Hidden Symmetries of 4D $\mathcal{N} = 2$ Gauge Theories

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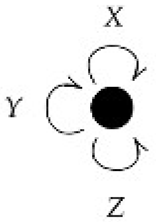
Main Points

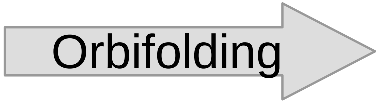
We study the global symmetries of the $\mathbb{Z}_2$-orbifold of $\mathcal{N} = 4$ Super-Yang-Mills theory and its marginal deformations. The process of orbifolding to obtain an $\mathcal{N} = 2$ theory would appear to break the SU(4) R-symmetry down to $SU(2) \times SU(2) \times U(1)$. We show that the broken generators can be recovered by moving beyond the Lie algebraic setting to that of a Lie algebroid. This remains true when marginally deforming away from the orbifold point by allowing the couplings

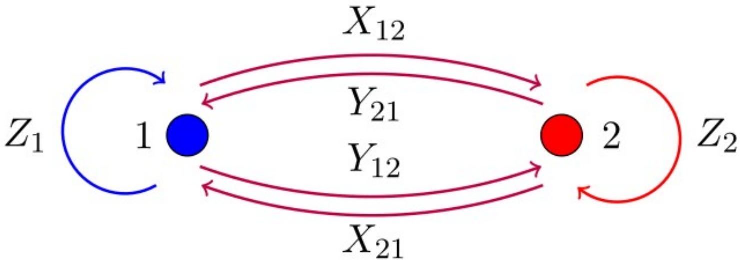
Results

- At the orbifold point define the “co-product” of all su(4) generators as Lie Algebroid.
- Show that the Lagrangian is invariant under the action of these R-symmetry generators.
- Use F-terms of marginally deformed theory to define a Drienfeld-like twist and deform the Lie Algebroid with it.
- Define coassociator and show that the deformed generators also leaves the deformed Lagrangian (scalar) invariant.
- Implications on the spectrum of the theory…

notions of symmetry. In particular, abandoning the requirement that all group operations can be composed leads to the concept of a groupoid (see [4, 5] for reviews).



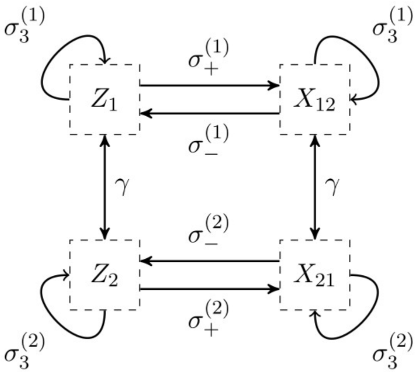




Gauge Group	SU(2N)		SU(N)xSU(N)
R-Symmetry	SU(4)		SU(2)xSU(2)xU(1)

$$X = \begin{pmatrix} 0 & X_{12} \\ X_{21} & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & Y_{12} \\ Y_{21} & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} Z_1 & 0 \\ 0 & Z_2 \end{pmatrix}$$

Broken Symmetries
In the Orbifold point
$g_1 = g_2$



$$\sigma_-^{(1)}(X_{12}) = Z_1 \ , \qquad \sigma_+^{(1)}(Z_1) = X_{12} \ ,$$

Broken and Unbroken Generators of su(4)

$$R_{(11)} = R_{(22)} = \{\mathcal{R}^1{}_1, \mathcal{R}^1{}_2, \mathcal{R}^2{}_1, \mathcal{R}^2{}_2, \mathcal{R}^3{}_3, \mathcal{R}^3{}_4, \mathcal{R}^4{}_3, \mathcal{R}^4{}_4\} \ ,$$
$$R^+_{(12)} = R^+_{(21)} = \{\mathcal{R}^3{}_1, \mathcal{R}^3{}_2, \mathcal{R}^1{}_4, \mathcal{R}^2{}_4\} \ ,$$
$$R^-_{(12)} = R^-_{(21)} = \{\mathcal{R}^1{}_3, \mathcal{R}^2{}_3, \mathcal{R}^4{}_1, \mathcal{R}^4{}_2\} \ .$$

$$[\mathcal{R}^a{}_b, \mathcal{R}^c{}_d] = \delta^c_b \mathcal{R}^a{}_d - \delta^a_d \mathcal{R}^c{}_b \ .$$

