THERMODINAMIC OF INTEGRABLE $\mathcal{N}=2$ THEORIES, SQUARED

BASED ON 2502.10356

WITH X. KERVYN & A. SFONDRINI

SUPERSTRINGS PROPAGATING IN $AdS_3 \times S^3 \times T^4$ ARE CLASSICALLY INTEGRABLE [A. CAGNAZZO, K. ZAREMBO '12]

$$S = \int d\tau \, d\sigma \left[-\frac{T}{2}\sqrt{-\gamma}\,\gamma^{ab} G_{\mu\nu} + \frac{K}{2\pi} \varepsilon^{ab} B_{\mu\nu} \right] \partial_a X^\mu \partial_b X^\nu$$

$+$ FERMIONS

THERE IS A FAMILY OF SUCH THEORIES SPANNED BY

$T \geq 0$ STRING TENSION

$K = 0, 1, 2$ QUANTISED AMOUNT OF B FIELD

$$\left(psu(1|1)_L \oplus psu(1|1)_R\right)^{\oplus 2}_{c.e}$$

SUSY
ALGEBRA

$$\{q, s\} = \frac{1}{2}(E+M) \qquad \{\tilde{q}, \tilde{s}\} = \frac{1}{2}(E-M)$$

$$\{q, \tilde{q}\} = C \qquad \{s, \tilde{s}\} = C^{\dagger}$$

CENTRAL
EXTENSION

Φ_m : BOSON

Ψ_m : FERMION

$$\begin{tikzcd} & \Phi_m \arrow[dl, shift right, "q, \tilde{S}"'] \\ \Psi_m \arrow[ur, shift right, "\tilde{q}, S"'] & \end{tikzcd}$$

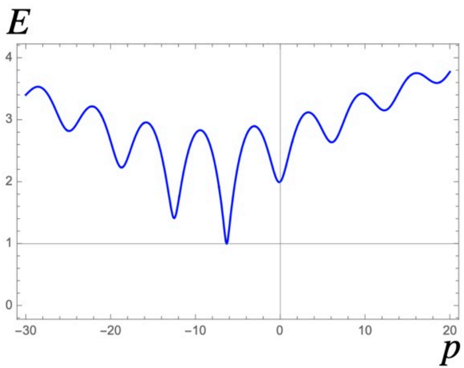
$$E_m(p) = \sqrt{\left(m + \frac{K}{2\pi} p\right)^2 + 4h^2 \sin^2\left(\frac{p}{2}\right)}$$

$m = 1, 2, \ldots$ $\qquad K \in \mathbb{N}$ $\qquad h \geq 0$ ($\sim$ 't HOOFT COUPLING)

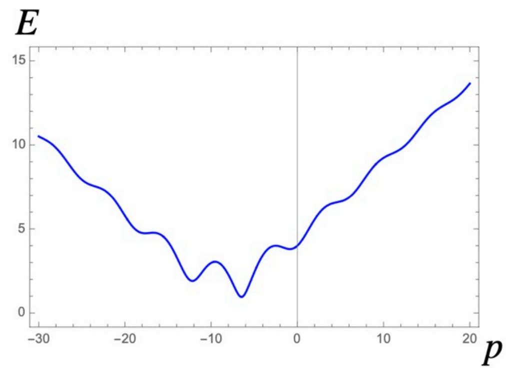
$$\oplus 2 \implies
\begin{tikzcd}
 & \Phi_m \otimes \Phi_m \arrow[dl, shift right, "{q^1,\ \tilde{S}^1}"'] \arrow[dr, shift left, "{q^2,\ \tilde{S}^2}"] & \\
\Psi_m \otimes \Phi_m \arrow[ur, shift right, "{\tilde{q}^1,\ S^1}"'] \arrow[dr, shift right] & & \Phi_m \otimes \Psi_m \arrow[ul, shift left, "{\tilde{q}^2,\ S^2}"] \arrow[dl, shift left] \\
 & \Psi_m \otimes \Psi_m \arrow[ul, shift right] \arrow[ur, shift left] &
\end{tikzcd}$$

$$q^1 = q \otimes \mathbb{1}, \quad q^2 = \mathbb{1} \otimes q, \quad \tilde{q}^1 = \tilde{q} \otimes \mathbb{1} \ \ldots$$

