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Fast calculation of the resistive wall wake sum for multi bunch beams

E. Vogel

Abstract

For the numerical study of the beam injection process into the LHC, several technical and beam dynamical effects have to be taken into account, e.g. closed orbit variations of the pre-accelerator SPS, ripples on extraction and injection kickers, optics of the LHC itself and the behaviour of its transverse feedback system. Prominent beam dynamical effects are coupled bunch instabilities caused for example by the vacuum chamber resistance.

This paper describes a fast approximate calculation of the beam deflection due to the wake field which is built up by the vacuum chamber resistance and caused by a large number of previous bunches and/or bunch passages. The code has been implemented into the framework of the CNGS extraction simulations [1] and the results are compared with measurements [2].

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1 INTRODUCTION

At large circular accelerators and storage rings, the resistance of the vacuum chamber leads to non-negligible transverse coupled bunch instabilities. Nonlinearity in the beam optics results in a damping mechanism (Landau damping) of single bunches. This mechanism suppresses coupled bunch instabilities up to certain beam intensities. For accelerators operating with higher intensities, coupled bunch feedback systems are used to provide damping of beam oscillations.

Coupled bunch feedback systems also damp beam oscillations appearing during injection. At injection the nonlinearity in the beam optics leads to (small) damping accompanied by growth of the transverse beam dimensions due to filamentation. This undesired beam blow-up reduces the performance, in particular at proton accelerators and storage rings because of the missing damping due to synchrotron radiation. A feedback system decreases the beam blow-up at injection.

Numerical studies are performed to examine these time dependent processes including beam instabilities and feedbacks as they are usually impervious to analytical approaches. In [1] a simulation concept has been developed for the study of transient effects of beam-feedback inter-

action. It models the beam by rigid bunch tracking. De-coherence and beam blow-up are taken into account in an analytic way. Particular attention is paid to accurate modelling of the technical behaviour of the feedback system. It has been shown that the technical behaviour of the feedback system dominates the bunch-to-bunch beam behaviour. The predictions made are confirmed by accelerator studies [2]. So far, coupled bunch instabilities have not been considered in the model.

At LHC injection it is expected that the resistance of the vacuum chamber will be the largest contribution to the coupling of transverse beam oscillations of the multi bunch beam. The coupling is described by ‘kicks’ mediated by ‘wake fields’ acting on subsequent bunches. In the following, a method is presented for a fast approximate calculation of the resistive wall wake kick, caused by a large number of previous bunches and/or bunch passages. The method can easily be included into the framework of [1].

In the accelerator study [2] we observed unstable behaviour of the CNGS beam at high energy without transverse feedback. The comparison with measurements gives proof of the predictions obtained by the simulation method presented.

2 WAKE FUNCTION AND COUPLING FACTOR

A point like (rigid) bunch, carrying N_b elementary charges e travelling with transverse offset x_0 through an accelerator beam pipe, trails a wake electromagnetic field behind. At each fraction Δl of the accelerator circumference $2\pi R$ this ‘wake field’ causes a change of the transverse momentum Δp_\perp of a second bunch, travelling with distance z behind the first particle, of [3]

$$\Delta p_\perp = -\frac{e^2}{c} N_b w_\perp(z) x_0.$$

Here, $w_\perp(z)$ is the ‘wake function’ and c the speed of light. The classical transverse resistive wall wake function

is given by¹ [5]

$$w_{\perp}(z) = -\frac{c \Delta l}{\pi^{2/3} b^3} \sqrt{\frac{Z_0}{\sigma_c}} \frac{1}{\sqrt{z}}$$

with the free space impedance $Z_0 \approx 377 \Omega$, the beam pipe radius b and the beam pipe conductivity σ_c . In the ultrarelativistic case $v_{\parallel} \approx c$ the momentum change results in a change of the particle trajectory corresponding to an angular kick

$$\varphi_{\text{kick}} = \frac{dx_1}{dl} = \frac{\Delta v_{\perp}}{c} = \frac{\Delta p_{\perp}}{m c} = -\frac{e^2}{E} N_b w_{\perp}(z) x_0,$$

where E is the particle total energy (per elementary charge).

