# String Journal Club 18-02-2025


**Abstract**

Effective string theory describes the physics of long confining strings in theories, like Yang-Mills theory, where the mass gap $M_{gap}^2$ is of the same order as the string tension $T$. In $2+1$ dimensions, there is a class of confining theories, including massive $QED_3$ as first analyzed by Polyakov, for which $M_{gap}^2 \ll T$. These theories are weakly coupled at low energies of order $M_{gap}$, and may be analyzed perturbatively. In this paper, we analyze the physics of strings in such theories, focusing on $QED_3$, at energies of order $M_{gap}$ (but still well below $\sqrt{T}$). We argue that the width of the string in these theories should be of order $1/M_{gap}$ independently of its length, as long as the string is not exponentially long. We also compute at leading order in perturbation theory the ground state energy of a confining string on a circle, and the scattering of Nambu-Goldstone bosons on the string worldsheet.

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*1. QED3 and Effective String Theory*

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*1. QED3 and Effective String Theory* *2. The regime of interest*

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*1. QED3 and Effective String Theory* *2. The regime of interest* *3. Applications*

# QED in d=2+1 and topological symmetry

- d=2+1 QFTs are important: Wick rotation → d=3 lab. experiments!

- QED3: lab for confinement

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu}^2 + \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi$$

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$$m_e \gg e^2$$

*«Coulomb phase»: the theory is always weakly coupled*

- QED3 + heavy electrons: below the electron mass, only photons propagate.

# QED in d=2+1 and topological symmetry

- The EFT below the electron mass scale will be a theory of:

$$F_{\mu\nu} = e\epsilon_{\mu\nu\rho}\partial^\rho\phi.$$

- QED3 has a **topological symmetry** $U(1)_{top}$ as well:

$$J_\mu = \epsilon_{\mu\nu\rho}F^{\nu\rho} \qquad Q_{\rm top} = \int d^2x \; J^0_{\rm top} = \frac{1}{2\pi}\int d^2x \; B$$

- After integrating out the heavy electrons:

$$Z = \int \mathcal{D}A_\mu \; \exp\left(-\int d^3x \; -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}\right) \longrightarrow \boxed{Z = \exp\left(-\int d^3x \; \frac{e^2}{8\pi^2}\,\partial_\mu\sigma\partial^\mu\sigma\right)}$$

$$U(1)_{\rm top} : \sigma \mapsto \sigma + \alpha$$

# Magnetic monopoles

- We can introduce **magnetic monopoles** $\mathcal{M}(x)$ such that:

$$U(1)_{\text{top}} : \mathcal{M}(x) \mapsto e^{i\alpha}\mathcal{M}(x) \qquad \frac{1}{2\pi}\int d^2x \; B = 1$$

- Inserting a monopole in the EFT means inserting in the P.I.:

$$\mathcal{M}(x) \sim e^{i\sigma(x)}$$

- Effect of a **«dilute» gas of monopoles**:

$$S_{\text{sg}} = \int dxdydt \left(\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}\left(1 - \cos(\beta\phi)\right)\right)$$

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$$m \ll e^2$$

$$S_{\text{sg}} = \int dxdydt \left(\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1-\cos(\beta\phi))\right)$$

