Strings from Feynman Diagrams

Rajesh Gopakumar[Φ], Rishabh Kaushik[S], Shota Komatsu[χ] ,
Edward A. Mazenc[ψ] & Debmalya Sarkar[S†]

[Φ,S,S†] *International Centre for Theoretical Sciences-TIFR,*
Shivakote, Hesaraghatta Hobli, Bengaluru North 560089, India.

[χ] *CERN, Theoretical Physics Department,*
CH-1211 Geneva 23, Switzerland.

[ψ] *Institut für Theoretische Physik, ETH Zürich,*
CH-8093 Zürich, Switzerland.

[S†] *Jefferson Physical Laboratory, Harvard University,*
Cambridge, MA 02138, USA.

String Journal Club, 4.2.2025

Till Bargheer

General Setting: Gauge/String duality

1. How does it work microscopically?

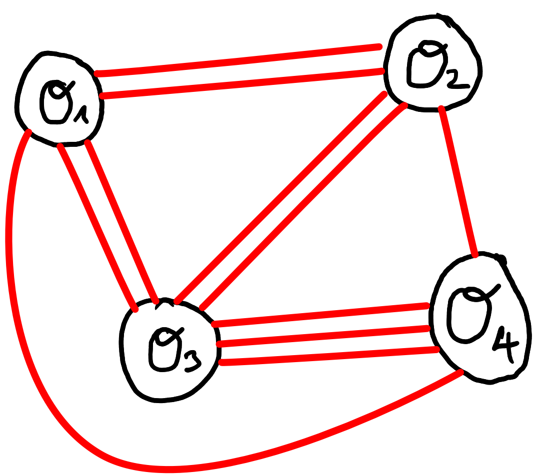
2. Answer to (1) ~ Proof of the duality

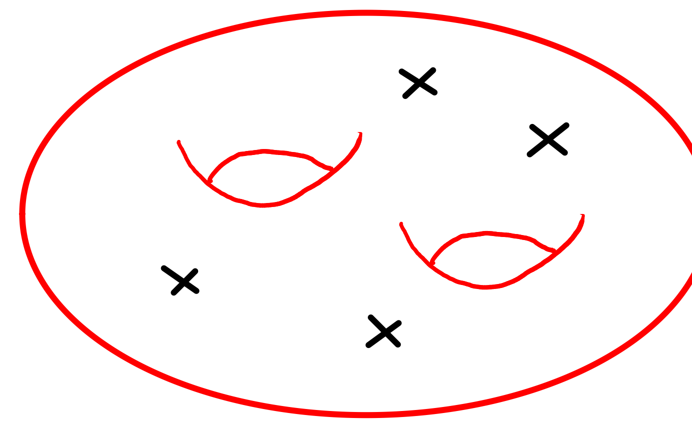
Central Question

Gauge Theory Correlators ⟷ ? ⟷ **String amplitudes**

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \sum_{\text{graphs}} \prod_{\text{propagators}} \frac{1}{x_{ij}^2}$$

