Quantum Groups as Global Symmetries

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Authors: Barak Gabai, Victor Gorbenko, Jiaxin Qiao, Bernardo Zan, Aleksandr Zhabin

← Today

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Quantum Groups as Global Symmetries II. Coulomb Gas Construction
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← Very technical but important

- What is the most general & useful notion of Symm?

→ Do not have to be groups [SUSY, non-inv, high forms ..]

→ Quantum groups are DEFORMATIONS OF LIE ALGEBRAS

GOAL: Properties of QFTs with Quantum Groups as Global symmetry

$\boxed{\text{IDEA}}$ SU(2) Global Symmetry

Degenerated energy levels

[$2\ell+1$ degeneracies]

e.g. $(+1, 0, -1)$ triplets.

Selection rules

$\langle O_i\, O_j\, O_k \rangle \propto \varepsilon_{ijk}$

$\Rightarrow \langle O\, O\, O \rangle = 0$

$U_q(su(2))$ Global Symmetry

SAME AS SU(2)

Degenerated energy levels

[$2\ell+1$ degen.]

DIFFERENT SEL. RULES

$\langle 0\ 0\ 0 \rangle \neq 0$

prediction of $U_q(su(2))$

$$\frac{\langle 0\,0\,0\rangle}{\langle +\,0\,-\rangle} = \frac{q^2-1}{q} \quad \left(\xrightarrow[q\to 1]{} 0\right)$$

$q \in \mathbb{C}$ $\qquad U_q(su(2))$ cont. def. of SU(2)

$U_q(su(2))$ Quantum Group.

sl_2 commutators: $[H,E] = 2E$, $[H,F] = 2F$, $[E,F] = H$

$U_q(sl_2)$ commutators:

$$q^{H} E q^{-H} = q^{2} E \qquad q^{H} F q^{-H} = q^{-2} F$$

$$[E,F] = \frac{q^{H} - q^{-H}}{q - q^{-1}}$$

- $q \to 1$ back to sl_2.

sl_2 $E_i(a \otimes b) = E_i(a) \otimes b + a \otimes E_i(b)$

coproduct: action on tensor product

$\Delta(H) = H \otimes 1 + 1 \otimes H$, $\Delta(E) = E \otimes 1 + q^{-H} \otimes E$, $\Delta(F) = F \otimes q^{H} + 1 \otimes F$

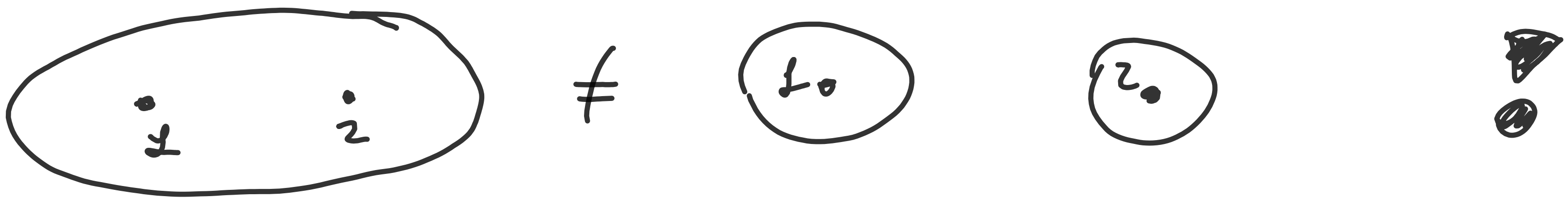
usual

q-def.

Action of Quantum Group Symm is NON-LOCAL

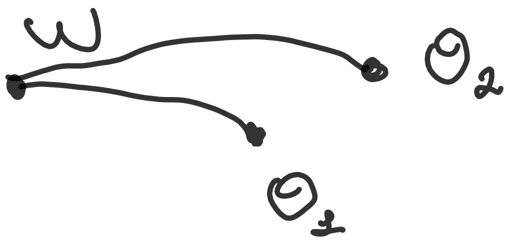
$$\Delta(E) = E \otimes 1 + q^{-H} \otimes E\,, \qquad \Delta(F) = F \otimes q^{H} + 1 \otimes F$$

Depends on the order!

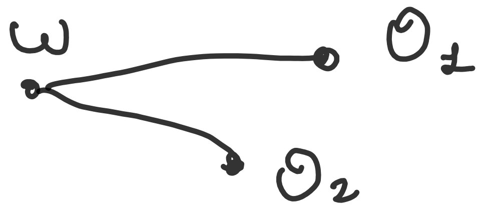


They suggest: QG ACTS ON DEFECT-ENDING OPS!

