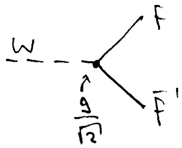
11.8 - 11.9. Test of the Standard Model

① Decay Rates of W and Z

a) W-Boson:



$$\boxed{\Gamma(W^- \to e^- \bar{\nu}_e) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

$\Gamma_{e\nu} = \Gamma_{\mu\nu} = \Gamma_{\tau\nu}$

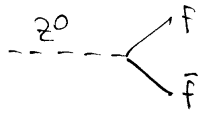
$\Gamma_{ud'} = \Gamma_{cs'} = \underset{\text{colour}}{3}\,\Gamma_{e\nu}$ w/o QCD-corrections!

$$\Gamma_{tot} = 9\Gamma_{e\nu} \approx 2047\text{ MeV}$$

$BR(W \to \text{hadrons}) \approx \frac{2}{3}$

$BR(W \to \text{leptons}) \approx \frac{1}{3}$

b) Z-Boson



$$\boxed{\Gamma(Z^0 \to \nu_e \bar{\nu}_e) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}} \approx 167\text{ MeV}$$

$$\Gamma_Z(Z^0 \to e^+e^-) = \left[(2\sin^2\theta_W - 1)^2 + 4\sin^4\theta_W\right] \cdot \frac{G_F M_Z^3}{12\pi\sqrt{2}} \approx 0.5\,\Gamma_{\nu\bar{\nu}} \approx 84\text{ MeV}$$

$$\Gamma(Z^0 \to u\bar{u}) = \underset{\text{color}}{3}\left(\frac{32}{9}x_W^2 - \frac{8}{3}x_W + 1\right) \cdot \frac{G_F M_Z^3}{12\pi\sqrt{2}} \approx 287\text{ MeV}$$

$$\Gamma(Z^0 \to d\bar{d}) = 3\left(\frac{8}{9}x_W^2 - \frac{4}{3}x_W + 1\right) \cdot \frac{G_F M_Z^3}{12\pi\sqrt{2}} \approx 370\text{ MeV}$$

Universality:

$\Gamma_{\nu_e\bar{\nu}_e} = \Gamma_{\nu_\mu\bar{\nu}_\mu} = \Gamma_{\nu_\tau\bar{\nu}_\tau}$; $\Gamma_{ee} = \Gamma_{\mu\mu} = \Gamma_{\tau\tau}$

$\Gamma_{d\bar{d}} = \Gamma_{s\bar{s}} = \Gamma_{b\bar{b}}$; $\Gamma_{u\bar{u}} = \Gamma_{c\bar{c}}$

$$\Gamma_{tot} = \sum \Gamma_{f\bar{f}} = 3\Gamma_{\nu\nu} + 3\Gamma_{ll} + 2\Gamma_{u\bar{u}} + 3\Gamma_{d\bar{d}} = 2437\text{ MeV}$$

($\Gamma_{tot} = 2495$ MeV with QCD, mass-corrections).

Z-Branching ratios:

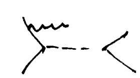
$\nu\bar{\nu}$	$\approx 20.6\,\%$
l^+l^-	$\approx 10.2\,\%$
$q\bar{q}$	$\approx 69.2\,\%$
	$100.0\,\%$

② Cross-section for $e^+e^- \to Z^0$

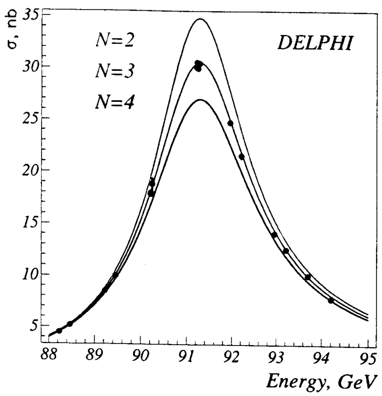
$$\sigma_{f\bar{f}}(s) = \frac{12\pi\,\Gamma_{ee}\Gamma_{f\bar{f}}}{M_Z^2}\;\frac{s}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

(comes from modified propagator: $\dfrac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M^2}}{q^2 - M^2 + \underline{i\Gamma_Z M}}$)

has to be modified by:

- $\alpha(m_Z) = \frac{1}{128}$.
- $m_Z^2 = \frac{m_W^2}{\cos^2\theta_W}(1 - \underline{\Delta s})$
- γ-Z-interference
- Initial state radiation (ISR) 

