

Constraints on RG Flows from Protected Operators

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Prelude: 2d
c-theorem

Energy-momentum 2-pt function: scale **s**

$$\langle T_{zz}(z, \bar{z}) T_{zz}(0) \rangle = \frac{1}{2(2\pi)^2 z^4} F(z\bar{z}s) .$$

At 2 RG fixed points: $\lim_{s \rightarrow 0} F(z\bar{z}s) = c_{UV},$

$$\lim_{s \rightarrow \infty} F(z\bar{z}s) = c_{IR} .$$

Prelude:
c-theorem

2d

$$\Theta = T_{\mu}^{\mu} = 4T_{z\bar{z}},$$

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+

$$\bar{\partial} T_{zz} = -\frac{1}{4} \partial \Theta,$$

=

Sum rule !

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$$\Delta c = 3\pi \int_{z \neq 0} d^2 z |z|^2 \langle \Theta(z, \bar{z}) \Theta(0) \rangle .$$

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> 0

> 0 (reflection
positivity)

C-theorem: $\delta c = c^{\text{UV}} - c^{\text{IR}} \geq 0 .$

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C-theorem: $\delta c = c^{\text{UV}} - c^{\text{IR}} \geq 0 .$

Does it work for other protected quantities?

Paper: focus on **protected scalars**

$$\langle \mathcal{O}(x) \overline{\mathcal{O}}(0) \rangle = \frac{C_{\Delta}(\Lambda|x|)}{|x|^{2\Delta}},$$

$$C_{\Delta}^{\text{UV}} = \lim_{|x| \rightarrow 0} C_{\Delta}(\Lambda|x|), \quad C_{\Delta}^{\text{IR}} = \lim_{|x| \rightarrow \infty} C_{\Delta}(\Lambda|x|).$$

Example: BPS operators in 4d N=2

Paper: focus on **protected scalars**:

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Is there a sum rule ?

$$\delta C_{\Delta} = C_{\Delta}^{\text{UV}} - C_{\Delta}^{\text{IR}} = \int d^d x \mathcal{D}_{\Delta} \langle \mathcal{O}(x) \overline{\mathcal{O}}(0) \rangle,$$

Schematic proof

$$\left(x^2 \square - 4\Delta \left(\Delta - \frac{d-2}{2} \right) \right) \langle \mathcal{O}(x) \overline{\mathcal{O}}(0) \rangle$$

Integrate:

$$\delta C_{\Delta} = \frac{\Gamma(\frac{d}{2})}{4\pi^{\frac{d}{2}}(2\Delta - \nu)} \int d^d x |x|^{2\Delta-d} (|x|^2 \square - 4\Delta (\Delta - \nu)) \langle \mathcal{O}(x) \overline{\mathcal{O}}(0) \rangle ,$$

Schematic proof:

$$\left(x^2 \square - 4\Delta \left(\Delta - \frac{d-2}{2} \right) \right) \langle \mathcal{O}(x) \overline{\mathcal{O}}(0) \rangle$$

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Not manifestly positive...

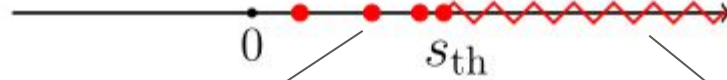
Use Kallen–Lehmann
decomposition

Free propagator

$$\langle \mathcal{O}(x) \mathcal{O}(0) \rangle = \int_0^\infty ds \rho(s) G_s(x) ,$$

The diagram illustrates the Kallen-Lehmann decomposition on a horizontal axis labeled s . The origin is marked as 0 . A point s_{th} is marked on the axis. To the left of s_{th} , the spectral density $\rho(s)$ is represented by a solid black line with several red dots, indicating discrete poles. To the right of s_{th} , the spectral density is represented by a red wavy line, indicating a continuous spectrum. Two lines with arrows point from the equations below to the corresponding parts of the diagram: one points to the discrete poles and the other points to the continuous spectrum.

$$\rho_{\text{sp}}(s) = \sum_i c_i \delta(s - m_i^2)$$
$$\rho_{\text{mp}}(s) = \sigma(s) \Theta(s - s_{\text{th}}) ,$$

$\lfloor s$ 

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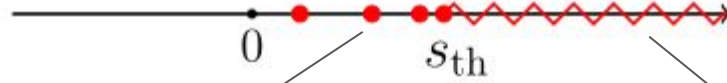
Sum rule + decomposition:

$$\delta C_{\Delta} = \frac{1}{4\Delta - 2\nu} \int_0^{\infty} dx x^{2\Delta-1} \int_{s_{\text{th}}}^{\infty} ds \sigma(s) (x^2 s - 4\Delta(\Delta - \nu)) \left(\frac{\sqrt{s}}{x}\right)^{\nu} K_{\nu}(\sqrt{s}x) .$$

(2 17)

No contribution from single particle states

$\lfloor s$



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$$\rho_{\text{mp}}(s) = \sigma(s) \Theta(s - s_{\text{th}}) ,$$

- δC_Δ depends on start of multi-particle states

- UV behaviour: $\rho(s) = C_\Delta^{\text{UV}} \frac{s^{\Delta - \frac{d}{2}}}{\Gamma(\Delta + 2\nu)} \left(1 + \frac{a_1}{s} + \frac{a_2}{s^2} + \dots \right)$

Plug back to sum rule

$$\delta C_{\Delta} = \frac{1}{4\Delta - 2\nu} \int_0^{\infty} dx x^{2\Delta-1} \int_{s_{\text{th}}}^{\infty} ds \sigma(s) (x^2 s - 4\Delta(\Delta - \nu)) \left(\frac{\sqrt{s}}{x}\right)^{\nu} K_{\nu}(\sqrt{s}x) .$$

(2 17)

Two possibilities:

1) $s = s_{\text{th}} > 0.$ \longrightarrow $\delta C_{\Delta} = C_{\Delta}^{\text{UV}}$

2) $s_{\text{th}} = 0,$ \longrightarrow Perturbative expansion diverges

But result finite

\longrightarrow Dominated by non-pert effects...

cannot conclude $\delta C_{\Delta} \geq 0.$

4d $N = 2$ theories (and type-B anomalies in general)

Conformal dimensions are integers

$$\chi_{ij} = \langle \mathcal{O}_i(1) \bar{\mathcal{O}}_j(0) \rangle$$

If marginal operators, [conformal manifold](#)

Elli and friends: $\nabla_\chi C_\Delta = 0$.

Use it to go close to a free point: $\delta C_\Delta = C_\Delta^{\text{UV}} - C_\Delta^{\text{IR}} \geq 0$.

Paper:

- 1) test sum rule in various examples
- 2) similar sum rule for conserved currents and stress tensor