Fragmentation functions as probes for the transversity distribution

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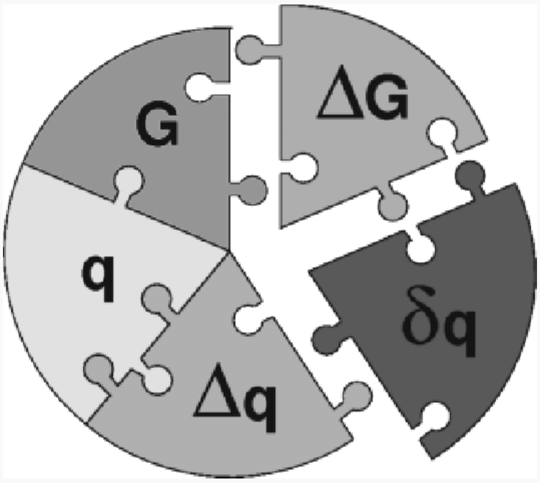
Outline

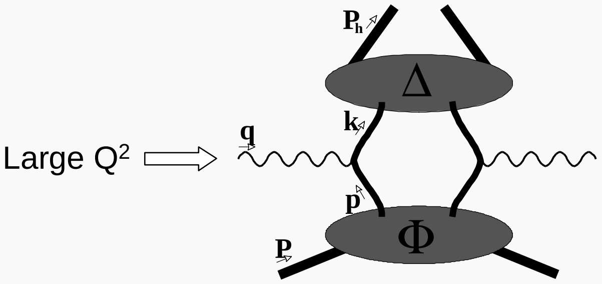
- The transversity distribution
- The Collins fragmentation function
- Polarized Λ fragmentation
- 2-hadron interference fragmentation
- Spin-1 fragmentation
- Conclusions

The puzzle of the distribution functions



Semi-inclusive DIS

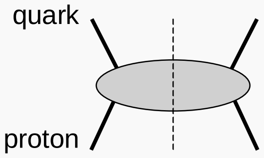
$$\mathbf{d}\sigma\ (\boldsymbol{l} + \boldsymbol{H} \to \boldsymbol{l}' + \boldsymbol{h} + \boldsymbol{X}) \propto \boldsymbol{L}_{\mu\nu} \boldsymbol{W}^{\mu\nu}$$



$$2\boldsymbol{M}\,\boldsymbol{W}^{\mu\nu} \propto \mathbf{Tr}[\Phi(x_B)\gamma^{\mu}\,\Delta(z_h)\gamma^{\nu}]$$

Different ways to see the distribution functions

1. Operator decomposition of the correlation function



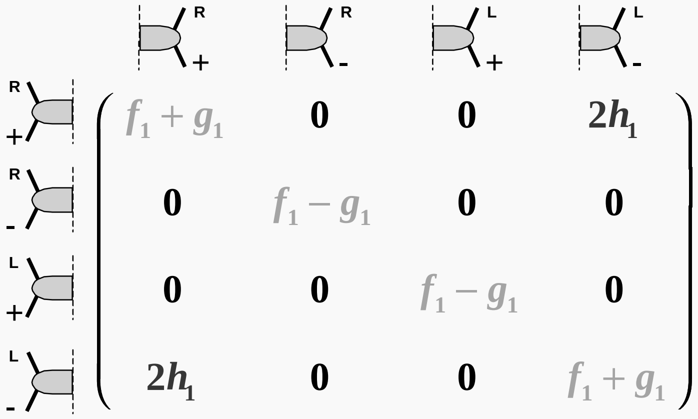
$$\Phi(x)=\frac{1}{2}\left\{\underset{q}{f_1(x)}+\underset{\Delta q}{g_1(x)}\gamma_5\, S_L+\underset{\delta q}{h_1(x)}\gamma_5\, \not{S}_T\right\}\gamma^-$$

NOTE: no intrinsic transverse momentum is included!