1. Non-perturbative correction to the EFT
2. The (dual) photon becomes massive with mass $m$

# The physics of the dual photon

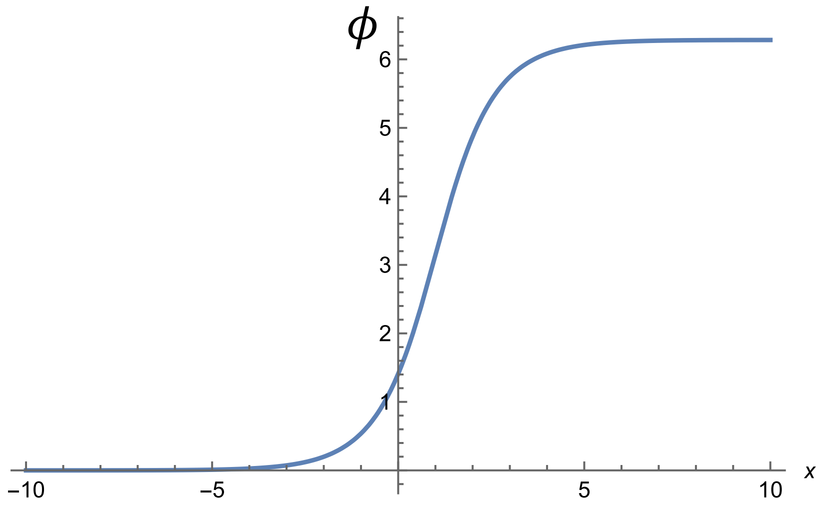
- At the end of the day, working with QED3(+monopoles) in the low energy limit means working with

$$S_{\rm sg} = \int dx dy dt \left( \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}\left(1-\cos(\beta\phi)\right)\right) \qquad \beta \equiv 2\pi/e \qquad m \ll e^2$$

- $e^- e^+$ pair at $x = x_0$, at infinite $y$ distance in the plane:

$$\phi_{\rm cl} = \frac{4}{\beta}\arctan\left(e^{m(x-x_0)}\right)$$

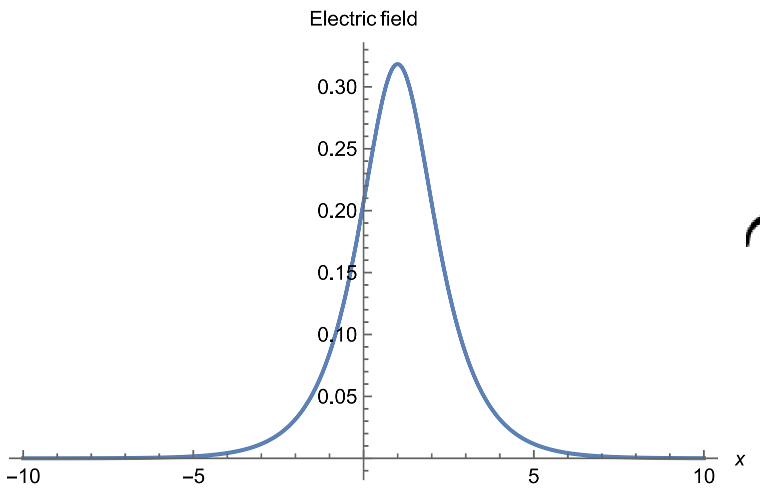
*«Domain wall» solitonic solution: x-axis translations spontaneously broken! → NGB*



# The flux tube, a.k.a. the effective string

$$E_y = F_{ty} = e\partial_x \phi_{\text{cl}} = \frac{e^2 m}{\pi} \frac{1}{\cosh(m(x - x_0))}$$

*«Tube» of collimated electric field connecting the pair of particles*




$\sim$


$$S_{\text{cl}} = -\frac{8m}{\beta^2} \int dydt$$

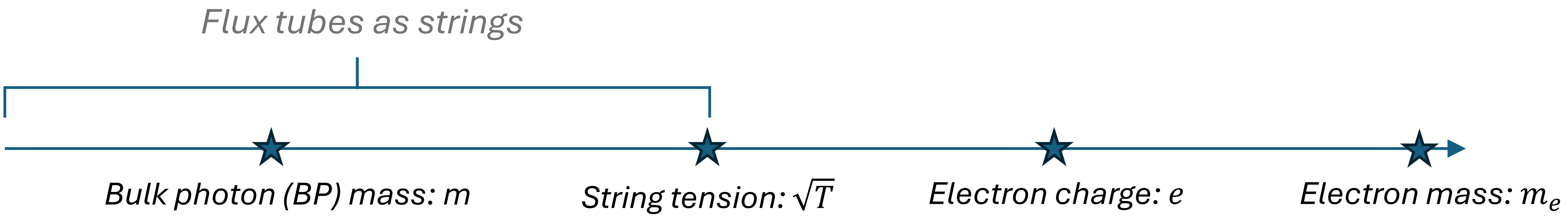
*Classical worldsheet action: we can use string theory techniques!*