Without coupling between the particles, both particles may perform betatron oscillations [6] with ‘frequency’ $q_0 = Q/R$, where the tune Q is the number of betatron oscillations during one revolution. In energy normalized coordinates

$$x_n(l) = \sqrt{\frac{\beta \gamma}{\beta_x(l)}} x(l),$$

these oscillations are harmonic. In this context β is the relativistic factor $\beta = (1 - 1/\gamma^2)^{1/2}$, $\gamma = E/E_0$, $E_0 = m_p c^2$ the particle rest energy and $\beta_x(l)$ the betatron amplitude function. The oscillations can be described using complex betatron amplitudes A_n :

$$x_n(l) = \text{Im } A_n(l) \quad \text{with} \quad A_n(l) = A_n(0) e^{i \Psi(l)} \quad (1)$$

A kick at position l_{kick} changing the particle trajectory by the angle φ_{kick} changes the real part of this amplitude

$$\begin{aligned} \text{Im } \Delta A_n(l_{\text{kick}}) &= 0 \\ \text{Re } \Delta A_n(l_{\text{kick}}) &= \sqrt{\beta \gamma} \sqrt{\beta_x(l_{\text{kick}})} \varphi_{\text{kick}}. \end{aligned}$$

Hence, at each fraction Δl of the accelerator circumference ending at l_{kick} , the wake field induced by the first particle results in a change of the real part of the betatron amplitude $A_{n,1}$ of the second particle

$$\begin{aligned} \text{Re } \Delta A_{n,1}(l_{\text{kick}}) \\ = -\beta_x(l_{\text{kick}}) \frac{e^2}{E} N_b w_{\perp}(z) \text{Im } A_{n,0}(l_{\text{kick}}) \end{aligned}$$

In the following, we assume that the effect of the resistive wall is equally spread around the accelerator circumference. The tune Q is usually a non-integer number. Over many turns, the oscillating particles have all amplitudes defined by the betatron amplitude function. Therefore, we may approximate the betatron amplitude function by its average value $\beta_x(l) \approx \bar{\beta}_x$. Using this approximation we can introduce the dimensionless coupling factor

$$K \equiv \bar{\beta}_x \frac{e^2}{E} N_b \frac{c 2\pi R}{\pi^{2/3} b^3} \sqrt{\frac{Z_0}{\sigma_c}} \frac{1}{\sqrt{2\pi R}} \quad (2)$$

¹Other wake functions controlling divergences in multi turn wake summations may be found in [4].

Table 1: Quantities for the case of fixed target beam in the CERN SPS at injection energy.

Symbol	Value	Quantity
E	14 GeV	proton total energy
$\bar{\beta}_x$	42 m	average β -function
N_b	$4.8 \cdot 10^9$	protons per bunch
h_b	4620	number of buckets
n_b	2×2100	number of bunches
$2\pi R$	6911.5 m	circumference
Q	26.62	horizontal tune
b	0.025 m	average pipe radius
σ_c	$1.4 \cdot 10^6 (\Omega \text{ m})^{-1}$	conductivity

by expressing lengths in units of the of the accelerator circumference. Then the above relation between the two bunches becomes

$$\text{Re } \Delta A_{n,1}(l_{\text{kick}}) = \frac{\Delta l}{2\pi R} K \sqrt{\frac{2\pi R}{z}} \text{Im } A_{n,0}(l_{\text{kick}}). \quad (3)$$

For example at the CERN SPS the beam pipe is made out of stainless steel and is not copper coated as in other accelerators of comparable size. As a consequence, the conductivity σ_c of the vacuum pipe is smaller and the coupling factor (2) larger. With the quantities given in Table 1 it is $K = 1.039 \times 10^{-5}$.

