String Journal Club 5-11-2024

Angular fractals in thermal QFT

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Abstract: We show that thermal effective field theory controls the long-distance expansion of the partition function of a d-dimensional QFT, with an insertion of any finite-order spatial isometry. Consequently, the thermal partition function on a sphere displays a fractal-like structure as a function of angular twist, reminiscent of the behavior of a modular form near the real line. As an example application, we find that for CFTs, the effective free energy of even-spin minus odd-spin operators at high temperature is smaller than the usual free energy by a factor of $1/2^d$. Near certain rational angles, the partition function receives subleading contributions from "Kaluza-Klein vortex defects" in the thermal EFT, which we classify. We illustrate our results with examples in free and holographic theories, and also discuss nonperturbative corrections from worldline instantons.

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1. Goal and main result

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1. *Goal and main result*

2. *Thermal EFT*

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1. *Goal and main result*
2. *Thermal EFT*
3. *Twisting the partition function*

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1. *Goal and main result*
2. *Thermal EFT*
3. *Twisting the partition function*
4. *Corrections*

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2. *Thermal EFT*
3. *Twisting the partition function*
4. *Corrections*
5. *Comment about holography*

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1. *Goal and main result*
2. *Thermal EFT*
3. *Twisting the partition function*
4. *Corrections*
5. *Comment about holography*
6. *Take-home message*

Goal and main result

- Part. function of a QFT on $S^1_\beta \times S^{d-1}_L$ + discrete isometry R in the th.dyn. limit?

$$\mathrm{Tr}\left[e^{-\beta H} R\right] \sim ? \qquad R^q = 1,\ L \to \infty$$

- An interesting example: odd spins vs even spins

$$\mathrm{Tr}\left[e^{-\beta H} (-1)^J\right] \sim ? \qquad R = e^{i\theta J},\ \theta = \pi$$

- For a general QFT on $S^1_\beta \times \mathcal{M}^{d-1}_L$:

$$-\log \mathrm{Tr}_{\mathcal{H}(\mathcal{M}_L)}\left[e^{-\beta H} R\right] \sim -\frac{1}{q} \log \mathrm{Tr}_{\mathcal{H}(\mathcal{M}_L)}\left[e^{-q\beta H}\right] + \text{topological} + \text{KK defects}$$

Thermal effective action

- Thermodynamic limit of a gapped QFT:

$$\mathrm{Tr}_{\mathcal{H}(\mathcal{M}_L)}[e^{-\beta H_L}] \sim e^{-S_{\mathrm{th}}[g,A,\phi]} + \text{nonperturbative in } 1/L$$

KK gauge field and dilaton

- For a CFT:

$$S_{\mathrm{th}} = \int \frac{d^{d-1}\vec{x}}{\beta^{d-1}} \sqrt{\widehat{g}} \left(-f + c_1 \beta^2 \widehat{R} + c_2 \beta^2 F^2 + \dots\right) + S_{\mathrm{anom}}.$$

Wilson coefficients!

Thermal effective action

- CFT with angular fugacities on $S^1_\beta \times S^{d-1}_L$:

$$\mathrm{Tr}\left[e^{-\beta(H - i\vec{\Omega}\cdot\vec{J})}\right]$$

$$\Downarrow$$

$$S_{\mathrm{th}} = \frac{\mathrm{vol}\, S^{d-1}}{\prod_{i=1}^{n}(1+\Omega_i^2)}\left[-fT^{d-1} + (d-2)\left((d-1)c_1 + \left(2c_1 + \frac{8}{d}c_2\right)\sum_{i=1}^{n}\Omega_i^2\right)T^{d-3} + \dots\right].$$

- Doesn't $\beta\Omega = \pi$ solve the problem?

$$\beta \to 0 \implies \Omega \to \infty\,!$$

Thermal effective action

- CFT with angular fugacities on $S^1_\beta \times S^{d-1}_L$:

$$\mathrm{Tr}\left[e^{-\beta(H - i\vec{\Omega}\cdot\vec{J})}\right]$$

$$S_{\mathrm{th}} = \frac{\mathrm{vol}\, S^{d-1}}{\prod_{i=1}^{n}(1+\Omega_i^2)}\left[-fT^{d-1} + (d-2)\left((d-1)c_1 + \left(2c_1 + \frac{8}{d}c_2\right)\sum_{i=1}^{n}\Omega_i^2\right)T^{d-3} + \dots\right].$$

- Doesn't $\beta\Omega = \pi$ solve the problem?

$$\beta \to 0 \implies \Omega \to \infty\,!$$

The EFT picture breaks down!

