String Journal Club 3-12-2024

UV complete local field theory of persistent symmetry breaking in 2+1 dimensions

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Spontaneous symmetry breaking can persist at all temperatures in certain biconical $O(N) \times \mathbb{Z}_2$ vector models when the underlying field theories are ultraviolet complete. So far, the existence of such theories has been established in fractional dimensions for local but nonunitary models or in 2+1 dimensions but for nonlocal models. Here, we study local models at zero and finite temperature directly in 2+1 dimensions employing functional methods. At zero temperature, we establish that our approach describes the quantum critical behaviour with high accuracy for all $N \geq 2$. We then exhibit the mechanism of discrete symmetry breaking from $O(N) \times \mathbb{Z}_2 \to O(N)$ for increasing temperature near the biconical critical point when N is finite but large. We calculate the corresponding finite-temperature phase diagram and further show that the Hohenberg-Mermin-Wagner theorem is fully respected within this approach, i.e., symmetry breaking only occurs in the $\mathbb{Z}_2$ sector. Finally, we determine the critical N above which this phenomenon can be observed to be $N_c \approx 15$.

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- *The central question and the past attempts*

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- *'Breakthrough' via numerical approach*

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- *The central question and the past attempts*
- *'Breakthrough' via numerical approach*
- *Take-home message*

Past attempts

Thermal order in conformal theories

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It is widely expected that at sufficiently high temperatures order is always lost, e.g., magnets lose their ferromagnetic properties. We pose the question of whether this is always the case in the context of quantum field theory in d space dimensions. More concretely, one can ask whether there exist critical points (CFTs) which break some global symmetry at arbitrary finite temperature. The most familiar CFTs do not exhibit symmetry breaking at finite temperature, and moreover, in the context of the AdS/CFT correspondence, critical points at finite temperature are described by an uncharged black brane which obeys a no-hair theorem. Yet, we show that there exist CFTs which have some of their internal symmetries broken at arbitrary finite temperature. Our main example is a vector model which we study both in the epsilon expansion and arbitrary rank as well as the large rank limit (and arbitrary dimension). The large rank limit of the vector model displays a conformal manifold, a moduli space of vacua, and a deformed moduli space of vacua at finite temperature. The appropriate Nambu-Goldstone bosons including the dilatonlike particle are identified. Using these tools we establish symmetry breaking at finite temperature for finite small ϵ. We also prove that a large class of other fixed points, which describe some of the most common quantum magnets, indeed behave as expected and do not break any global symmetry at finite temperature. We discuss some of the consequences of finite temperature symmetry breaking for the spectrum of local operators. Finally, we propose a class of fixed points which appear to be possible candidates for finite temperature symmetry breaking in $d = 2$.

Symmetry Breaking at All Temperatures

Noam Chai,[1] Soumyadeep Chaudhuri,[1] Changha Choi,[2] Zohar Komargodski,[2] Eliezer Rabinovici,[1] and Michael Smolkin[1]

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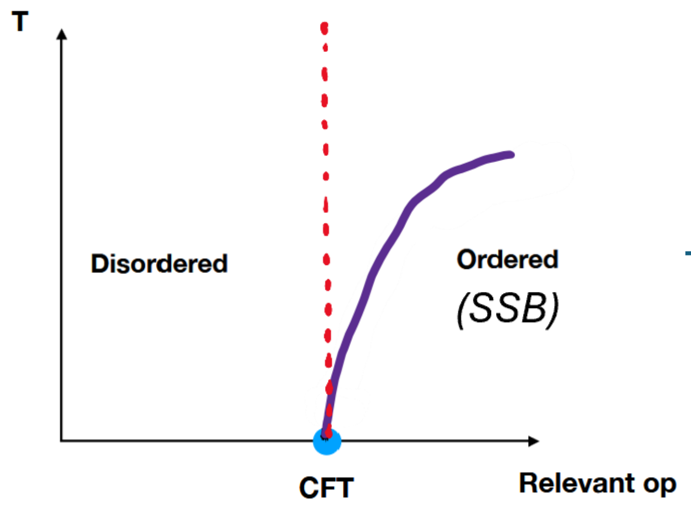
We explore the existence of conformal field theories that persistently break a global symmetry at finite temperature. We identify vector models in $(3 - \epsilon)$ spatial dimensions that have internal symmetries broken at any temperature. We study these systems in the small ϵ regime and in the large rank limit. The latter displays a conformal manifold and a moduli space of vacua deformed at finite temperature. We touch upon a candidate in $d = 2$ dimensions.