$$\mathcal{A}_n \sim \sum_{\text{genus } g} \int_{\mathcal{M}_{g,n}} e^{-S}$$



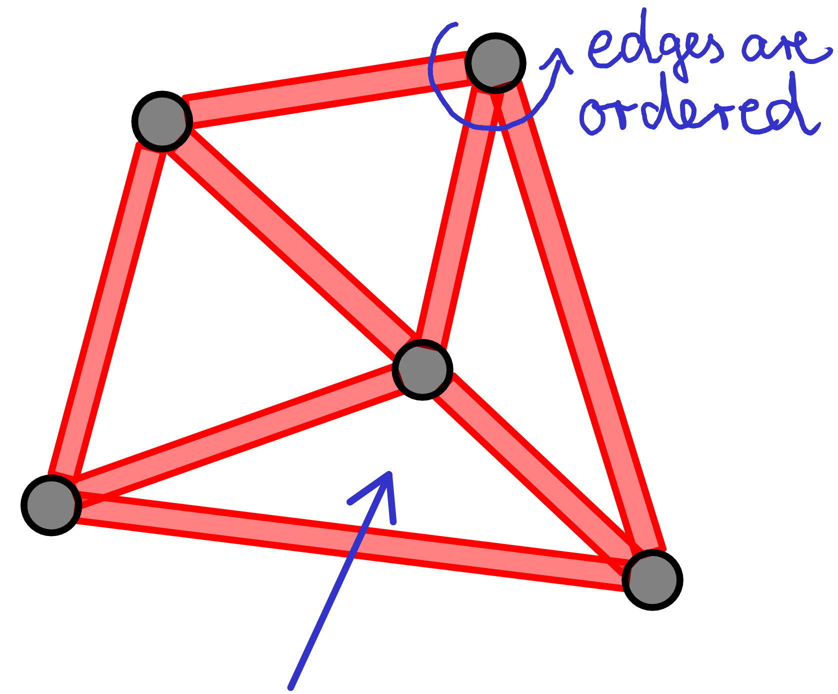


$\lambda = 0$ ⟷ $\lambda = \infty$

Central Concept

Ribbon Graph:

- regular graph
- edges are strips/ribbons
- ordering of edges @ vertices
- defines oriented surface



Can associate a disc to each face
--> defines a surface

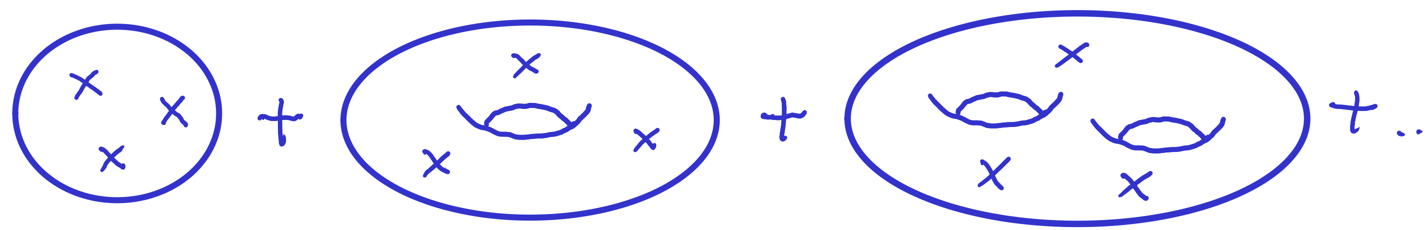
Gauge Theory

Gauge theory graphs at large N are naturally ribbon graphs:

- Adjoint field propagators (double lines) are ribbons.
- Faces are color traces (each face = factor N).
- Single-trace operators are vertices (ordering from trace).

→ 't Hooft expansion (1974):

$\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle =$



String Theory?

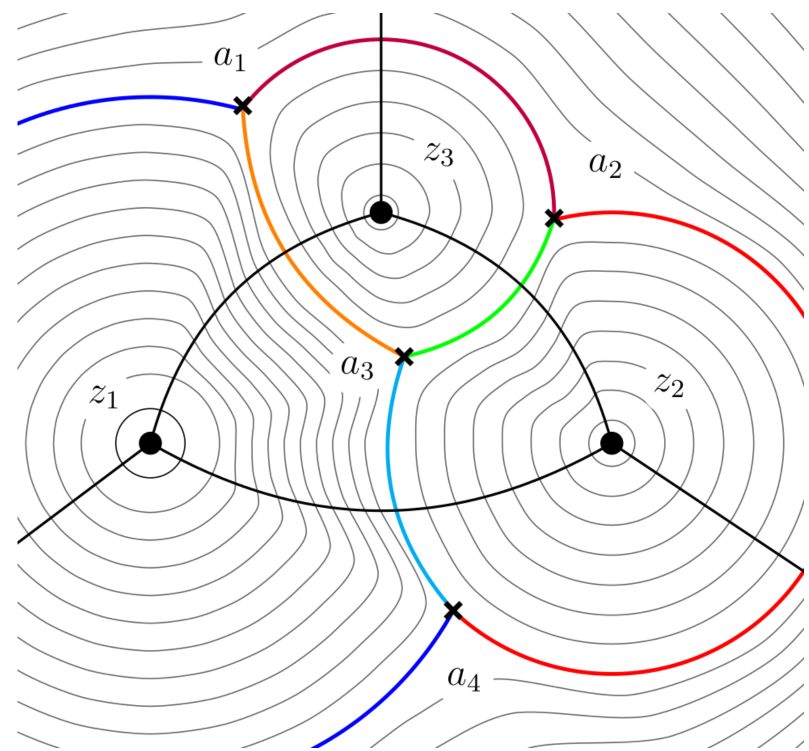
Recall:

$$\mathcal{A}_n \sim \sum_g \int_{\mathcal{M}_{g,n}}$$

Each point in the moduli space M(g,n) =
Riemann surface (genus, punctures, complex structure)

There are coordinates on M(g,n) in terms of ribbon graphs! (Strebel 1984)

- Given a Riemann surface, can define a unique quadratic differential on it
- Has quadratic poles (punctures)
- Around each pole, differential defines:
  - Vertical trajectories: Connect different punctures
  - Horizontal trajectories: Closed contours around punctures
- Horizontal trajectories of neighboring punctures separated by critical graph = "Strebel graph"



Strebel graph = Metric ribbon graph

Gauge vs String

One-to-one map:

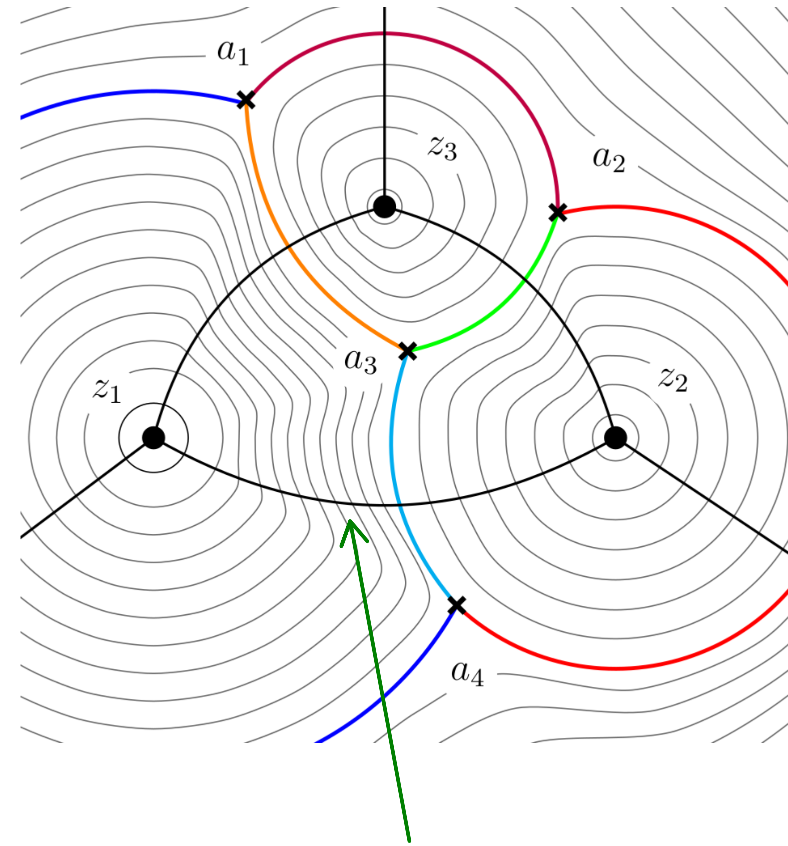
Point in M(g,n) $\longleftrightarrow$ Metric ribbon graph (n faces)

$$\int_{\mathcal{M}_{g,n}} e^{-S} = \sum_{\substack{\text{Strebel}\\ \text{graphs}}} \int_{\text{lengths}} e^{-S}$$

$$\stackrel{?}{=} \sum_{\text{graphs}} \prod_{\text{propagators}} \frac{1}{x_{ij}^2} = \langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle$$

Punctures are insertion points of external strings

$\longrightarrow$ Horizontal trajectories are lines of constant τ



Gauge theory graphs are dual to Strebel graph.