Twisting the partition function

- In 2d the problem is solved thanks to modularity:

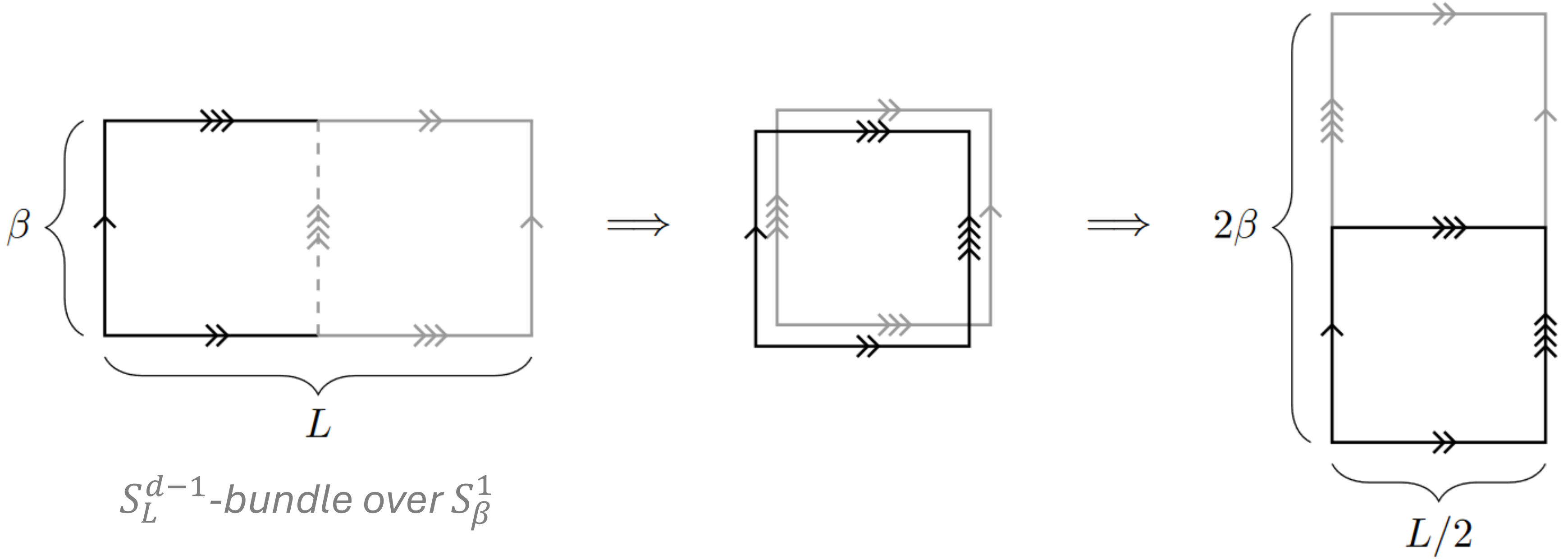
$$\mathrm{Tr}\left[e^{-\beta(H-i\Omega J)}\right] \sim \exp\left[\frac{4\pi^2}{\beta(1+\Omega^2)}\frac{c}{12}\right]$$

$$\mathrm{Tr}\left[e^{-\beta(H-i\Omega J)}(-1)^J\right] \sim \exp\left[\frac{1}{4}\frac{4\pi^2}{\beta(1+\Omega^2)}\frac{c}{12}\right] \qquad \gamma = \pm\begin{pmatrix}-1 & 0\\ 2 & -1\end{pmatrix} \in \mathrm{PSL}(2,\mathbb{Z})$$

$$\mathrm{Tr}\left[e^{-\beta(H-i\Omega J)}e^{2\pi i\frac{p}{q}J}\right] \sim \exp\left[\frac{1}{q^2}\frac{4\pi^2}{\beta(1+\Omega^2)}\frac{c}{12}\right] \qquad \gamma = \pm\begin{pmatrix}-(p^{-1})_q & b\\ q & -p\end{pmatrix} \in \mathrm{PSL}(2,\mathbb{Z})$$

Can we replicate the pattern without modularity in higher dimension?

