1. MSSM Higgs sector @ tree-level

2 Higgs doublets → 5 physical Higgs bosons

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \phi_i + i\chi_i) \end{pmatrix}$$

$$V_{THDM} = \frac{1}{2} m_1^2 \Phi_1^\dagger \Phi_1 + \frac{1}{2} m_2^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \Phi_1^\dagger \Phi_2 + \frac{1}{4}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{4}\lambda_2 (\Phi_2^\dagger \Phi_2)^2$$
$$+ \frac{1}{2}\lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \frac{1}{2}\lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2$$
$$+ \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + h.c.$$

$$\Downarrow \text{SUSY}$$

$$V_{MSSM} = V_{THDM}\left(\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2),\ \lambda_3 = \frac{1}{4}(g^2 - g'^2),\ \lambda_4 = -\frac{1}{2}g^2,\ \lambda_5 = \lambda_6 = \lambda_7 = 0\right)$$

⇔ CP-conservation at the tree-level

[MSSM Yukawa sektor ≙ THDM type II, Φ_1 couples to leptons and down-type quarks, Φ_2 to up-type quarks)

Rotate to mass eigenstates

$$\begin{pmatrix} h \\ H \\ A \\ G \end{pmatrix} = U_n \begin{pmatrix} \phi_1 \\ \phi_2 \\ \chi_1 \\ \chi_2 \end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = U_c \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix},$$

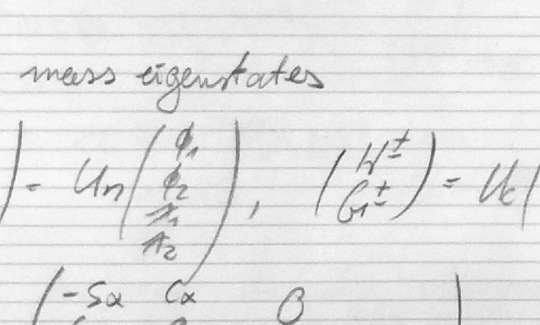
$h, H \to$ CP-even
$A, G \to$ CP-odd
⇔ 5 physical Higgs bosons

$$U_n = \begin{pmatrix} -s_\alpha & c_\alpha & & \\ c_\alpha & s_\alpha & & 0 \\ & 0 & -s_{\beta_n} & c_{\beta_n} \\ & & c_{\beta_n} & s_{\beta_n} \end{pmatrix}, \quad U_c = \begin{pmatrix} -s_{\beta_c} & c_{\beta_c} \\ c_{\beta_c} & s_{\beta_c} \end{pmatrix}$$

Minimize potential, choose M_A and $t_\beta = v_2/v_1$ as input parameters

$$\Rightarrow m_{h,H}^2 = \frac{1}{2}\left(M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4 M_A^2 M_Z^2 c_{2\beta}^2}\right), \quad m_{H^\pm}^2 = M_A^2 + M_W^2$$

and $\beta_n = \beta_c = \beta$, $\frac{t_{2\alpha}}{t_{2\beta}} = \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$ $\left(-\frac{\pi}{2} < \alpha < 0\right)$



decoupling limit

$$\boxed{m_h^2 \underset{M_A \gg M_Z}{=} M_Z^2 c_{2\beta}^2 \leq M_Z^2}$$

⇒ Quantum corrections play vital role to reach $M_h \sim 125$ GeV

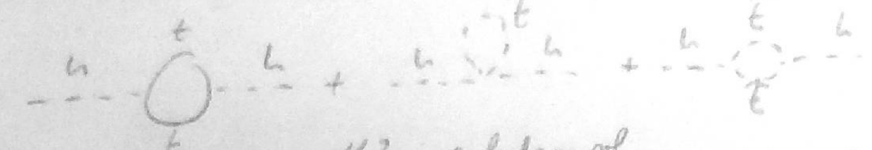
2. Fixed-order approach

$\Rightarrow$ Evaluate quantum corrections by calculating self-energy corrections

@1L: dominant contribution from (s)top sector

Stop mixing param./M_S

$m_{\tilde t_2}^2 - m_{\tilde t_1}^2 = 2 m_t X_t$

$\hat\Sigma_{hh}^{1L}(p^2) \approx$ 

stop mass scale

If $M_A^2 \gg M_Z^2$: physical mass M_h^2 is solution of

$p^2 - m_h^2 + \hat\Sigma_{hh}(p^2) = 0 \quad \rightarrow \quad M_h^2 = m_h^2 + 6 k v^2 y_t^4 \left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \hat X_t^2 - \frac{1}{12} \hat X_t^4 \right]$

(comment on Higgs being SM-like)

$\hookrightarrow$ paint X_t plot

if $M_A^2 \sim M_Z^2 \Rightarrow$ mixing between tree-level mass eigenstates:

~~solve~~ find poles of $\Gamma_{hHA}^{\text{...}}(p^2) = \begin{pmatrix} p^2 - m_h^2 + \hat\Sigma_{hh}(p^2) & \hat\Sigma_{hH}(p^2) & \hat\Sigma_{hA}(p^2) \\ \hat\Sigma_{hH}(p^2) & p^2 - m_H^2 + \hat\Sigma_{HH}(p^2) & \hat\Sigma_{HA}(p^2) \\ \hat\Sigma_{hA}(p^2) & \hat\Sigma_{HA}(p^2) & p^2 - m_A^2 + \hat\Sigma_{AA}(p^2) \end{pmatrix}$

if all input parameters are real: $\hat\Sigma_{hA}(p^2) = \hat\Sigma_{HA}(p^2) = 0$ ($\rightarrow$ renormalize M_A on-shell)

otherwise: full mixing between $h, H, A \Rightarrow$ mass eigenstates h_1, h_2, h_3

($\rightarrow$ renormalize $M_{H^\pm}$ on-shell)

Current status: full 1L corrections, $\mathcal{O}(\alpha_t\alpha_s, \alpha_b\alpha_s, \alpha_t^2, \alpha_b^2, \alpha_t\alpha_b, \alpha_s\alpha)$ 2L $+ \mathcal{O}(\alpha_s^2\alpha_t)$ $_{p^2=0}$

3L corrections / Challenges: # of involved scales, regularization, evaluation of loop integrals

e.g. $M_{SUSY} = 1\,\text{TeV}$, $\hat X_t^{OS} = 2$, $t_\beta = 10$

tree-level: $M_h = 89.4$ GeV) + 54%

1L $\mathcal{O}(M_t^4)$: $M_h = 137.4$ GeV) − 1%

1L $q/\tilde q$: $M_h = 135.9$ GeV) − 2%

full 1L: $M_h = 133.2$ GeV) − 5%

$\mathcal{O}(\alpha_s\alpha_t, \alpha_s\alpha_b, \alpha_{t,b}^2, \ldots)$: $M_h = 126.2$ GeV) − 1.5%

+ log resum: $M_h = 124.3$ GeV

vs. $M_h^{exp} = 125.09 \pm 0.21$ (stat.) ± 0.11 (sys.)

$\hookrightarrow \Delta M_h^{exp} \sim 0.3\%$

Advantage of FO approach: Allows to capture all contributions at a given order

Disadvantage " " " : If $M_{SUSY} \gg M_t$ large logarithms spoil convergence of perturbative expansion

3 EFT approach

Simplest scenario: all non-SM particles at common scale M_{susy}
↳ integrate them out below M_{susy} → SM as EFT

MSSM
——— M_{susy} → $\lambda(M_{susy}) = \frac{1}{4}(g^2+g'^2)c_{2\beta}^2 + \Delta\lambda$

SM
run to M_t, where physical Higgs mass is calculated:
——— M_t
$$p^2 - (m_h^{SM})^2 + \hat{\Sigma}_{hh}^{SM}(p^2) = 0$$
with $(m_h^{SM})^2 = \lambda(M_t)\, v^2$

RGE running ↔ resummation of logarithms

Needed ingredients:
- extraction of SM $\overline{MS}$ couplings @ M_t
- high energy threshold corrections
- SM Higgs self-energy
- RGEs

Current status:
- SM (LL, NLL, NNLL (gl. limit), partial N^3LL)
- SM + EWinos (")
- SM + EWinos + $\tilde{g}$ (no N^3LL)
- THDM (LL, NLL, partial NNLL)
- THDM + EWinos (")
- THDM + EWinos + $\tilde{g}$ (")

MSSM
———
THDM + $\tilde{g}$ + EWinos
e.g. $M_{\tilde{g}}$ ———
THDM + EWinos
M_A ———
SM + EWinos
M_χ ———
SM
M_t ———

Only one paper including dimension 6 operators
↳ suppressed $\mathcal{O}(v/M_{susy})$ terms often missed
↳ EFT approach imprecise for low SUSY scales

4. Hybrid approach

Fixed-order approach: precise for low SUSY scales, not precise for high SUSY scales

EFT approach: not precise " " " ", precise " " " "

↳ Combine both approaches for precise prediction also for intermediary scales. ⇒ hybrid approach

Currently two different approaches:

a) FeynHiggs approach

1. Obtain FO result, $\hat{\Sigma}_{hh}^{MSSM}$, and EFT result, $\lambda(M_t)v^2$.
2. Subtract logarithms and non-log terms already contained in $\hat{\Sigma}_{hh}^{MSSM}$
3. Add both: $\hat{\Sigma}_{hh}^{hybrid}(p^2) = \hat{\Sigma}_{hh}^{MSSM}(p^2)\Big|_{sub} - \lambda(M_t)v^2$