*Tension of the string*

# Effective string theory (EST)

- We have a classical, stable string configuration with $m \ll \sqrt{T}$
- The worldsheet action rules the NGB behaviour. It is universal below the scale $\sqrt{T}$, in absence of other scales.

$$S_{\text{EST}} = \int d^2\sigma[-T\sqrt{-\det(g)} + c\sqrt{-\det(g)}R[g]^2 + \text{higher derivative terms}],$$



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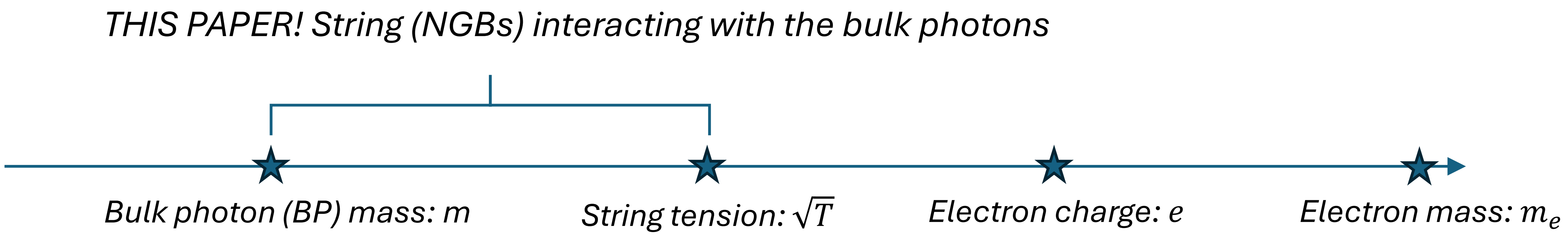


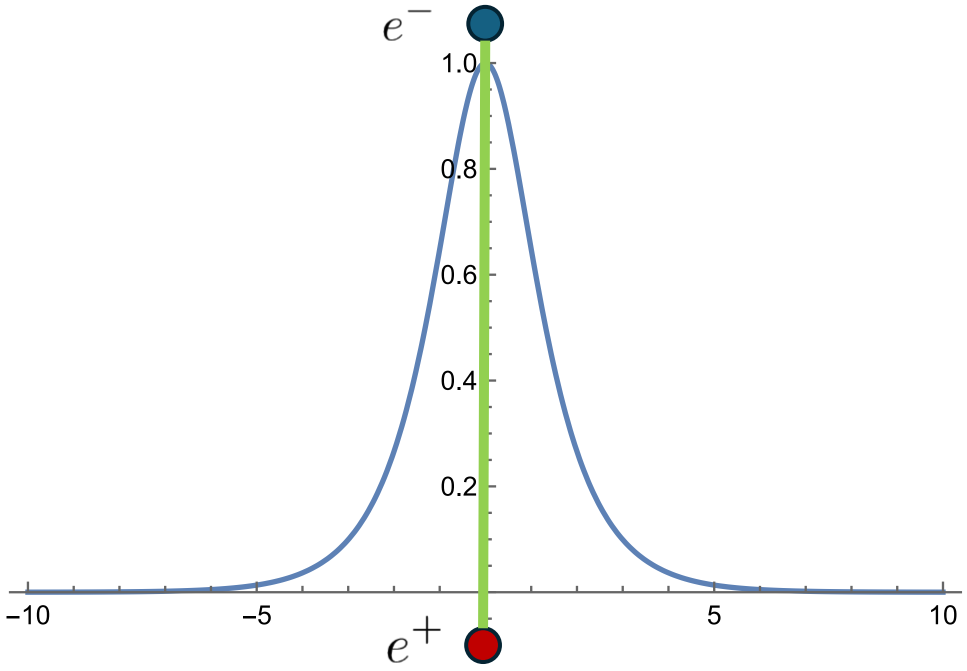
# The string and the photons: semiclassical

