Energy Correlators in Perturbative Quantum Gravity

E. Hermann, M. Kologlu , I. Moult

Alessio Miscioscia, DESY, theory group, string theory Journal Club 2025

Energy Correlators in Perturbative Quantum Gravity

Enrico Herrmann,[1] **Murat Kologlu,**[2] **Ian Moult**[2]

[1] *Mani L. Bhaumik Institute for Theoretical Physics, University of California at Los Angeles, Los Angeles, CA 90095, USA*

[2] *Department of Physics, Yale University, New Haven, CT 06511*

E-mail: eh10@g.ucla.edu, murat.kologlu@yale.edu, ian.moult@yale.edu

Abstract: Despite tremendous progress in our understanding of scattering amplitudes in perturbative (super-) gravity, much less is known about other asymptotic observables, such as correlation functions of detector operators. In this paper, we initiate the study of detector operators and their correlation functions in perturbative quantum gravity. Inspired by recent progress in field theory, we introduce a broad class of new asymptotic observables in gravity. We outline how correlation functions of detector operators can be efficiently computed from squared, state-summed amplitudes, allowing us to harness the wealth of perturbative scattering amplitude data to explore these observables. We then compute the two-point correlator of energy detectors in the annihilation of two scalars into gravitons, in Einstein gravity minimally coupled to a massive scalar field. We study the kinematic limits of this correlator, finding that it is finite in the collinear limit, and exhibits a soft divergence in the back-to-back limit, as expected from the understanding of the factorization of gravitational amplitudes in the soft and collinear limits. Our results offer a first exploration into the structure of detector operators and their correlators in perturbative quantum gravity, and we outline numerous directions for future study.

What is an energy correlator?

QFT: an energy detector.[1] Consider a calorimeter cell in an idealized collider experiment.

1: The calorimeter cell sits asymptotically far away at a particular angular position $\widehat{\mathbf{n}}$ on the celestial sphere S^{d-2} surrounding the experiment. In this idealized context, we imagine

2: it is placed there for all time, collecting radiation escaping to asymptotic infinity along its angular direction, and recording its energy. One way to define it is as the following

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What this corresponds to?

$$\mathcal{E}(\widehat{\mathbf{n}}) \propto \int_0^\infty dt \lim_{r\to\infty} r^{d-2}\, \widehat{\mathbf{n}}^i\, T_{0i}(t, r\,\widehat{\mathbf{n}}),$$

Working time of detector

Stress tensor

Are they interesting?

1: Phenomenological applications (e.g. old but very active story in QCD)

2: Understanding CFTs (light-ray operators)

3: Understanding QFTs (null energy conditions, positivity theorems, causality, …)

4: Understanding QFTs in non-vacuum states (e.g. heavy states)

5: Understanding quantum gravity ?

Are they interesting in gravity?

	Local observables	Asymptotic observables	Energy correlators
QFTs	✓	✓	✓
CFTs	✓	✗	✓
Gravity	✗	✓	✓

How to compute EC in gravity?

$$= \frac{1}{N}\langle\Psi|\mathcal{D}_{J_{L1}}(z_1)\cdots\mathcal{D}_{J_{Ln}}(z_n)|\Psi\rangle = \frac{1}{N}\sum_X \int \left[\prod_{a=1}^{n} \hat{d}^d q_a\, V_{J_{La}}(q_a, z_a)\right]\left[\prod_{b\in X}\hat{d}^d q_b\, W(q_b)\right] |\langle q_1,\ldots,q_n;X|\Psi\rangle|^2$$

Detector vertex (universal)

$$V_{J_L}(z;p) = \int_0^\infty d\beta \beta^{-J_L-1}\delta^d(p-\beta z)\,,$$

Free propagator

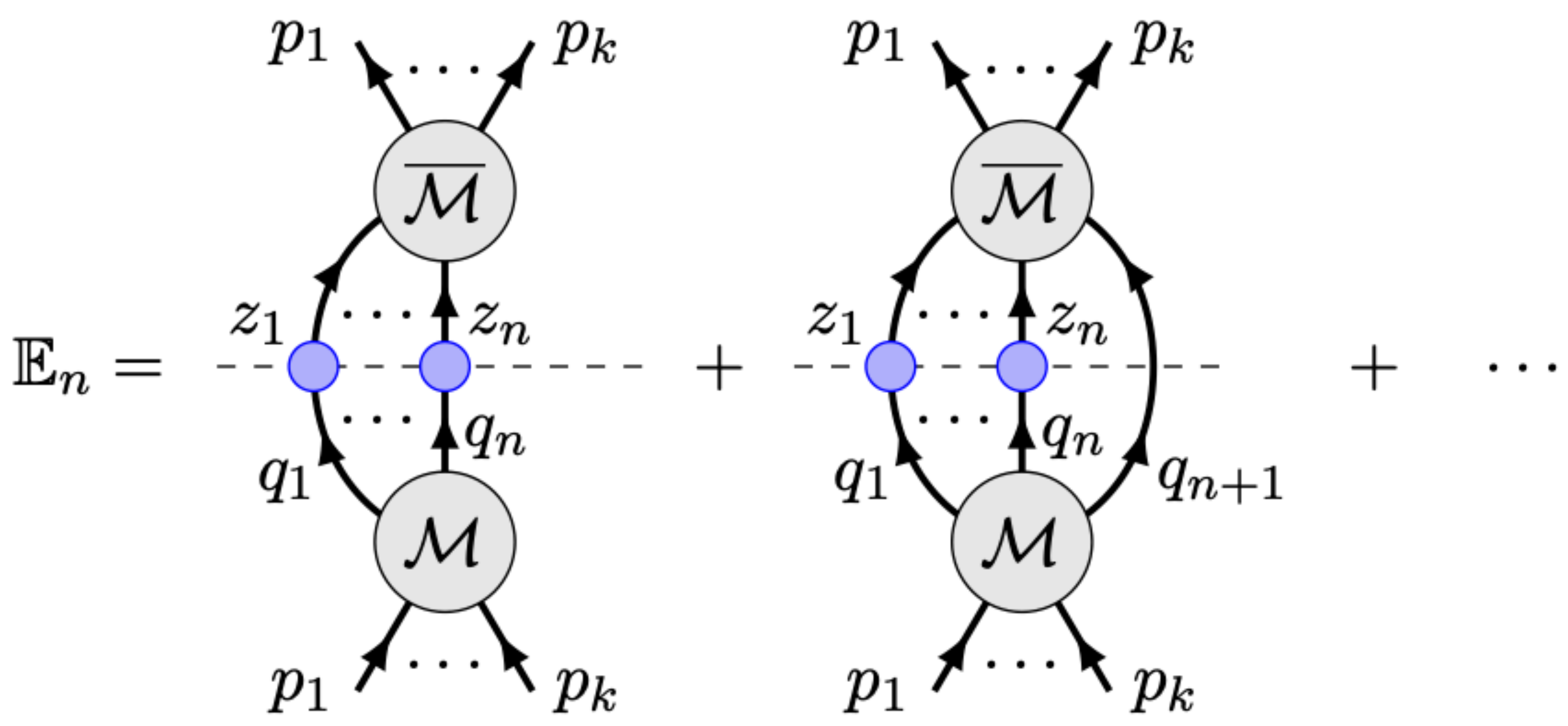
“Form factor”