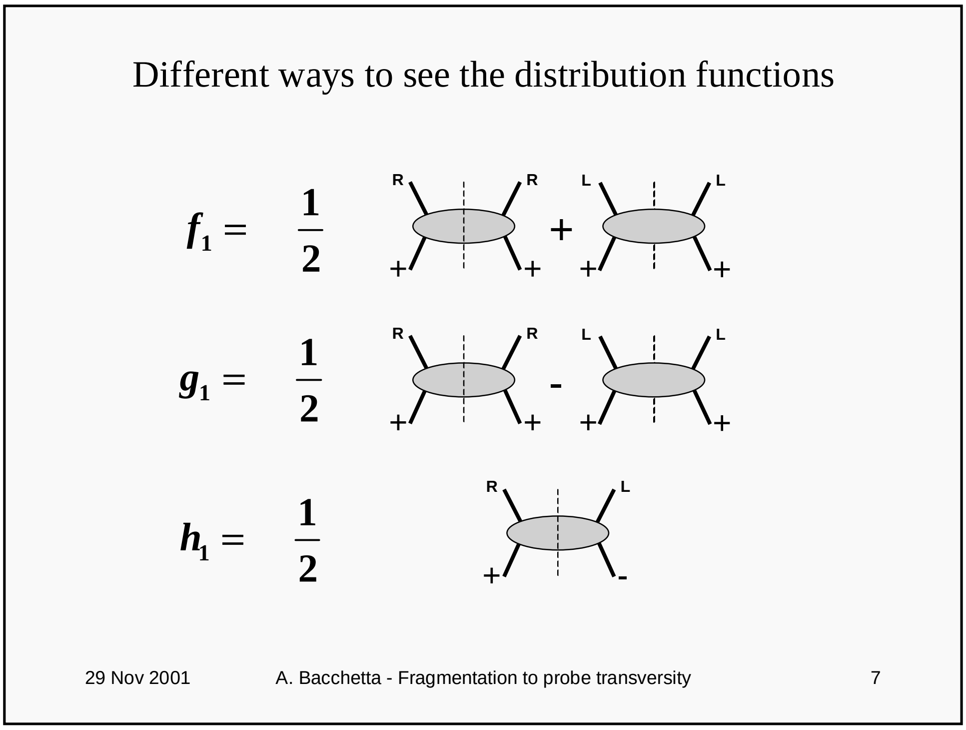
Different ways to see the distribution functions

3. Matrix elements in hadron spin space ⊗ parton chirality space

A.B, M. Boglione, A. Henneman, P. Mulders, PRL 85 (2000)



$$\begin{array}{c|cccc}
 & R+ & R- & L+ & L- \\
\hline
R+ & f_1+g_1 & 0 & 0 & 2h_1 \\
R- & 0 & f_1-g_1 & 0 & 0 \\
L+ & 0 & 0 & f_1-g_1 & 0 \\
L- & 2h_1 & 0 & 0 & f_1+g_1
\end{array}$$