- Perturbing the classical configuration:

$$\phi(x,y,t) = \phi_{\rm cl}(x) + \delta\phi(x,y,t) = \frac{4}{\beta}\,\mathrm{arctan}\left(e^{mx}\right) + \delta\phi(x,y,t)$$

- The **NGB** solution:

$$\phi_0(x) \equiv \sqrt{\frac{m}{2}}\,\mathrm{sech}(mx)$$



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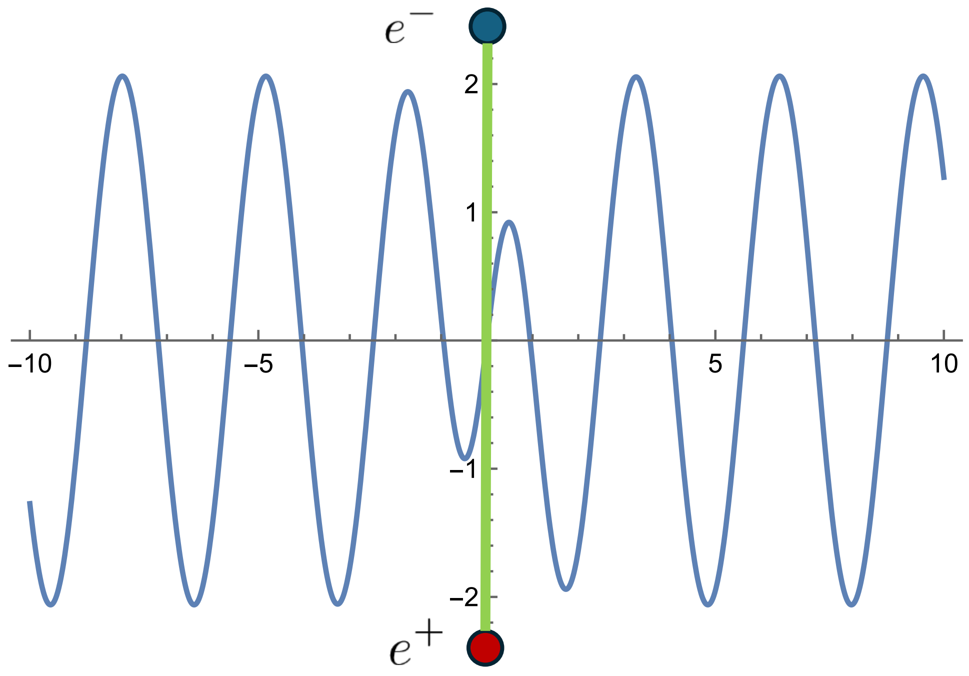
- The BP solution:

$$c_1 e^{-ipx}\left(\frac{ip}{m} + \tanh(mx)\right) + c_2 e^{ipx}\left(-\frac{ip}{m} + \tanh(mx)\right)$$