If:

$$|\Psi\rangle = |\mathcal{O}(E)\rangle \equiv \int d^dx\; e^{-i\,Ex^0 - x_E^2/\sigma^2}\,\mathcal{O}(x)|0\rangle\,,$$

How to compute EC in gravity?

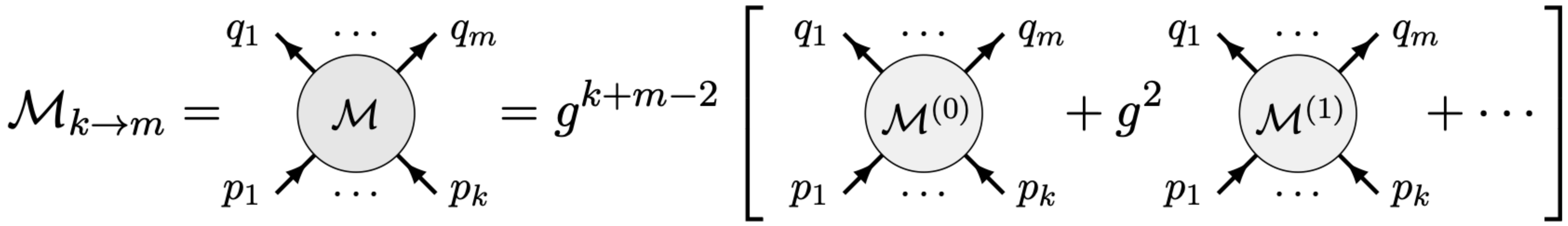
$$= \frac{1}{N}\langle\Psi|\mathcal{D}_{J_{L1}}(z_1)\cdots\mathcal{D}_{J_{Ln}}(z_n)|\Psi\rangle = \frac{1}{N_k}\sum_X \int \left[\prod_{a=1}^{n} \hat{d}^d q_a\, V_{J_{La}}(q_a, z_a)\right]\left[\prod_{b\in X} \hat{d}^d q_b\, W(q_b)\right] \times$$
$$\hat{\delta}^d(q_1 + \ldots + q_n + Q_X - P)\,\left|\mathcal{M}_{k\to n+|X|}(p_i, q_j)\right|^2,$$



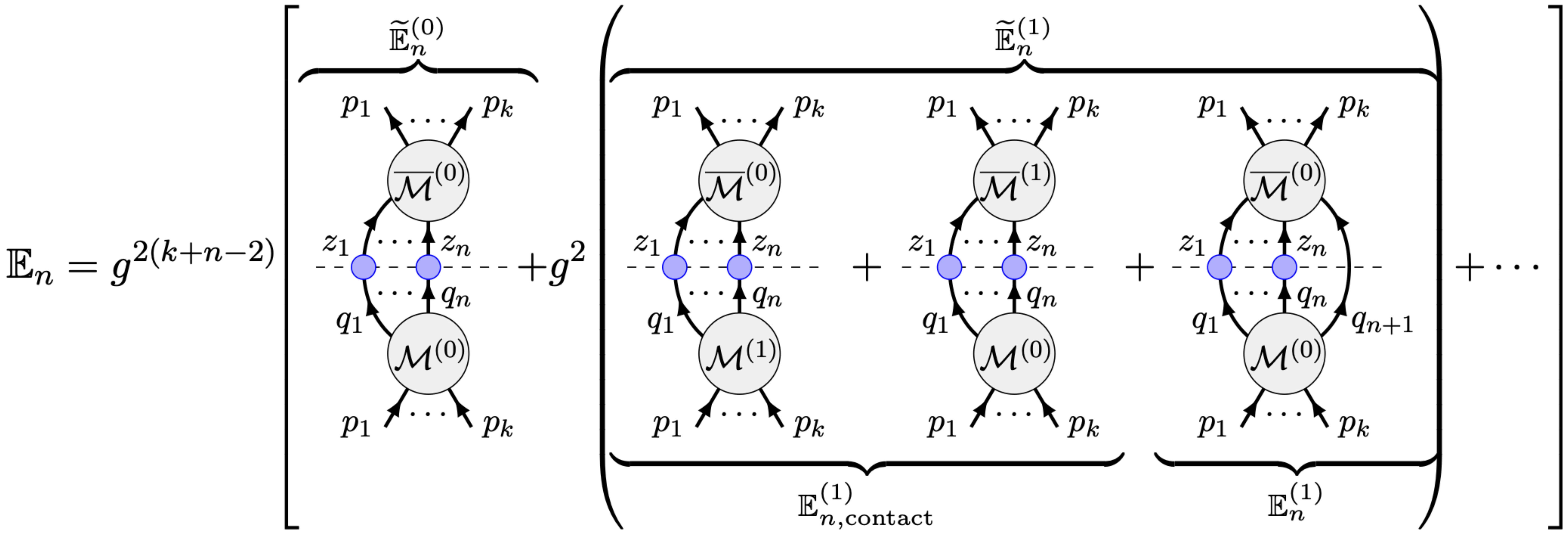
To compute the perturbative square S-matrix elements one can use e.g. double copies

