External leg corrections as an origin of large logarithms

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Talk outline

1. Introduction
2. External leg corrections as an origin of large logarithms
3. Applications
4. Conclusion

Introduction

Motivation

- BSM physics needed to explain e.g. Dark Matter, baryon asymmetry, etc.
- Many BSM models predict extended scalar sector (e.g. singlet extensions, 2HDM, MSSM).
- If a new particle is discovered, its characterization will be of foremost interest.
- Precise theory predictions will be needed to discriminate between different possible realization of BSM physics $\Rightarrow$ calculation of loop corrections crucial.
- One of the main challenges: large logarithms.

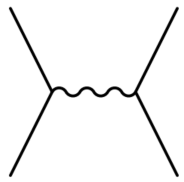
This talk: identify a new source of large logarithms related to external leg corrections.

Large logarithms in precision predictions

- In many calculations, large logarithms appear spoiling the reliability of the perturbative expansion.
- Different types of large logarithms are known (non-comprehensive list):
  1. Logarithms containing heavy mass scale appearing in prediction of low-scale observable
     - E.g. prediction of SM-like Higgs mass in the MSSM $\rightarrow \ln\frac{M_{SUSY}^2}{m_t^2}$.
     - Resum by integrating out the heavy particles.
  2. Logarithms involving light quark mass
     - E.g. heavy Higgs boson decay to fermions.
     - Resum large logarithms by involving Yukawa coupling to heavy mass scale.
  3. Sudakov logarithms
     - E.g. thrust T distribution in QCD $\rightarrow$ $\ln^2(1-T)$ appears in integrated distribution.
     - Resum by exponentiation or using Soft Collinear Field Theory (SCET).

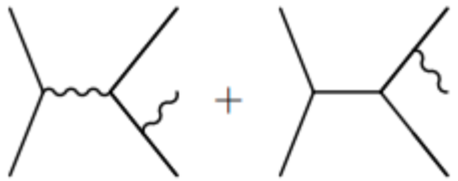
Recap: infrared divergencies

- Consider e.g. $e^+e^- \to \mu^+\mu^-$.



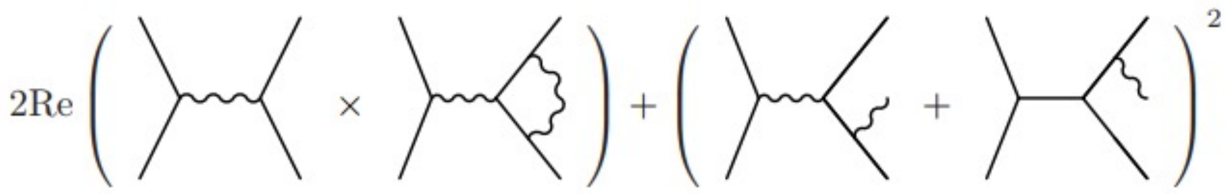
- 1L virtual QED correction is IR divergent; i.e., contains $\ln^2 \frac{m_\gamma^2}{Q^2}$ and $\ln^1 \frac{m_\gamma^2}{Q^2}$ terms (m_γ: photon mass, Q: ren. scale).

- Real emission contribution of photons with energy below detector resolution also contains IR divergencies.



$$\Rightarrow \quad \sigma_{2\to 2} = \sigma\left(e^+e^- \to \mu^+\mu^-\right) + \sigma\left(e^+e^- \to \mu^+\mu^-\gamma\right)\Big|_{E_\gamma < E_{\rm res} \text{ or } \theta_{\gamma\mu} < \theta_{\rm res}}$$

- Sum of virtual and real corrections IR finite.



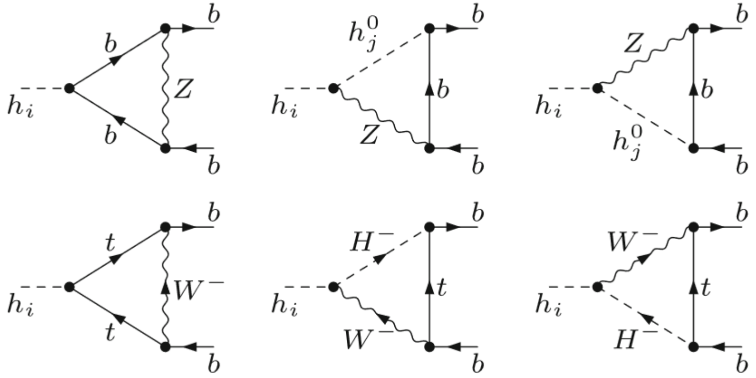
$\Rightarrow$ IR finite.

Electroweak Sudakov logarithms

- Sudakov logarithms also appear in electroweak corrections in the form

$$\sim \frac{g^2}{16\pi^2}\ln^2\frac{M_V^2}{s} \quad \text{and} \quad \sim \frac{g^2}{16\pi^2}\ln\frac{M_V^2}{s}$$ where M_V is a gauge or Higgs boson mass.

- Example: heavy Higgs boson decay into $b\bar{b}$ (see e.g. [Domingo,Paßehr, 1907.05468]).



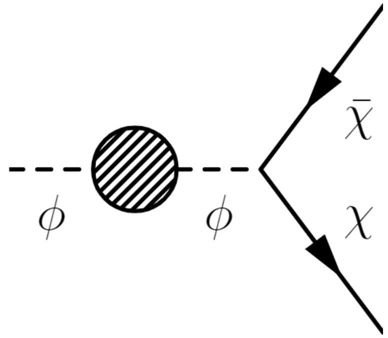
$$\frac{\Delta\Gamma^{\rm DL}}{\Gamma^{\rm Born}}[h_i \to b\bar{b}] \simeq -\frac{1}{48\,\pi^2}\left[\frac{5}{12}\,g_1^2 + \frac{9}{4}\,g_2^2 - \frac{2}{3}\,e^2\right]\ln^2\frac{M_V^2}{M_{h_i}^2}$$

- Sudakov logarithms related to infrared limit ($M_V \to 0$); cancel in combined $h_i \to b\bar{b}$, $h_i \to b\bar{b} + Z/h_j$, $h_i \to t\bar{b} + W^-$ amplitude.
- If additional $Z/h_j/W$ radiation can be resolved analytically → large logarithms remain in result.

→ This talk: Sudakov-like logarithms arising from external leg corrections.

External leg corrections: LSZ factor

- Need to ensure that external particles have correct OS properties ⇒ **LSZ formalism**!



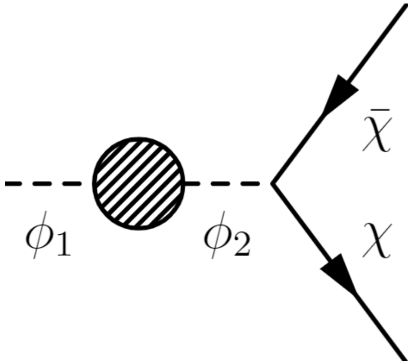
- For non-mixing particles, this accounts to multiplying the amplitude by factors of $\sqrt{Z_\phi}$ for every external particle ϕ,

$$\sqrt{Z_\phi} = \frac{1}{\sqrt{1+\hat{\Sigma}'_{\phi\phi}\left(\mathcal{M}_\phi^2\right)}},$$

where $\hat{\Sigma}'_{\phi\phi}$ is the momentum derivative of the $\phi\phi$ self energy.

External leg corrections: Z-matrix formalism

[Fuchs,Weiglein,1610.06193]



In general, we also need to consider mixing:

$$\hat{\Gamma}_{\phi_a^{\text{physical}}} = \sum_j \hat{\mathbf{Z}}_{aj} \hat{\Gamma}_{\phi_j}$$

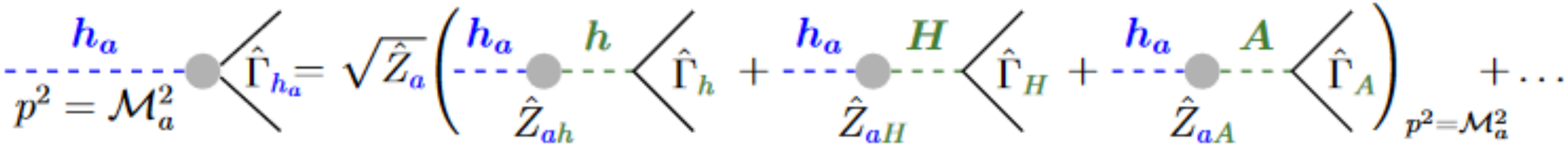
With $\hat{\mathbf{Z}}_{aj} = \sqrt{\hat{Z}_i^a \hat{Z}_{ij}^a}$ and $\hat{Z}_i^a = \frac{1}{1 + \hat{\Sigma}_{ii}^{\text{eff}\,\prime}(p^2 = \mathcal{M}_a^2)}$, $\hat{Z}_{ij}^a = \left.\frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)}\right|_{p^2 = \mathcal{M}_a^2}$

Δ_{ij} is the ij element of the propagator matrix, $\mathcal{M}_a^2$ is the complex pole and

$$\hat{\Sigma}_{ii}^{\text{eff}}(p^2) = \hat{\Sigma}_{ii}(p^2) + \frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)} \hat{\Sigma}_{jj}(p^2) + \frac{\Delta_{ik}(p^2)}{\Delta_{ii}(p^2)} \hat{\Sigma}_{kk}(p^2)$$

for three particles i, j, k.

e.g. for three Higgs boson h, H, A:



External leg corrections as a source of large logarithms

Toy model

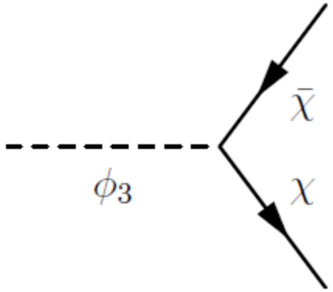
- Three real scalars and one Dirac fermion: ϕ_1, ϕ_2, ϕ_3 and χ
- $\mathbb{Z}_2$ symmetry: $\phi_1 \rightarrow -\phi_1, \phi_2 \rightarrow -\phi_2,\ \phi_3 \rightarrow \phi_3, \chi \rightarrow \chi$

$$
\begin{aligned}
\mathcal{L} = & \frac{1}{2}\sum_{i=3}^{3}(\partial_\mu\phi_i\partial^\mu\phi_i - m_i^2\phi_i^2) \\
& - \frac{1}{2}A_{113}\phi_1^2\phi_3 - A_{123}\phi_1\phi_2\phi_3 - \frac{1}{2}A_{223}\phi_2^2\phi_3 - \frac{1}{6}A_{333}\phi_3^3 \\
& - \frac{1}{24}\lambda_{1111}\phi_1^4 - \frac{1}{6}\lambda_{1112}\phi_1^3\phi_2 - \frac{1}{4}\lambda_{1122}\phi_1^2\phi_2^2 - \frac{1}{6}\lambda_{1222}\phi_1\phi_2^3 - \frac{1}{24}\lambda_{2222}\phi_2^4 \\
& - \frac{1}{4}\lambda_{1133}\phi_1^2\phi_3^2 - \frac{1}{2}\lambda_{1233}\phi_1\phi_2\phi_3^2 - \frac{1}{4}\lambda_{2233}\phi_2^2\phi_3^2 - \frac{1}{24}\lambda_{3333}\phi_3^4 \\
& + \bar{\chi}(i\partial\!\!\!/ - m_\chi)\chi + y_3\phi_3\bar{\chi}\chi\,,
\end{aligned}
$$

- For the present study, we are mainly interested in the trilinear couplings (especially A_{123}).