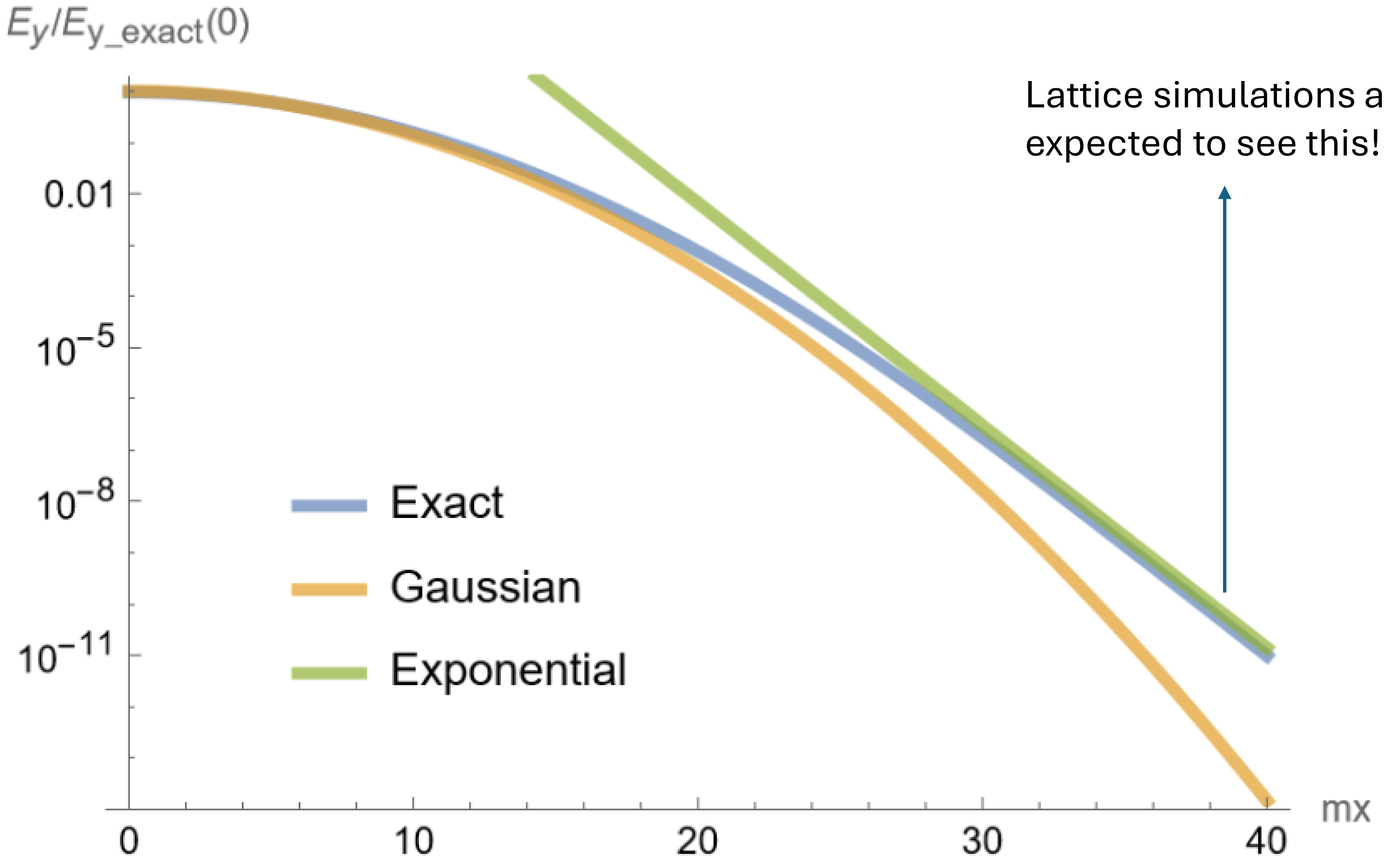
# The width of the string

- How «thick» is the string?

  - EST predicts a Gaussian profile, with a width $\propto \log(length) \times T^{-1}$
  - The classical solution predicts an exponential decay, independent of the length, with a width $\propto m^{-1}$

  - Solution: convolution!

