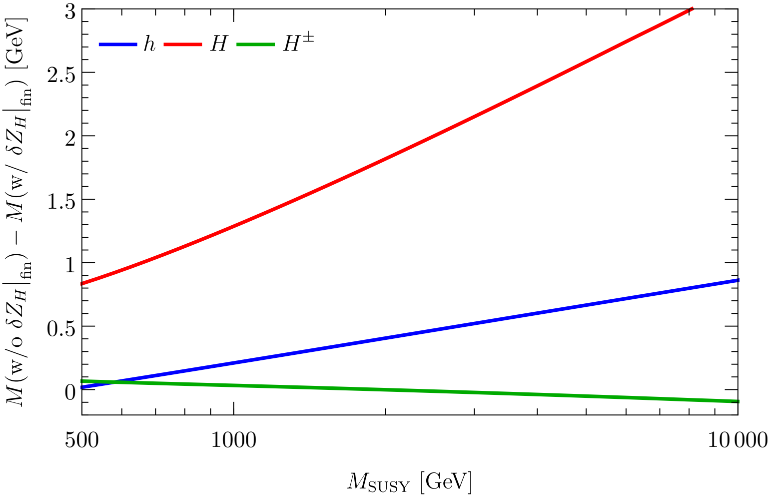


Compared three different methods to determine Higgs pole masses:

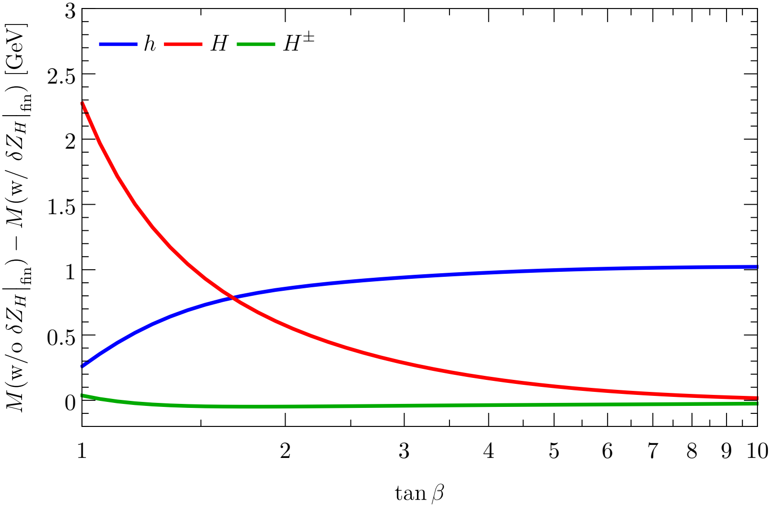
1. Numerial pole determination:
   - Conceptionally easy ✓
   - Incomplete cancellation of higher order "field-normalization-like" terms ✗
2. Fixed-order pole determination:
   - Complete cancellation ✓
   - Can lead to jumps in Higgs mass predictions ✗
3. Numerial pole det. with finite field renormalization:
   - Complete cancellation ✓
   - No jumps ✓
   - Better agreement with pure EFT calculations for high scales ✓



num pole det w/o $\delta Z_H|_{\text{fin}}$
FO pole det
num pole det w/ $\delta Z_H|_{\text{fin}}$
solid: $\mu_R = M_t$
dashed: $\mu_R = M_{\text{SUSY}}$
M_h [GeV]
145
140
135
130
125
120
500
1000
10 000
20 000
M_{SUSY} [GeV]



h H $H^\pm$
$M(\mathrm{w/o}\ \delta Z_H|_{\mathrm{fin}}) - M(\mathrm{w/}\ \delta Z_H|_{\mathrm{fin}})$ [GeV]
3
2.5
2
1.5
1
0.5
0
500
1000
10 000
M_{SUSY} [GeV]



h
H
$H^{\pm}$
$M(\text{w/o } \delta Z_H|_{\text{fin}}) - M(\text{w/ } \delta Z_H|_{\text{fin}})$ [GeV]
0
0.5
1
1.5
2
2.5
3
1
2
3
4
5
6
7
8
9
10
$\tan\beta$