How to compute EC in gravity?

Perturbation theory is:

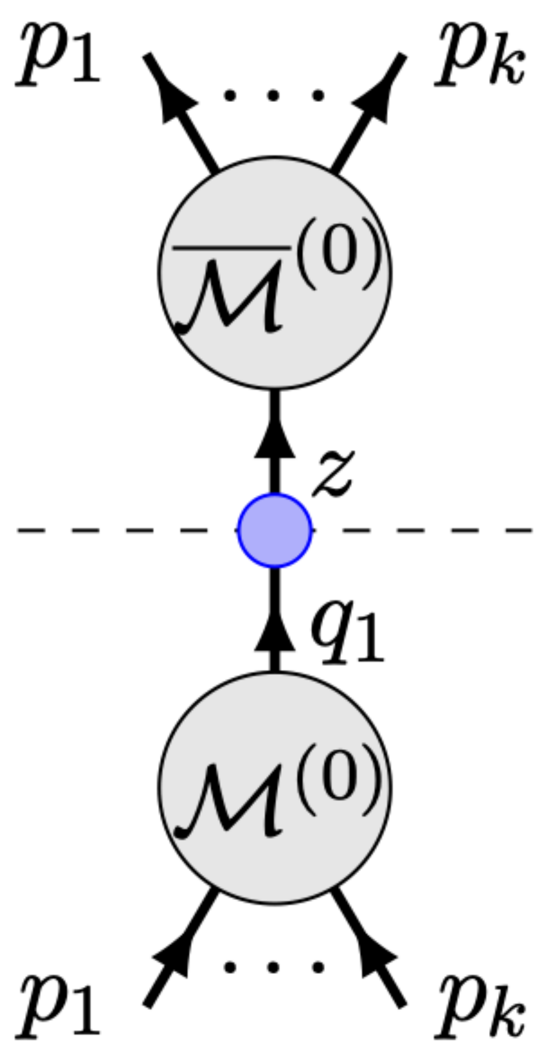


Therefore



Example 1-pt function

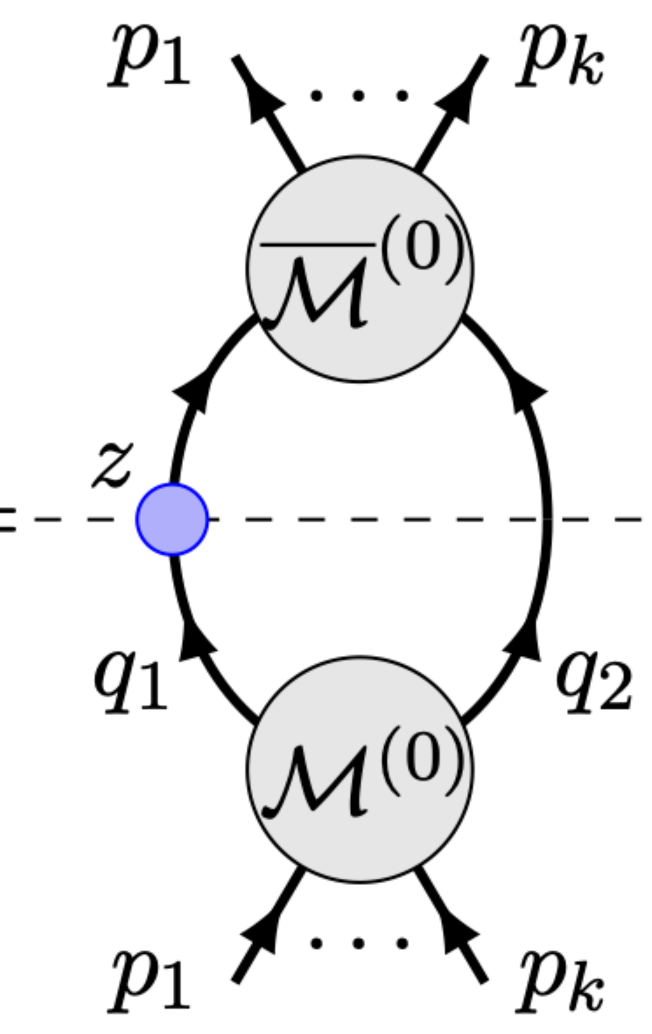
$$\widetilde{\mathbb{E}}_1^{(0)}(p_i;z) =$$



