Greensfunktion und Lösung der Wellengleichung.

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) G(\bar{r}-\bar{r}'; t-t') = -\delta(\bar{r}-\bar{r}')\,\delta(t-t').$$

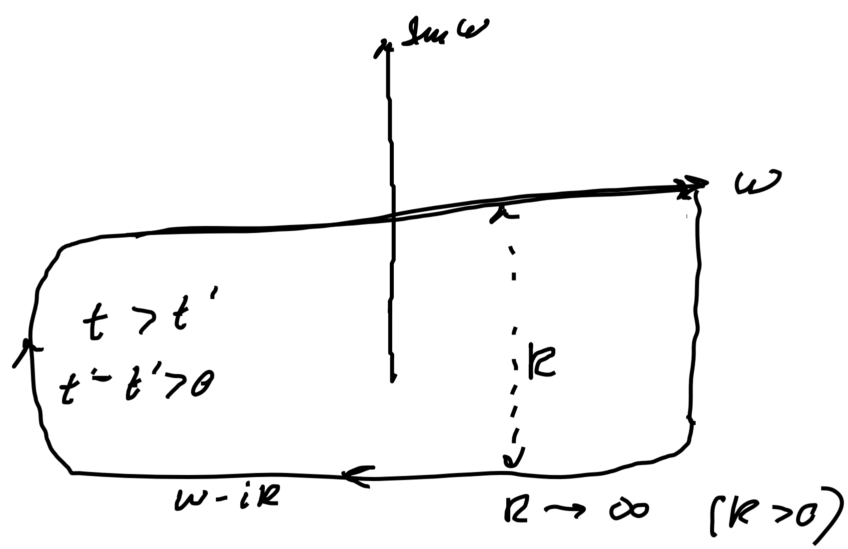
F.T. $$\tilde{G}(\bar{k},\omega) = \frac{1}{k^2 - \frac{\omega^2}{c^2}}$$

$$G(\bar{r}-\bar{r}', t-t') = \int \frac{d\omega}{2\pi} \int \frac{d^3\bar{k}}{(2\pi)^3} \, \frac{e^{i\bar{k}(\bar{r}-\bar{r}')}\, e^{-i\omega(t-t')}}{k^2 - \frac{\omega^2}{c^2}}$$

$k = \pm \frac{\omega}{c}$ – Singularität.

$$G(t-t') \overset{!}{=} 0, \quad t < t'$$

$$\int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{k^2 - \omega^2/c^2}$$

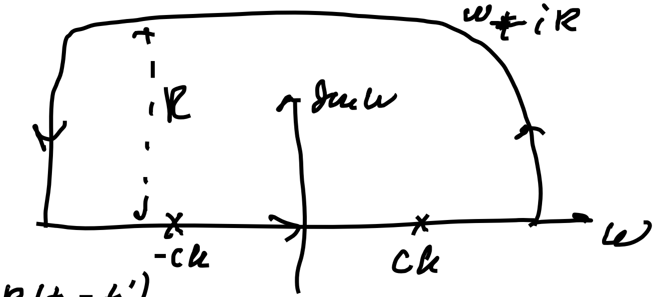


$$e^{-i(\omega - iR)(t-t')} =$$

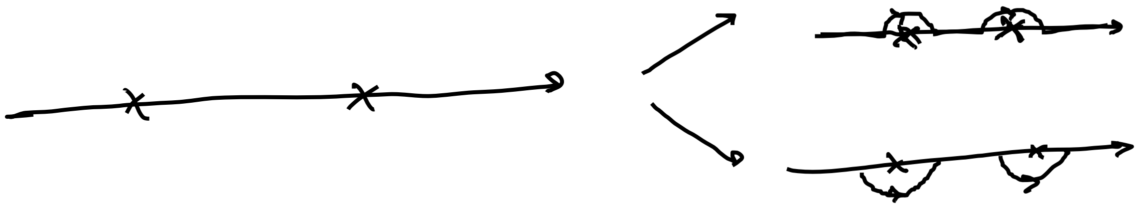
$$= e^{-i\omega(t-t')}\, e^{-R(t-t')} \longmapsto 0$$

$R \to +\infty, \ t - t' > 0$

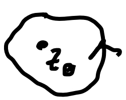
$t - t' < 0 \qquad (t < t')$

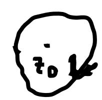


$$e^{-i(\omega + iR)(t-t')} = e^{-i\omega(t-t')}\, e^{R(t-t')} \xrightarrow[R\to+\infty,\ t-t'<0]{} 0$$

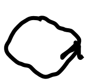


$$\oint \frac{f(z)\,dz}{z-z_0} = 2\pi i\, f(z_0) \qquad\qquad \oint \frac{f(z)\,dz}{z-z_0} = -2\pi i\, f(z_0)$$

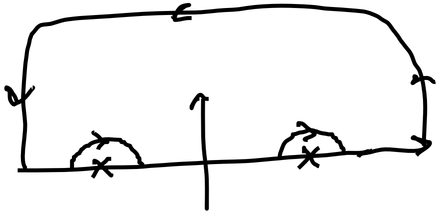


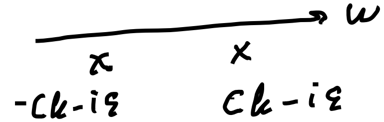


$$\oint f(z)\,dz = 0 \quad \text{oder} \quad \oint \frac{f(z)\,dz}{z-z_0} = 0$$



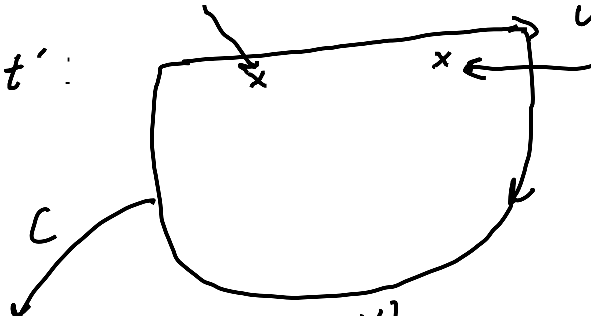
$$t<t': \quad \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{k^2-\frac{\omega^2}{c^2}} \overset{!}{=} 0$$