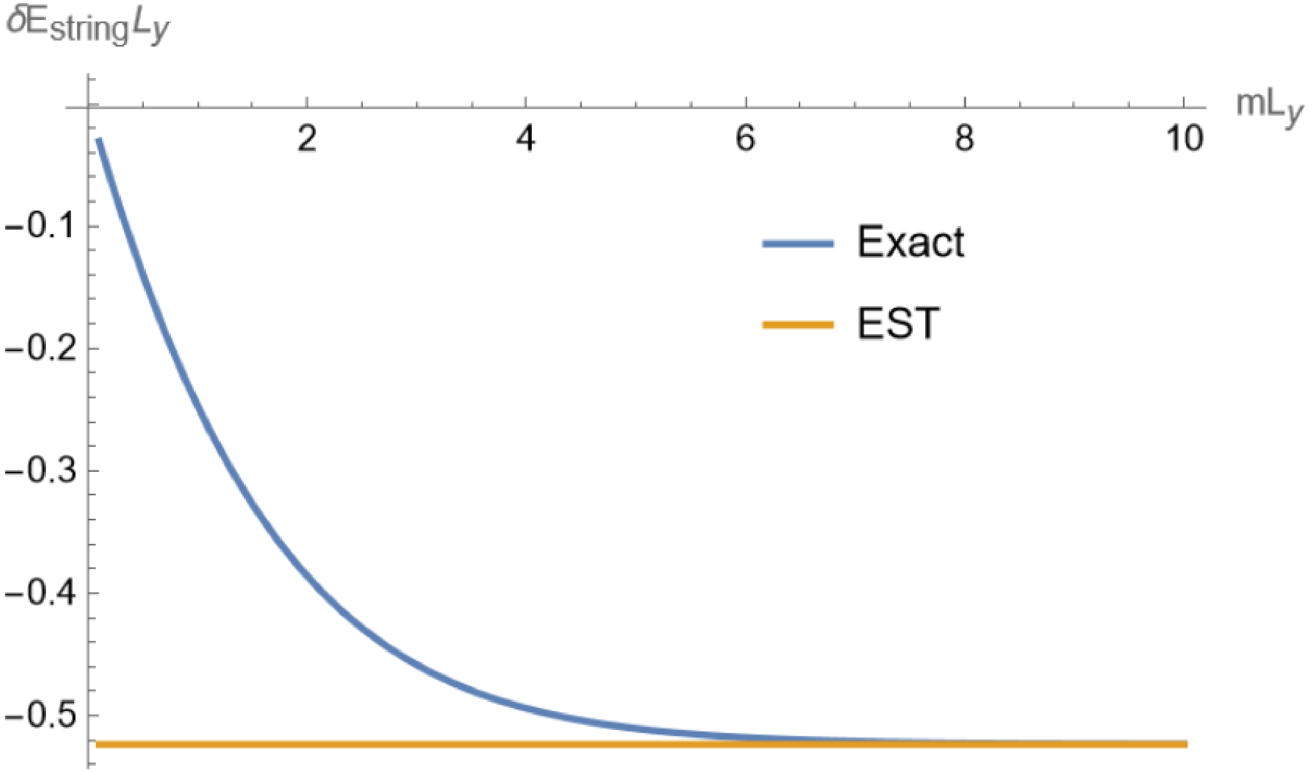
# The width of the string




Lattice simulations are expected to see this!

# The energy of the ground state

- For a **finite-sized string**, the corrections to the ground state energy can be computed perturbatively



# The scattering NGB+NGB→BP

- It is possible to study many **scattering processess** involving NGBs and BPs.