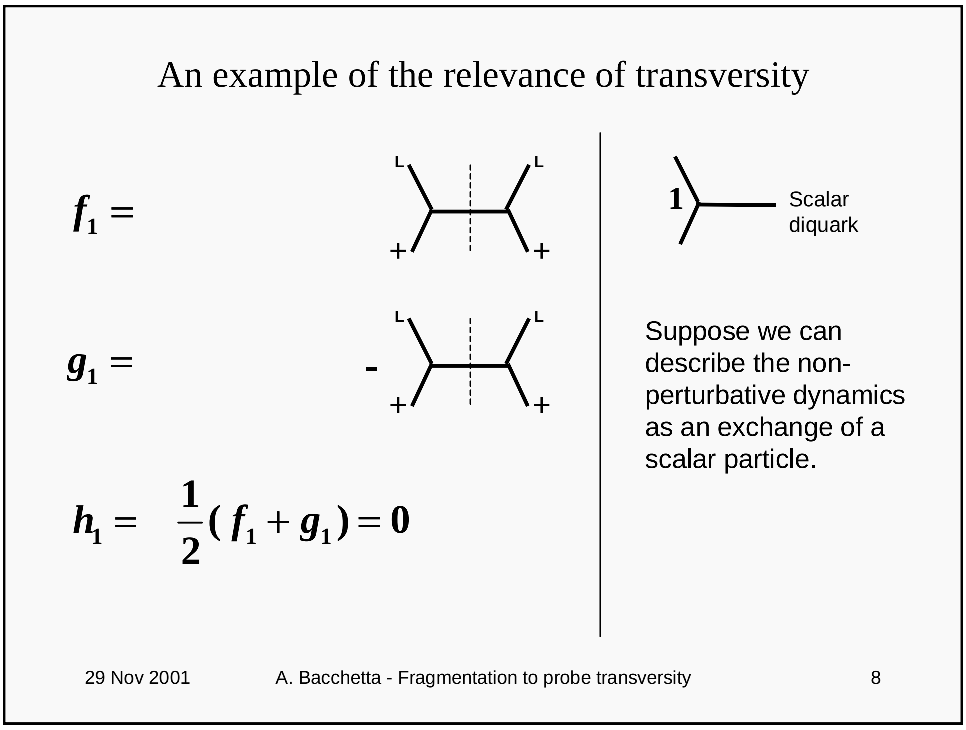


Different ways to see the distribution functions

$f_1 = \frac{1}{2}$ +

$g_1 = \frac{1}{2}$ -

$h_1 = \frac{1}{2}$



An example of the relevance of transversity

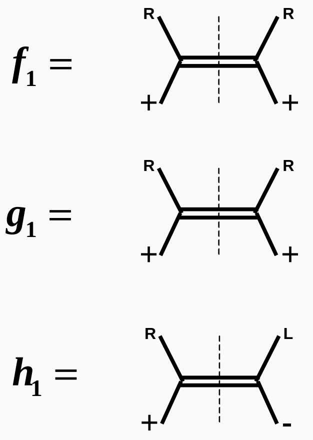
$f_1 =$

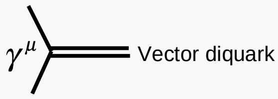
$g_1 =$ -

$$h_1 = \frac{1}{2}(f_1 + g_1) = 0$$

Suppose we can describe the non-perturbative dynamics as an exchange of a scalar particle.

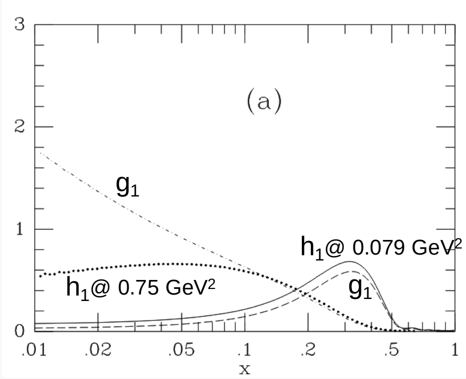
An example of the relevance of transversity





Suppose we can describe the non-perturbative dynamics as an exchange of a vector particle.

