Drude model

kinetic theory : classical electron gas

Ion / atom fixed

Basic assumptions :

1) no $e^- e^-$ interactions (except collisions)
2) mean free time τ indep of velocity and position (τ : mean time before collision)
3) thermal equilibrium reached through collision w/ atoms (before equilibrium $v(x) = v(T(x))$

explains : * metal vs insulator
* electric transport (current)
* thermal conductivity

result:

friction due to collision

$$\frac{d}{dt}\langle \vec{p}(t)\rangle = q\left(\vec{E} + \frac{\langle p\rangle}{m}\wedge B\right) - \frac{\langle p(t)\rangle}{\tau}$$

$$\vec{J} = \underbrace{\frac{n q^2 \tau}{m}}_{\sigma}\vec{E}$$

current

q: charge

m: mass

n: number density $\left(\sim \frac{N}{V}\right)$

Maxwell-Ampère

$$\vec{\nabla}^2\vec{E} - \mu_r\mu_0\,\varepsilon_r\,\varepsilon_0\,\frac{\partial^2}{\partial t^2}\vec{E} = \mu_r\mu_0\frac{\partial \vec{J}}{\partial t}$$

$n_r = \sqrt{\varepsilon_r\mu_r}$ $\quad$ $n_0 = \sqrt{\varepsilon_0\mu_0}$

Bloch theorem

Let a perfect crystal with lattice vector $\vec{a}_i$ and reciprocal lattice vectors $\vec{b}_i$ (recall $\vec{a}_i \cdot \vec{b}_j = 2\pi \delta_{ij}$)

<u>def:</u> A Bloch function is a function of the form

$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{x}} \, u_k(\vec{x})$$

<u>Thm:</u> I) Bloch functions are eigenfunctions of the Hamiltonian

II) $u_{\vec{k}}(\vec{x}) = u_k(\vec{x} + \vec{a}_i)$

Let $\hat{T}_{\vec{n}}$ the translation operator

$$\hat{T}_{\vec{n}} f(\vec{x}) = f(\vec{x} + n^i \vec{a}_i)$$

Lemma: if is an eigenfunction of $\hat{T}_{\vec{n}}$

φ is a Bloch function

$$\hat{T}_{\vec{n}} f(\vec{x}) = e^{2\pi i n^i \theta_i} f(\vec{x})$$

define $\vec{k} = \theta^i \vec{b}_i$ and $U(\vec{x}) = e^{-ik\cancel{x}} \psi(\vec{x})$

$$T_n U(\vec{x}) = U(\vec{x} + n^i \vec{a}_i)$$

$$= e^{-i\vec{k}\vec{x} - i n^i \vec{a}_i \cdot \vec{k}} \psi(\vec{x} + n^i \vec{a}_i)$$

$$= e^{-i\vec{k}\vec{x} - i n^i \vec{a}_i \cdot b_j \theta^j} \psi(\vec{x} + n^i \vec{a}_i)$$

$$= e^{-i\vec{k}\vec{x} - i 2\pi n^i \theta_i} \psi(\vec{x} + n^i \vec{a}_i)$$

$$= e^{-ikx} e^{-2\pi i \theta_i n^i} e^{2\pi i n^i \theta_i} \psi(\vec{x}) = U(\vec{x})$$

Proof of Bloch theorem

$\hat{T}_{\vec{n}}$ is a symmetry of the system, meaning

$[\hat{T}_{\vec{n}}, \hat{H}] = 0$

By the spectral theorem, $\exists$ a common eigen basis $\psi_{\vec{k}}$ st

$$\hat{T}_{\vec{n}} \psi_k(\vec{x}) = C_{\vec{n},\vec{k}} \psi_k(\vec{x})$$

$$H \psi_{\vec{k}}(\vec{x}) = E_{\vec{k}} \psi_{\vec{k}}(\vec{x})$$

By lemma, ψ_k are Block functions