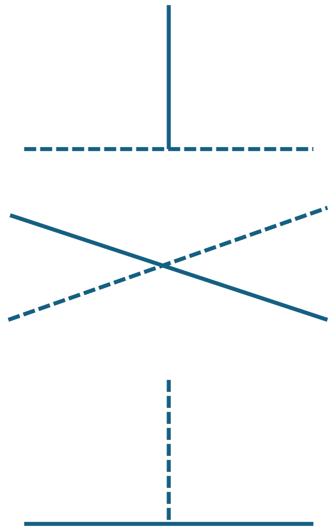
$$\mathcal{V}_1 \equiv \frac{m\beta}{4} \int d^2\sigma (\partial_a X_0)^2 \int dx \,\mathrm{sech}(mx) \tanh(mx) \delta\phi_{\mathrm{bulk}}.$$

$$\mathcal{V}_2 \equiv \frac{\beta^2}{16m} \int d^2\sigma (\partial_a X_0)^2 \int dx \, (\partial_x \delta\phi_{\mathrm{bulk}})^2$$

$$\mathcal{V}_3 \equiv -\frac{\beta}{\sqrt{8m}} \int d^2\sigma \partial_a \boxed{X_0} \int dx \partial_x \delta\phi_{\mathrm{bulk}} \partial_a \boxed{\delta\phi_{\mathrm{bulk}}}$$

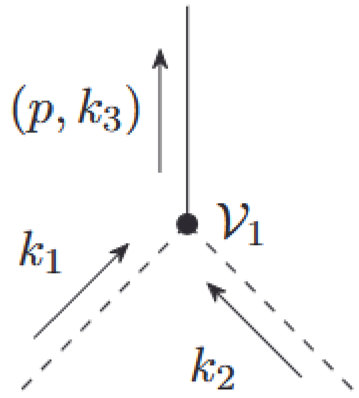
*NGB (transverse position of the string)*

*Bulk photon*

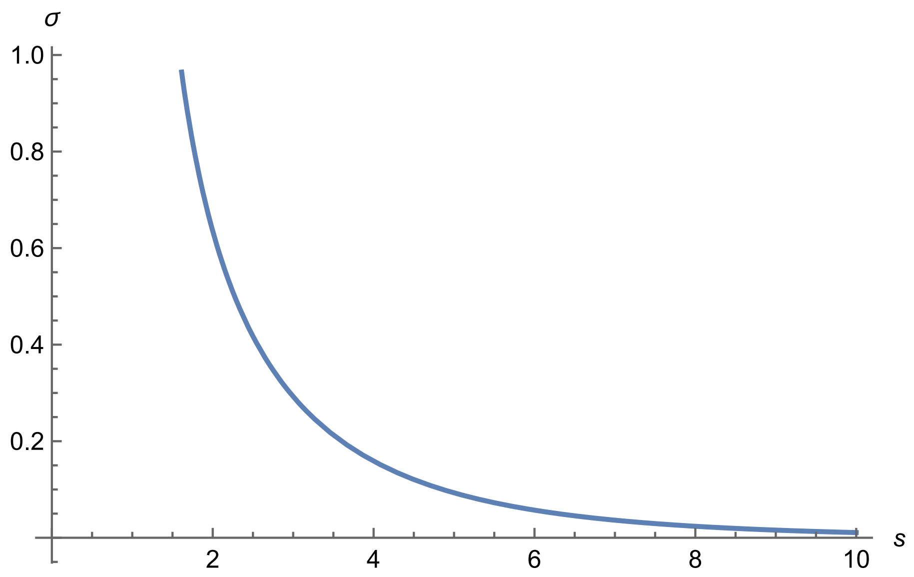


# The scattering NGB+NGB→BP

$$\langle X_0(-k_1)X_0(-k_2)\delta\phi_{\mathrm{bulk}}(p,k_3)\rangle_{\mathrm{tree}} =$$




$$\sigma_{1\mathrm{BP}} = \frac{\pi^2\beta^2 s^2}{128 m^2\sqrt{s-m^2}}\,\mathrm{sech}\left(\frac{\pi\sqrt{s-m^2}}{2m}\right)^2$$



# Take-home message

- Theories with confinement allow for classical solutions where flux tubes connect particle-antiparticle pairs.

- The flux tubes can be studied using string theory.

- In QED3, an interesting regime exists where the string coexist with a bulk massive photon.

- The paper studies many aspects of this interaction.