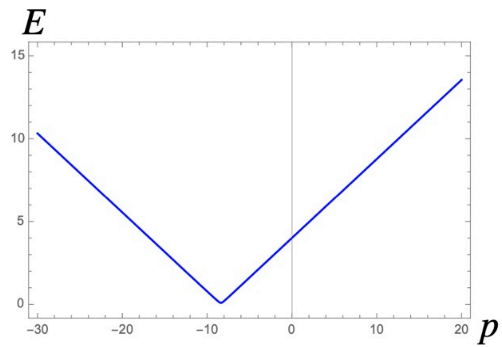
$$E_m(p) = \sqrt{\left(m + \frac{K}{2\pi}\,p\right)^2 + 4h^2 \sin^2\left(\frac{p}{2}\right)}$$



$h = 4$

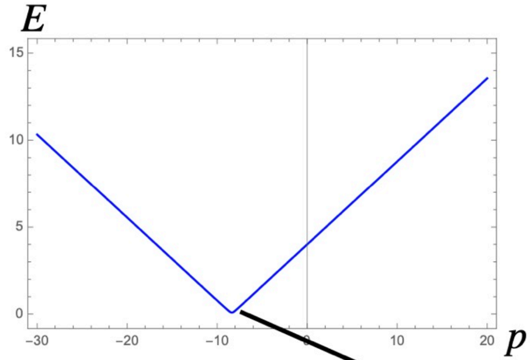


$h = 1.5$



$h = 0.05$

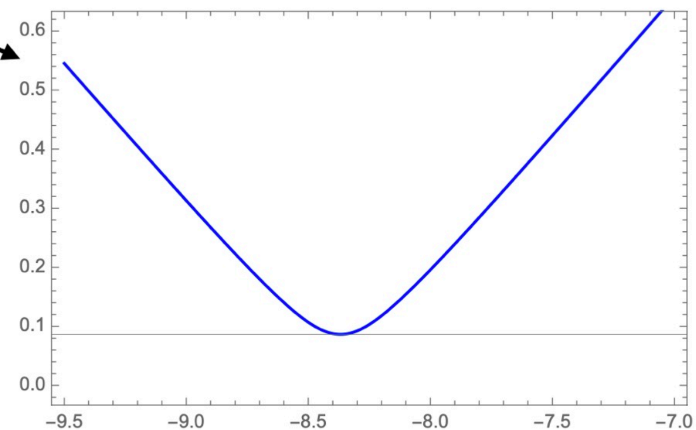
IF $h \ll 1 \Rightarrow p_{MIN} \simeq -\frac{2\pi}{K}\, m$



$$h = 0.05$$

Zoom in

$$p = -\frac{2\pi}{k} m + hq$$



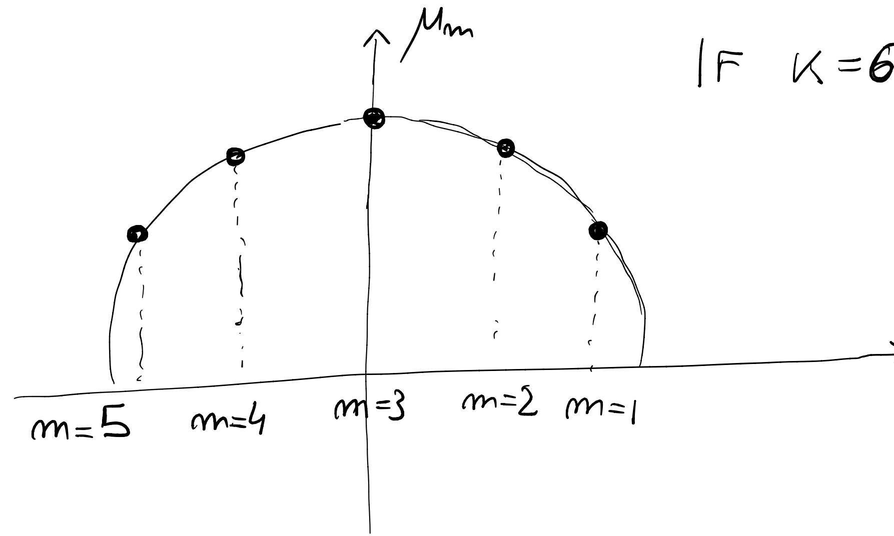
$$E_m(P) \longrightarrow \mu_m \, \mathrm{ch}\,\theta$$