Idea: Each propagator must map to strip of worldsheet

$\longrightarrow$ Strebel lengths $\simeq$ # of crossing propagators

How can this work?

$$\int_{\mathcal{M}_{g,n}} e^{-S} = \sum_{\substack{\text{Strebel} \\ \text{graphs}}} \int_{\text{lengths}} e^{-S} \stackrel{?}{=} \sum_{\text{graphs}} \prod_{\text{propagators}} \frac{1}{x_{ij}^2} = \langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle$$

Idea: Each propagator must map to strip of worldsheet

→ Strebel lengths $\simeq$ # of crossing propagators

→ String path integral must localize on integer Strebel lengths in the deep quantum regime $\lambda \approx 0$

→ String action e^{-S} must become product of propagators

→ Gauge theory loops must "diffuse" the moduli space discretization

This paper

$$\left\langle \prod_{i=1}^{n_1} \frac{1}{k_i} \mathrm{Tr}\Big[\left(Y_1 \cdot \Phi(w_1)\right)^{k_i} \Big] \prod_{j=1}^{n_2} \frac{1}{l_j} \mathrm{Tr}\Big[\left(Y_2 \cdot \Phi(w_2)\right)^{l_j} \Big] \right\rangle^{c}_{\mathcal{N}=4}$$

N=4 SYM
1/2 BPS operators

$$=$$

$$\sum_{g\geq 0} N^{2-2g-(n_1+n_2)} \sum_{\substack{\text{Feyn. Diags} \\ \text{of genus g}}} \frac{1}{\mathrm{Aut(Diag)}} \left[\frac{\lambda Y_1 \cdot Y_2}{|w_1 - w_2|^2} \right]^{\#Edges}$$

Takeaway #1

$$g_{st} = \frac{1}{N}$$

Takeaway #2

Each FD = a Specific Worldsheet

Takeaway #3

Weight of FD $\leftrightarrow$ Action of String

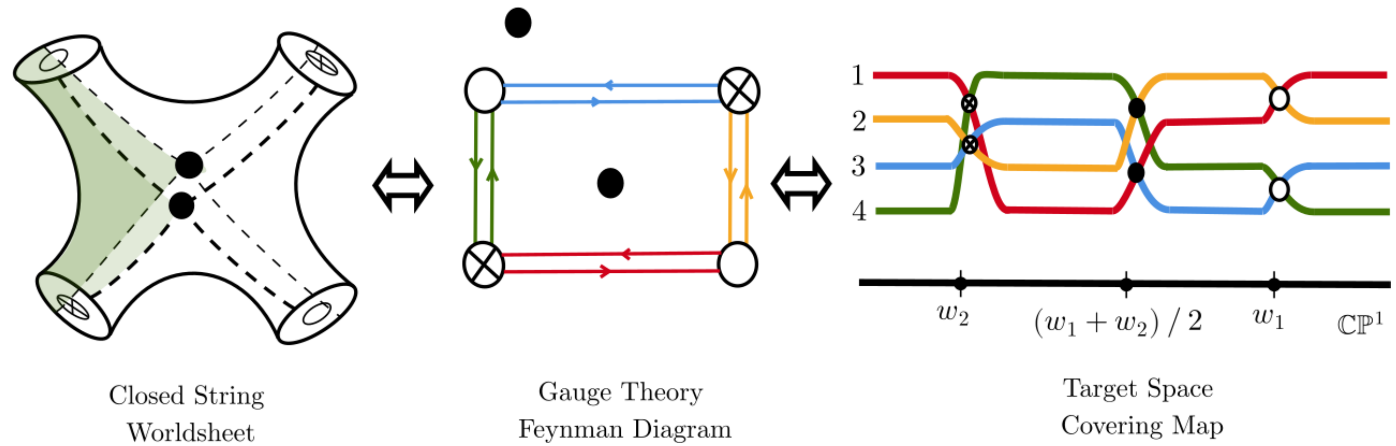
$$\sum_{g\geq 0} g_{st}^{2g-2+(n_1+n_2)} \int_{\mathcal{M}_{g,\, n_1+n_2}} \sum_{\substack{\mathcal{N}=4 \\ \text{genus g Feyn. Diags}}} \delta\begin{pmatrix} \text{Strebel Graph of} \\ \text{Worldsheet} \end{pmatrix} = \begin{pmatrix} \text{Dual Skeleton Graph of} \\ \text{Feyn.Diag} \end{pmatrix} e^{-S_{NG}[X_{FD}]}$$