Twisting the partition function



S_L^{d-1}-bundle over S_β^1

$S_{q\beta}^1$-bundle over S_L^{d-1}/Z_q

$$-\log \mathrm{Tr}\left[e^{-\beta H} R\right]$$

$$-\frac{1}{q}\log \mathrm{Tr}\left[e^{-q\beta H}\right] + \text{topological}$$

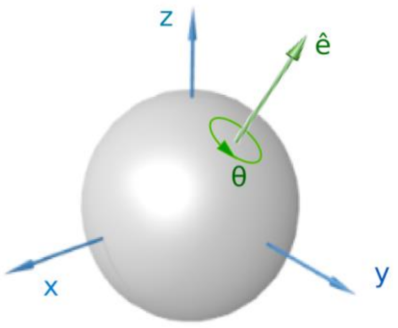
Corrections

- Topological corrections: new, non-trivial bundle! Chern-Simons terms in the Thermal EFT can be sensitive to the features of the bundle

$$-\frac{1}{q}\log \mathrm{Tr}\left[e^{-q\beta H}\right] + \text{topological}$$

- KK vortices: if the action of the isometry R is not free, there are fixed points

Defect $\mathfrak{D}_i$ from the thermal EFT point of view!



$$-\frac{1}{q}\log \mathrm{Tr}\left[e^{-q\beta H}\right] + \text{topological} + \sum_{\mathfrak{D}_i} S_{\mathfrak{D}_i}$$

Comment about holography

- High temperature on the boundary → AdS BH thermodynamics
- Folding trick on the boundary → New geometry!

$$\Sigma_{\beta,\vec{\Omega}} \Rightarrow \Sigma_{q\beta,\vec{\Omega}}/\mathbb{Z}_q$$

Free action of R: smooth solution

Non-free action of R: orbifold singularity!

Take-home message

- ‘Twisted physics’ at high T ⟺ ‘untwisted physics’ at lower T

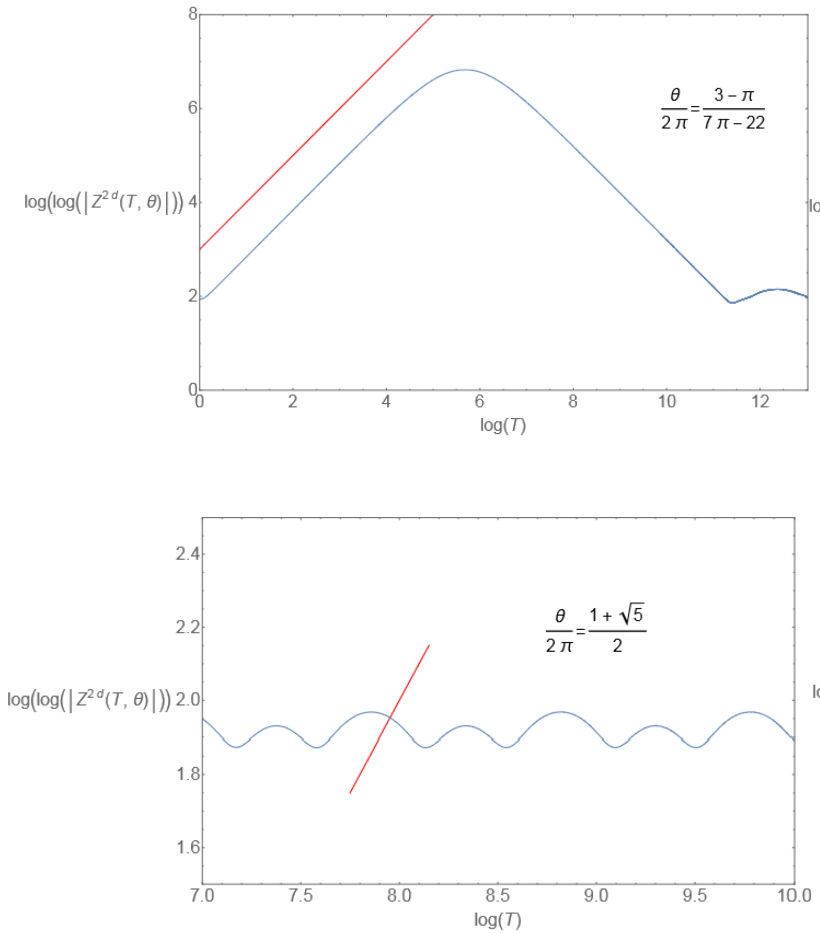
→Interesting consequences for strongly coupled theories?