Introduce reference point ω arbitrary



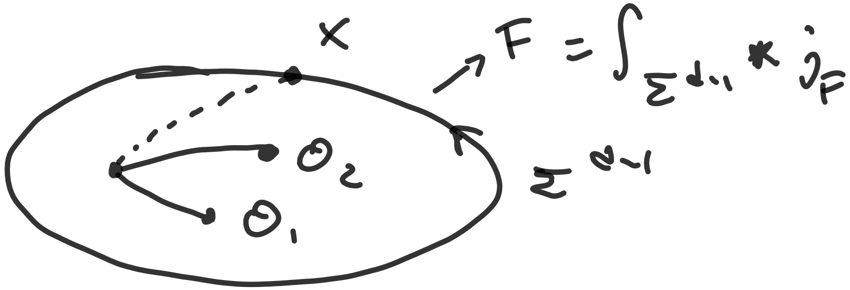
: $O_1 O_2 \neq O_2 O_1$



there are global symmetry currents

$$\mathcal{H} = \int_{\Sigma^{d-1}} * j_H, \quad F = \int_{\Sigma^{d-1}} * j_F, \quad E = \int_{\Sigma^{d-1}} * j_E$$

$F(\mathcal{O}_1, \mathcal{O}_2)$:



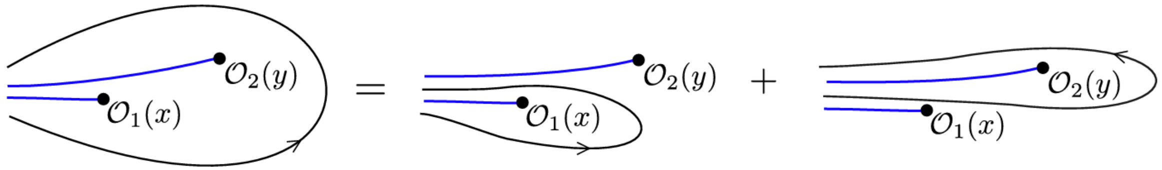
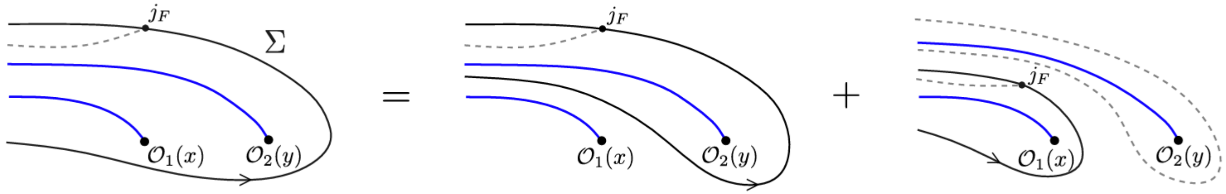


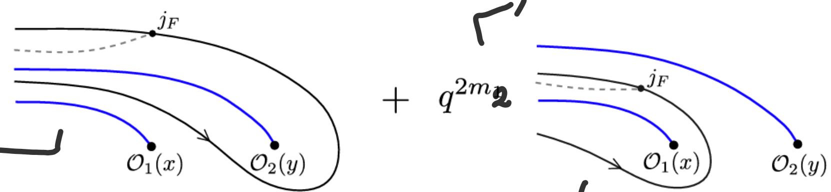
Figure 5: The action of ordinary global symmetry on defect-ending operators.

[Consistent with QG action]



$2m_2 = \mathcal{H}(\mathcal{O}_2)$

$$F(\mathcal{O}_1, \mathcal{O}_2) = F(\mathcal{O}_1) \otimes q^{\mathcal{H}(\mathcal{O}_2)} + 1 \otimes F(\mathcal{O}_2) =$$



CONSTRAINTS OF GLOBAL QUANTUM GROUP SYMMETRIES

WARD IDENTITIES:

$$\langle X (O_1 \cdots O_n) \rangle = 0 \qquad X = E, F, H$$

E.g. for $\frac{1}{2}$ rep of $U_q(su(2))$:

$$\langle F (O_+(x) O_+(y)) \rangle = q^{-1/2} \langle O_+(x) O_-(y) \rangle + q^{1/2} \langle O_-(x) O_+(y) \rangle = 0$$

CORR. FUNCTIONS:

$$\langle O^{m_1}_{\ell_1} O^{m_2}_{\ell_2} O^{m_3}_{\ell_3} \rangle \propto \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}_q$$

3j - symbol

They study XXZ_q CFT

known to have defect-ending op Lorem.

UV complete by spin-chain XXZ_q

for $R_0 = 0$ and $\sigma_i = \sigma_{L-i}$ $\Rightarrow$ XXZ

$$\mathcal{H} = Lq - \sum_{i=1}^{L-1} R_i - R_0,$$

where R_i for $i \in 1, \ldots, L-1$ are defined as

$$R_i = \frac{1}{2}\left[\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \frac{q+q^{-1}}{2}(\sigma_i^z \sigma_{i+1}^z + 1) - \frac{q-q^{-1}}{2}(\sigma_i^z - \sigma_{i+1}^z - 2)\right]$$

$$R_0 \equiv R_L = G\, R_{L-1}\, G^{-1}, \qquad R_L = R_1 \cdots R_{L-1}$$

It has $U_q(SU(2))$ Global symmetry.

They compute OPE, level etc... with Coulomb gas and check all the general results