The $\phi_3 \to \bar{\chi}\chi$ decay process

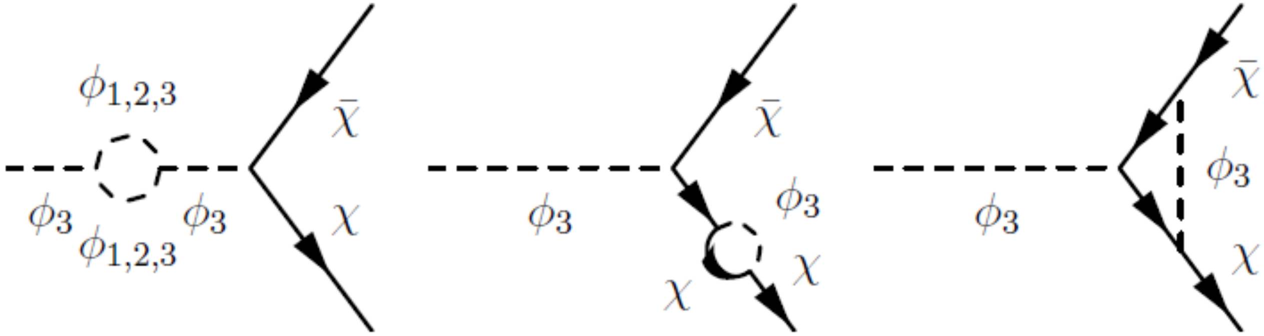
Tree-level:



$$\Gamma^0(\phi_3 \to \bar{\chi}\chi) = \frac{1}{8\pi} m_3 \left(1 - \frac{4m_\chi^2}{m_3^2}\right)^{3/2} \; y_3^2$$

One-loop virtual:

$(k \equiv (4\pi)^{-2})$



$$\Delta\hat{\Gamma}^{(1)}_{\phi_3\to\bar{\chi}\chi} \supset -\frac{1}{2} k y_3 \mathrm{Re}\Big[(A_{113})^2 \frac{d}{dp^2} B_0(p^2, m_1^2, m_1^2) + 2(A_{123})^2 \frac{d}{dp^2} B_0(p^2, m_1^2, m_2^2)$$
$$+ (A_{223})^2 \frac{d}{dp^2} B_0(p^2, m_2^2, m_2^2) + (A_{333})^2 \frac{d}{dp^2} B_0(p^2, m_3^2, m_3^2)\Big]\Big|_{p^2=m_3^2}$$
$$+ \cdots ,$$

No mixing & no vertex corrections proportional to A_{ijk} !

Infrared limits

1. ϕ_2 and ϕ_3 are almost mass-degenerate, ϕ_1 is light ($m_2 \to m_3,\ m_1 \to 0$)

$$\frac{d}{dp^2} B_0(p^2, m_1^2, m_2^2)\big|_{p^2=m_3^2} = \frac{1}{m_3^2}\left(\frac{1}{2}\ln\frac{m_3^2}{m_1^2} - 1 + \mathcal{O}(\epsilon)\right)$$

with $\epsilon \equiv m_3^2 - m_2^2$ and $m_1^2 \sim \epsilon$.

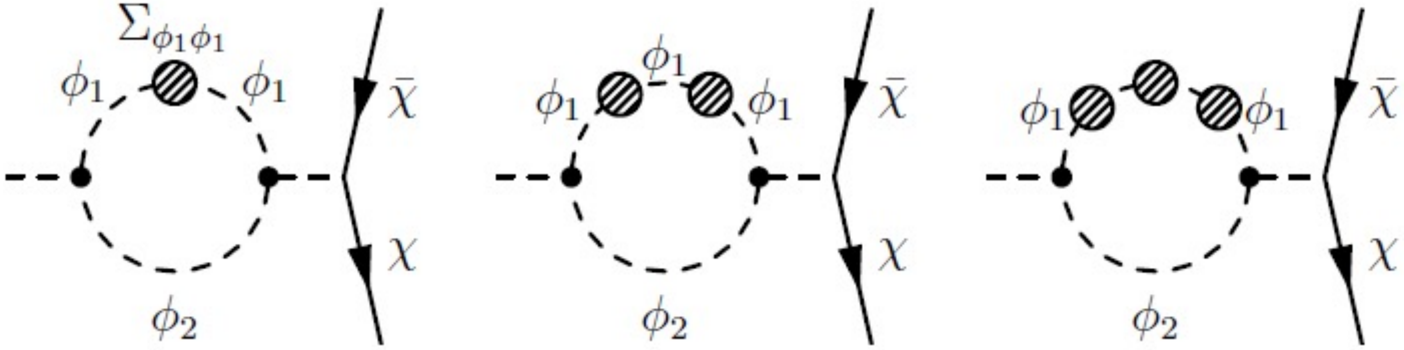
2. ϕ_2 and ϕ_3 are almost mass-degenerate, ϕ_1 is massless ($m_1 = 0, m_2 \to m_3$)

$$\frac{d}{dp^2} B_0(p^2, 0, m_2^2)\big|_{p^2=m_3^2} = \frac{1}{m_3^2}\left(\ln\frac{m_3^2}{\epsilon} - 1 + \mathcal{O}(\epsilon)\right)$$

→ Infrared divergencies appear in external leg corrections

Regulating the IR divergency I: resummation of ϕ_1 contributions

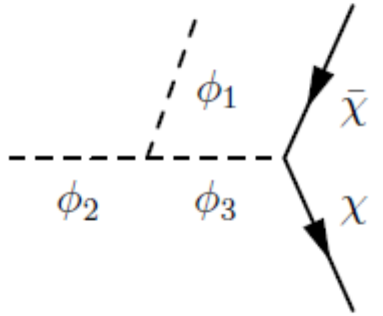
Idea: give ϕ_1 an effective mass by resuming ϕ_1 self-energy insertions (like for the Goldstone boson catastrophe).



$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) \supset \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi}) \cdot \left[1 + k\frac{(A_{123})^2}{m_3^2}\left(\frac{1}{2}\ln\frac{\Delta m_1^2}{m_3^2} + 1\right)\right] \quad \text{with} \quad \Delta m_{\phi_1}^2 = \hat{\Sigma}_{11}^{(1)}(p^2 = 0)$$

→ IR divergence regulated, but physical interpretation unclear.

Regulating the IR divergency II: soft ϕ_1 radiation



Include soft ϕ_1 radiation (here: $m_1 \neq 0$ with $m_2 = m_3$; $\epsilon \neq 0$ with $m_1 = 0$ case follows analogously):

$$\Gamma^{(0)}(\phi_2 \to \chi\bar{\chi}\phi_1)\big|^{\text{soft}} = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi}) \cdot k \frac{(A_{123})^2}{m_3^2}\Big[-\frac{E_\ell}{\sqrt{E_\ell^2 + m_1^2}} - \frac{1}{2}\ln m_1^2 + \ln(E_\ell + \sqrt{E_\ell^2 + m_1^2})\Big]$$

$$= \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi}) \cdot k \frac{(A_{123})^2}{m_3^2}\Big[-1 - \frac{1}{2}\ln m_1^2 + \ln(2E_\ell) + \mathcal{O}(m_1)\Big]$$

⇒ sum of virtual and real corrections is infrared finite:

$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) + \Gamma^{(0)}(\phi_2 \to \chi\bar{\chi}\phi_1)\big|^{\text{soft}} = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi}) \cdot \Big[1 + k \frac{(A_{123})^2}{m_3^2} \ln \frac{2E_\ell}{m_3}\Big] + \cdots,$$

detector resolution

Infrared divergencies are regulated with clear physical interpretation!

The appearance of large logarithms

If the mass of ϕ_1 is large enough (or the mass difference ϵ), $\phi_3 \to \chi\overline{\chi}$ and $\phi_3 \to \chi\overline{\chi}\phi_1$ processes can be distinguished experimentally.

Then, we will have terms like

$$\frac{A_{123}^2}{m_3^2} \ln \frac{m_3^2}{m_1^2}$$

appearing in our amplitude.

For many BSM theories trilinear couplings are of the order of the BSM mass scale ($A_{123} \sim m_3$).

→ Large unsuppressed logarithms appear in the prediction of the decay width!

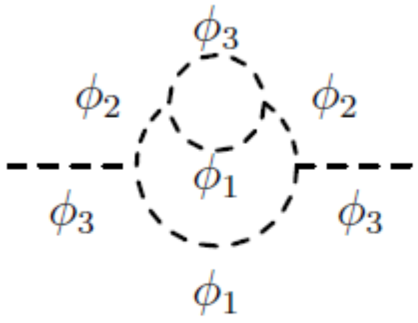
How large is the impact of these logarithms at higher orders?

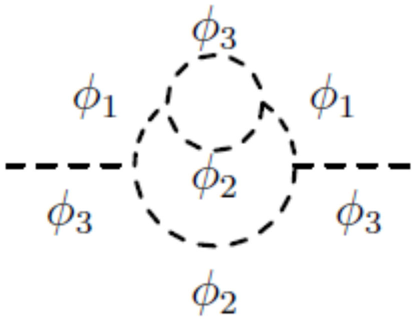
External leg corrections at the 2L level I

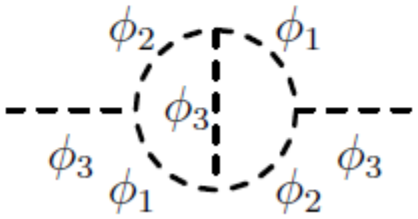
- Resummation could be feasible using SCET approach (see e.g. [Alte,König,Neubert,1902.04593]).
- We take a more direct approach by explicitly evaluating 2L corrections.