Model calculation of h_1

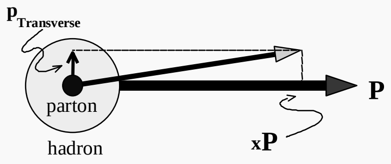


S. Scopetta, V. Vento,PLB 424 (1997)

for a nice review on the transversity see also

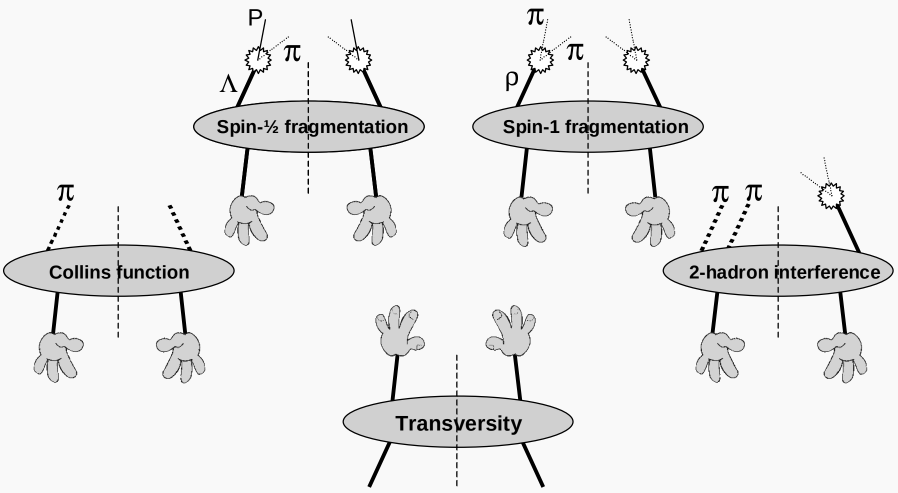
V. Barone, A. Drago, P. Ratcliffe, hep-ph/0104283

Inclusion of intrinsic transverse momentum



There are many more independent distribution functions and the puzzle becomes much bigger...

A chiral partner for transversity

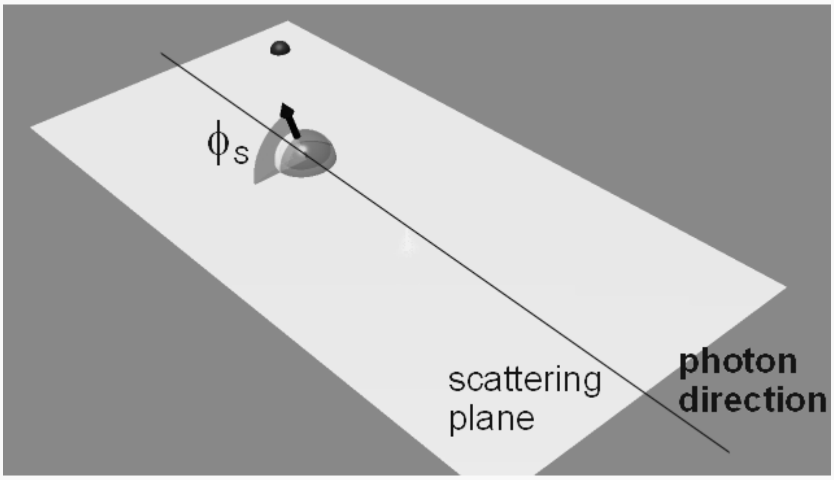


One-hadron transverse spin asymmetries

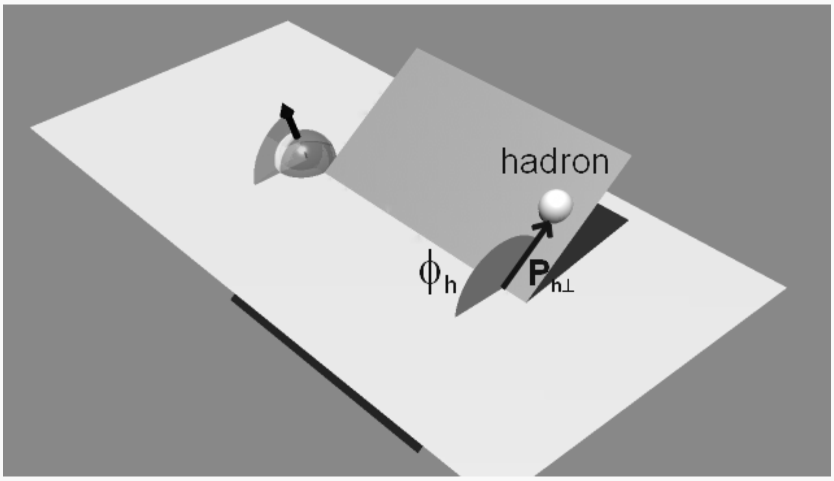
$$A_T = \frac{d^6\sigma^{\uparrow} - d^6\sigma^{\downarrow}}{d^6\sigma^{\uparrow} + d^6\sigma^{\downarrow}}$$

$$d^6\sigma = \frac{d^6\sigma}{dx\,dy\,dz\,d\left|\vec{P}_{h\perp}\right|d\phi_h\,d\phi_S}$$