Earlier papers:

- Gopakumar: From Free Fields to AdS I, II, III (2003-2005)
- Razamat: On a worldsheet dual of the Gaussian matrix model (2008)
- Eberhardt, Gaberdiel, Gopakumar: The worldsheet dual to the symmetric product CFT (2018)

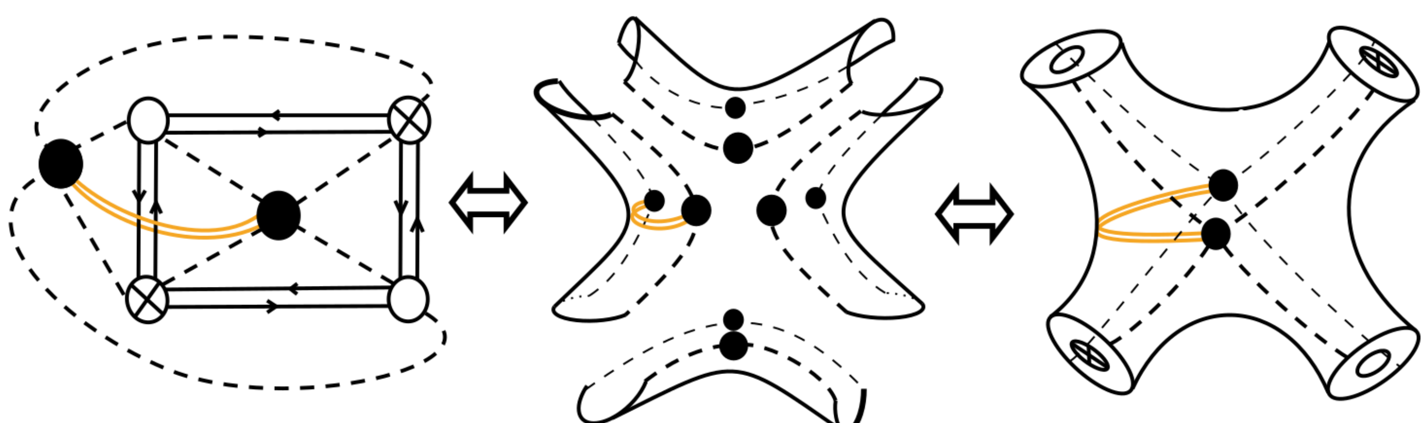
Feynman diagram --> branched covering X of Riemann sphere (Belyi map)
= embedding of worldsheet in target space (CP1 on AdS boundary)

String action: Nambu-Goto action S(X)



Discrete moduli space: Belyi maps only exist on discrete points (Belyi 1980)
--> string path integral must localize on such branched coverings

Worldsheet reconstructed microscopically:
Each propagator becomes a strip of worldsheet



Comments

No explicit worldsheet theory that localizes on Belyi maps,

But 2 papers announced: Gaberdiel/Knighton/Naderi & Komatsu/Maity

What about interactions?

Hexagon form factors inserted into graph faces