$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\Big\{1 - \mathrm{Re}\hat{\Sigma}^{(1)\prime}_{33}(m^2) - \mathrm{Re}\hat{\Sigma}^{(2)\prime}_{33}(m^2) + \left(\mathrm{Re}\hat{\Sigma}^{(1)\prime}_{33}(m^2)\right)^2 + \left(\mathrm{Im}\hat{\Sigma}^{(1)\prime}_{33}(m^2)\right)^2 + \mathcal{O}(k^3)\Big\}$$

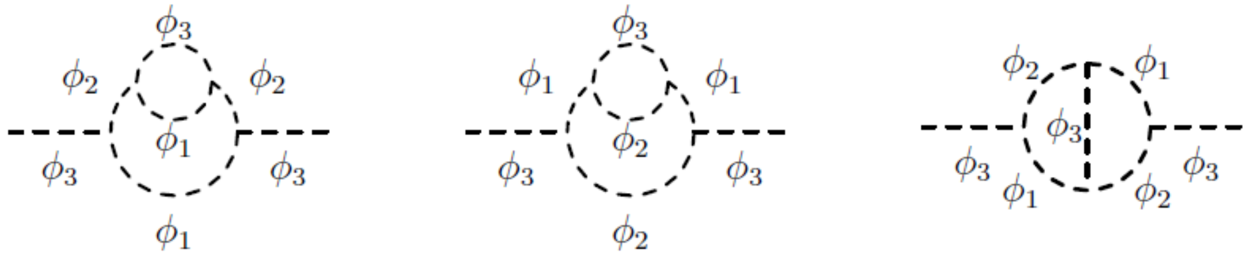
with the two-loop diagrams (including only corrections leading in powers of A_{123})







External leg corrections at the 2L level II



With $m_1^2 = \epsilon, m_2^2 = m_3^2 = m^2$ we obtain

$$\hat{\Sigma}_{33}^{(1)}(p^2) = k(A_{123})^2 B_0(p^2, \epsilon, m^2)\,,$$

$$\hat{\Sigma}_{33}^{(2,\text{ genuine})}(p^2) = k^2(A_{123})^4 \big[T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon) + T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2) + T_{12345}(p^2, m^2, \epsilon, m^2, \epsilon, m^2)\big].$$

T_{11234} and T_{12345} are the finite parts of

$$\mathbf{T}_{11234}(p^2, x, y, z, u, v) \equiv$$
$$\equiv \mathcal{C}^2 \int\int \frac{d^d q_1 d^d q_2}{(q_1^2 - x)(q_1^2 - y)((q_1 + p)^2 - z)((q_1 - q_2)^2 - u)(q_2^2 - v)}\,,$$
$$\mathbf{T}_{12345}(p^2, x, y, z, u, v) \equiv$$
$$\equiv \mathcal{C}^2 \int\int \frac{d^d q_1 d^d q_2}{(q_1^2 - x)((q_1 + p)^2 - y)((q_1 - q_2)^2 - z)(q_2^2 - u)((q_2 + p)^2 - v)}\,,$$

Evaluation of 2L integrals

- 2L integrals can be evaluated numerically using e.g. TSIL [Martin,Robertson,0501132].
- We want to extract the large logarithms ⇒ analytic expansion in infrared limits.

  (using expressions from [Martin,Robertsion,0312092,0307101,0501132])

- Example result for T_{11234} (with $m_1^2 = \epsilon, m_2^2 = m_3^2 = m^2$):

$$\frac{d}{dp^2} T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2)\Big|_{p^2=m^2} =$$

$$= -\frac{\overline{\ln} m^2}{2m^2\epsilon} + \frac{3\pi \overline{\ln} m^2}{8m^3\sqrt{\epsilon}} + \frac{-50 + 6\pi^2 + 3\overline{\ln}\epsilon - 12\overline{\ln} m^2 + 18\overline{\ln}\epsilon\overline{\ln} m^2 - 18\overline{\ln}^2 m^2}{36m^4}$$

- Terms of $\mathcal{O}(1/\epsilon, 1/\sqrt{\epsilon})$ appear!

($\overline{\ln} x = \ln x / Q^2$ and ren. scale Q)

$\overline{MS}$ 2L result

(for $m_1^2 = \epsilon, m_2^2 = m_3^2 = m^2$)

We obtain

$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\Bigg\{1 - \frac{k(A_{123})^2}{m^2}\left[\frac{1}{2}\ln\frac{m^2}{\epsilon} - 1\right] + \frac{k^2(A_{123})^4}{m^4}\Bigg[\frac{m^2\overline{\ln}m^2}{2\epsilon} - \frac{m\pi(4+\overline{\ln}m^2)}{8\sqrt{\epsilon}} + \frac{17}{9} - \frac{\pi^2}{8} + \frac{1}{8}\ln^2\frac{m^2}{\epsilon} + \frac{1}{6}\overline{\ln}\epsilon + \frac{1}{12}\overline{\ln}m^2 + \pi^2\ln 2 - \frac{3}{2}\zeta(3)\Bigg]\Bigg\}.$$

Terms enhanced by $\mathcal{O}(1/_\epsilon, 1/_{\sqrt{\epsilon}})$ appear in result! Can we absorb them into the renormalization of the masses and A_{123}?

$$\hat{\Sigma}_{33}^{(2,\text{ subloop})}(p^2) = k(A_{123})^2\Bigg[\left(\frac{2\delta^{(1)}A_{123}}{A_{123}} + \delta^{(1)}Z_3\right)B_0(p^2, m_1^2, m_2^2) + \delta^{(1)}m_1^2\frac{\partial}{\partial m_1^2}B_0(p^2, m_1^2, m_2^2) + \delta^{(1)}m_2^2\frac{\partial}{\partial m_2^2}B_0(p^2, m_1^2, m_2^2)\Bigg]$$

Mass renormalization

Renormalize m_1 and m_2 in the OS scheme:

$$\delta^{(1)} m_1^2 = k(A_{123})^2 \mathrm{Re} B_0(m_1^2, m_2^2, m_3^2)$$
$$\delta^{(1)} m_2^2 = k(A_{123})^2 \mathrm{Re} B_0(m_2^2, m_1^2, m_3^2)$$

→ Cancels $\mathcal{O}({}^1/_\epsilon\, , {}^1/_{\sqrt{\epsilon}})$ terms!

OS mass renormalization essential to avoid unphysically large corrections!

Similar issues are known to appear e.g. in the MSSM: non-decoupling of gluino corrections.

(see e.g. [9812472, 0105096,1606.09213, 1912.04199, 1912.10002])

Renormalization of A_{123}

- Three options for renormalization of A_{123} (CT is scale independent at leading order in A_{123}):
  - A_{123} $\overline{MS}$:

$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\left\{1 - \frac{k(A_{123})^2}{m^2}\left[\frac{1}{2}\ln\frac{m^2}{\epsilon} - 1\right] + \frac{k^2(A_{123})^4}{m^4}\left[\frac{1}{8}\ln^2\frac{m^2}{\epsilon} - \frac{7}{6}\ln\frac{m^2}{\epsilon} + \frac{71}{36} - \frac{3\pi^2}{8} + \pi^2\ln 2 - \frac{3}{2}\zeta(3)\right]\right\}$$

  - A_{123} OS via $\phi_2 \to \phi_1\phi_3$ amplitude:

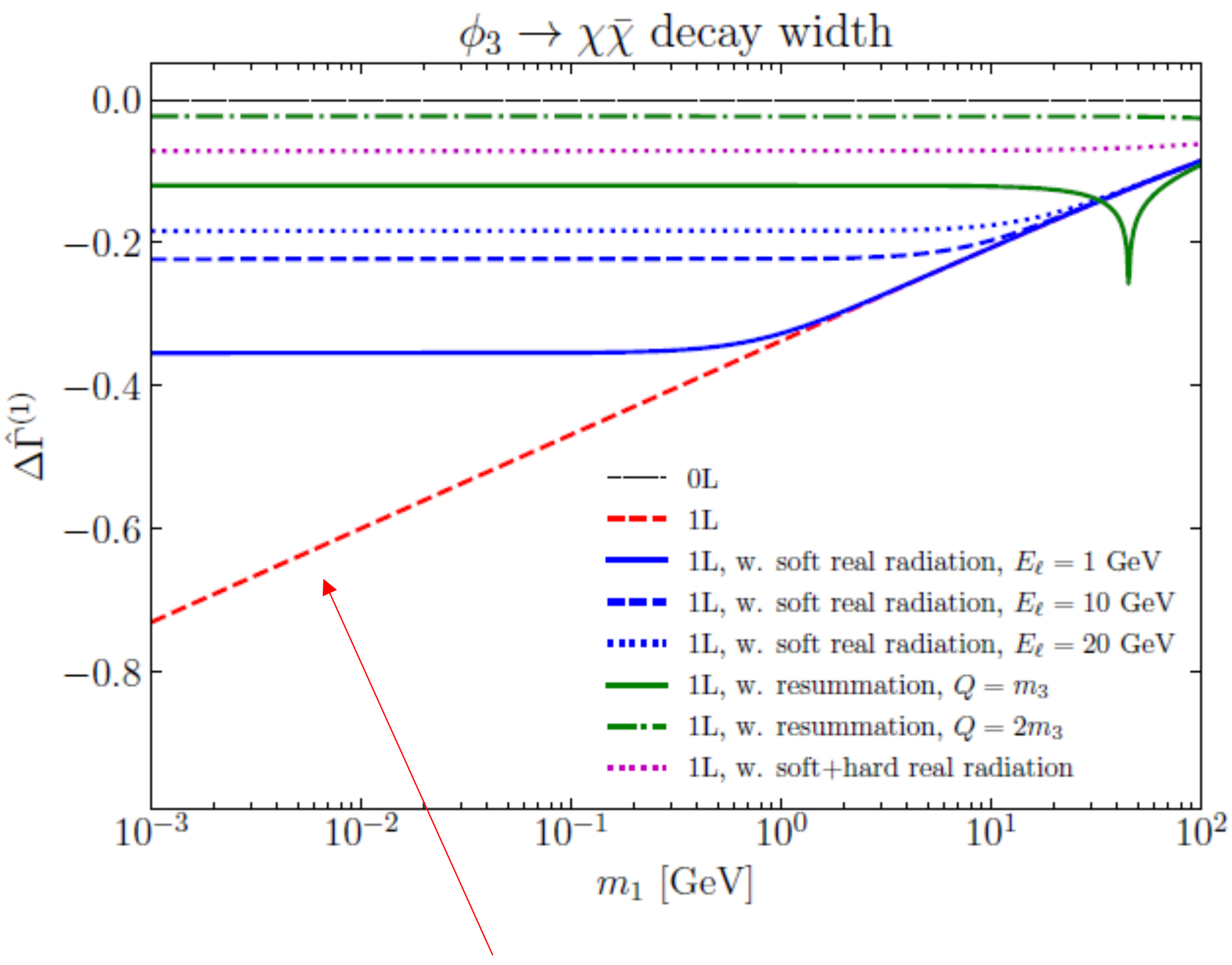
$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\left\{1 - \frac{k(A_{123})^2}{m^2}\left[\frac{1}{2}\ln\frac{m^2}{\epsilon} - 1\right] + \frac{k^2(A_{123})^4}{m^4}\left[\frac{1}{8}\ln^2\frac{m^2}{\epsilon} - \frac{31}{24}\ln\frac{m^2}{\epsilon} + \frac{19}{18} - \frac{3\pi^2}{8} + \pi^2\ln 2 - \frac{3}{2}\zeta(3)\right]\right\}$$