SSB and thermal CFTs

- Naive expectation: at high temperature disordered states (high entropy) dominate!
- UV incomplete theories: we can witness intermediate symmetry breaking (Rochelle salt, Helium-3, etc…)
- However, there is no theorem at infinite volume for a UV complete QFT → CFT at finite temperature!



Most common phase diagram
'magnets lose their magnetization at high T'

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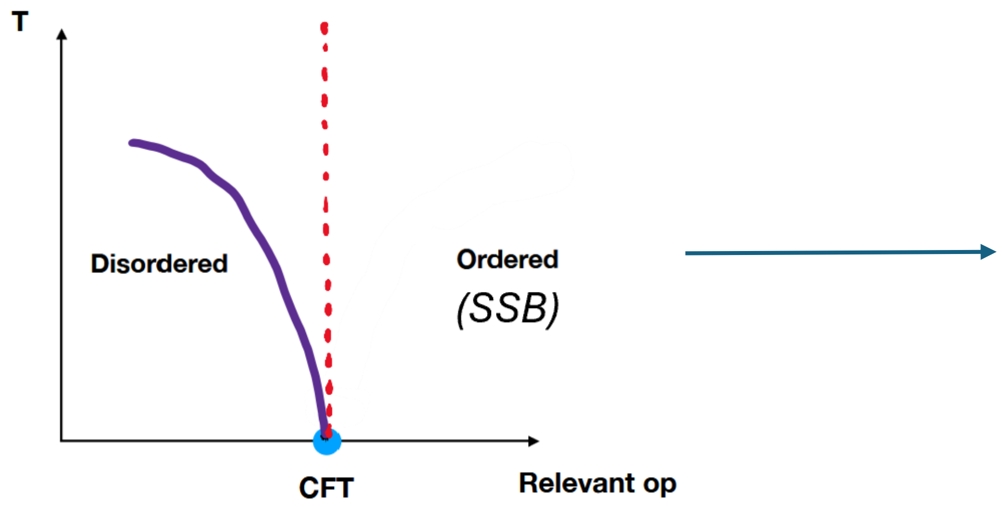

'magnets keep *their magnetization at high T'*

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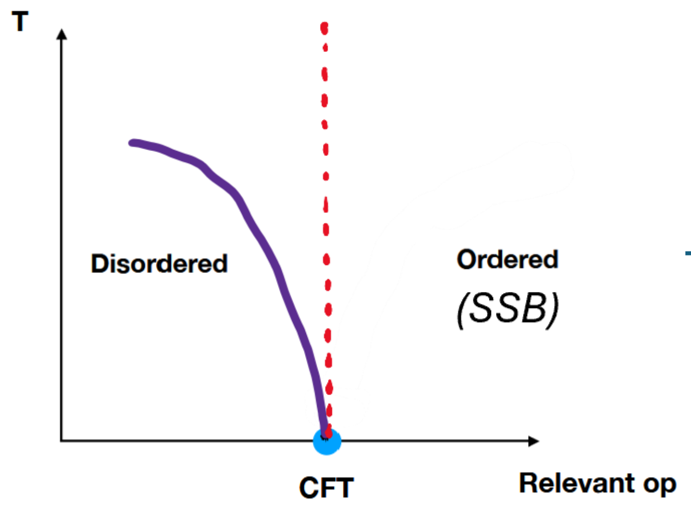
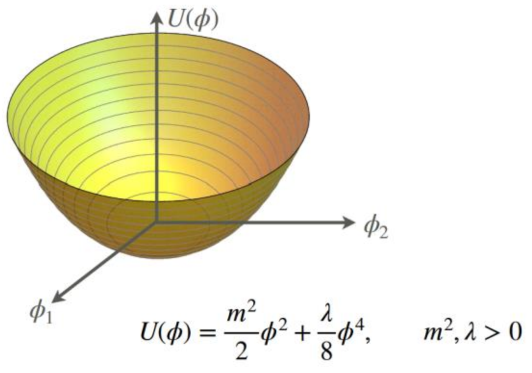