- Powerful predictions in CFTs (explicit form for the Thermal EFT is available)

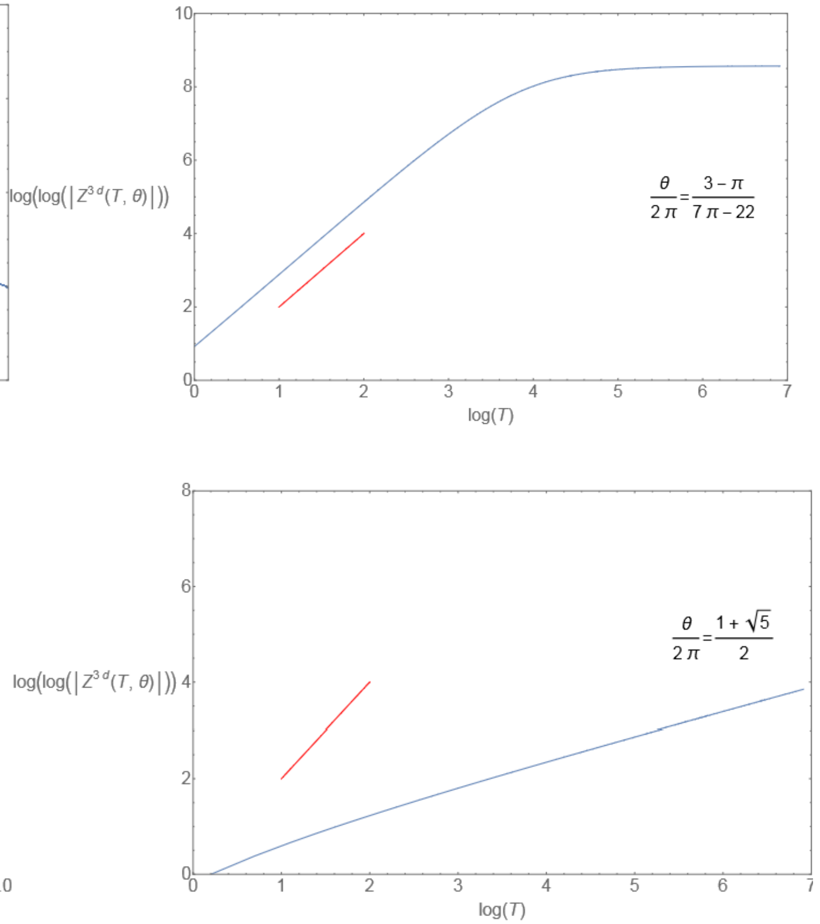
$$\log \mathrm{Tr}[e^{-\beta(H-i\vec{\Omega}\cdot\vec{J})}R] \sim \frac{1}{q^d}\frac{\mathrm{vol}S^{d-1}}{\prod_{i=1}^{n}(1+\Omega_i^2)}\frac{f}{\beta^{d-1}}+\dots$$

- The same Wilson coefficients describe twisted and untwisted partition functions

Bonus: irrational angles



(a) 24 free bosons in 2d.



(b) A free boson in 3d.