Constraints on RG Flows from Protected Operators

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Prelude: 2d c-theorem

Energy-momentum 2-pt function: scale s

$$\langle T_{zz}(z,\bar{z}) T_{zz}(0)\rangle = \frac{1}{2(2\pi)^2 z^4} F(z\bar{z}s) \ .$$

At 2 RG fixed points:

$$\lim_{s\to 0} F(z\bar{z}s) = c_{UV} ,$$

$$\lim_{s\to \infty} F(z\bar{z}s) = c_{IR} \ .$$

Prelude: 2d c-theorem

$$\Theta = T_\mu{}^\mu = 4T_{z\bar{z}},$$

Energy-momentum 2-pt function: scale s

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+

$$\bar{\partial} T_{zz} = -\frac{1}{4}\partial\Theta,$$

=

Sum rule !

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$$\Delta c = 3\pi\int_{z\neq 0} d^2z\,|z|^2\langle\Theta(z,\bar{z})\Theta(0)\rangle \ .$$

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> 0

> 0 (reflection positivity)

C-theorem: $\delta c = c^{\mathrm{UV}} - c^{\mathrm{IR}} \geq 0 \, .$

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C-theorem: $\delta c = c^{\mathrm{UV}} - c^{\mathrm{IR}} \geq 0 \, .$

Does it work for other protected quantities?

Paper: focus on protected scalars

$$\langle \mathcal{O}(x)\overline{\mathcal{O}}(0)\rangle = \frac{C_\Delta(\Lambda|x|)}{|x|^{2\Delta}} \,,$$

$$C_\Delta^{\rm UV} = \lim_{|x|\to 0} C_\Delta(\Lambda|x|) \,, \qquad C_\Delta^{\rm IR} = \lim_{|x|\to \infty} C_\Delta(\Lambda|x|) \,.$$

Example: BPS operators in 4d N=2

Paper: focus on protected scalars:

$$\langle \mathcal{O}(x)\overline{\mathcal{O}}(0)\rangle = \frac{C_\Delta(\Lambda|x|)}{|x|^{2\Delta}} ,$$

$$C_\Delta^{\mathrm{UV}} = \lim_{|x|\to 0} C_\Delta(\Lambda|x|) , \qquad C_\Delta^{\mathrm{IR}} = \lim_{|x|\to\infty} C_\Delta(\Lambda|x|) .$$

Is there a sum rule ?

$$\delta C_\Delta = C_\Delta^{\mathrm{UV}} - C_\Delta^{\mathrm{IR}} = \int \mathrm{d}^d x \, \mathcal{D}_\Delta \langle \mathcal{O}(x)\overline{\mathcal{O}}(0)\rangle ,$$

Schematic proof

$$\left(x^2\Box - 4\Delta\left(\Delta - \frac{d-2}{2}\right)\right)\langle\mathcal{O}(x)\overline{\mathcal{O}}(0)\rangle$$

Integrate:

$$\delta C_\Delta = \frac{\Gamma(\frac{d}{2})}{4\pi^{\frac{d}{2}}(2\Delta-\nu)}\int \mathrm{d}^d x\ |x|^{2\Delta-d}\left(|x|^2\Box - 4\Delta\left(\Delta-\nu\right)\right)\langle\mathcal{O}(x)\overline{\mathcal{O}}(0)\rangle\,,$$

Schematic proof:

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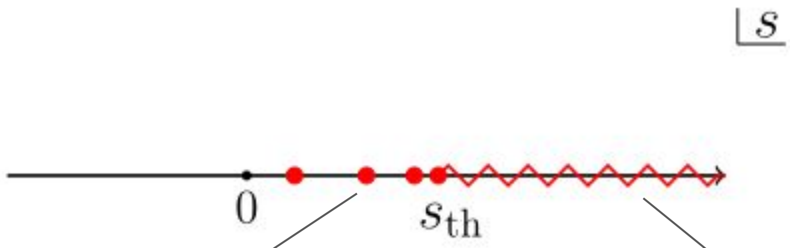
Not manifestly positive…