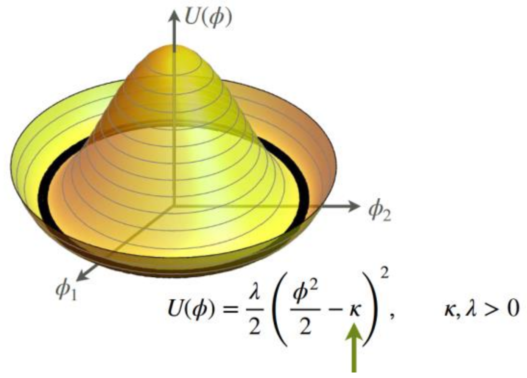
'magnets keep their magnetization at high T'

Are there unitary, local, nontrivial CFTs which break a global symmetry at finite temperature?

An example: vector models

- Many quantum magnets are described by the $O(N)$ vector model





- The one-loop thermal mass is positive defined → no SSB!

- In general, no theorem says that the thermal mass should be positive!

$Biconical\ vector\ models$

- These are $O(m) \times O(N-m)$ vector models in $d = 4 - \varepsilon$

$$V = \lambda_{11}(\phi_1^2)^2 + 2\lambda_{12}(\phi_1^2)(\phi_2^2) + \lambda_{22}(\phi_2^2)^2 \ .$$

- Equal rank: no SSB

- **Different rank**: study at large N

  →**Biconical fixed point** with $\lambda_{12} < 0$

  →Vacuum: $\phi_1^2 = 0, \phi_2^2 \neq 0$! **We have SSB!**

The candidate model

- Symmetry breaking: $O(n) \times O(p) \to O(n-1) \times O(p)$

- Cannot hold until $\varepsilon = 1$, i.e., $d = 2 + 1$ (no continuous symmetry breaking in 2d), but $Z_2 \times O(p) \to O(p)$ is allowed!

The candidate is the $Z_2 \times O(N)$ vector model at the biconical fixed point!

State of the art before 2409.10606

- A candidate for SSB at high temperature has been found

$$S=\int d^D x\Big(\frac{1}{2}(\partial\phi)^2+\frac{1}{2}(\partial\chi)^2+U[\rho_\phi,\rho_\chi]\Big) \qquad U=m_\phi^2\rho_\phi+m_\chi^2\rho_\chi+\frac{\lambda_\phi}{2}\rho_\phi^2+\frac{\lambda_\chi}{2}\rho_\chi^2+\lambda_{\phi\chi}\rho_\phi\rho_\chi$$

$$\rho_\phi=\tfrac{1}{2}\phi_a\phi_a \text{ and } \rho_\chi=\tfrac{1}{2}\chi^2$$

- It has only been studied for large N and in fractional dimension

Could it be studied directly in $d=2+1$?

Can we find at which N the SSB arises?

Numerically solving the RG flow

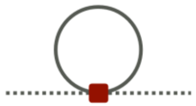
- Solution: solve explicitly the RG flow equations near the biconical fixed point!