Scattering plane angle



Hadron production angle



Unpolarized part

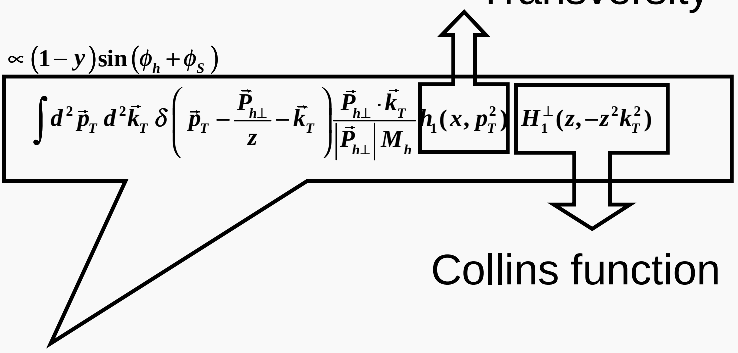
$$d^6\sigma^{\uparrow} + d^6\sigma^{\downarrow} \propto \left(1 - y + \frac{y^2}{2}\right) \int d^2\vec{p}_T \, d^2\vec{k}_T \, \delta\left(\vec{p}_T - \frac{\vec{P}_{h\perp}}{z} - \vec{k}_T\right) f_1(x, p_T^2) \, D_1(z, -z^2 k_T^2)$$

We can perform the integration over three variables

$$\int d\left|\vec{P}_{h\perp}\right| d\phi_h \, d\phi_S \, \left(d^6\sigma^{\uparrow} + d^6\sigma^{\downarrow}\right) \propto 2\pi \left(1 - y + \frac{y^2}{2}\right) \boxed{f_1(x) \, D_1(z)}$$

Factorized

Polarized part



Transversity

$$d^6\sigma^{\uparrow} - d^6\sigma^{\downarrow} \propto (1-y)\sin(\phi_h + \phi_S)$$

$$\int d^2\vec{p}_T\, d^2\vec{k}_T\, \delta\left(\vec{p}_T - \frac{\vec{P}_{h\perp}}{z} - \vec{k}_T\right) \frac{\vec{P}_{h\perp}\cdot\vec{k}_T}{|\vec{P}_{h\perp}|\, M_h} \boxed{h_1(x, p_T^2)}\, \boxed{H_1^{\perp}(z, -z^2 k_T^2)}$$

Collins function

The transversity appears in a complicated convolution with the Collins function

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Factorizing the polarized part

$$d^6\sigma^{\uparrow} - d^6\sigma^{\downarrow} \propto (1-y)\sin(\phi_h + \phi_S)$$

$$\int d^2\vec{p}_T\, d^2\vec{k}_T\, \delta\left(\vec{p}_T - \frac{\vec{P}_{h\perp}}{z} - \vec{k}_T\right) \frac{\vec{P}_{h\perp}\cdot\vec{k}_T}{|\vec{P}_{h\perp}|\, M_h} h_1(x, p_T^2)\, H_1^{\perp}(z, -z^2 k_T^2)$$

The only rigorous way to get a factorized expression is by weighting the integral

$$\int d|\vec{P}_{h\perp}|\, d\phi_h\, d\phi_S \sin(\phi_h + \phi_S)\frac{|\vec{P}_{h\perp}|}{M_h}\left(d^6\sigma^{\uparrow} - d^6\sigma^{\downarrow}\right)$$

$$\propto 2\pi(1-y)\, h_1(x)\, z^3 \int d^2\vec{k}_T\, \frac{k_T^2}{2M_h^2} H_1^{\perp}(z, -z^2 k_T^2)$$

$$= 2\pi(1-y)\boxed{h_1(x)\, z\, H_1^{\perp(1)}(z)}$$

Factorized

The factorized expression should not be spoiled by perturbative corrections

A. Henneman, D. Boer, P. Mulders, hep-ph/0104271 (to appear in NPB)

Azymuthal asymmetry weighted with $\left|\vec{P}_{h\perp}\right|$

$$A_T \left\langle \boxed{\frac{\left|\vec{P}_{h\perp}\right|}{M_h}\sin\left(\phi_h+\phi_S\right)} \right\rangle (x,y,z) = \frac{\left(1-y\right)}{\left(1-y+\frac{y^2}{2}\right)} \frac{h_1(x)\, z\, H_1^{\perp(1)}(z)}{f_1(x)\, D_1(z)}$$