3 WAKE SUM

Consider a circular accelerator with M possible bunch positions (buckets) with only two subsequent ones occupied by rigid bunches of equal intensity. If multi turn effects are neglected, the second bunch obtains a kick caused by the previous one at each fraction Δl of the accelerator circumference ending at l_{kick} . From (3) we obtain the effect on the second bunch

$$\text{Re } \Delta A_1(l_{\text{kick}}) = \frac{\Delta l}{2\pi R} K \sqrt{M} \sqrt{\frac{2\pi R}{z M}} \text{Im } A_0(l_{\text{kick}}).$$

If there are more bunches the wake field is built up by all passing bunches. The build-up depends only on the values of the betatron amplitudes of the bunches at the times they pass and not from the fact that they pass several times. Hence, a bunch obtains a kick at the accelerator position l_{kick} according to

$$\text{Re } \Delta A_N(l_{\text{kick}}) = \frac{\Delta l}{2\pi R} K \sqrt{M} \sum_{n=1}^N \frac{\text{Im } A_{n-1}(l_{\text{kick}})}{\sqrt{n}}, \quad (4)$$

where N is the number of all (equidistant) previous bunch passages and $A_{n-1}(l_{\text{kick}})$ the corresponding betatron amplitudes. N/M is the number of revolutions since the first bunch passage.

Each time a bunch passes the position l_{kick} we have to calculate this ‘wake sum’ (4) in a tracking simulation. Because of the slow vanishing factor $1/\sqrt{n}$ (for $n \rightarrow \infty$) a direct implementation of the sum results usually in too many numerical operations, in particular for simulations of accelerators with a large number of bunch positions. Several codes have been developed for a fast calculation of this sum (see for example [7]). The next paragraph describes an alternative method.

4 DEVIATIONS BUNDLE ALGORITHM

In order to perform the sum (4) up to N previous bunch passages directly we push at each bunch passage the deviation $x = \text{Im } A(l_{\text{kick}})$ into a queue x_n with $N + 1$ memory positions ($0 \leq n \leq N$). A constant table r_n , consisting of N elements, contains the numerical values of $1/\sqrt{n}$ starting with $n = 1$ up to $n = N$. The kick which has to be applied to the passing bunch is then

$$\text{Re } \Delta A_N(l_{\text{kick}}) = \frac{\Delta l}{2\pi R} K \sqrt{M} \sum_{n=1}^N x_n r_n.$$

Figure 1 shows this algorithm in the form of a block diagram. By calculating the wake sum in this way we perform N multiplications and N summations.

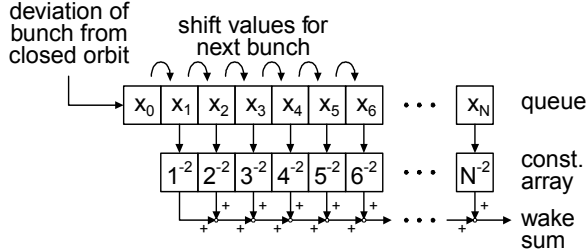


Figure 1: Code for performing the wake sum with a single queue.

The variation of the values from element to element in the constant table r_n becomes smaller and smaller for large n . As an example the values vary at $n = 4096$ only by $1 - r_{4097}/r_{4096} = 1.22 \times 10^{-4}$. For saving computation time, we may successively form bundles of values by summing them up and pushing the result in a second queue to multiply them with common table values:

For summing up the first 4096 values, we use the algorithm shown in Figure 1. In parallel we bundle successive the 64 successive values and push them into the second queue X_n . With these, we analogously form a second sum using a constant table R_n containing the numerical values of $1/\sqrt{64n - 63}$ for $n > 64$. The first 64 values of R_n have to be zero to avoid a doubling of the sum performed by the first queue. With a length of 2048 for R_n we calculate the wake sum of the 2^{17} last bunch passages, instead of 2^{12} by the use of the first queue only. The number of numerical operations is only increased by 50%.