  - Choose A_{123} counterterm such that $\ln^2\epsilon$ in $\hat{\Gamma}(\phi_3 \to \chi\bar{\chi})$ cancels ("no-log-sq" scheme):

$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\left\{1 - \frac{k(A_{123})^2}{m^2}\left[\frac{1}{2}\ln\frac{m^2}{\epsilon} - 1\right] + \frac{k^2(A_{123})^4}{m^4}\left[-\frac{11}{12}\ln\frac{m^2}{\epsilon} + \frac{71}{36} - \frac{3\pi^2}{8} + \pi^2\ln 2 - \frac{3}{2}\zeta(3)\right]\right\}$$

Numerical analysis – 1L level

($m_2 = m_3 = 1$ TeV, $A_{123} = 3$ TeV)

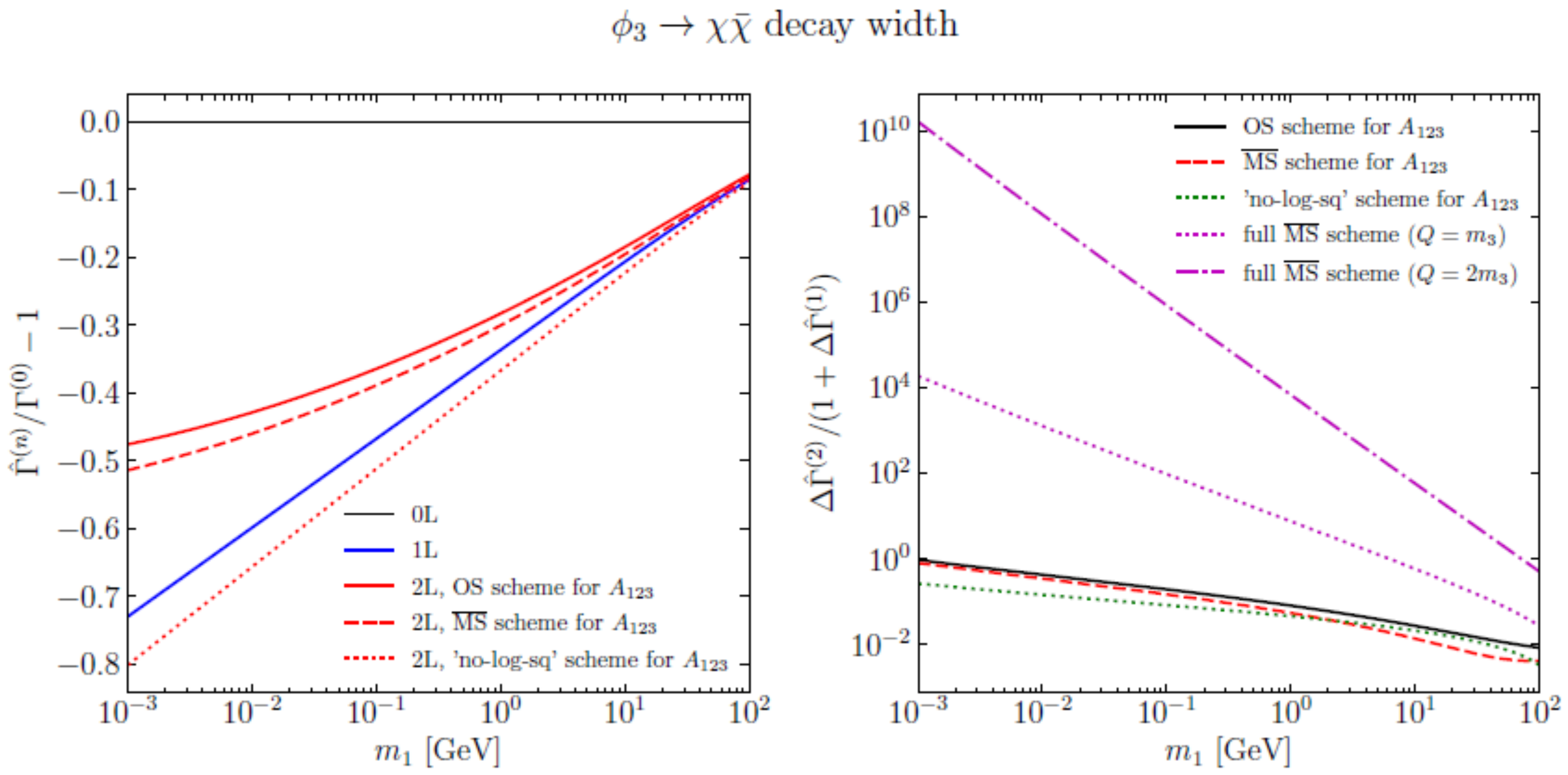


Large logarithm if no real radiation is included.

- If ϕ_1 radiation can be resolved experimentally, large 1L corrections are possible!
- Resumming ϕ_1 contributions results in substantial scale dependence (also no clear physical interpretation).

Numerical analysis – 2L level

($m_2 = m_3 = 1$ TeV, $A_{123} = 3$ TeV)

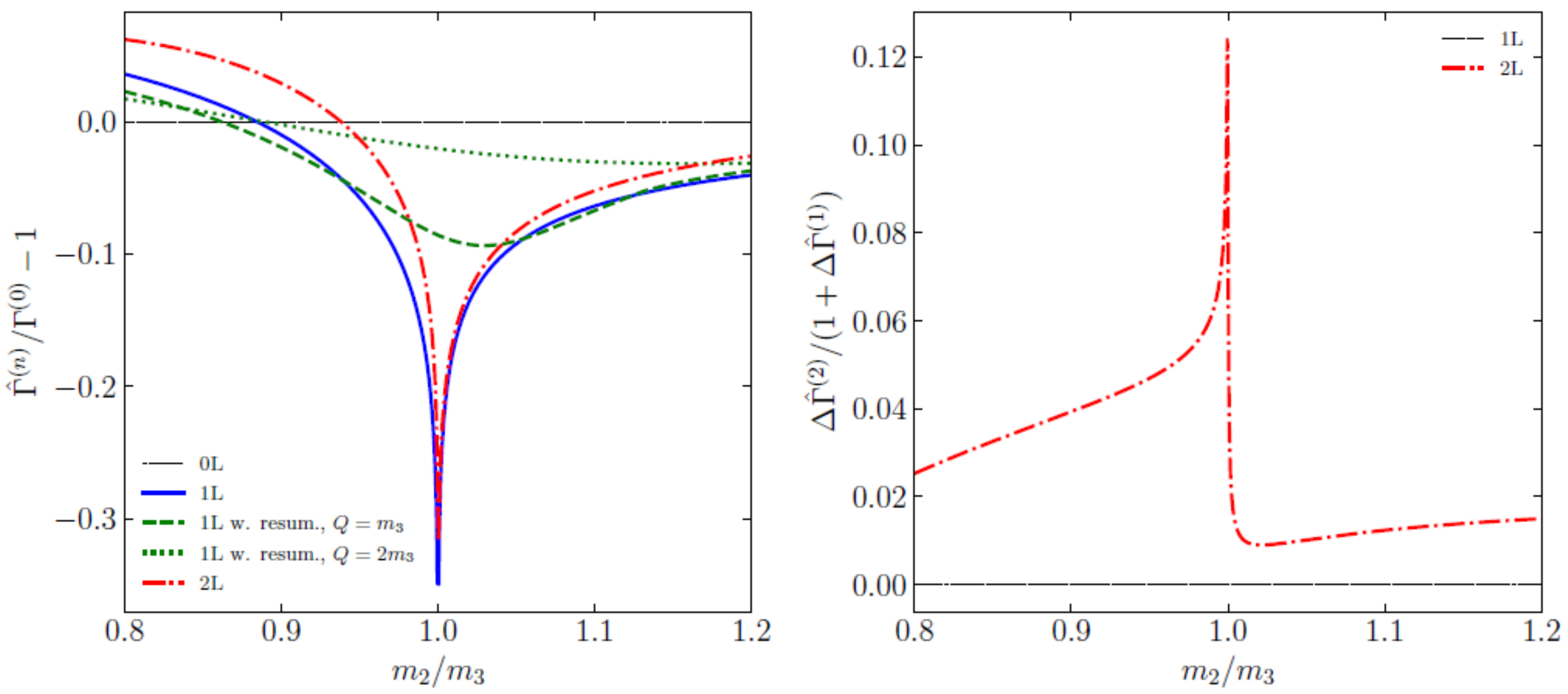


2L corrections can have substantial impact close to IR limit.
Only moderate differences between A_{123} schemes.

Numerical analysis – 2L level

($m_1 = 0$ TeV, $m_3 = 0.5$ TeV, $A_{123}^{\overline{MS}} = 1.5$ TeV)

$\phi_3 \to \chi\bar{\chi}$ decay width



2L corrections can have substantial impact close to IR limit.

Applications

Stop-Higgs couplings in the MSSM

Higgs bosons: $\mathcal{CP}$-even h, H bosons, $\mathcal{CP}$-odd A boson, charged $H^{\pm}$ bosons.

For simplicity: neglect all contributions proportional to the electroweak gauge couplings.

Then, the stop mass matrix is given by ($X_t = A_t - \mu / \tan\beta$)

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 & m_t X_t \\ m_t X_t & m_{\tilde{t}_R}^2 + m_t^2 \end{pmatrix}$$

In the **unbroken** phase of the theory ($\boldsymbol{v} = \mathbf{0} \rightarrow m_t = 0$), the stops do not mix ($\tilde{t}_L$ and $\tilde{t}_R$ are mass eigenstates).

