Irreversibility of quantum field theory in de Sitter: the C, F and A theorems

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Irreversibility of quantum field theory in de Sitter: the C, F and A theorems

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We prove the irreversibility of the renormalization group for quantum field theory in de Sitter space in $d = 2$, 3 and 4 space-time dimensions. The proof is based on strong subadditivity of the entanglement entropy, de Sitter invariance, and the Markov property of conformal field theory.

Recap of irreversibility in Mink

In 2d we have that:

$$c \sim \langle TT \rangle_{CFT} \;, f_{TTT}$$

$$\Delta c = \int x^2 \; \langle TT \rangle \; d^2x \geq 0$$

$$c \sim \int_{s^2} d^2x \; \langle T^\mu_\mu \rangle$$

[Zamolodchikov; 1986] [Cardy; 1988]

And in higher d?

Cardy proposal:

$$c^{(d)} \sim \int_{s^d} d^dx \; \langle T^\mu_\mu \rangle$$

[Cardy; 1989]

Recap of irreversibility in Mink

Cardy proposal: $$c^{(d)} \sim \int_{s^d} d^d x \ \langle T^\mu_\mu \rangle$$

- Komargodski Schwimmer:

[Komargodski Schwimmer; 2011]

$$a \sim \int_{s^d} d^4 x \ \langle T^\mu_\mu \rangle \qquad \Delta a \sim \int_{S^4} \frac{\sigma}{s^2} ds \geq 0$$

- Klebanov Pufu at all:

[Klebanov Pufu Safdi; 2011]

$$F \sim \int_{s^3} d^3 x \ \langle T^\mu_\mu \rangle \qquad \Delta F \geq 0$$

Recap of irreversibility in Mink

Alternative realization:

- ANEC: [Hartman Mathys; 2023] [Hartman Mathys; 2023]

$$a^{(4d)}, c^{(2d)} \sim \langle \Psi | \mathscr{E} | \Psi \rangle \geq 0$$

$$\mathscr{E} \sim \int du \; T^{uu} \qquad |\psi\rangle \sim \int d^{dx} T^{\mu}_{\mu} |0\rangle$$

- Entanglement entropy (EE): [Casini Huerta; 2009] [Casini Salazar Landea Torroba; 2023]

Strong subadditivity of EE + QNEC ⟶ F,a,c,g theorems!!!

Entanglement entropy in dS

Metric:

$$\mathrm{d}s_d^2 = \frac{\ell^2}{\cos^2 T}\left(-\mathrm{d}T^2 + \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\Omega_{d-2}^2\right),$$

Action:

$$S_{\mathrm{QFT}} = S_{\mathrm{CFT}} + \int \mathrm{d}^d x\,\sqrt{-g}\,\lambda_I \phi_I$$

EE:

$$S(R) = \mu_{d-2}\,R^{d-2} + \mu_{d-4}\,R^{d-4} + \dots + \begin{cases} (-)^{\frac{d-2}{2}} 4\,A\,\log(R/\epsilon) & d \text{ even} \\ (-)^{\frac{d-1}{2}} F & d \text{ odd} \end{cases}$$

$$\mu_k \sim \epsilon^{-k}$$

Purely divergent: so we consider

$$\Delta S(R) = S_{QFT}(R) - S_{UV}(R)$$

Entanglement entropy in dS

EE:

$$\mu_k \sim \epsilon^{-k}$$

$$S(R) = \quad \mu_{d-2}\, R^{d-2} + \mu_{d-4}\, R^{d-4} + \dots + \begin{cases} (-)^{\frac{d-2}{2}} 4\, A \, \log(R/\epsilon) & d \text{ even} \\ (-)^{\frac{d-1}{2}} F & d \text{ odd} \end{cases}$$

Purely divergent: so we consider:

$$\Delta S(R) = S_{QFT}(R) - S_{UV}(R)$$

Very similar to flat space but now R cannot be ∞: physically because there is an maximum entangling distance scale $R = \ell$, corresponding to the largest causally accessible region to an observer

Entanglement entropy in dS

EE:

$$S(R) = \quad \mu_{d-2}\, R^{d-2} + \mu_{d-4}\, R^{d-4} + \dots \\ + \begin{cases} (-)^{\frac{d-2}{2}} 4\, A \log(R/\epsilon) & d \text{ even} \\ (-)^{\frac{d-1}{2}} F & d \text{ odd} \end{cases}$$

$$\mu_k \sim \epsilon^{-k}$$

Purely divergent: so we consider:

$$\Delta S(R) = S_{QFT}(R) - S_{UV}(R)$$

We expand the EE using boosts transformations

$$0 = \int_{\Omega_1} S_1\, \delta\gamma_1 + \frac{1}{2}\int_{\Omega_1,\Omega_2} S_{12}\, \delta\gamma_1 \delta\gamma_2 + \dots\ ,$$

$$S_1(\hat{x}_1) = \frac{\delta S}{\delta\gamma(\hat{x}_1)}\Big|_{\gamma_0},\ S_{12}(\hat{x}_1,\hat{x}_2) = \frac{\delta^2 S}{\delta\gamma(\hat{x}_1)\delta\gamma(\hat{x}_2)}\Big|_{\gamma_0},$$

Entanglement entropy in dS

We expand the EE using boosts transformations

$$0 = \int_{\Omega_1} S_1\, \delta\gamma_1 + \frac{1}{2}\int_{\Omega_1,\Omega_2} S_{12}\, \delta\gamma_1 \delta\gamma_2 + \ldots\,,$$

$$S_1(\hat{x}_1) = \frac{\delta S}{\delta\gamma(\hat{x}_1)}\Big|_{\gamma_0},\; S_{12}(\hat{x}_1,\hat{x}_2) = \frac{\delta^2 S}{\delta\gamma(\hat{x}_1)\delta\gamma(\hat{x}_2)}\Big|_{\gamma_0},$$

Using with Strong subadittivity of EE $S(A) + S(B) \geq S(A \cup B) + S(A \cap B)$

We get

$$R\Delta S''(R) - (d-3)\Delta S'(R) \leq 0\,.$$

Entanglement entropy in dS

$$R\Delta S''(R) - (d-3)\Delta S'(R) \leq 0\,.$$

$$\downarrow$$

- $d = 2$ $\Delta C(R) \leq 0$
- $d = 3$ $\Delta F(R) \leq 0$
- $d = 4$ $\Delta A(R) \leq 0$

Conclusion

$$\Delta C, \Delta F, \Delta A \leq 0$$

Irreversibility of RG flows in dS!

Outlook

- Connection with ANEC?
- Higher dimensional proof?;
- Calculate examples (e.g. numerically) in interacting theory;


Thank you


GROUP DISCUSSION