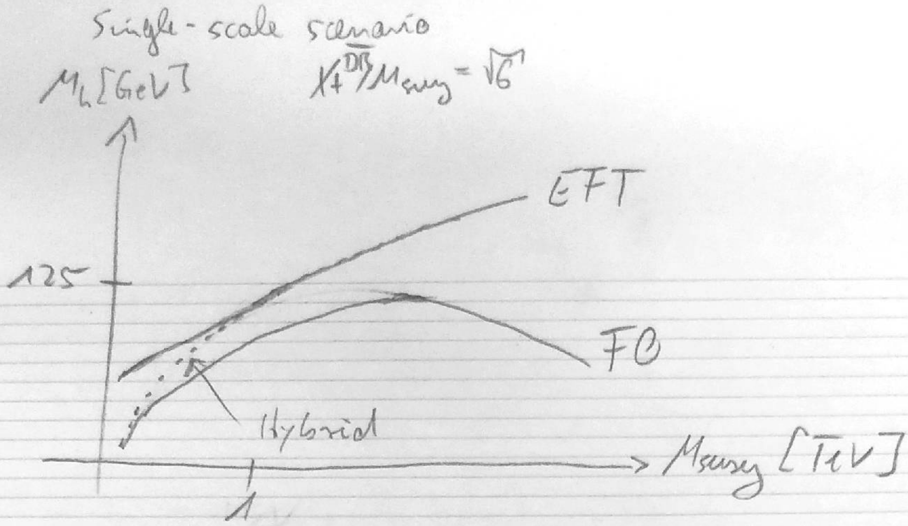
b) FlexibleEFTHiggs

1. Calculate fixed order predictions @ M_{SUSY}, $(M_h^2)_{MSSM}$, ~~MSSM and SM~~ and SM Higgs corrections
2. $(M_h^2)_{SM} = \lambda(M_S)v^2(M_S) + (\Delta M_h^2)_{SM}$

   ↳ $\lambda(M_S) = \frac{1}{v^2(M_S)}\left((M_h^2)_{MSSM} - (\Delta M_h^2)_{MSSM}\right)$

   Includes $O(v/M_{SUSY})$ in threshold correction
3. Proceed as in pure EFT calculation

	FeynHiggs	FlexibleEFTHiggs
Precise for low, intermediary and high SUSY scales	✓	✓
Precision	LL, NLL, $O(\alpha_s, \alpha_t)$ NNLL	LL, NLL
Low-energy EFTs	SM($+\tilde{\chi}, +\tilde{g}$) THDM($+\tilde{\chi}, \tilde{g}$)	SM ($+\tilde{\chi}, \tilde{g}$) (+ beyond MSSM) ↳ easily extendable to any model featuring SM-like Higgs sector for low scales



in single scale scenario suppressed terms are negligible in phenomenologically interesting region. Can be different e.g. $M_A \sim M_t$

5. Pole determination

For $M_A \gg M_Z$, we have to solve $p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) = 0$

Up to 2L level solution reads: $M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \underbrace{\hat{\Sigma}_{hh}^{(2)}(m_h^2)}_{\text{gaugeless limit}} + \underbrace{\hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\,\hat{\Sigma}_{hh}^{(1)}(m_h^2)}_{\text{fully available}}$

$\rightarrow$ problem: there is a cancellation of $\hat{\Sigma}_{hh}^{(1),\text{heavy}\,\prime}(m_h^2)$ with parts of subloop renormalization in $\hat{\Sigma}_{hh}^{(2)}(m_h^2)$ ~~due to~~ (origin: decoupling theorem)

$\rightarrow$ possible solution:
- include $\Sigma' \Sigma$ term also only in gaugeless limit
- truncate ~~at~~ expansion @ 2L

What happens for $M_A \sim M_Z$ (CP-conserving case)?

Have to solve: $\det\begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2) \end{pmatrix} = 0$

$$\Rightarrow M_h^2 = m_h^2 - \hat{\Sigma}_{hh}^{(1)}(m_h^2) - \hat{\Sigma}_{hh}^{(2)}(m_h^2) + \hat{\Sigma}_{hh}^{(1)\prime}(m_h^2)\,\hat{\Sigma}_{hh}^{(1)}(m_h^2) + \frac{\left(\hat{\Sigma}_{hH}^{(1)}(m_h^2)\right)^2}{m_h^2 - m_H^2}$$

Can we again use the same solution as in $M_A \gg M_Z$ case? $\cancel{\checkmark}$

For $M_A \sim M_Z \rightarrow m_h^2 \sim m_H^2 \Rightarrow$ last term gets large

$\hookrightarrow$ truncation @ 2L $\rightarrow$ ~~negative~~ very small or even tachyonic Higgs mass for low M_A (and t_β)

- Origin of problem: Momentum dependence induced by heavy particles

Should not have physical effect according to decoupling theorem!

$$\Sigma(p^2) = \Sigma^{light}(p^2) + \Sigma^{heavy}(p^2) = \quad \leftarrow \text{behaviour of loop integrals in } \cancel{\text{high}} \text{ heavy mass limit}$$

$$= \Sigma^{light}(p^2) + \Sigma^{heavy}(m^2) + \Sigma^{heavy\prime}(m^2)(p^2 - m^2)$$

$$\hat{\Sigma}(p^2) = \Sigma(p^2) - \delta m^2 + \delta Z (p^2 - m^2)$$

↳ momentum dependence ~~of~~ induced by heavy contributions behaves like field renormalization ⇒ can be absorbed by a redefinition of fields

⇒ dependence completely removed from calculation

⇒ No cancellation between different pieces in pole determination procedure

⇒ Determine poles numerically!