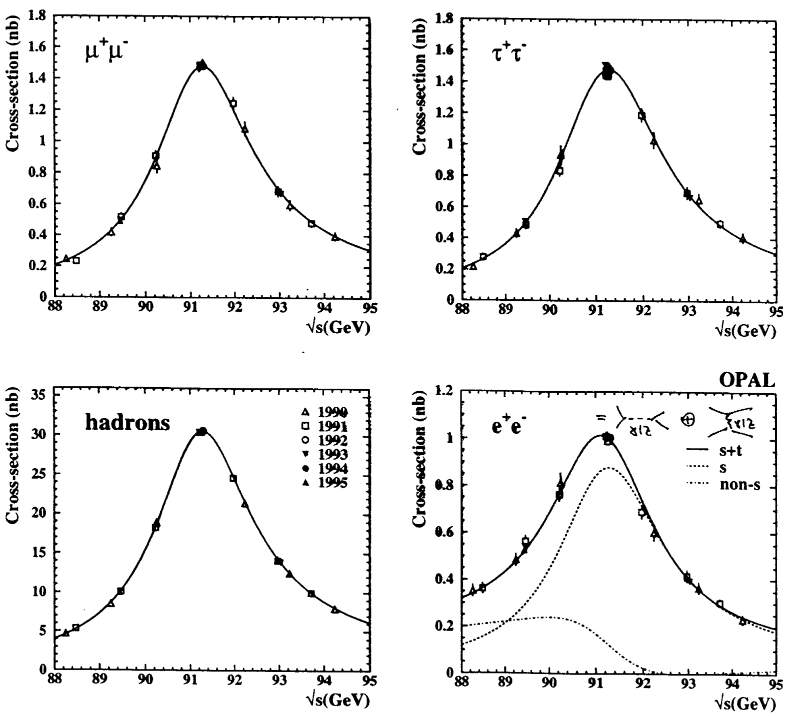
$\Rightarrow$ measure number of light neutrinos:



$\Rightarrow$ measurement prefers

$N_\nu = 3$ clearly!

Universality:



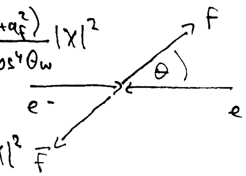
③ Extraction of Vector- and Axialvector Couplings

a) Angular Distribution:

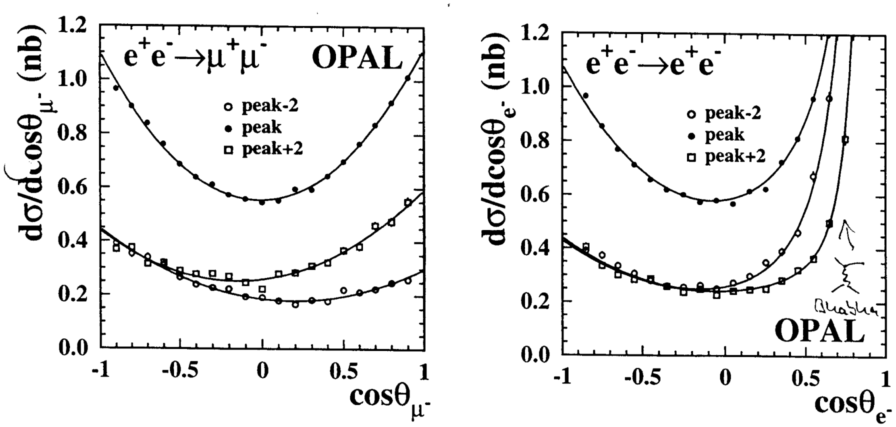
$$\frac{d\sigma}{d\Omega}(e^+e^- \to f\bar{f}) = \frac{\alpha^2}{4s}\left[F_1(s)\underbrace{(1+\cos^2\theta)}_{\text{even}} + 2F_2(s)\underbrace{\cos\theta}_{\text{odd}}\right]$$

$$F_1(s) = Q_f^2 - \frac{Q_f v_f v_e}{2\sin^2\theta_W\cos^2\theta_W}\,\mathrm{Re}(\chi) + \frac{(v_e^2+a_e^2)(v_f^2+a_f^2)}{16\sin^4\theta_W\cos^4\theta_W}|\chi|^2$$

$$F_2(s) = \frac{Q_f a_f a_e}{2\sin^2\theta_W\cos^2\theta_W}\,\mathrm{Re}(\chi) + \frac{v_e a_e v_f a_f}{4\sin^2\theta_W\cos^4\theta_W}|\chi|^2$$



and $\chi(s) = \dfrac{s}{s - M_Z^2 + iM_Z\Gamma_Z}$



$e^+e^- \to \tau^+\tau^-$ looks like $\mu^+\mu^-$

Extract $\sin^2\Theta_W$ from Forward-Backward-Asymmetry:

$$A_{FB} = \frac{N(\theta < 90^\circ) - N(\theta > 90^\circ)}{N(\theta < 90^\circ) + N(\theta > 90^\circ)}$$

Theory: $A_{FB} = \dfrac{3F_2(s)}{4F_1(s)} \Rightarrow \sin^2\Theta_W$

b) $\sin^2\Theta_W$ from τ-Polarisation

c) $\sin^2\Theta_W$ from A_{LR} (Left-Right-Asymmetry) with polarized e^- beam (SLC).