$$= \frac{1}{N_k} \int \hat{d}^d q_1 \, V_{J_L}(q_1, z) \left|\mathcal{M}^{(0)}_{k\to 1}(p_i, q_1)\right|^2 \hat{\delta}^d(q_1 - P)$$

$$= \frac{1}{N_k} \left|\mathcal{M}^{(0)}_{k\to 1}(p_i, q_1{=}P)\right|^2 V_{J_L}(P, z) \,.$$

$$\mathbb{E}_1^{(1)}(p_i;z) =$$



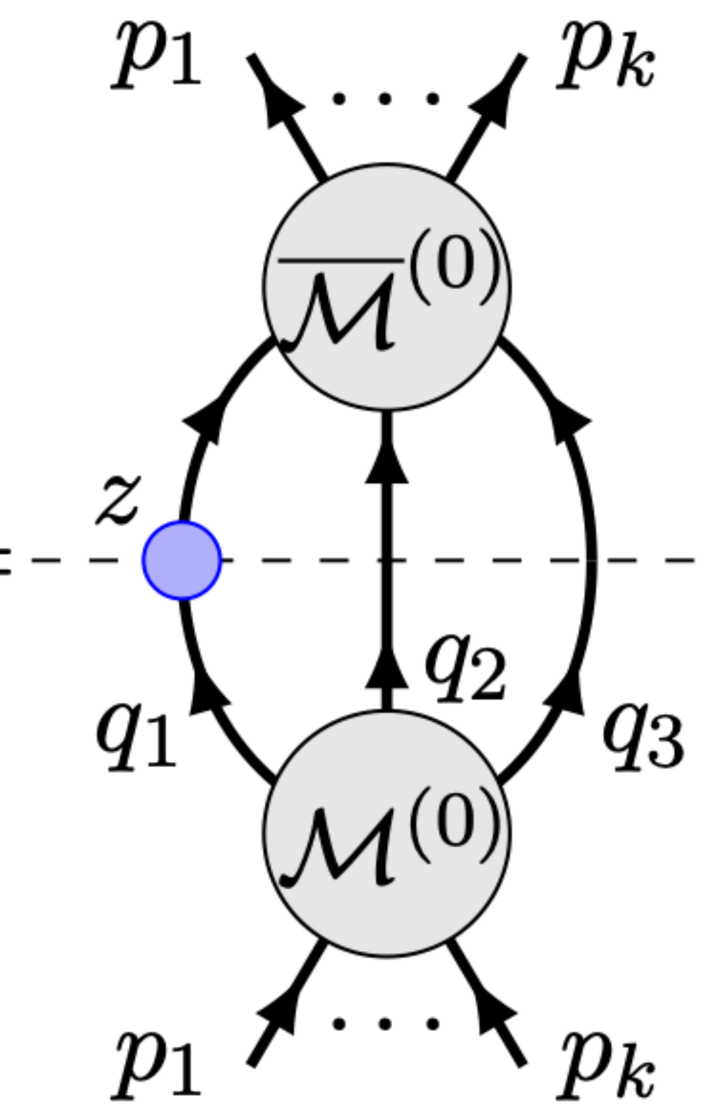
$$= \frac{1}{N_k} \int \prod_{a=1}^{2} \hat{d}^d q_a \, V_{J_L}(q_1, z) \, \hat{\delta}^+(q_2^2) \left|\mathcal{M}^{(0)}_{k\to 2}(p_i, q_1, q_2)\right|^2 \hat{\delta}^d(q_1{+}q_2{-}P)$$

$$= \frac{1}{N_k \, (2\pi)^d} \int_0^\infty d\beta \beta^{-J_L-1} \hat{\delta}^+((P{-}\beta z)^2) \left|\mathcal{M}^{(0)}_{k\to 2}(p_i, \beta z, P{-}\beta z)\right|^2 , \tag{3.23}$$

Example 1-pt function

Both the terms before do not include “loops” (momenta integration), but the next term requires an integration:

$$\mathbb{E}_1^{(2)}(p_i;z) =$$



$$= \frac{1}{N_k}\int\prod_{a=1}^{3}\hat{d}^d q_a\, V_{J_L}(q_1,z)\hat{\delta}^+(q_2^2)\hat{\delta}^+(q_3^2) \times \left|\mathcal{M}^{(0)}_{k\to 3}(p_i,q_1,q_2,q_3)\right|^2\hat{\delta}^d(q_1{+}q_2{+}q_3{-}P) \tag{3.27}$$

$$= \frac{1}{N_k\,(2\pi)^d}\int_0^\infty d\beta\beta^{-J_L-1}\int\hat{d}^d q_2\hat{\delta}^+(q_2^2)\hat{\delta}^+((P{-}\beta z-q_2)^2) \times \left|\mathcal{M}^{(0)}_{k\to 3}(p_i,q_1{=}\beta z,q_2,q_3{=}P{-}\beta z{-}q_2)\right|^2.$$