θ: RAPIDITY OF A RELATIVISTIC THEORY

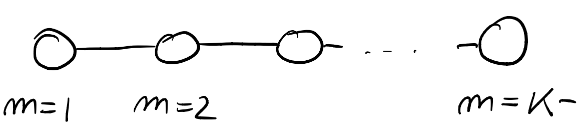
$$\mu_m = 2h \sin\left(\frac{\pi}{K} m\right)$$

$$m = 1, 2, \ldots, K-1$$

IF $K=6$



SPECTRUM OF INTEGRABLE TODA THEORIES OF A_{K-1} TYPE



ZF ALGEBRA

$\phi_m = (1, 0) \qquad \psi_m = (0, 1)$

$$S_{m_1 m_2}(\theta) = f_{m_1 m_2}(\theta) \begin{pmatrix} A(\theta) & 0 & 0 & 0 \\ 0 & C(\theta) & D(\theta) & 0 \\ 0 & B(\theta) & E(\theta) & 0 \\ 0 & 0 & 0 & F(\theta) \end{pmatrix}$$

DRESSING FACTOR

$$A(\theta) = 1, \quad B(\theta) = \frac{\operatorname{sh}\left(\frac{\theta}{2} - \frac{i\pi}{2K}(m_1 - m_2)\right)}{\operatorname{sh}\left(\frac{\theta}{2} + \frac{i\pi}{2K}(m_1 + m_2)\right)}, \quad \ldots$$

$$f_{m_1 m_2}(\theta) = \frac{R\left(\theta - \frac{i\pi}{\kappa}(m_1+m_2)\right) R\left(\theta + \frac{i\pi}{\kappa}(m_1+m_2)\right)}{R\left(\theta - \frac{i\pi}{\kappa}(m_1-m_2)\right) R\left(\theta + \frac{i\pi}{\kappa}(m_1-m_2)\right)}$$

$$R(\theta) = \frac{G\left(1 - \frac{\theta}{2\pi i}\right)}{G\left(1 + \frac{\theta}{2\pi i}\right)}$$

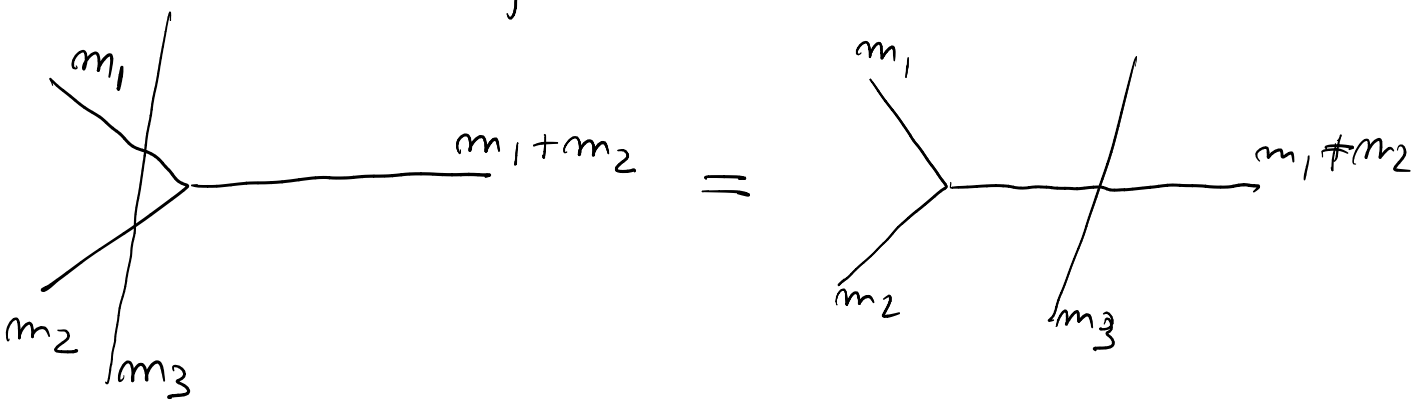
BARNES GAMMA FUNCTIONS

$+$ CDD

f SATISFIES ALL THE PROPERTIES OF THE BOOTSTRAP

- $$f_{m_1 m_2}(\theta)\, f_{m_2 m_1}(-\theta) = 1$$

- $$f_{m_1 m_3}\left(\theta - \frac{i\pi}{K} m_2\right) f_{m_2 m_3}\left(\theta + \frac{i\pi}{K} m_1\right) = f_{m_1+m_2, m_3}(\theta)$$



WHAT WE DID IN THE PAPER

- PUT THE SYSTEM AT FINITE VOLUME AND DERIVE ITS TBA EQUATIONS AND Y SYSTEM.