In this approximations, the stop-Higgs couplings are given by ($Y_t = A_t + \mu \tan\beta$)

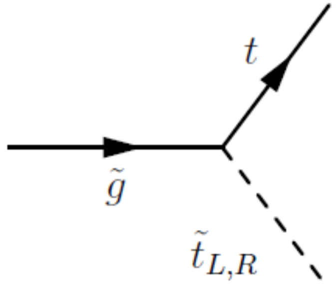
$$\begin{aligned} c(H\tilde{t}_L\tilde{t}_L) &= c(H\tilde{t}_R\tilde{t}_R) = c(A\tilde{t}_L\tilde{t}_L) = c(A\tilde{t}_R\tilde{t}_R) = 0 \\ c(H\tilde{t}_L\tilde{t}_R) &= -\frac{1}{\sqrt{2}} h_t c_\beta Y_t, \\ c(A\tilde{t}_L\tilde{t}_R) &= -c(A\tilde{t}_R\tilde{t}_L) = \frac{1}{\sqrt{2}} h_t c_\beta Y_t, \\ c(H^+\tilde{t}_R\tilde{b}_R) &= c(H^+\tilde{t}_L\tilde{b}_L) = c(H^+\tilde{t}_L\tilde{b}_R) = 0, \\ c(H^+\tilde{t}_R\tilde{b}_L) &= -h_t c_\beta Y_t, \end{aligned}$$

$$\begin{aligned} c(h\tilde{t}_L\tilde{t}_L) &= c(h\tilde{t}_R\tilde{t}_R) = c(G\tilde{t}_L\tilde{t}_L) = c(G\tilde{t}_R\tilde{t}_R) = 0, \\ c(h\tilde{t}_L\tilde{t}_R) &= \frac{1}{\sqrt{2}} h_t s_\beta X_t, \\ c(G\tilde{t}_L\tilde{t}_R) &= -c(G\tilde{t}_R\tilde{t}_L) = \frac{1}{\sqrt{2}} h_t s_\beta X_t, \\ c(G^+\tilde{t}_R\tilde{b}_R) &= c(G^+\tilde{t}_L\tilde{b}_L) = c(G^+\tilde{t}_L\tilde{b}_R) = 0, \\ c(G^+\tilde{t}_R\tilde{b}_L) &= -h_t s_\beta X_t. \end{aligned}$$

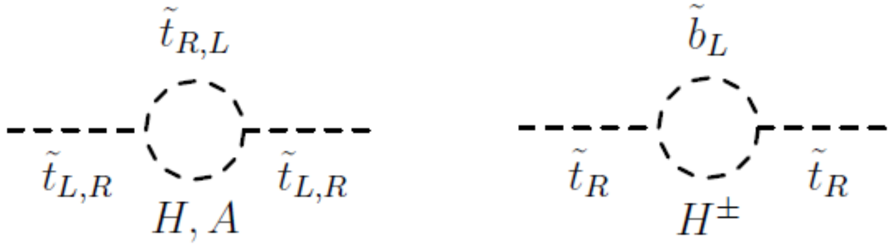
h_t: top-Yukawa coupling,
$\tan\beta$: ratio of vevs
$c_\beta \equiv \cos\beta$,
$s_\beta \equiv \sin\beta$

Note:
no couplings involving two identical stops.

Gluino decay in the MSSM: Y_t terms



Consider first corrections leading corrections in Y_t:



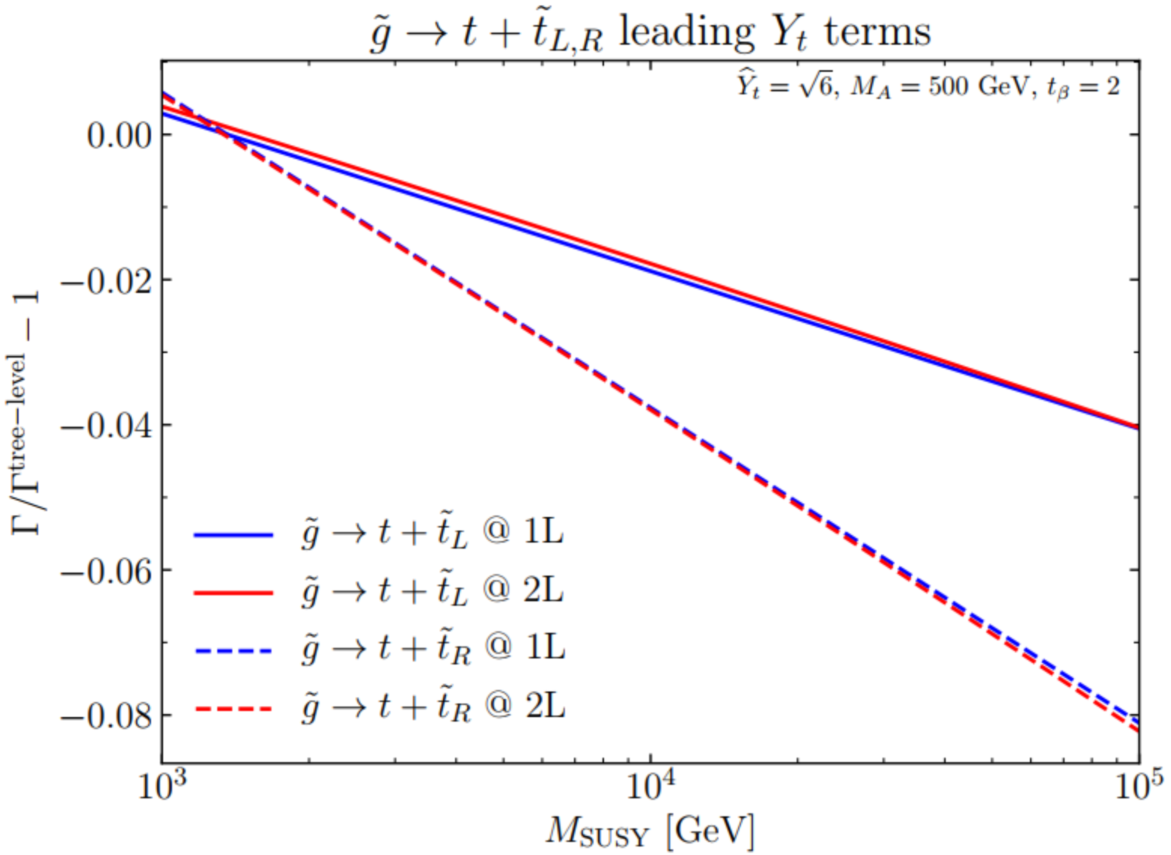
Non-SM Higgs bosons $H, A, H^\pm$ have the mass m_A, which plays the role of m_1 in the toy model (and $m_{\tilde{t}_{L,R}} \leftrightarrow m_{2,3}$).

Assuming $m_{\tilde{t}_R} = m_{\tilde{t}_L} = M_{SUSY}$ and renormalising all masses and Y_t on-shell, we obtain ($\hat{Y}_t \equiv Y_t/M_{SUSY} \sim \mathcal{O}(1)$)

$$\begin{aligned}\hat{\Gamma}_{\tilde{g}\to t+\tilde{t}_L} &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_L}\Big\{1 - \mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L}) - \mathrm{Re}\hat{\Sigma}^{(2)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L}) \\ &\qquad + \Big(\mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L})\Big)^2 + \Big(\mathrm{Im}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L})\Big)^2 + \mathcal{O}(k^3)\Big\} \\ &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_L}\Big\{1 - k h_t^2 c_\beta^2 \widehat{Y}_t^2\left[\frac{1}{2}\ln\frac{M^2_{\mathrm{SUSY}}}{m_A^2} - 1\right] \\ &\qquad - k^2 h_t^4 c_\beta^4 \widehat{Y}_t^4\left[\frac{1}{4}\ln^2\frac{M^2_{\mathrm{SUSY}}}{m_A^2} - 2\ln\frac{M_{\mathrm{SUSY}}}{m_A^2} + \frac{11}{12}\pi^2 - \frac{35}{12}\right] \\ &\qquad + \mathcal{O}\left(\frac{m_A}{M_{\mathrm{SUSY}}}\right) + \mathcal{O}(k^3)\Big\},\end{aligned}$$

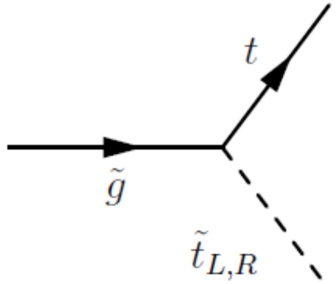
$$\begin{aligned}\hat{\Gamma}_{\tilde{g}\to t+\tilde{t}_R} &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_R}\Big\{1 - \mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R}) - \mathrm{Re}\hat{\Sigma}^{(2)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R}) \\ &\qquad + \Big(\mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R})\Big)^2 + \Big(\mathrm{Im}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R})\Big)^2 + \mathcal{O}(k^3)\Big\} \\ &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_R}\Big\{1 - k h_t^2 c_\beta^2 \widehat{Y}_t^2\left[\ln\frac{M^2_{\mathrm{SUSY}}}{m_A^2} - 2\right] \\ &\qquad - k^2 h_t^4 c_\beta^4 \widehat{Y}_t^4\left[\frac{1}{4}\ln^2\frac{M^2_{\mathrm{SUSY}}}{m_A^2} - 2\ln\frac{M^2_{\mathrm{SUSY}}}{m_A^2} + \frac{17}{12}\pi^2 - \frac{47}{6}\right]\Bigg] \\ &\qquad + \mathcal{O}\left(\frac{m_A}{M_{\mathrm{SUSY}}}\right) + \mathcal{O}(k^3)\Big\}.\end{aligned}$$

Gluino decay in the MSSM: Y_t terms

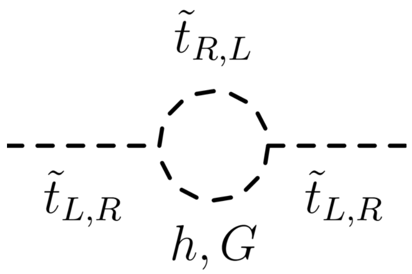


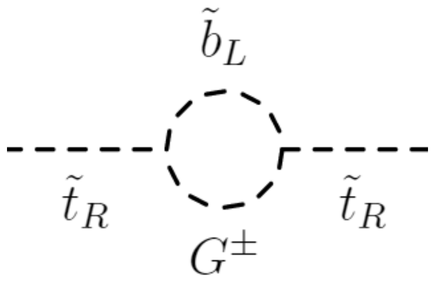
- We set $M_A = 500$ GeV $\Rightarrow \tilde{g} \to t + \tilde{t}_{L,R}$ probably distinguishable from $\tilde{g} \to t + \tilde{t}_{L,R} + H, A, H^{\pm}$.
- Large logarithms have sizeable impact at the one-loop level (i.e., for right-handed stop); two-loop corrections only moderate.
- Suggests that fixed-order treatment is sufficient.