Simplifications arise in collinear limits…

Scalar minimally coupled with (EH) gravity

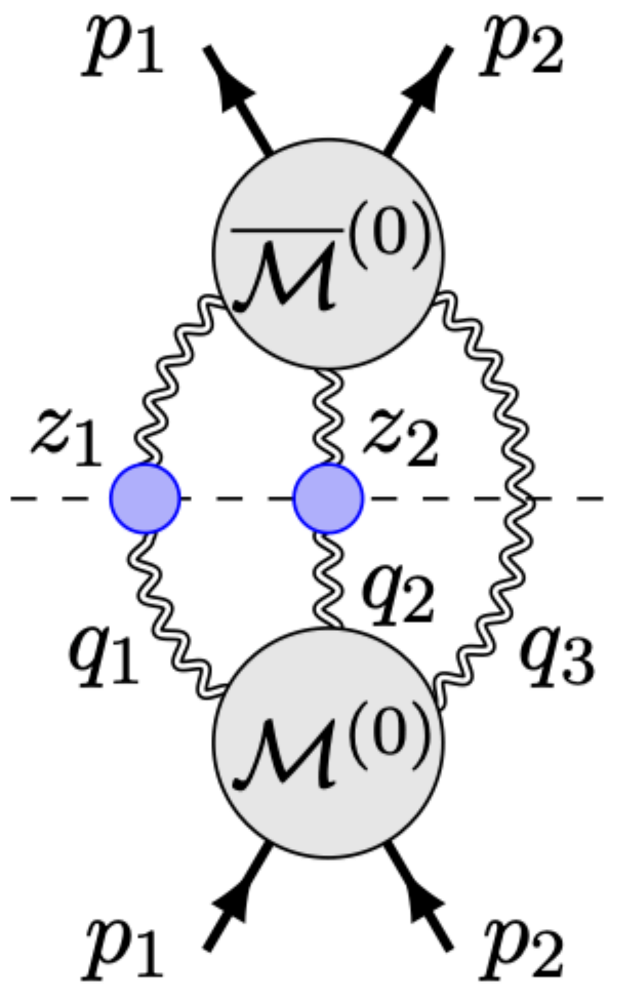
$$S_{\mathrm{EH}+\Phi} = \int d^d x \sqrt{-\mathfrak{g}} \left(\frac{1}{16\pi G_N} R + \frac{1}{2} \Phi \left(\Box - m^2 \right) \Phi \right) ,$$

$\kappa^2 = 32\pi G_N$

$$\mathfrak{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\mathbb{E}_2(p_1, p_2; z_1, z_2) = \frac{1}{N_2} \langle p_1, p_2 | \mathcal{D}_{J_{L1}}(z_1) \mathcal{D}_{J_{L2}}(z_2) | p_1, p_2 \rangle = \kappa^4 \, \widetilde{\mathbb{E}}_2^{(0)} + \kappa^6 \left(\mathbb{E}^{(1)}_{2,\mathrm{contact}} + \mathbb{E}_2^{(1)} \right) + \mathcal{O}(\kappa^8) ,$$

$\mathbb{E}_2^{(1)}(p_1, p_2; z_1, z_2) =$



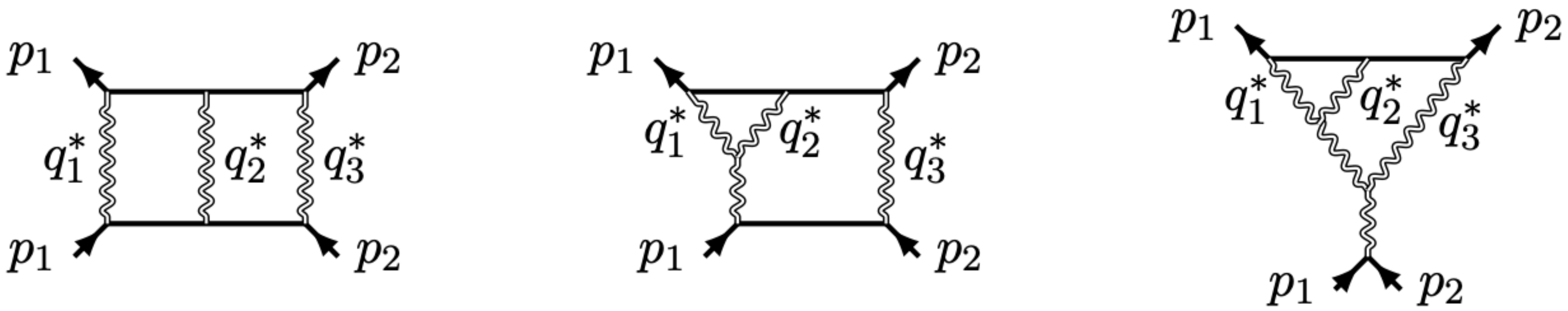
$$= \frac{1}{N_2 \, (2\pi)^{2d-1}} \frac{1}{P \cdot P} \left[\frac{2P \cdot z_1}{P \cdot P} \right]^{J_{L1}} \left[\frac{2P \cdot z_2}{P \cdot P} \right]^{J_{L2}} \qquad (5.3)$$

$$\times \int_0^1 d\alpha \, \alpha^{-J_{L1}-1} \, (1-\alpha)^{-J_{L2}-1} \, (1-\alpha\zeta)^{J_{L2}} \left| \mathcal{M}^{(0)}_{2\to 3}(p_1, p_2; q_i^*) \right|^2 .$$

$$N_2 = 4\sqrt{(p_1 \cdot p_2)^2 - m^4}$$

Scalar minimally coupled with (EH) gravity

We need to construct the coupling $\Phi\Phi \to hhh \Rightarrow$ 15 cubic diagrams



