Jackiw-Teitelboim Gravity as a Noncritical String

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Jackiw Teitelboim (JT) gravity has proven to be an excellent tool for investigating aspects of quantum gravity and black hole physics. In recent years, the study of JT gravity and its deformations has helped us learn about the different contributions of geometries in the gravitational path integral, the quantum gravity Hilbert space, the space-time factorization problem, the role of averaging in holography, the black hole information paradox, and the matrix models. All this motivates the exploration of the JT gravity in different setups, with and without matter. Here, we consider JT gravity conformally coupled to Liouville field theory and matter fields. This model admits to be interpreted as a non-critical string theory on a linear dilaton background with a tachyonic Liouville potential along a null direction. The constant curvature constraint of JT gravity results in a neutralization of the Liouville mode, which makes it possible to compute the four-point correlation function of the theory analytically. Here we give the explicit derivation of the four-point function and briefly comment on its properties, such as monodromy invariance, crossing symmetry, factorization, and limits.

A few questions

- What incarnation of JT gravity is studied?
- Which string theory is it related to?
- How are the two theories identified?
- Why is this useful?

➢What incarnation of JT gravity is studied?

$$S = S_{JT} + S_L + S_{\text{matter}} + S_{\text{ghost}} \,, \qquad (1)$$

with the JT action [1, 2]

$$S_{JT} = \frac{1}{2\pi}\int_M d^2x\sqrt{\hat{g}}\,\varphi\,(\hat{R}+\Lambda) + \frac{\theta}{4\pi}\int_M d^2x\sqrt{\hat{g}}\,\hat{R} \qquad (2)$$

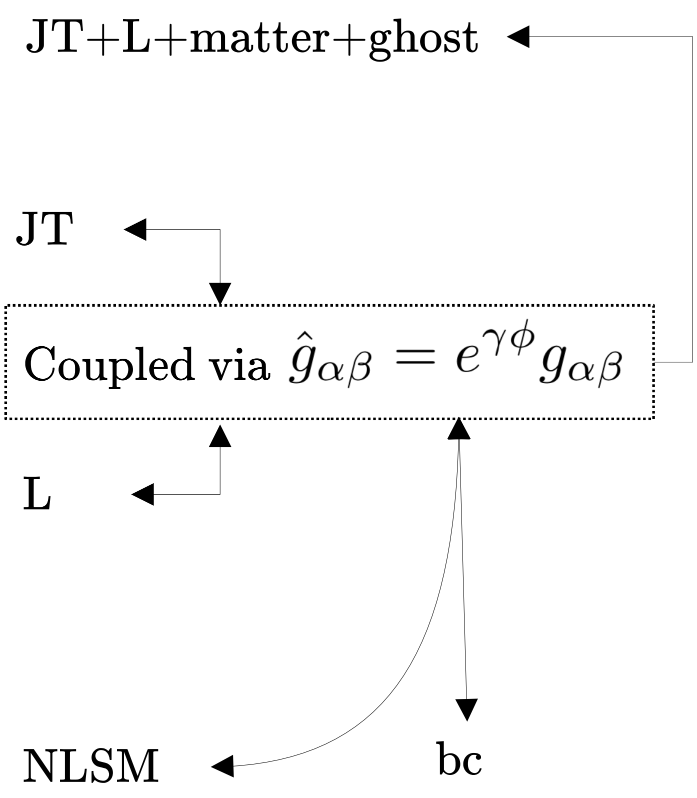
coupled to the Liouville action [27]

$$S_L = \frac{1}{2\pi}\int_M d^2x\sqrt{g}\,g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{Q}{2\pi}\int_M d^2x\sqrt{g}\,R\,\phi + \mu\int_M d^2x\sqrt{g}\,e^{\gamma\phi} \qquad (3)$$

together with matter fields

$$S_{\text{matter}} = \frac{1}{4\pi}\int_M d^2x\sqrt{g}\,g^{\alpha\beta}\partial_\alpha X^a\partial_\beta X^b\delta_{ab} \,, \qquad (4)$$

where $\alpha, \beta = 0, 1$ and $a, b = 2, 3, ..., D-1$. Matter content is given by $D-2$ space-like scalar fields X^a. The coupling constants θ, μ, Q, γ are real, with $\gamma \neq -1$. Re



$$S_{\text{ghost}} = \frac{1}{2\pi}\int_M d^2x\sqrt{g}\,g^{\alpha\beta}c^\rho\nabla_\alpha b_{\beta\rho}$$

➢Which string theory is it related to?

$$S = S_0 + S_I + \theta\chi(M) + S_{\text{ghost}} \tag{9}$$

with the Gaussian contribution

$$S_0 = \frac{1}{4\pi}\int_M d^2z\left(\eta_{\mu\nu}\partial X^\mu\bar{\partial}X^\nu + RQ_0X^0 + RQ_1X^1\right),$$

and the interaction term

$$S_I = \frac{1}{4\pi}\int_M d^2z\left(\hat{\Lambda}X^1 - \xi\hat{\Lambda}X^0 + 4\pi\mu\right)e^{2b(X^0+X^1)},$$

where $\eta_{\mu\nu}$ denotes the mostly-plus, D-dimensional Minkowski metric ($\mu,\nu = 0,1,2,...,D-1$). Also, we have rescaled the cosmological constant as