Algebroid must obey graded structure

$$[(\text{unbroken}), (\text{unbroken})] = (\text{unbroken}),$$
$$[(\text{broken}), (\text{unbroken})] = (\text{broken}),$$
$$[(\text{broken}), (\text{broken})] = (\text{unbroken}) \ .$$

We define a coproduct that extend the action of broken/unbroken generators to

$$\mathcal{V}_{11} = \left\{ \{Z_1, \bar{Z}_1\}, \{X_{12}X_{21}, Z_1Z_1, \cdots\}, \{X_{12}X_{21}Z_1, Z_1Z_1Z_1, \cdots\}, \cdots \right\}, \tag{3.2}$$

$$\mathcal{V}_{12} = \left\{ \{X_{12}, \bar{X}_{12}, Y_{12}, \bar{Y}_{12}\}, \{X_{12}Z_2, Z_1X_{12} \cdots\}, \{X_{12}X_{21}X_{12}, Z_1X_{12}Z_2, \cdots\}, \cdots \right\},$$

$$\Delta_\circ(\mathcal{R}^a{}_b) := \mathbb{1} \otimes \mathcal{R}^a{}_b + \mathcal{R}^a{}_b \otimes \Omega^a{}_b,$$

γ : 1 ↔ 2

$$\Omega^a{}_b = \begin{cases} \mathbb{1}, & \text{if } \mathcal{R}^a{}_b \text{ is unbroken} \\ \gamma, & \text{if } \mathcal{R}^a{}_b \text{ is broken} \end{cases} .$$

$$\Delta_\circ^{(L)}(\mathcal{R}^a{}_b) = \sum_{\ell=1}^{L} \Big(\mathbb{1} \otimes \cdots \otimes \mathbb{1} \otimes \overset{\ell}{\mathcal{R}^a{}_b} \otimes \Omega^a{}_b \otimes \cdots \otimes \Omega^a{}_b \Big)$$

$$\mathcal{R}^3{}_2 \triangleright \frac{1}{g} |\mathcal{W}_1\rangle_\circ = 0$$

Results: $\mathcal{R}^a{}_b \triangleright |\mathcal{L}_{K,1}\rangle_\circ = X_{12}\bar{X}_{21}$ +.. $= 0$

.

$$\mathcal{R}^a{}_b \triangleright |\mathcal{V}(g,g)\rangle = 0$$

$$\mathcal{F} = \kappa^{-\frac{s}{2}} \otimes \kappa^{-\frac{s}{2}} \ , \tag{5.1}$$

where we have introduced the $\mathbb{Z}_2$ element s, whose definition on site ℓ is

$$s(\ell) = \begin{cases} 1 & \text{if the first gauge index on site } \ell \text{ is } 1 \\ \text{-1} & \text{if the first gauge index on site } \ell \text{ is } 2 \end{cases} \ . \tag{5.2}$$

Since the $\mathbb{Z}_2$ generator γ flips both gauge indices at a given site, s does not commute with γ but rather we have

$$s\gamma = -\gamma s \ . \tag{5.3}$$

It is easy to check that this twist correctly leads to the F-term XZ quantum plane:

$$\mathcal{F} \triangleright (X_{12}Z_2 - Z_1X_{12}) = X_{12}Z_2 - \frac{1}{\kappa}Z_1X_{12} \ , \tag{5.4}$$

while also giving $\mathcal{F} \triangleright (X_{12}X_{21}) = X_{12}X_{21}$ and $\mathcal{F} \triangleright (Z_1Z_1) = \kappa^{-1}\ Z_1Z_1$.