Scalar minimally coupled with (EH) gravity

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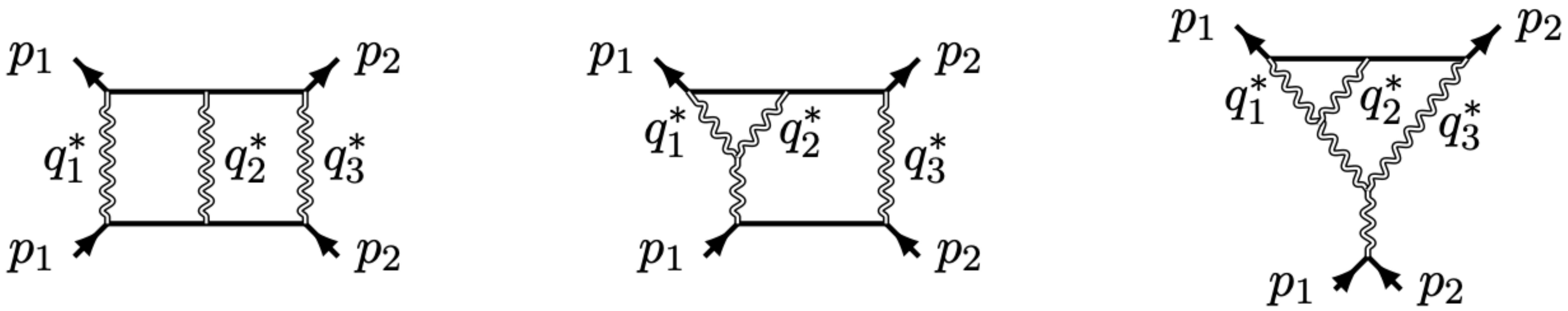


Calculations…

Scalar minimally coupled with (EH) gravity

$$\mathcal{G}^{(1)}_{\rm EEC}(\zeta,\chi_1,\chi_2,x) = r^{(0)}(\zeta,\chi_1,\chi_2,x) + \sum_{i=1}^{7} r^{(i)}(\zeta,\chi_1,\chi_2,x) \times f^{(i)}(\zeta,\chi_1,\chi_2)\,,$$

$$f^{(1)}(\zeta,\chi_1,\chi_2) = \frac{\arctan\left[\frac{\chi_2-\chi_1+2\zeta\chi_1}{\sqrt{\Delta}}\right] - \arctan\left[\frac{\chi_2-\chi_1-2\zeta}{\sqrt{\Delta}}\right]}{\sqrt{\Delta}}\,,$$

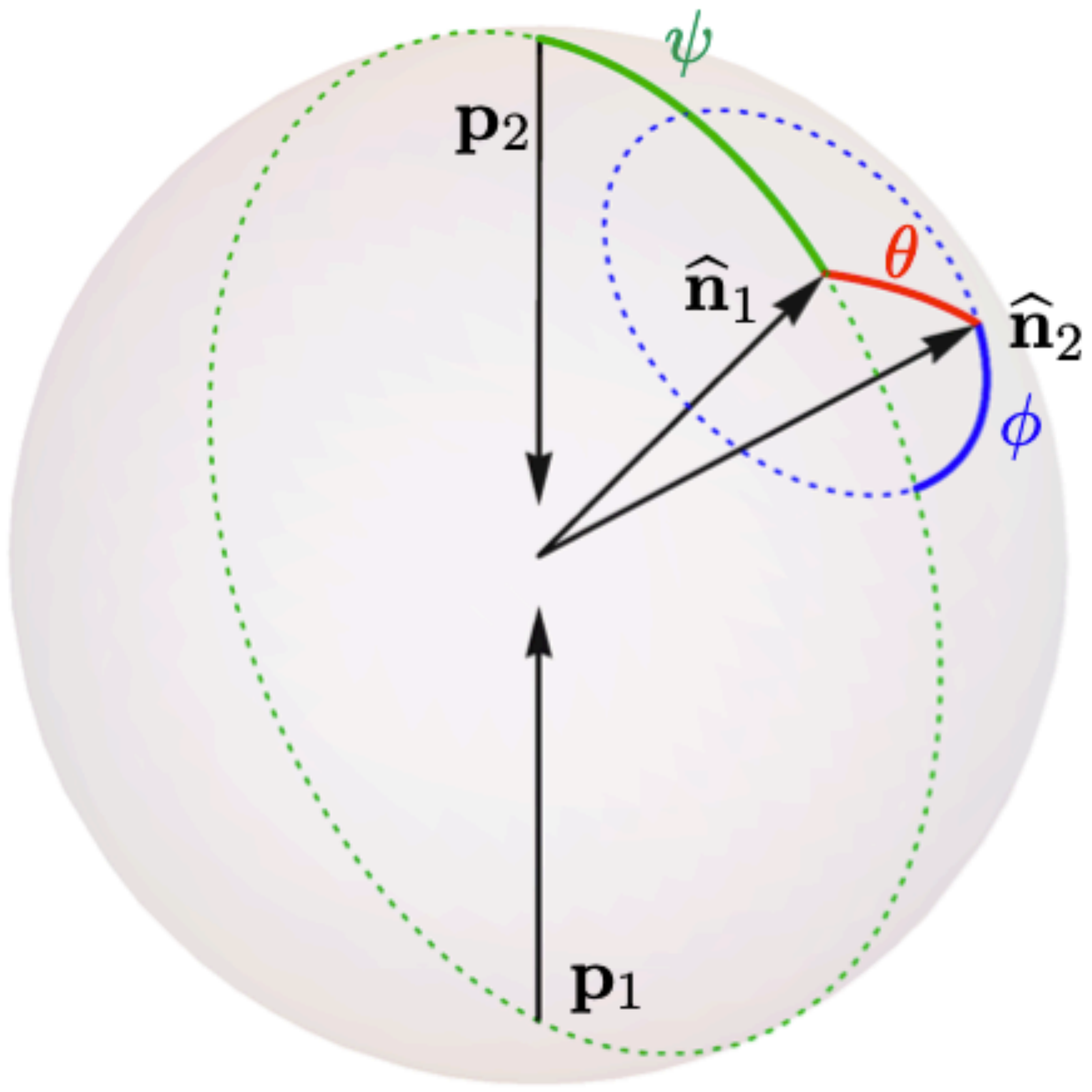
$$f^{(2)}(\zeta,\chi_1,\chi_2) = \frac{\arctan\left[\frac{\chi_2-\chi_1+2\zeta}{\sqrt{\Delta}}\right] - \arctan\left[\frac{\chi_2-\chi_1-2\zeta}{\sqrt{\Delta}}\right]}{\sqrt{\Delta}}\,,$$