- RG 'time': $t = log\frac{k}{\Lambda}$, with k RG scale, Λ UV cutoff

- Select a temperature + a vacuum and solve the flow for $k \to 0$

Running of the thermal mass

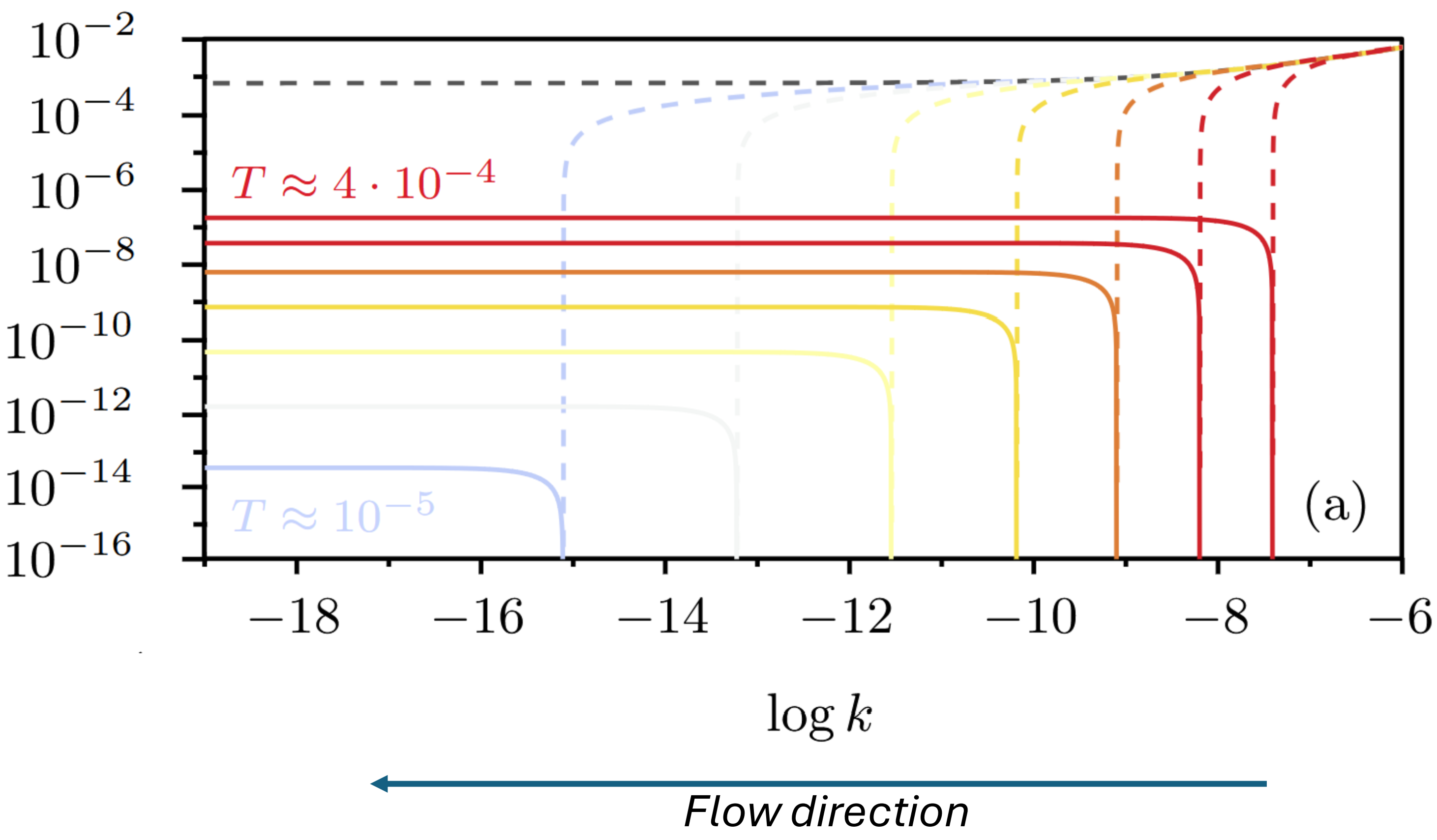
$$\partial_t m_\chi^2 = -\frac{k^4 a_D T}{3\pi^2}\left(\frac{3\lambda_\chi}{(k^2+m_\chi^2)^2} + \frac{N\lambda_{\phi\chi}}{(k^2+m_\phi^2)^2}\right)$$ *+ subleading in k*