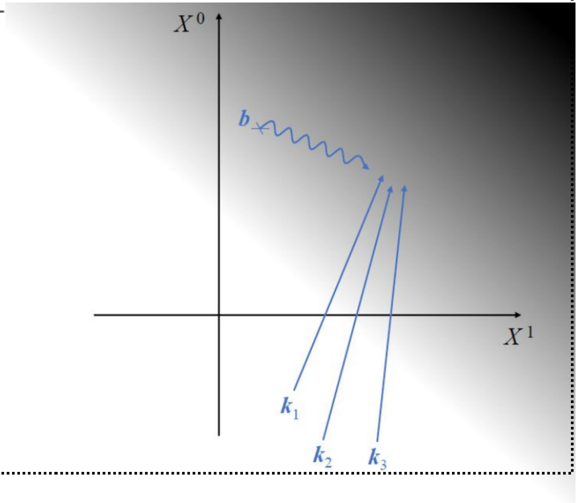
$$\hat{\Lambda} = \sqrt{\frac{2}{1+\gamma}}\Lambda, \tag{10}$$

Parameters: $Q_0, Q_1, \xi, b, (\mu, \theta)$

Here, we will consider the theory with $\Lambda = 0$. For the case $\Lambda \neq 0$ see reference [26]. For $\Lambda = 0$, the action above can interpreted as a string worldsheet σ-model in the D-dimensional dilaton-tachyon background

$$\Phi(X) = \frac{1}{2}Q_\mu X^\mu + \frac{\theta}{2}, \qquad T(X) = \pi\mu\, e^{2b_\nu X^\nu} \tag{16}$$

with $Q_\mu = Q_0\,\delta^0_\mu + Q_1\,\delta^1_\mu$, $b_\nu = b\,(\delta^0_\nu + \delta^1_\nu)$ with $\mu,\nu = 0,1,2,...,D-1$ and $\alpha' = 2$. This describes a flat, linear dilaton configuration with a Liouville type potential along the light-like direction X^+



➢How are the two theories identified?

Field identification

$$\phi = \frac{X^1 + X^0}{\sqrt{2(1+\gamma)}}\,, \qquad \varphi = \frac{X^1 - \xi X^0}{\sqrt{2(1+\gamma)}}$$

Remember from first slide:

$$\hat{g}_{\alpha\beta} = e^{\gamma\phi} g_{\alpha\beta}$$

Parameter identification

$$\xi = 1 + 2\gamma^{-1}$$

$$2b = \frac{\gamma}{\sqrt{2(1+\gamma)}}$$

$$Q_0 = \sqrt{\frac{2}{1+\gamma}}(Q - 1 - 2\gamma^{-1})$$

$$Q_1 = \sqrt{\frac{2}{1+\gamma}}(Q + 1)\,.$$

Weyl anomaly

$$\frac{c - D}{24} = \frac{\gamma Q - 1}{\gamma^2}$$

$$D = 2 \qquad Q = \gamma + \gamma^{-1}$$

➤Why is this useful?

$$S \supset \frac{1}{4\pi}\int_M d^2z\,(\,\partial X^-\bar{\partial}X^+ + 4\pi\mu\, e^{2bX^+}\,)$$

Reduced to free Bosons plus light-like Liouville interaction.

More specifically, we will compute correlation functions of vertex operators of the form

$$\Phi_{k_j}(z_j) = e^{2ik^j_\mu X^\mu(z_j)}\,, \tag{21}$$

with $j = 1, 2, ..., N$ and $\mu = 0, 1, ..., D-1$. These are

$$\left\langle \prod_{j=1}^N \Phi_{k_j}(z_j)\right\rangle = \int \mathcal{D}X\, e^{-S}\prod_{j=1}^N e^{2ik^j_{\mu_j}X^{\mu_j}(z_j)} \tag{22}$$

Task: compute resonant correlators of vertex operators.

$$n\,b + i\sum_{j=1}^N k^j_- = Q_-\,.$$

Solution: Integrate out zero-mode of the Liouville field to reduce to integrated N+n point correlator of free fields

Since the screening operators do not talk to each other, the integrals factorise!

A few answers

- What incarnation of JT gravity is studied?