$$f^{(3)}(\zeta,\chi_1,\chi_2) = \log(1-\chi_1)\,, \qquad f^{(4)}(\zeta,\chi_1,\chi_2) = \log(1+\chi_1)\,,$$

$$f^{(5)}(\zeta,\chi_1,\chi_2) = \log(1+\chi_2)\,, \qquad f^{(6)}(\zeta,\chi_1,\chi_2) = \log(1-\chi_2)\,,$$

$$f^{(7)}(\zeta,\chi_1,\chi_2) = \log(1-\zeta)\,.$$

$$\Delta = 4\zeta(1-\zeta) + 2\chi_1\chi_2(1-2\zeta) - \chi_1^2 - \chi_2^2\,.$$

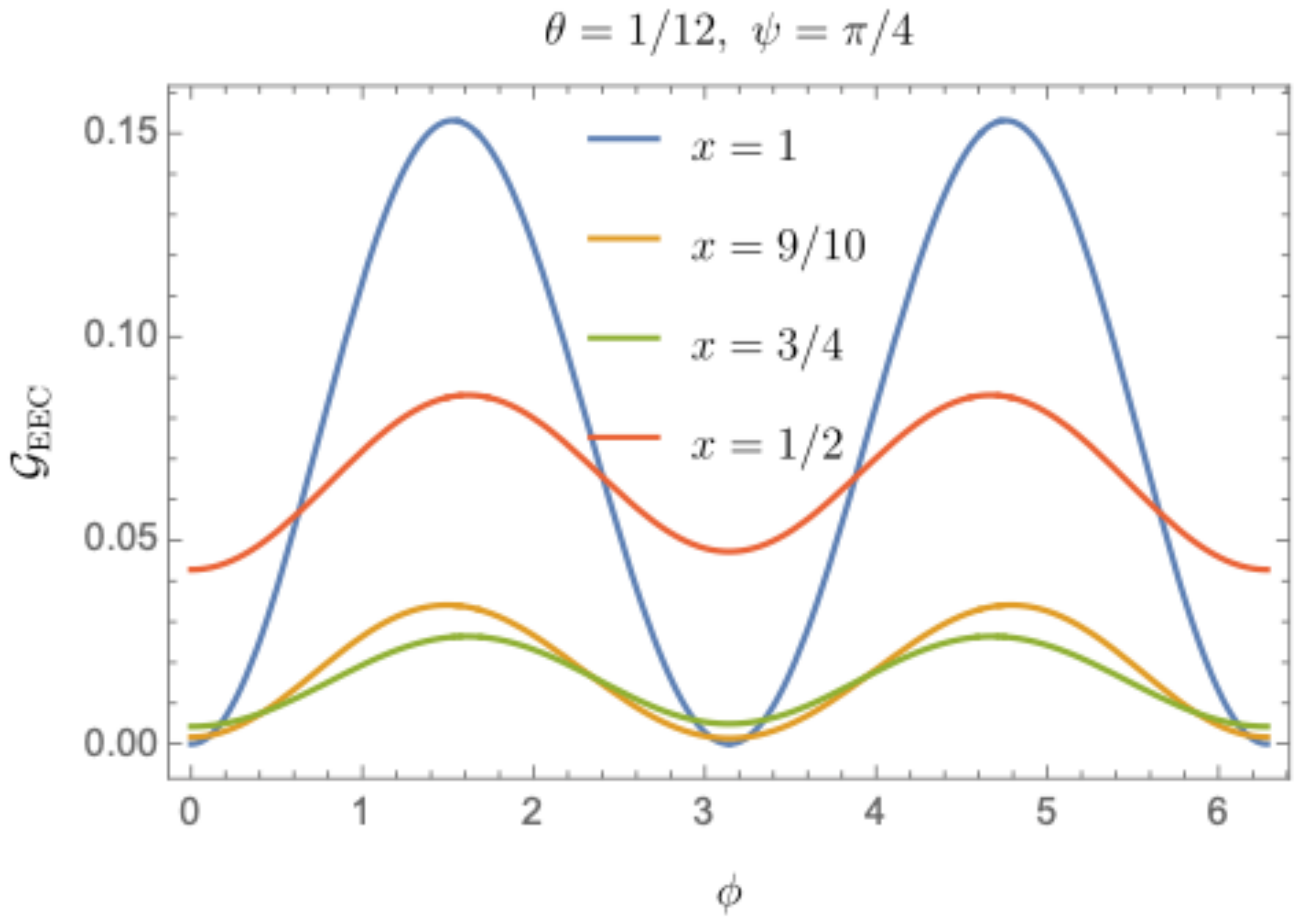


$$\zeta = \frac{1}{2}\left[1-\cos\theta\right]\,, \quad \chi_1 = x\cos\psi\,, \quad \chi_2 = x\left[\cos\theta\cos\psi - \cos\phi\sin\theta\sin\psi\right]$$

Scalar minimally coupled with (EH) gravity

Collinear limit (massless) $\theta \to 0$

$$\mathcal{G}^{(1)}_{\text{EEC}}(\theta \to 0, \psi, \phi, x = 1) = \frac{11}{18} \sin^4(\psi) \sin^2(\phi) + \mathcal{O}(\theta)\,.$$



Finite correlator in this limit!!