Weighting factor in the numerator

Factorizing the polarized part

$$d^6\sigma^{\uparrow} - d^6\sigma^{\downarrow} \propto \left(1-y\right)\sin\left(\phi_h+\phi_S\right)$$
$$\int d^2\vec{p}_T\, d^2\vec{k}_T\, \delta\left(\vec{p}_T - \frac{\vec{P}_{h\perp}}{z} - \vec{k}_T\right) \frac{\vec{P}_{h\perp}\cdot\vec{k}_T}{\left|\vec{P}_{h\perp}\right| M_h} h_1(x,p_T^2)\, H_1^{\perp}(z,-z^2k_T^2)$$

If we don’t weight with $\left|\vec{P}_{h\perp}\right|$, we have to make simplifying assumptions on the intrinsic transverse momentum distribution. For instance:

$$h_1(x,p_T^2) \to h_1(x)\frac{\delta(p_T^2)}{\pi}$$

$$\int d^2\vec{P}_{h\perp}\, d\phi_S\, \sin\left(\phi_h+\phi_S\right)\left(d^6\sigma^{\uparrow} - d^6\sigma^{\downarrow}\right)$$
$$\propto 2\pi\left(1-y\right)h_1(x)z^2\int d^2\vec{k}_T\, \frac{\left|\vec{k}_T\right|}{2M_h} H_1^{\perp}(z,-z^2k_T^2)$$
$$= 2\pi\left(1-y\right)\boxed{h_1(x)\ H_1^{\perp(1/2)}(z)}$$

Factorized, but with problems

Azymuthal asymmetry

$$A_T \left\langle \sin(\phi_h+\phi_S)\right\rangle(x,y,z) \approx \frac{(1-y)}{\left(1-y+\frac{y^2}{2}\right)} \frac{h_1(x)\,H_1^{\perp(1/2)}(z)}{f_1(x)\,D_1(z)}$$

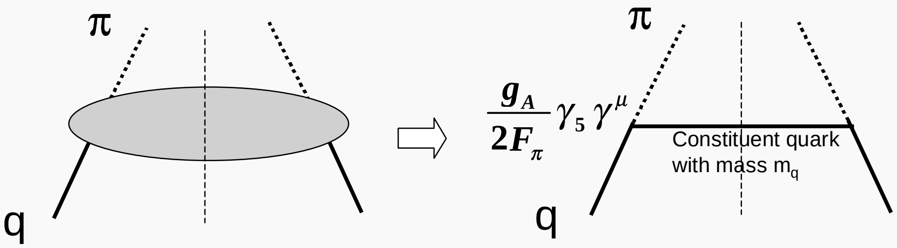
The approximate factorization is spoiled by perturbative corrections and requires the introduction of Sudakov factors in the evolution of the asymmetry.

D. Boer, NPB 603 (2000)

A model for the Collins function

The fragmentation process is modeled in a simple way

A.B., R. Kundu, A. Metz, in preparation

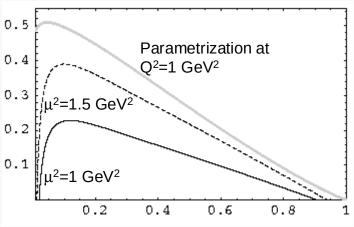


Quark with virtuality μ^2

Model results for D_1

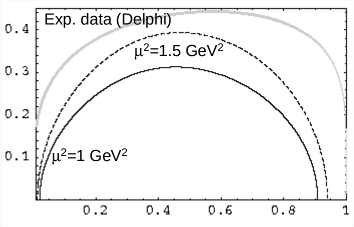
The results for the unpolarized quantities are nice, although they depend on the virtuality of the fragmenting quark