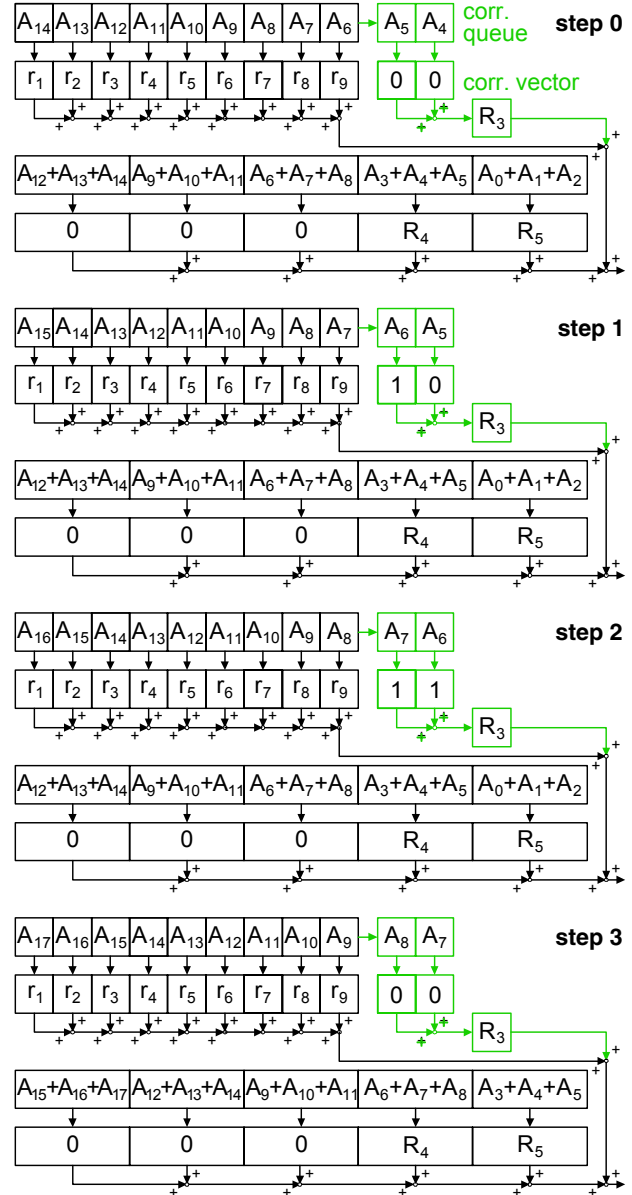


Figure 2: Queue overlap problem and cure by an additional overlap correction queue (green).

Care must be taken regarding the different rates at which the values are pushed into both queues. This causes a matching problem between the end of the first queue and the values calculated from the second queue. For demonstrating this in detail, Figure 2 shows step by step the situation for two queues with 9 and 5 elements, whereas the second queue contains bundles of three values.

At ‘step 0’ (see Figure 2) the second queue is completely filled with bundles of three from value A_{14} down to A_0 . The first queue contains A_{14} down to A_6 . The values A_5 down to A_0 have already left the queue while reaching the end. By performing the wake summations we obtain a result containing all values from A_0 to A_{14} .

At the next step (step 1 in Figure 2), the new value A_{15}

is pushed into the first queue and the value A_6 leaves the queue. There are not yet three new values, consequently no new bundle is pushed into the second queue. If we perform the wake summations the value A_6 is not contained in the result. In the next step (step 2 in Figure 2) the value A_7 is also lost.

One step later (step 3 in Figure 2) a new bundle of three is pushed into the second queue and the values A_6 and A_7 are back in the wake sum via the second queue. Hence, values are lost at steps which are not multiples of three.

This problem can be cured by an ‘overlap correction queue’ in which we always push the element leaving the first queue. The length of this correction queue has to be one less than the number of bundled values (two in Figure 2). An ‘overlap correction vector’ has to contain zeros at steps which are multiples of the number of bundled values. In between, it is filled up step by step with the value ‘1’. By forming the scalar product of the correction queue and the correction vector, we obtain at each step a sum of the lost values. We multiply the result with the last constant value mapped out at the second queue (R_3 in Figure 2) and obtain the necessary correction.

A quick check of an implementation may be performed with a single value pushed at different times (with respect to the bundling times) into the algorithm. The output value has to decay like $1/\sqrt{n}$, without gap at the time the value leaves the first queue.