Gluino decay in the MSSM: X_t terms



Next, consider corrections leading corrections in X_t:





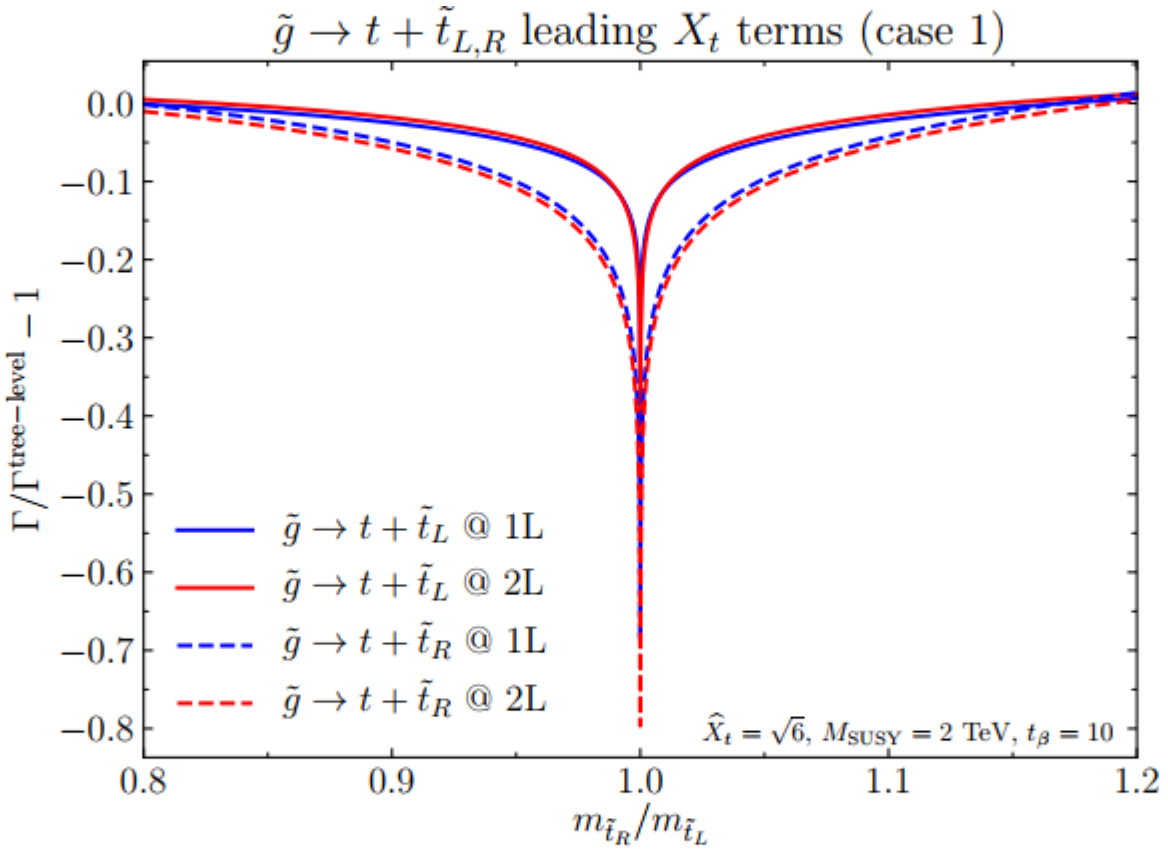
In the gaugeless limit, SM-like scalars $h, G, G^{\pm}$ are massless and $\epsilon = m^2_{\tilde{t}_R} - m^2_{\tilde{t}_L}$.

Renormalizing all masses and X_t in the OS scheme, we obtain ($\hat{X}_t \equiv X_t / M_{SUSY} \sim \mathcal{O}(1)$)

$$\begin{aligned}\hat{\Gamma}_{\tilde{g}\to t+\tilde{t}_L} &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_L}\Big\{1 - \mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L}) - \mathrm{Re}\hat{\Sigma}^{(2)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L}) \\ &\quad + \Big(\mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L})\Big)^2 + \Big(\mathrm{Im}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_L\tilde{t}_L}(m^2_{\tilde{t}_L})\Big)^2 + \mathcal{O}(k^3)\Big\} = \\ &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_L}\Big\{1 - k h_t^2 s_\beta^2 \widehat{X}_t^2\left[\ln\frac{M^2_{\mathrm{SUSY}}}{\epsilon} - 1\right] \\ &\quad - k^2 h_t^4 s_\beta^4 \widehat{X}_t^4\left[\ln^2\frac{M^2_{\mathrm{SUSY}}}{\epsilon} - \frac{15}{4}\ln\frac{M^2_{\mathrm{SUSY}}}{\epsilon} + \frac{1}{2}\ln\frac{m^2_{\mathrm{IR}}}{\epsilon} + \frac{1}{6}\pi^2 - \frac{35}{12}\right] \\ &\quad + \mathcal{O}\left(\frac{\epsilon}{M_{\mathrm{SUSY}}}\right) + \mathcal{O}(k^3)\Big\},\end{aligned}$$

$$\begin{aligned}\hat{\Gamma}_{\tilde{g}\to t+\tilde{t}_R} &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_R}\Big\{1 - \mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R}) - \mathrm{Re}\hat{\Sigma}^{(2)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R}) \\ &\quad + \Big(\mathrm{Re}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R})\Big)^2 + \Big(\mathrm{Im}\hat{\Sigma}^{(1)\prime}_{\tilde{t}_R\tilde{t}_R}(m^2_{\tilde{t}_R})\Big)^2 + \mathcal{O}(k^3)\Big\} = \\ &= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_R}\Big\{1 - 2k h_t^2 s_\beta^2 \widehat{X}_t^2\left[\ln\frac{M^2_{\mathrm{SUSY}}}{\epsilon} - 1\right] \\ &\quad - k^2 h_t^4 s_\beta^4 \widehat{X}_t^4\left[\ln^2\frac{M^2_{\mathrm{SUSY}}}{\epsilon} - \frac{7}{2}\ln\frac{M^2_{\mathrm{SUSY}}}{\epsilon} + \frac{1}{4}\ln\frac{m^2_{\mathrm{IR}}}{\epsilon} + \frac{5}{3}\pi^2 - \frac{47}{6}\right] \\ &\quad + \mathcal{O}\left(\frac{\epsilon}{M_{\mathrm{SUSY}}}\right) + \mathcal{O}(k^3)\Big\},\end{aligned}$$

Gluino decay in the MSSM: X_t terms



- Large logarithms have sizeable impact at the one-loop level close to IR limit; two-loop corrections only moderate.

- Suggests that fixed-order treatment is sufficient.

Gluino decay in the MSSM: X_t terms ($v \neq 0$)

We can also consider leading corrections in X_t for $v \neq 0$ (assuming $m_{\tilde{t}_R} = m_{\tilde{t}_L}$):

- stops mix → $\tilde{t}_1$ and $\tilde{t}_2$ mass eigenstates,
- $m^2_{\tilde{t}_1} = M^2_{SUSY} + m_t^2 - m_t X_t$ and $m^2_{\tilde{t}_2} = M^2_{SUSY} + m_t^2 + m_t X_t$
- For $M_{SUSY} \gg m_t$, stop mass difference $\epsilon = 2 m_t X_t$ will be small with respect to M^2_{SUSY}.

$$\hat{\Sigma}^{(1)}_{\tilde{t}_1\tilde{t}_1}(p^2) = \hat{\Sigma}^{(1)}_{\tilde{t}_2\tilde{t}_2}(p^2) = \frac{1}{2} k h_t^2 s_\beta^2 X_t^2 \Big[B_0(p^2, m^2_{\rm IR}, M^2_{\rm SUSY}) + B_0(p^2, m^2_{\rm IR}, M^2_{\rm SUSY} - m_t X_t + m_t^2) + B_0(p^2, m^2_{\rm IR}, M^2_{\rm SUSY} + m_t X_t + m_t^2) \Big]$$

$$\hat{\Sigma}^{(1)}_{\tilde{t}_1\tilde{t}_2}(p^2) = \hat{\Sigma}^{(1)}_{\tilde{t}_2\tilde{t}_1}(p^2) = \frac{1}{2} k h_t^2 s_\beta^2 X_t^2 B_0(p^2, m^2_{\rm IR}, M^2_{\rm SUSY}) \,,$$

$$c(h\tilde{t}_1\tilde{t}_1) = -c(h\tilde{t}_2\tilde{t}_2) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(h\tilde{t}_1\tilde{t}_2) = c(h\tilde{t}_2\tilde{t}_1) = 0,$$
$$c(G\tilde{t}_1\tilde{t}_1) = c(G\tilde{t}_2\tilde{t}_2) = 0,$$
$$c(G\tilde{t}_1\tilde{t}_2) = -c(G\tilde{t}_2\tilde{t}_1) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_1\tilde{b}_1) = c(G^+\tilde{t}_2\tilde{b}_1) = -\frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_1\tilde{b}_2) = c(G^+\tilde{t}_2\tilde{b}_2) = 0.$$

- Additional infrared divergency because of couplings involving two identical stops.

  ⇒ need to introduce infrared regulator mass m^2_{IR}.

Gluino decay in the MSSM: X_t terms ($v \neq 0$)

Virtual amplitude:

$$\hat{\Gamma}_{\tilde{g}\to t+\tilde{t}_1} = \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_1}\left[1 - \mathrm{Re}\frac{\partial}{\partial p^2}\hat{\Sigma}^{(1)}_{\tilde{t}_1\tilde{t}_1}(p^2)\big|_{p^2=m^2_{\tilde{t}_1}}\right] - 2\frac{\mathrm{Re}\hat{\Sigma}^{(1)}_{\tilde{t}_1\tilde{t}_2}(m^2_{\tilde{t}_1})}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}\cdot\Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_2} =$$

$$= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_1}\left[1 - \frac{1}{2}kh_t^2 s_\beta^2 \widehat{X}_t^2\left(\ln\frac{M^2_{\mathrm{SUSY}}}{m_t^2} + \frac{1}{2}\ln\frac{M^2_{\mathrm{SUSY}}}{m^2_{\mathrm{IR}}} - 3 - \ln 2 - 2\ln|\widehat{X}_t|\right)\right]$$

$$- \frac{1}{4}kh_t^2 s_\beta^2 \widehat{X}_t^2\left(\ln\frac{M^2_{\mathrm{SUSY}}}{m_t^2} - 2\ln|\widehat{X}_t|\right)\cdot\Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_2},$$

$$\hat{\Gamma}_{\tilde{g}\to t+\tilde{t}_2} = \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_2}\left[1 - \mathrm{Re}\frac{\partial}{\partial p^2}\hat{\Sigma}^{(1)}_{\tilde{t}_2\tilde{t}_2}(p^2)\big|_{p^2=m^2_{\tilde{t}_2}}\right] - 2\frac{\mathrm{Re}\hat{\Sigma}^{(1)}_{\tilde{t}_2\tilde{t}_1}(m^2_{\tilde{t}_2})}{m^2_{\tilde{t}_2}-m^2_{\tilde{t}_1}}\cdot\Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_1} =$$

$$= \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_2}\left[1 - \frac{1}{2}kh_t^2 s_\beta^2 \widehat{X}_t^2\left(\ln\frac{M^2_{\mathrm{SUSY}}}{m_t^2} + \frac{1}{2}\ln\frac{M^2_{\mathrm{SUSY}}}{m^2_{\mathrm{IR}}} - 3 - \ln 2 - 2\ln|\widehat{X}_t|\right)\right]$$