Qualitative picture neglecting denominators:

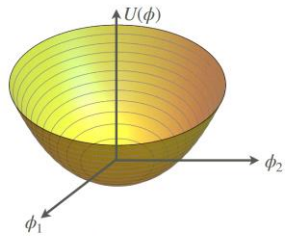
- $\lambda_\chi > 0$ for a bounded potential
- $\lambda_{\phi\chi} < 0$ at the biconical fixed point
- For some $\mathrm{N} > \frac{3\lambda_\chi}{|\lambda_{\phi\chi}|}$ the flow can make m_χ^2 decrease!

$O(N)$ bath: symmetry restoration

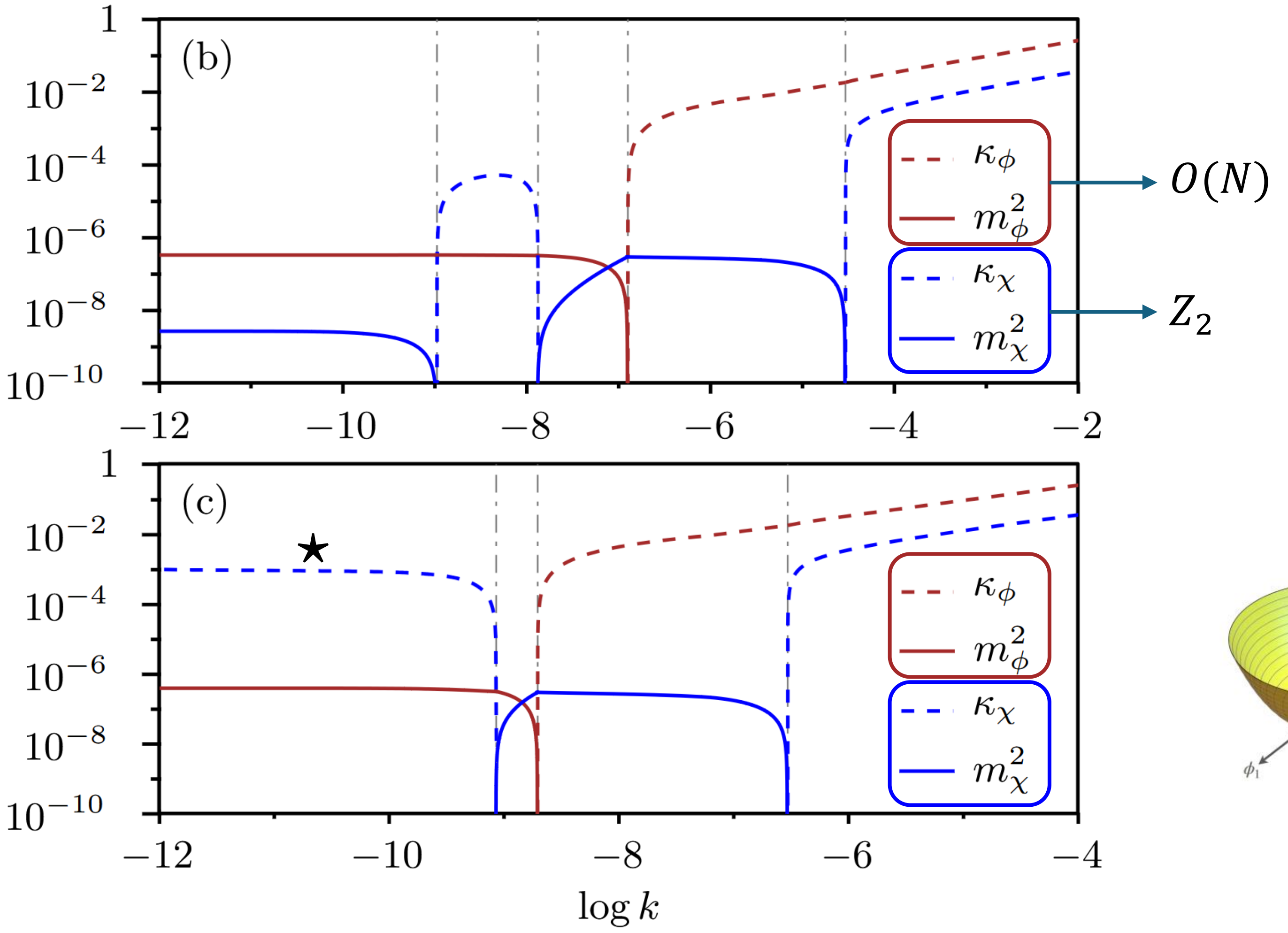


Symmetry restoration at every temperature!