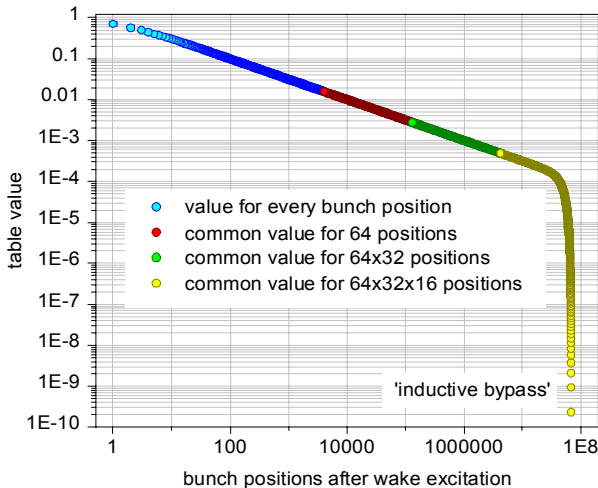


Figure 3: Values in the constant arrays used in the code implemented.

For wake sums containing several thousand bunches and several thousand turns we can build up cascades of this procedure. This means we set up a third queue which bundles values of the second queue and so on. The code, implemented in Simulink [8] for the study of beam injection and extraction at CERN accelerators, contains four queues with lengths 4096, 2048, 2048 and 2048 containing bundles of 64, 32 ($\times 64$) and 16 ($\times 32 \times 64$) each from the previous queue. In this way the wake sum considers $2^{26} \approx 67 \times 10^6$

bunch passages.

For obtaining a smooth transition to a vanishing kick strength at the end of the last queue, the constant array of the last queue goes faster than $1/\sqrt{n}$ against zero (Figure 3). This represents an ad hoc implementation of a so-called ‘inductive bypass’ [4]. The motivation for the work presented is the examination of transient injection processes in connection with feedbacks. Hence, such very long time effects are not further discussed here.

The code does not depend on the number of bucket positions in an accelerator. Transitions from queue to queue are usually not synchronous with the accelerator revolutions. Therefore, the small systematic errors caused by the bundling are expected to average out over many turns.

5 COMPARISON WITH MEASUREMENTS

CNGS beam will be ejected from the SPS in two batches causing residual oscillations by kicker ripples on the second batch. This second batch continues to circulate for 50 ms (about 2170 turns) after the first batch has been extracted. In an accelerator study we have proven experimentally that the oscillations can be damped by the feedback system [2].

To refine our understanding of the transient processes, we also switched off the transverse feedback at high energy (400 GeV). At nominal intensity for CNGS operation ($2.4 \cdot 10^{13}$ protons/batch), we observed without feedback a growth of the betatron oscillation amplitudes at the second batch after the extraction kick for the first batch. Due to increased beam loss, the beam was dumped before the time of the second extraction.

The CNGS extraction kicker ripple causes initial betatron oscillation amplitudes in the second batch with a significant variation from bunch to bunch (Figure 4). From the measured amplitudes the extraction kicker ripple can be reconstructed. The reconstructed kicker ripple applied in the simulation [1] results in the oscillation amplitudes indicated in Figure 4 in blue. Before the extraction kick, there are neither strong wake fields caused by betatron oscillations in the experiment nor in the simulation. Hence, we have similar initial conditions, i.e. no wake fields.

To save computation time we apply the resistive wall wake kick only once per revolution. This lumping of the resistive wall impedance at one point ($\Delta l \rightarrow 2\pi R$) is justified for small coupling factors [9]. To preserve the bunch-to-bunch phase relations due to the coupling, we use in the simulation the complex betatron oscillation amplitudes for the summation and apply the complex kick

$$\Delta A_N(l_{\text{kick}}) = -\frac{i}{2} K \sqrt{M} \sum_{n=1}^N \frac{A_{n-1}(l_{\text{kick}})}{\sqrt{n}}, \quad (5)$$

instead of (4).