$D_1(z)$

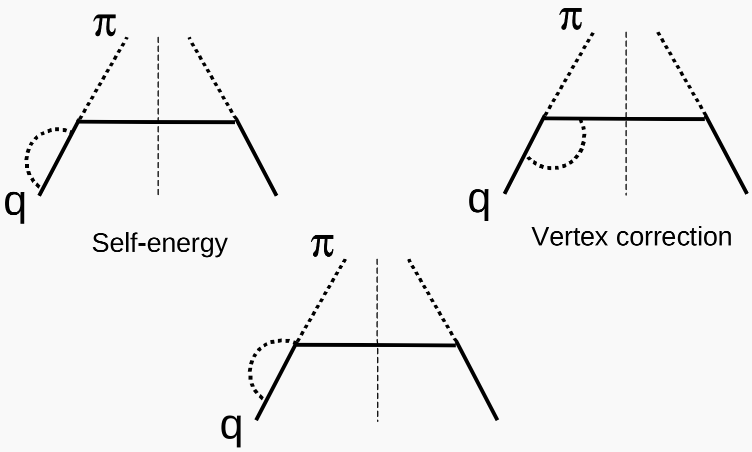


g_A=1 GeV, m_q=0.3 GeV

Average transverse momentum



One-loop corrections to obtain the Collins function



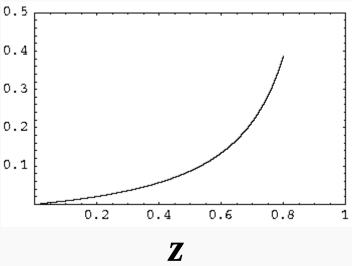
Self-energy

Vertex correction

Contact interaction

Results for the Collins function

$H_1^{\perp}(z)\big/D_1(z)$



The ratio with D_1 does not depend strongly on μ^2

The dependence on the constituent quark mass and on the coupling constant is weak

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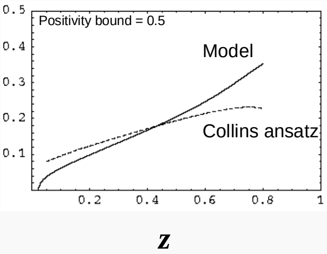
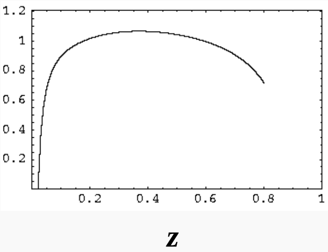
Results for moments of the Collins function

$H_1^{\perp(1/2)}(z)\big/D_1(z)$

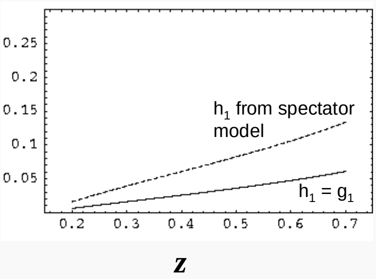


$H_1^{\perp(1)}(z)\big/D_1(z)$

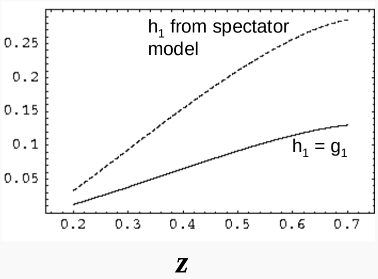


Results for the asymmetries

$A_T \left\langle \sin\left(\phi_h + \phi_S\right)\right\rangle$



$A_T \left\langle \left|\vec{P}_{h\perp}\right| / M_h \sin\left(\phi_h + \phi_S\right)\right\rangle$