 $\Leftrightarrow$ 

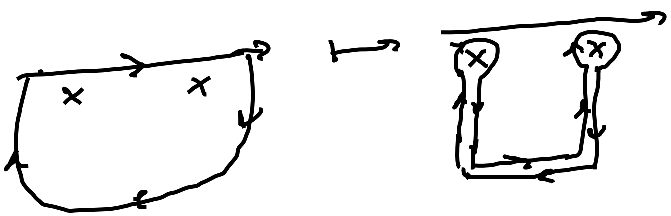
$(\varepsilon \to +0)$

$$\omega = -kc - i\varepsilon \Leftrightarrow k + \frac{\omega}{c} + i\varepsilon = 0$$

$t>t'$: 

$$\omega = kc - i\varepsilon \Leftrightarrow k - \frac{\omega}{c} - i\varepsilon = 0$$

$$\oint_C \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\left(k-\frac{\omega}{c}-i\varepsilon\right)\left(k+\frac{\omega}{c}+i\varepsilon\right)} = \int \frac{d\omega}{2\pi} c^2 \frac{e^{-i\omega(t-t')}}{(\omega+kc+i\varepsilon)(kc-\omega-i\varepsilon)} =$$



$$= -c^2 \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{(\omega+kc+i\varepsilon)(\omega-kc+i\varepsilon)} =$$

$$= -\frac{c^2}{2\pi}(-2\pi i)\left\{ \frac{e^{ikc(t-t')}}{-2kc} + \frac{e^{-ikc(t-t')}}{2kc} \right\} =$$

$$= i\,\frac{c}{k}\,\frac{e^{-ikc(t-t')} - e^{ikc(t-t')}}{2} = \frac{c}{k}\sin[kc(t-t')].$$

$t>t'$: $\quad G(\vec r-\vec r\,';t-t') = \int \frac{d^3\vec k}{(2\pi)^3}\frac{c}{k}\sin[kc(t-t')]\,e^{i\vec k(\vec r-\vec r\,')} =$

$$= \int \frac{d^3\vec k}{(2\pi)^3}\frac{c}{2ik}\left(e^{ikc(t-t')}-e^{-ikc(t-t')}\right)e^{i\vec k(\vec r-\vec r\,')}$$

Winkelintegration:

$$\int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi\, e^{ik|\vec r-\vec r\,'|\cos\theta} = \left|\begin{array}{l}|\vec r-\vec r\,'|\equiv R\\ x=\cos\theta\end{array}\right| =$$

$$= 2\pi\int_{-1}^{1} dx\, e^{ikRx} = 2\pi \frac{e^{ikRx}}{ikR}\Big|_{-1}^{1} = 2\pi\frac{e^{ikR}-e^{-ikR}}{ikR} = \frac{4\pi}{kR}\sin(kR)$$

$$t-t'\equiv\tau,\quad \vec r-\vec r\,'\equiv\vec R$$

$$G(\vec R,\tau) = \frac{1}{(2\pi)^3}\int_0^\infty k^2 dk\, \frac{4\pi}{kR}\sin(kR)\cdot\frac{c}{k}\sin(kc\tau) =$$

$$= \frac{1}{2\pi^2}\frac{c}{R}\int_0^\infty dk\,\sin(kR)\sin(kc\tau) = \frac{1}{2\pi^2}\frac{c}{R}\frac{1}{2}\int_{-\infty}^{\infty} dk\,\sin(kR)\sin(kc\tau) =$$

$$= \frac{1}{4\pi^2}\frac{c}{R}\frac{1}{(2i)^2}\int_{-\infty}^{\infty} dk\left\{e^{ik(R+c\tau)}+e^{-ik(R+c\tau)}-e^{ik(R-c\tau)}-e^{-ik(R-c\tau)}\right\} =$$

$$= \Big| x = ck \Big| = -\frac{1}{4\pi^2 R}\,\frac{1}{2}\int_{-\infty}^{\infty} dx \left\{ e^{ix(\tau + R/c)} - e^{ix(R/c - \tau)} \right\} =$$

$$= \frac{2\pi}{8\pi^2 R}\left\{ \delta\left(\frac{R}{c} - \tau\right) - \delta\left(\frac{R}{c} + \tau\right) \right\}.$$

$$R \geq 0\,, \quad \tau = t - t' > 0 \quad \Rightarrow \quad \delta\Big(\underbrace{\frac{R}{c} + \tau}_{>0}\Big) \equiv 0$$

$\tau > 0$

$$G(\vec{R}, \tau) = \frac{1}{4\pi R}\,\delta\left(\frac{R}{c} - \tau\right) \quad \text{– retardierte Greensfunktion}$$

$$G^R(\vec{r} - \vec{r}\,'; t - t') = \frac{\delta\left(t - t' - \frac{|\vec{r} - \vec{r}\,'|}{c}\right)}{4\pi |\vec{r} - \vec{r}\,'|}.$$

$$t_r = t - \frac{|\vec{r} - \vec{r}\,'|}{c} \quad \text{– retardierte Zeit.}$$