Use Kahlen–Lehmann decomposition

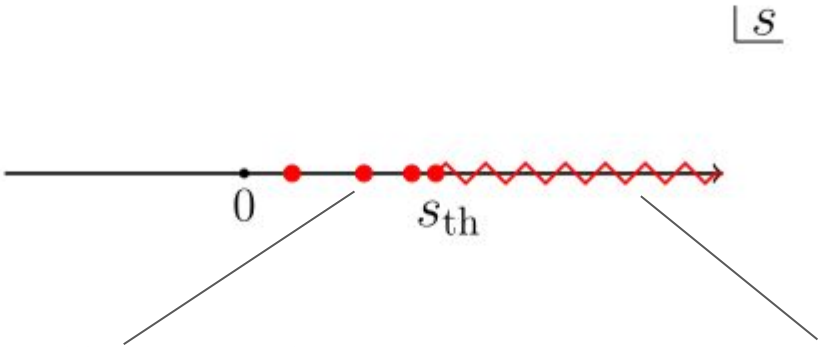
Free propagator

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \int_0^\infty ds\ \rho(s) G_s(x)\ ,$$



$$\rho_{\mathrm{sp}}(s) = \sum_i c_i \delta(s - m_i^2)$$

$$\rho_{\mathrm{mp}}(s) = \sigma(s)\Theta(s - s_{\mathrm{th}})\ ,$$



$$\rho_{\mathrm{sp}}(s) = \sum_i c_i \delta(s - m_i^2)$$

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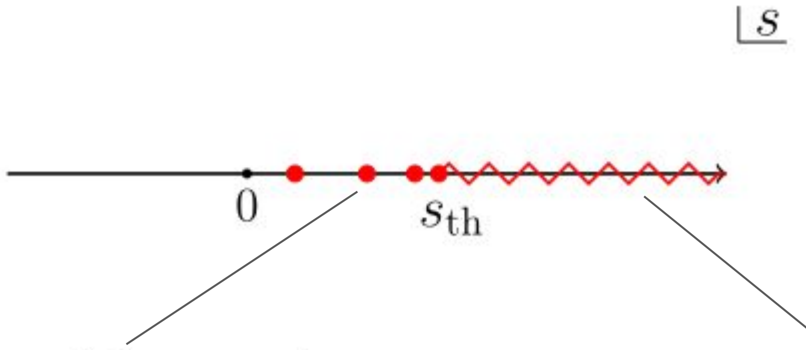
Sum rule + decomposition:

$$\delta C_\Delta = \frac{1}{4\Delta - 2\nu} \int_0^\infty dx\, x^{2\Delta - 1} \int_{s_{\mathrm{th}}}^\infty ds\, \sigma(s)\, (x^2 s - 4\Delta(\Delta - \nu)) \left(\frac{\sqrt{s}}{x}\right)^\nu \mathrm{K}_\nu(\sqrt{s}x) \ . \tag{3.17}$$

No contribution from single particle states



$$\rho_{\text{sp}}(s) = \sum_i c_i \delta(s - m_i^2)$$

$$\rho_{\text{mp}}(s) = \sigma(s)\Theta(s - s_{\text{th}}) \ ,$$

- δC_Δ depends on start of multi-particle states

- UV behaviour: $\rho(s) = C_\Delta^{\text{UV}} \dfrac{s^{\Delta - \frac{d}{2}}}{\Gamma(\Delta + 2\nu)} \left(1 + \dfrac{a_1}{s} + \dfrac{a_2}{s^2} + \dots\right)$

Plug back to sum rule

$$\delta C_\Delta = \frac{1}{4\Delta - 2\nu} \int_0^\infty dx\, x^{2\Delta-1} \int_{s_{\rm th}}^\infty ds\, \sigma(s)\, \left(x^2 s - 4\Delta(\Delta - \nu)\right) \left(\frac{\sqrt{s}}{x}\right)^\nu \mathrm{K}_\nu(\sqrt{s}x)\ . \quad (3.17$$

Two possibilities:

1) $s = s_{\rm th} > 0.$ ⟶ $\delta C_\Delta = C_\Delta^{\rm UV}$

2) $s_{\rm th} = 0,$ ⟶ Perturbative expansion diverges

But result finite

—> Dominated by non-pert effects...

cannot conclude $\delta C_\Delta \geq 0.$

4d N = 2 theories (and type-B anomalies in general)

Conformal dimensions are integers

$$\chi_{ij} = \langle \mathcal{O}_i(1)\overline{\mathcal{O}}_j(0)\rangle$$

If marginal operators, conformal manifold

Elli and friends: $$\nabla_\chi C_\Delta = 0 \, .$$

Use it to go close to a free point: $$\delta C_\Delta = C_\Delta^{\mathrm{UV}} - C_\Delta^{\mathrm{IR}} \geq 0 \, .$$

Paper:
1) test sum rule in various examples
2) similar sum rule for conserved currents and stress tensor