  Y_p $\quad p = 1, \dots, K-1.$ $\quad$ PHYSICAL Y FUNCTIONS

  Y_a $\quad a = \uparrow, \downarrow.$ $\quad$ AUXILIARY Y FUNCTIONS

- SEARCH FOR CONSTANT SOLUTIONS TO Y SYSTEM
  ($\Rightarrow$ UV CENTRAL CHARGE)

IF SPECTRUM $\Phi_p\,,\ \Psi_p \qquad p = 1, \dots, K-1$

$$Y_p^2 = \frac{(1+Y_{p+1})(1+Y_{p-1})}{\prod_{a=\uparrow,\downarrow}\left(1+\frac{1}{Y_a}\right)^{1\cdot I_{pa}}} \qquad I_{pa} = \delta_{p1}\,\delta_{a\uparrow} + \delta_{p,K-1}\,\delta_{a\downarrow}$$

$$Y_\uparrow^2 = (1+Y_1)^{-1}\left(1+\frac{1}{Y_\uparrow}\right)^{-1}\left(\frac{1+\frac{1}{Y_\uparrow}}{1+\frac{1}{Y_\downarrow}}\right)^{\frac{1}{K}}$$

$$Y_\downarrow^2 = (1+Y_{K-1})^{-1}\left(1+\frac{1}{Y_\downarrow}\right)^{-1}\left(\frac{1+\frac{1}{Y_\downarrow}}{1+\frac{1}{Y_\uparrow}}\right)^{\frac{1}{K}}$$

THEN

$$Y_P = \frac{\sin\left(\frac{(2P+3)\pi}{2(K+1)}\right)\sin\left(\frac{(2P-1)\pi}{2(K+1)}\right)}{\sin^2\left(\frac{\pi}{K+1}\right)}$$

$$Y_\uparrow = Y_\downarrow = \frac{\sin\left(\frac{\pi}{2(K+1)}\right)}{\sin\left(\frac{3\pi}{2(K+1)}\right)}$$

$$C = 3\,\frac{(K-1)}{(K+1)}$$

A_{K-1} TYPE $N=2$ MINIMAL MODELS PERTURBED BY MOST RELEVANT OPERATOR

[P. FENDLEY, K. INTRIGILATOR '92]

IF SPECTRUM $\quad \Phi_P \otimes \Phi_P, \; \Phi_P \otimes \Psi_P, \; \Psi_P \otimes \Phi_P, \; \Psi_P \otimes \Psi_P$

$$Y_p^2 = \frac{(1+Y_{p+1})(1+Y_{p-1})}{\prod_{a=\uparrow,\downarrow}\left(1+\frac{1}{Y_a}\right)^{2I_{pa}}}$$

$$I_{pa} = \delta_{p1}\,\delta_{a\uparrow} + \delta_{p,K-1}\,\delta_{a\downarrow}$$

$$Y_\uparrow^2 = (1+Y_1)^{-1}\left(1+\frac{1}{Y_\uparrow}\right)^{-2}\left(\frac{1+\frac{1}{Y_\uparrow}}{1+\frac{1}{Y_\downarrow}}\right)^{\frac{2}{K}}$$

$$Y_\downarrow^2 = (1+Y_{K-1})^{-1}\left(1+\frac{1}{Y_\downarrow}\right)^{-2}\left(\frac{1+\frac{1}{Y_\downarrow}}{1+\frac{1}{Y_\uparrow}}\right)^{\frac{2}{K}}$$

THEN

$$Y_p = \frac{\sin\left(\frac{\pi}{k}(p-1)\right)\sin\left(\frac{\pi}{k}(p+1)\right)}{\sin^2\frac{\pi}{k}} \qquad p = 1, \dots, k-1$$

$$Y_\uparrow = Y_\downarrow = 0$$

$$C = 6\left(1 - \frac{1}{k}\right)$$

WHAT DOES CORRESPOND TO ?

OUTLOOK AND REMARKS

- DOES THE TBA CONTAIN ANY INFORMATION ON THE THEORY BEFORE THE LIMIT ?

- FINITE VOLUME = DOUBLE WICK ROTATION

$$p \to i\tilde{E}$$

$$E \to i\tilde{p}$$

FOR A RELATIVISTIC THEORY: $E^2 - p^2 = m^2 \longrightarrow \tilde{E}^2 - \tilde{p}^2 = m^2$

$$\theta \longrightarrow \theta + \frac{i\pi}{2}$$

- S MATRIX OF DIFFERENCE FORM

$$\Rightarrow S(\theta_1 - \theta_2) \longrightarrow S(\theta_1 - \theta_2)$$

- IN THE FULL MODEL THE S-MATRIX IS NOT INVARIANT.

  DOES THE LIMIT COMMUTE WITH DOUBLE WICK ROTATION?

- CAN WE PERFORM THE SAME LIMIT ON SIMILAR MODELS AS

$$AdS_3 \times S^3 \times S^3 \times S^1 \quad ?$$

Thank you