Turning to the coproduct of the broken generators, writing $\sigma^+ = \mathcal{R}^3{}_2$ and $\sigma^- = \mathcal{R}^2{}_3$ we have

$$\begin{aligned} \Delta_\kappa(\sigma^\pm) &= \mathcal{F}_{12}\Delta_\circ(\sigma^\pm)\mathcal{F}_{12}^{-1} = \kappa^{-\frac{s}{2}} \otimes \kappa^{-\frac{s}{2}}(\mathbb{1} \otimes \sigma^\pm + \sigma^\pm \otimes \gamma)\kappa^{\frac{s}{2}} \otimes \kappa^{\frac{s}{2}} \\ &= \mathbb{1} \otimes \sigma^\pm + \sigma^\pm \otimes \kappa^{-\frac{s}{2}}\gamma\kappa^{\frac{s}{2}} = \mathbb{1} \otimes \sigma^\pm + \sigma^\pm \otimes \gamma\kappa^s \\ &= \mathbb{1} \otimes \sigma^\pm + \sigma^\pm \otimes K, \end{aligned} \tag{5.5}$$

Spectrum and Future Directions

		primary of
$(\mathbf{1},\mathbf{1})_2$	$\|0,0,+2\rangle$	$\mathcal{E}_{2(0,0)}$
$(\mathbf{1},\mathbf{1})_{-2}$	$\|0,0,-2\rangle$	$\bar{\mathcal{E}}_{-2(0,0)}$
$(\mathbf{1},\mathbf{1})_0$	$\|0,0,0\rangle$	$\hat{\mathcal{C}}_{0(0,0)}$
$(\mathbf{2},\mathbf{2})_1$	$\|\pm\frac{1}{2},\pm\frac{1}{2},+1\rangle$	$\mathcal{D}^{(\pm\frac{1}{2})}_{\frac{1}{2}(0,0)}$
$(\mathbf{2},\mathbf{2})_{-1}$	$\|\pm\frac{1}{2},\pm\frac{1}{2},-1\rangle$	$\bar{\mathcal{D}}^{(\pm\frac{1}{2})}_{-\frac{1}{2}(0,0)}$
$(\mathbf{3},\mathbf{3})_0$	$\|\pm 1,\pm 1,0\rangle, \|0,0,0\rangle$	$\mathcal{B}_1$

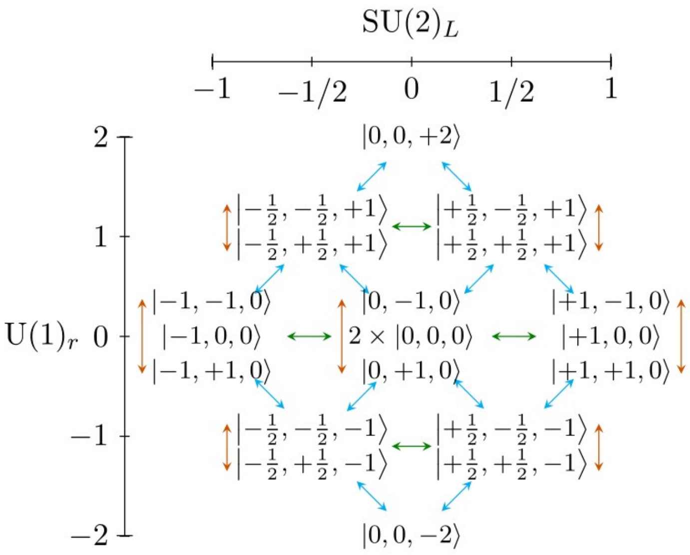


Figure 5: *Depiction of the open* $\mathbf{20}'$ *multiplet, with the action of the broken R-symmetry generators as dotted blue arrows and the unbroken* $\mathrm{SU}(2)_L$ *as solid green arrows. The states present at each node of this diagram are connected via the action of the unbroken* $\mathrm{SU}(2)_R$.

For future, fill the gaps and generalize to $\mathbb{Z}_N$ orbifolds...