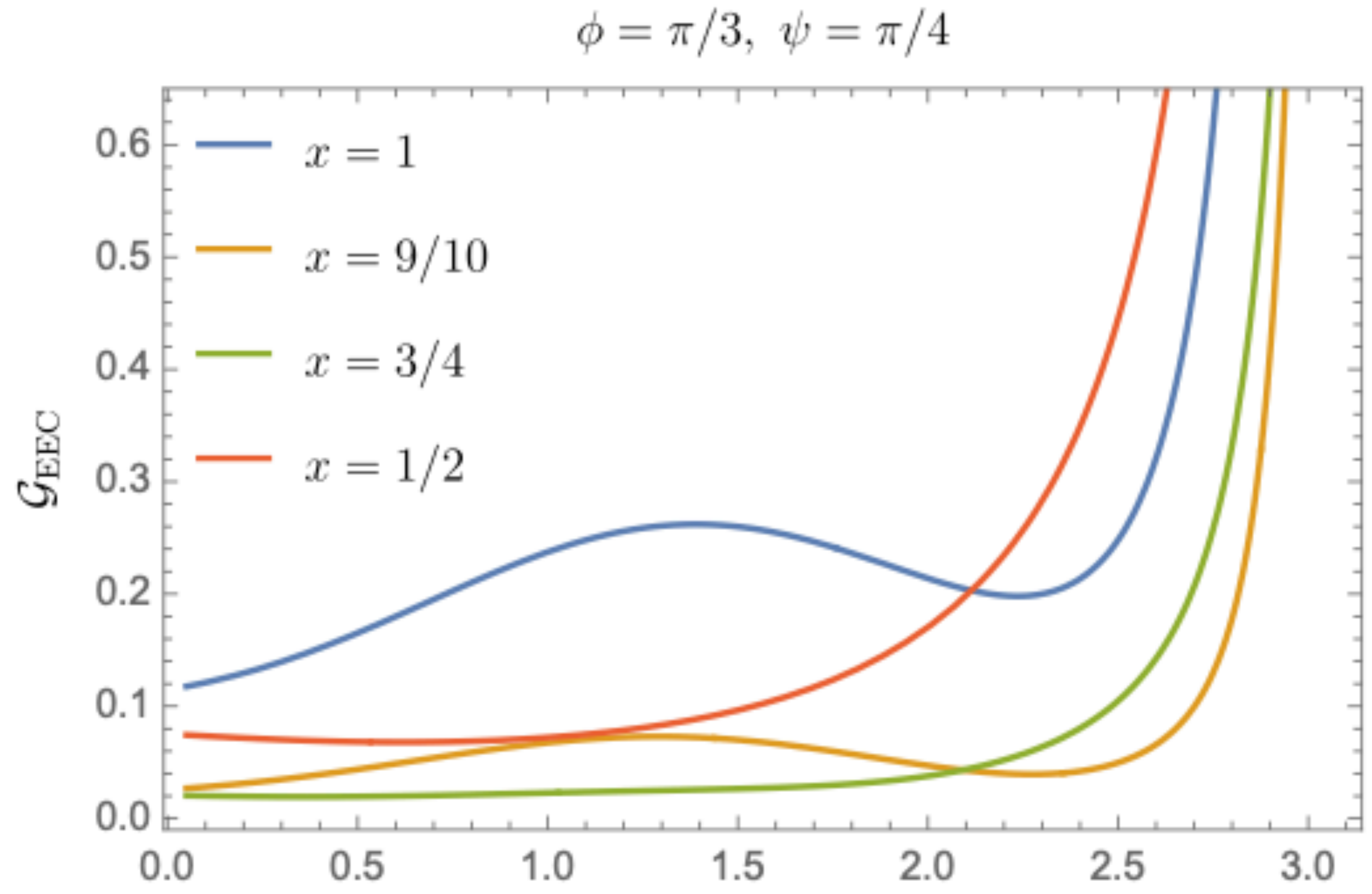
Different from Gauge theories ($\sim \theta^{-1}$), but in agreement with amplitude factorization

Scalar minimally coupled with (EH) gravity

Back to back (massless) $\theta \to \pi$

$$\mathcal{G}^{(1)}_{\rm EEC}(\theta \to \pi, \psi, \phi, x) \sim \frac{b(\psi,\phi,x)}{(\theta-\pi)^2} + \mathcal{O}\left(\frac{1}{(\theta-\pi)}\right) ,$$

$$b(\psi,\phi,x=1) = \frac{s_\psi^4 \left(2c_\psi \,\text{arctanh}\,(c_\psi)\, s_\phi^2 + s_{2\phi}\, \text{arctan}(\cot\phi)\right)}{c_\phi^2 + \cot^2\psi} .$$



Pole in this limit!!

Different from Gauge theories (double log governed by cusp anomalous dimension), but in agreement with amplitude factorization

Conclusion and future

We have perturbative setting for energy correlators in PQG and first example

What's next?

OPE of detector operators in gravity

The space of detector operators in (quantum) gravity

Perturbative structure of multi-point correlators in supergravity

Asymptotic symmetries and their interplay with detector operators

Detectors in nontrivial spacetimes and nonperturbative effects in quantum gravity

Conformal collider bounds in gravitational theories

Detector operators in flat space from the flat space limit of AdS/CFT

This is only my (personal) selection…

Thank You