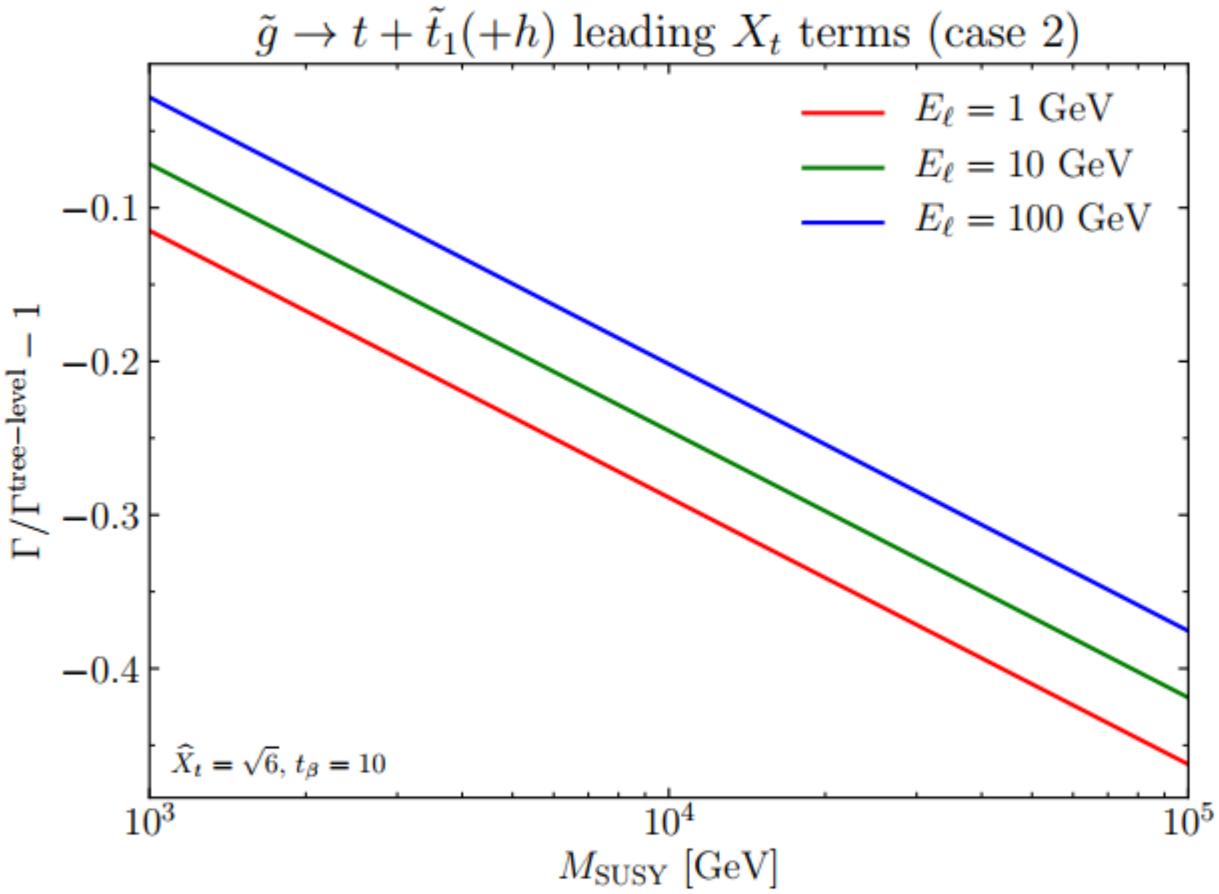
$$- \frac{1}{4}kh_t^2 s_\beta^2 \widehat{X}_t^2\left(\ln\frac{M^2_{\mathrm{SUSY}}}{m_t^2} - 2\ln|\widehat{X}_t|\right)\cdot\Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_1}.$$

Real emission amplitude:

$$\Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_{1,2}+h} = \Gamma^{(0)}_{\tilde{g}\to t+\tilde{t}_{1,2}}\cdot\frac{1}{2}kh_t s_\beta \widehat{X}_t^2\left[\frac{1}{2}\ln\frac{E_\ell^2}{m^2_{\mathrm{IR}}} - 1 + \ln 2\right]$$

Note:
Real emission of h boson does not affect large logarithms.

Gluino decay in the MSSM: X_t terms ($v \neq 0$)



Large logarithms are not an artifact of assuming $v = 0$, but also appear in the broken phase ($v \neq 0$).

Heavy Higgs decay in the N2HDM

- Extend SM Higgs sector by second doublet as well as a real singlet Φ_S:

$$\begin{aligned}V^{(0)} &= m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \text{h.c.}\right)\\ &+ \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{1}{2}\lambda_5\left((\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right)\\ &+ \frac{1}{2}m_S^2\Phi_S^2 + \frac{1}{6}a_S\Phi_S^3 + \frac{1}{24}\lambda_S|\Phi_S|^4 + \frac{1}{2}a_{1S}|\Phi_1|^2\Phi_S + \frac{1}{2}a_{2S}|\Phi_2|^2\Phi_S\\ &+ \frac{1}{6}\lambda_{1S}|\Phi_1|^2\Phi_S^2 + \frac{1}{2}\lambda_{2S}|\Phi_2|^2\Phi_S^2\,.\end{aligned}$$

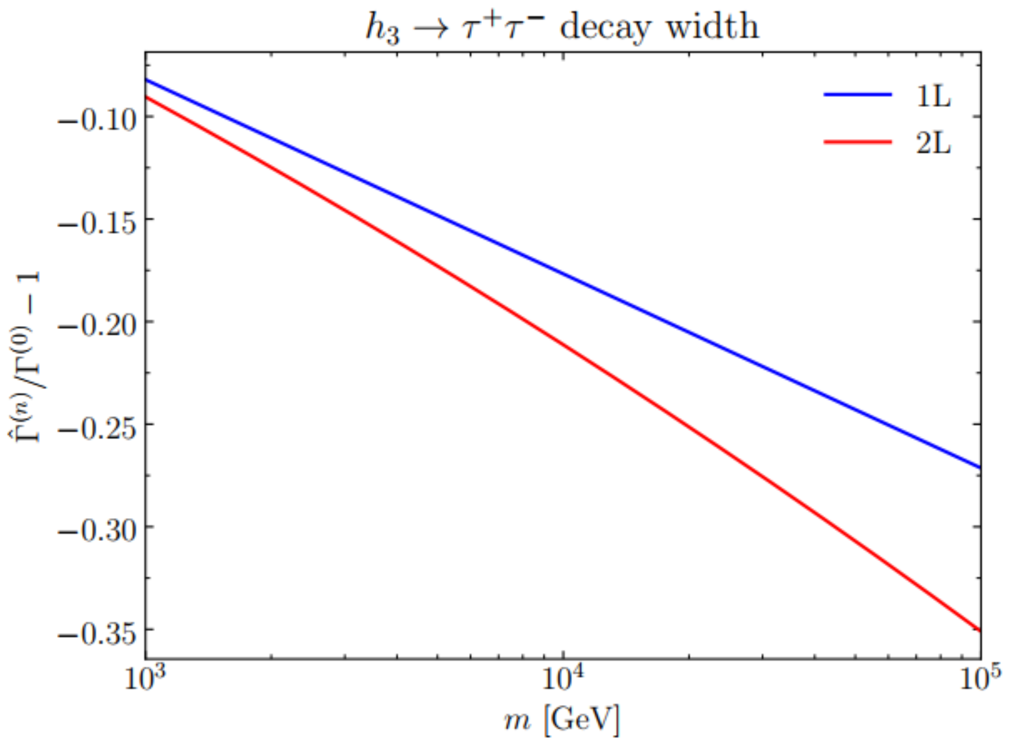
$$X_a \equiv \frac{a_{1S} - a_{2S}}{4}\,,\quad Y_a \equiv \frac{a_{1S}s_\beta^2 + a_{2S}c_\beta^2}{4}\,,\quad Z_a \equiv \frac{a_S}{4} - Y_a$$

- No $\mathbb{Z}_2$ symmetry transforming Φ_S imposed ⇒ trilinear couplings.

- Consider decay of heaviest three $\mathcal{CP}$-even mass eigenstate h_3 (which is mostly doublet-like) to tau leptons.

- $h_3 \to \tau^+\tau^-$ decay width will receive external leg corrections proportional to trilinear couplings; focus on X_a here.

$h_3 \to \tau^+\tau^-$ decay in the N2HDM



Considered scenario with

$$m_{h_1}^2 \sim m_{h_2}^2 \sim m_G^2 \sim m_{G^\pm}^2 \sim \epsilon, \quad m_{h_3}^2 = m_A^2 = m_{H^\pm}^2 = m^2$$

and chose

$$\tan\beta = 1.26, \sin\alpha_3 = 0.94\,, X_a = 3m\,.$$

⇒ Sizeable one- and two-loop corrections.

Conclusions

Conclusions

- If a new BSM particle is discovered, precise theoretical predictions will be a crucial to unravel its nature.
- Identified **new source of large Sudakov-like logarithmic contributions**:
  - Appear on **external legs** of heavy scalar particles.
  - At least one light scalar particle needs to present.
  - Large **trilinear coupling** between scalars needed.
- Discussed toy model containing one light and two heavy scalars at the one- and two-loop level:
  - Occurrence of large logarithms related to **infrared limit**.
  - Infrared divergencies can be regulated by including radiation of the light scalar particle.
  - If additional radiation can be resolved experimentally → large logarithms appear.
  - On-shell renormalization of masses crucial at the 2L level.
- Exemplary applications: gluino decay in the MSSM, heavy Higgs decay in the N2HDM
  - Found sizeable 1L corrections; only moderate 2L effects → no resummation needed.

Thanks for your attention!