Figure 5 shows the growth rates within the first 1000 turns after the extraction kick from the measurement and

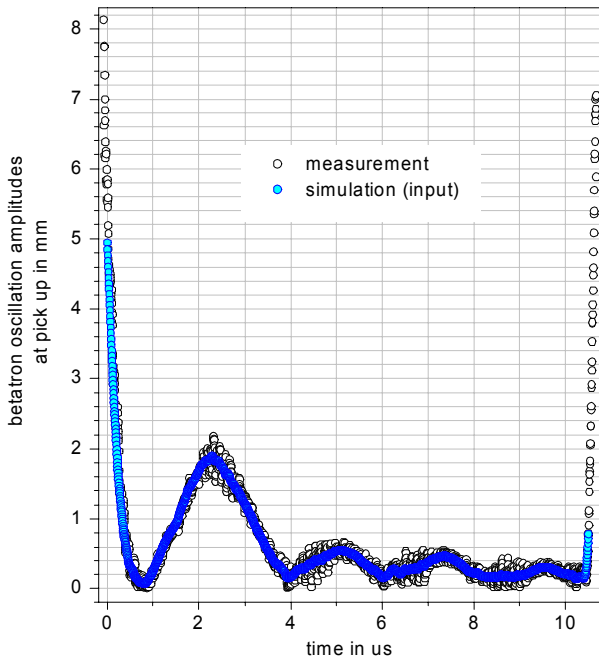


Figure 4: Betatron oscillation amplitudes in the second CNGS batch, measured at the pick-up directly after the first extraction kick.

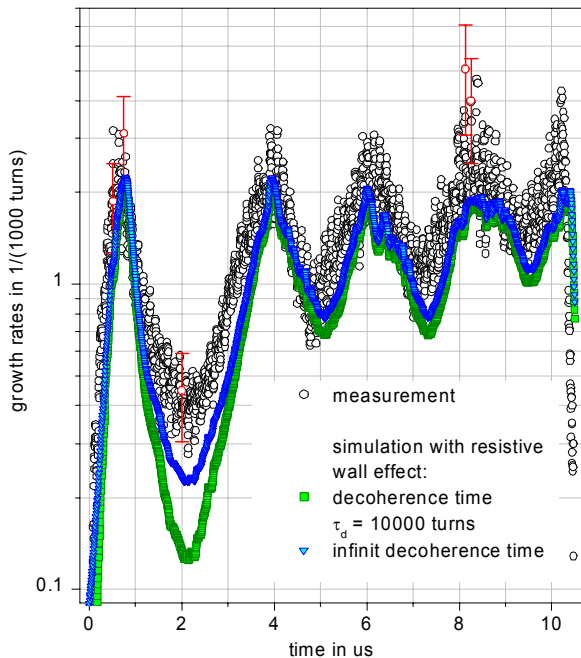


Figure 5: Instability growth rates measured during the CNGS extraction test (beam intensity: $2.34 (\pm 2) \cdot 10^{13}$ protons/batch) and the result of simulations considering the beam pipe resistance as the only impedance.

from the simulation. Bunches with large initial oscillation amplitudes give rise to enhanced wake fields acting

on bunches with small initial amplitudes. Therefore, the instability growth rate shows a significant change from bunch to bunch. This change is well reproduced by the simulation. The real accelerator shows larger growth rates than the simulation. This is due to the fact that impedances other than the beam pipe resistance are not taken into account in the simulation so far.

6 SUMMARY AND OUTLOOK

Starting from basic considerations on the wake function and betatron oscillations described by complex amplitudes, a factor is obtained completely describing the kick suffered by a bunch from the resistive wall wake caused by a previous bunch. The expansion of this treatment to multiple previous bunches and/or bunch passages results in the ‘resistive wall wake sum’.

Due to the $1/\sqrt{n}$ dependence of the resistive wall wake the factors in the wake sum vanish only slowly for large n . By bundling the bunch deviations and calculating only the average wake of the bundles for large n , the computation time is drastically reduced. Cascades of this approach allow the treatment of extremely high numbers of previous bunches and/or bunch passages, e.g. $2^{26} \approx 67 \times 10^6$.

The bundling process causes a systematic varying buffer overlap. To avoid computation errors caused by these variations a correction method has been developed.

Results from simulations of CNGS beam at extraction energy using the deviations bundle algorithm are in good agreement with the measurements. In particular the bunch-to-bunch variations of the growth rates are well reproduced. The growth rates of the simulations are smaller than the measured growth rates. This is due to the fact that the simulations did not consider impedances other than the vacuum chamber resistance.

When applied to the case of the LHC the use of this algorithm will improve the predictions for the bunch-to-bunch emittance variation at injection.

7 ACKNOWLEDGMENTS

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