$m_\phi^2 > 0$ *for any* T

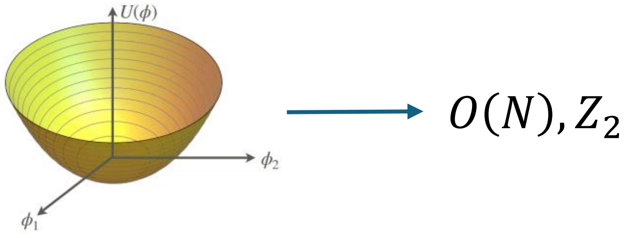


Z_2 symm: persistent breaking

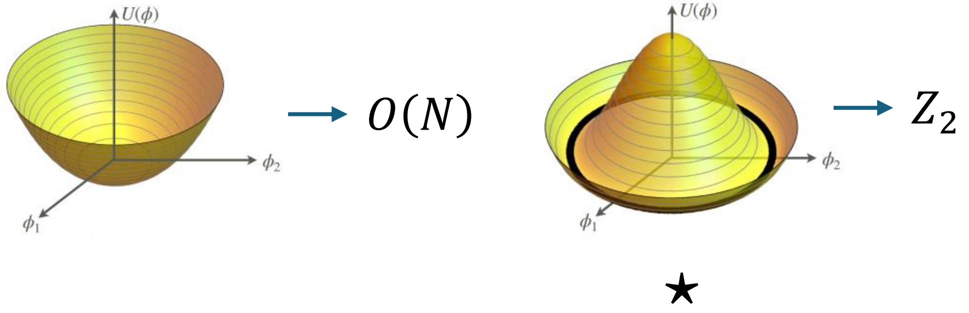


Flow direction

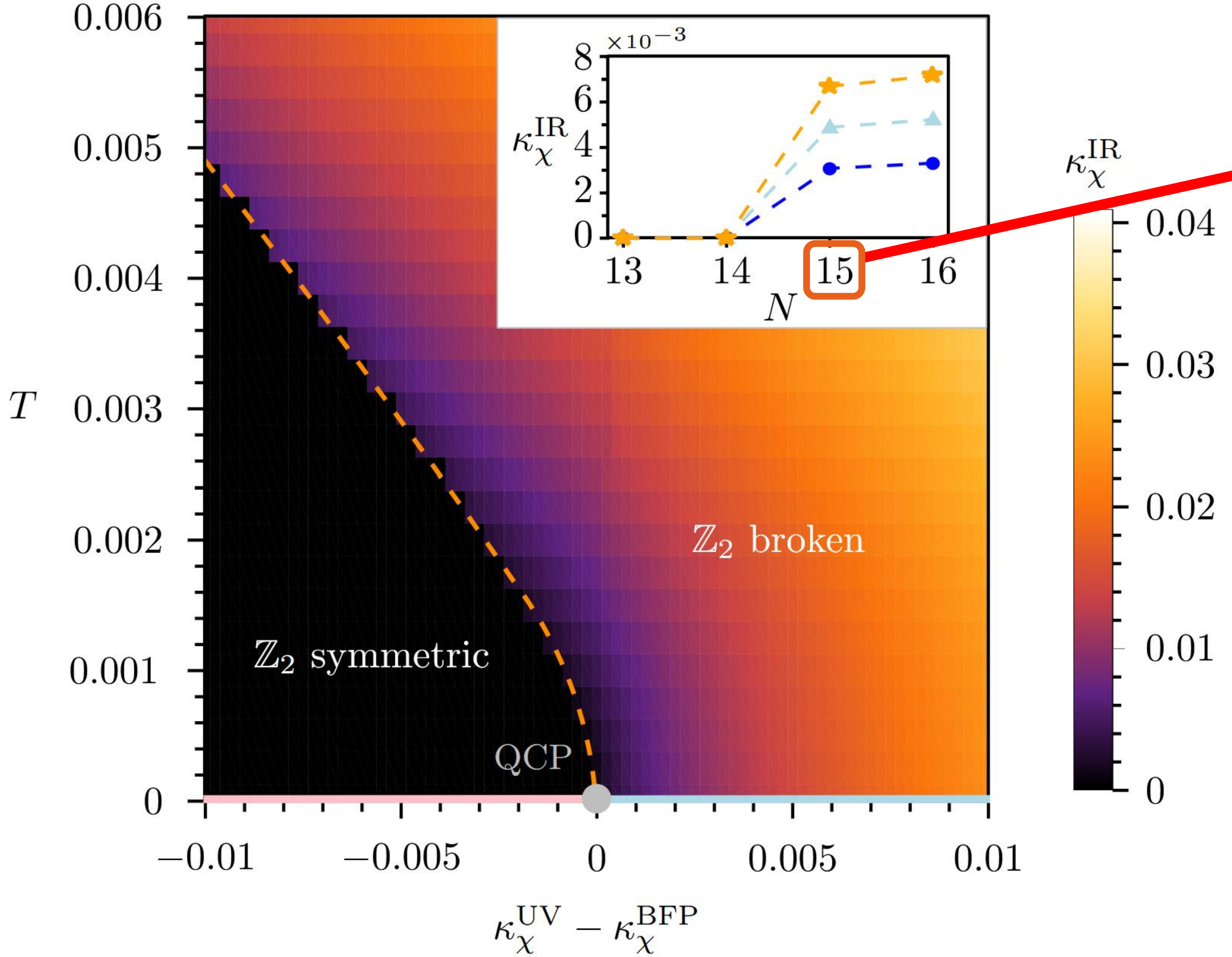
Symmetry restoration



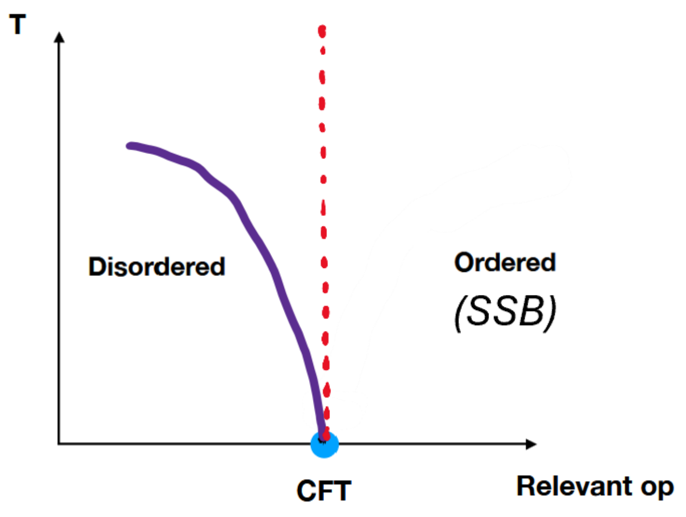
Persistent symmetry breaking



Phase diagram



Critical rank value: pesistent breaking starting from the $O(15) \times Z_2$ model



Take-home message

- In nature, we witness intermediate SSB. These systems are not described by UV complete models.

- In a thermal CFT, all temperatures are equal, so SSB either doesn't exist or it exists at all temperatures.

- Theoretically, it is possible to witness SSB at high T in $Z_2 \times O(N)$ models for N large enough.

  → 'Ising-like' magnets in a '$O(N)$ bath'