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$$\hat{g}_{\alpha\beta} = e^{\gamma\phi} g_{\alpha\beta}$$

- Which string theory is it related to?

$$S = S_0 + S_I + \theta\chi(M) + S_{\text{ghost}}$$

$$\Phi(X) = \frac{1}{2} Q_\mu X^\mu + \frac{\theta}{2}$$

$$T(X) = \pi\mu\, e^{2b_\nu X^\nu}$$

- How are the two theories identified?

$$\phi = \frac{X^1 + X^0}{\sqrt{2(1+\gamma)}}, \qquad \varphi = \frac{X^1 - \xi X^0}{\sqrt{2(1+\gamma)}}$$

- Why is this useful?

$$\Big\langle \prod_{j=1}^{3} \Phi_{k_j}(z_j) \Big\rangle_{\rm R} = \frac{(-\pi\mu)^n}{b\,\Gamma(n+1)}\, e^{-2\theta} \Big(I_3\Big)^n \times \delta\Big(\sum_{j=1}^{3} k_+^j + \frac{i}{2b}\Big)\, \delta\Big(\sum_{j=1}^{3} k_a^j\Big)$$

$$\Big\langle \prod_{j=1}^{4} \Phi_{k_j}(z_j) \Big\rangle_{\rm R} = \frac{(-\pi\mu)^n}{b\,\Gamma(n+1)}\, e^{-2\theta}\, |z|^{4k_\mu^1 k^{2\,\mu}} |1-z|^{4k_\mu^2 k^{3\,\mu}} \times \Big(I_4(z)\Big)^n \delta\Big(\sum_{j=1}^{4} k_+^j + \frac{i}{2b}\Big)\delta\Big(\sum_{j=1}^{4} k_a^j\Big) \qquad (28)$$

A little bit of context: Is there something new?

Classical Gravity Coupled to Liouville Theory

Francisco D. Mazzitelli[1]

Noureddine Mohammedi[2] (1991)

We consider the two dimensional Jackiw-Teitelboim model of gravity. We first couple the model to the Liouville action and c scalar fields and show, treating the combined system as a non linear sigma model, that the resulting theory can be interpreted as a critical string moving in a target space of dimension $D = c + 2$. We then analyse perturbatively a generalised model containing a kinetic term and an arbitrary potential for the auxiliary field. We use the background field method and work with covariant gauges. We show that the renormalisability of the theory depends on the form of the potential. For a general potential, the theory can be renormalised as a non linear sigma model. In the particular case of a Liouville-like potential, the theory is renormalisable in the usual sense.

Brief comments on Jackiw-Teitelboim gravity coupled to Liouville theory

Gastón E. Giribet (2003)

Jackiw-Teitelboim gravity with non-vanishing cosmological constant coupled to Liouville theory is considered as a non-critical string on d dimensional flat spacetime. It is discussed how the presence of cosmological constant yields additional constraints on the parameter space of the theory, even when the conformal anomaly is independent of the cosmological constant. Such constraints agree with the necessary conditions for the tachyon field to be a primary –prelogarithmic– operator of the worldsheet conformal field theory. Thus, the linearized tachyon field equation allows to impose the diagonal condition for the interaction term. We analyze the neutralization of the Liouville mode induced by the coupling to the Jackiw-Teitelboim Lagrangian. The free field prescription leads to obtain explicit expressions for three-point correlation functions for the case of vanishing cosmological constant in terms of a product of Shapiro-Virasoro integrals. This is a consequence of the mentioned neutralization effect.

➢Full expression for four-point function

$$\left\langle \prod_{j=1}^{4} \Phi_{k_j}(z_j) \right\rangle_{\mathrm{R}} = \frac{(-\pi\mu)^n}{b\,\Gamma(n+1)}\, e^{-2\theta}\, |z|^{4k^1_\mu k^{2\,\mu}} |1-z|^{4k^2_\mu k^{3\,\mu}}$$

$$\times \Big(I_4(z)\Big)^n \delta\Big(\sum_{j=1}^{4} k^j_+ + \frac{i}{2b}\Big)\delta\Big(\sum_{j=1}^{4} k^j_a\Big) \qquad (28)$$

$$I_4(z) = C_1(k^j_+)\left| {}_2F_1\left[\begin{matrix}-4ibk^2_+,\ 1+4ibk^4_+ \\ -4ib(k^1_+ + k^2_+)\end{matrix}; z\right]\right|^2$$

$$+\,C_2(k^j_+)\left| z^{1+4ib(k^1_+ + k^2_+)}\,{}_2F_1\left[\begin{matrix}-4ibk^3_+,\ 1+4ibk^1_+ \\ -4ib(k^3_+ + k^4_+)\end{matrix}; z\right]\right|^2$$

with the coefficients

$$C_1(k^j_+) \;=\; \frac{\Gamma(1+4ib(k^1_+ + k^2_+))\Gamma(1+4ibk^3_+)\Gamma(1+4ibk^4_+)}{\Gamma(-4ib(k^1_+ + k^2_+))\Gamma(-4ibk^3_+)\Gamma(-4ibk^4_+)}$$

$$C_2(k^j_+) \;=\; \frac{\Gamma(1+4ib(k^3_+ + k^4_+))\Gamma(1+4ibk^1_+)\Gamma(1+4ibk^2_+))}{\Gamma(-4ib(k^3_+ + k^4_+))\Gamma(-4ibk^1_+)\Gamma(-4ibk^2_+)}$$