Mass configuration 1

$$\frac{d}{dp^2}T_{11234}(p^2,m^2,m^2,\epsilon,m^2,\epsilon)\Big|_{p^2=m^2} =$$

$$=\frac{\pi(2-\overline{\ln}m^2)}{4\sqrt{\epsilon}m^3}+\frac{-6\overline{\ln}\epsilon\overline{\ln}m^2-3\overline{\ln}^2\epsilon+24\overline{\ln}\epsilon+9\overline{\ln}^2m^2-24\overline{\ln}m^2-\pi^2}{24m^4},$$

$$\frac{d}{dp^2}T_{11234}(p^2,\epsilon,\epsilon,m^2,m^2,m^2)\Big|_{p^2=m^2} =$$

$$=-\frac{\overline{\ln}m^2}{2m^2\epsilon}+\frac{3\pi\overline{\ln}m^2}{8m^3\sqrt{\epsilon}}+\frac{-50+6\pi^2+3\overline{\ln}\epsilon-12\overline{\ln}m^2+18\overline{\ln}\epsilon\overline{\ln}m^2-18\overline{\ln}^2m^2}{36m^4},$$

$$\frac{d}{dp^2}T_{12345}(p^2,m^2,\epsilon,m^2,\epsilon,m^2)\Big|_{p^2=m^2} =$$

$$=\frac{1}{4m^4}\left[2+\ln\frac{m^2}{\epsilon}+\ln^2\frac{m^2}{\epsilon}\right]-\frac{\pi^2\ln 2-3/2\zeta(3)}{m^4}.$$

Integral	Numerical results TSIL	Approx. $\mathcal{O}(\epsilon^0)$
$m^4\frac{d}{dp^2}T_{11234}(p^2,m^2,m^2,\epsilon,m^2,\epsilon)\Big\vert_{p^2=m^2}$	85.552342	85.606671
$m^4\frac{d}{dp^2}T_{11234}(p^2,\epsilon,\epsilon,m^2,m^2,m^2)\Big\vert_{p^2=m^2}$	-3387.9644	-3387.9533
$m^4\frac{d}{dp^2}T_{12345}(p^2,m^2,\epsilon,m^2,\epsilon,m^2)\Big\vert_{p^2=m^2}$	21.636871	21.274760

Mass configuration 2

$$\frac{d}{dp^2}T_{11234}(p^2, m^2+\epsilon, m^2+\epsilon, 0, m^2, 0)\Big|_{p^2=m^2} =$$

$$= \frac{2-\overline{\ln} m^2}{m^2\epsilon} + \frac{-\pi^2 + 6\overline{\ln}\epsilon - 3\overline{\ln}^2\epsilon - 6\overline{\ln} m^2 + 3\overline{\ln}^2 m^2}{6m^4} + \mathcal{O}(\epsilon)\,,$$

$$\frac{d}{dp^2}T_{11234}(p^2, m^2, m^2, 0, m^2+\epsilon, 0)\Big|_{p^2=m^2+\epsilon} =$$

$$= \frac{\overline{\ln} m^2 - 2}{m^2\epsilon} + \frac{2\pi^2 + 18 + 6i\pi + (6-6i\pi)\overline{\ln}\epsilon - 3\overline{\ln}^2\epsilon - 12\overline{\ln} m^2 + 3\overline{\ln}^2 m^2}{6m^4} + \mathcal{O}(\epsilon)\,,$$

$$\frac{d}{dp^2}T_{12345}(p^2, m^2+\epsilon, 0, m^2, 0, m^2+\epsilon)\Big|_{p^2=m^2} =$$

$$= \frac{1}{m^4}\left[\pi^2\left(\frac{1}{4} - \ln 2\right) + \frac{3}{2}\zeta(3) + \ln\frac{m^2}{\epsilon} + \ln^2\frac{m^2}{\epsilon}\right] + \mathcal{O}(\epsilon)\,,$$

$$\frac{d}{dp^2}T_{12345}(p^2, m^2, 0, m^2+\epsilon, 0, m^2)\Big|_{p^2=m^2+\epsilon} =$$

$$= \frac{1}{m^4}\left[-\pi^2\left(\frac{3}{4} + \ln 2\right) + \frac{3}{2}\zeta(3) + i\pi + (1+2i\pi)\ln\frac{m^2}{\epsilon} + \ln^2\frac{m^2}{\epsilon}\right] + \mathcal{O}(\epsilon)\,.$$

$$\frac{d}{dp^2}T_{11234}(p^2, m_1^2, m_1^2, m^2+\epsilon, m^2+\epsilon, m^2)\Big|_{p^2=m^2}$$

$$= -\frac{\overline{\ln} m^2}{2m^2 m_1^2} + \frac{3\pi\overline{\ln} m^2}{8m^3 m_1} + \frac{-50 + 6\pi^2 + 3\overline{\ln} m_1^2 - 12\overline{\ln} m^2 + 18\overline{\ln} m_1^2\overline{\ln} m^2 - 18\overline{\ln}^2 m^2}{36m^4}$$
$$+ \frac{\epsilon}{m^2}\left[\frac{\pi\overline{\ln} m^2}{8mm_1^3} - \frac{1+2\overline{\ln} m^2}{4m^2m_1^2} + \frac{\pi(40+27\overline{\ln} m^2)}{192m^3m_1} - \frac{23 + 90\overline{\ln} m^2 - 42\overline{\ln} m_1^2}{144m^4}\right] + \mathcal{O}(\epsilon^2)\,,$$

$$\frac{d}{dp^2}T_{11234}(p^2, m_1^2, m_1^2, m^2, m^2, m^2+\epsilon)\Big|_{p^2=m^2+\epsilon}$$

$$= -\frac{\overline{\ln} m^2}{2m^2 m_1^2} + \frac{3\pi\overline{\ln} m^2}{8m^3 m_1} + \frac{-50 + 6\pi^2 + 3\overline{\ln} m_1^2 - 12\overline{\ln} m^2 + 18\overline{\ln} m_1^2\overline{\ln} m^2 - 18\overline{\ln}^2 m^2}{36m^4}$$
$$+ \frac{\epsilon}{m^2}\left[\frac{\pi\overline{\ln} m^2}{8mm_1^3} - \frac{3}{4m^2m_1^2} + \frac{\pi(-112+81\overline{\ln} m^2)}{192m^3m_1} + \frac{329 - 48\pi^2 - 138\overline{\ln} m^2 + 144\overline{\ln}^2 m^2 + 90\overline{\ln} m_1^2 - 144\overline{\ln} m^2\overline{\ln} m_1^2}{144m^4}\right]$$
$$+ \mathcal{O}(\epsilon^2)\,,$$

Integral	Numerical results TSIL	Expansion
$m^4\frac{d}{dp^2}T_{11234}(p^2, m^2+\epsilon, m^2+\epsilon, 0, m^2, 0)\Big\|_{p^2=m^2}$	-13022.295	-13021.642
$m^4\frac{d}{dp^2}T_{11234}(p^2, m_1^2, m_1^2, m^2+\epsilon, m^2+\epsilon, m^2)\Big\|_{p^2=m^2}$	-3361.5011	-3361.3207
$m^4\frac{d}{dp^2}T_{12345}(p^2, m^2+\epsilon, 0, m^2, 0, m^2+\epsilon)\Big\|_{p^2=m^2}$	91.482800	91.470115

N2HDM: analytic results

$$\hat{\Sigma}^{(2)\prime}_{h_3h_3}(m^2)\Big|^{\mathcal{O}(s^4_{\alpha_3})} =$$

$$= k^2 X_a^4 c^4_{\alpha_3} s^4_{2\beta} s^4_{\alpha_3} \Big\{ 16 \frac{\partial}{\partial p^2} T_{12345}(p^2, m^2, \epsilon, m^2, \epsilon, m^2)$$
$$+ 16 \frac{\partial}{\partial p^2} T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon)$$
$$+ 8 \frac{\partial}{\partial p^2} T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2)$$
$$+ 16 \frac{\partial}{\partial p^2} C_0(0, p^2, p^2, m^2, m^2, \epsilon) B_0(m^2, \epsilon, m^2)$$
$$+ 8 \frac{\partial}{\partial p^2} C_0(0, p^2, p^2, m^2, \epsilon, \epsilon) B_0(\epsilon, m^2, m^2)$$
$$+ 8 B_0'(p^2, m^2, \epsilon) \times \Big[C_0(m^2, \epsilon, m^2, m^2, m^2, \epsilon) + 4 B_0'(m^2, \epsilon, m^2)$$
$$+ B_0'(\epsilon, m^2, m^2) \Big] \Big\} \Big|_{p^2=m^2}$$

$$= \frac{2k^2 X_a^4 c^4_{\alpha_3} s^4_{2\beta} s^4_{\alpha_3}}{m^4} \Big[\frac{121}{9} + 4\sqrt{3}\pi + \frac{7\pi^2}{3} - 8\pi^2 \ln 2 + \frac{2}{3}\left(21 + \sqrt{3}\pi\right) \ln \frac{\epsilon}{m^2}$$
$$+ 5 \ln^2 \frac{\epsilon}{m^2} + 12\zeta(3) \Big] .$$

$$\hat{\Sigma}^{(2)\prime}_{h_3h_3}(m^2)\Big|^{\mathcal{O}(s^2_{\alpha_3})} =$$

$$= k^2 X_a^4 c^4_{\alpha_3} s^4_{2\beta} s^2_{\alpha_3} \Big\{ 12 \frac{\partial}{\partial p^2} T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon)$$
$$+ 12 \frac{\partial}{\partial p^2} C_0(0, p^2, p^2, m^2, m^2, \epsilon) B_0(m^2, \epsilon, m^2)$$
$$+ 6 B_0'(p^2, m^2, \epsilon) \times \Big[C_0(m^2, \epsilon, m^2, m^2, m^2, \epsilon) + 8 B_0'(m^2, \epsilon, m^2)$$
$$+ B_0'(\epsilon, m^2, m^2) \Big] \Big\} \Big|_{p^2=m^2}$$

$$= \frac{k^2 X_a^4 c^4_{\alpha_3} s^4_{2\beta} s^2_{\alpha_3}}{2m^4} \Big[94 + 5\pi^2 + 4\sqrt{3}\pi + \left(95 + 2\sqrt{3}\pi\right) \ln \frac{\epsilon}{m^2} + 21 \ln^2 \frac{\epsilon}{m^2} \Big] .$$

$$\hat{\Sigma}^{(2)\prime}_{h_3h_3}(m^2)\Big|^{\mathcal{O}(s^0_{\alpha_3})} = 3k^2 X_a^4 c^4_{\alpha_3} s^4_{2\beta} \Big\{ \frac{\partial}{\partial p^2} T_{12345}(p^2, m^2, \epsilon, m^2, \epsilon, m^2)$$
$$+ \frac{\partial}{\partial p^2} T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon)$$
$$+ \frac{\partial}{\partial p^2} T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2)$$
$$+ \frac{\partial}{\partial p^2} C_0(0, p^2, p^2, m^2, m^2, \epsilon) B_0(m^2, \epsilon, m^2)$$
$$+ \frac{\partial}{\partial p^2} C_0(0, p^2, p^2, m^2, \epsilon, \epsilon) B_0(\epsilon, m^2, m^2)$$
$$+ 6 B_0'(p^2, m^2, \epsilon) B_0'(m^2, \epsilon, m^2) \Big\} \Big|_{p^2=m^2}$$

$$= \frac{k^2 X_a^4 c^4_{\alpha_3} s^4_{2\beta}}{4m^4} \Big[\frac{181}{3} + \frac{9}{2}\pi^2 - 12\pi^2 \ln 2 + 70 \ln \frac{\epsilon}{m^2}$$
$$+ \frac{39}{2} \ln^2 \frac{\epsilon}{m^2} + 18\zeta(3) \Big] .$$