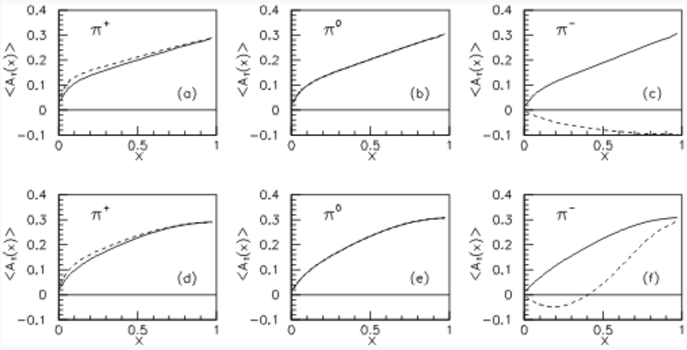


Non-weighted asymmetry, with assumption on intrinsic transverse momentum

Weighted asymmetry

A different estimate

$A_T \left\langle \left|\vec{P}_{h\perp}\right| / M_h \sin\left(\phi_h + \phi_S\right)\right\rangle$



Diquark model

pQCD based analysis

B.-Q. Ma, I. Schmidt, J.-J. Tang, hep-ph/0110324, to appear in PRD

Summary of Collins function

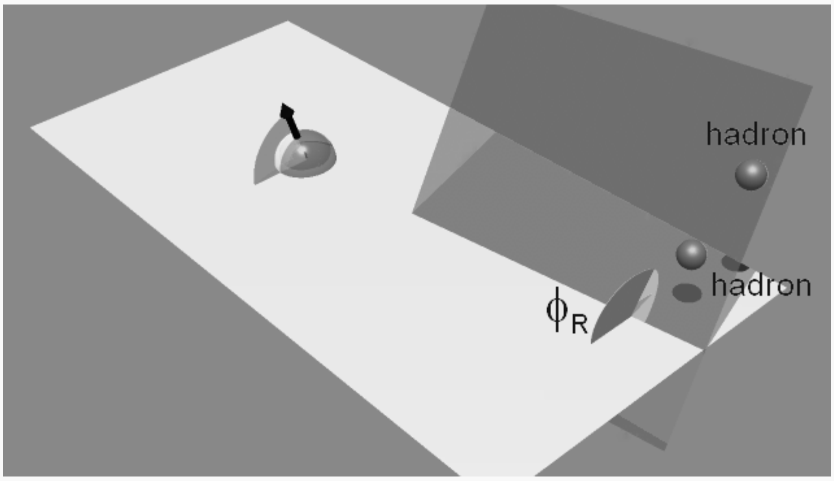
- It seems to be a very nice tool to probe transversity.
- Estimates suggest a 10%-25% effect.
- The weighted asymmetry is theoretically preferable to the non-weighted one.
- The evolution of this function is under study.

Two-hadrons transverse spin asymmetries

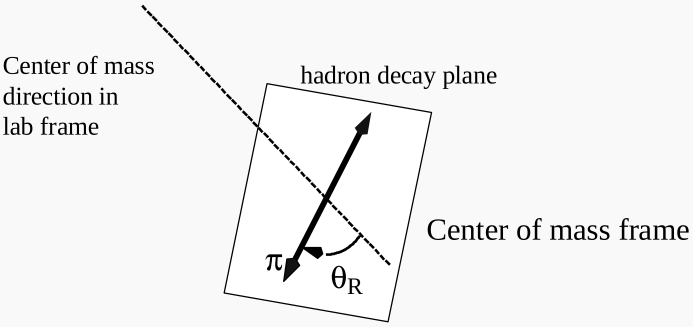
$$A_T = \frac{d^7\sigma^{\uparrow} - d^7\sigma^{\downarrow}}{d^7\sigma^{\uparrow} + d^7\sigma^{\downarrow}}$$

$$d^7\sigma = \frac{d^7\sigma}{dx\,dy\,dz\,dM^2\,d\theta_R\,d\phi_R\,d\phi_S}$$

Hadron pair azimuthal angle



Center of mass angle



Center of mass direction in lab frame

hadron decay plane

Center of mass frame

Polarized Λ production

$$d^7\sigma^\uparrow + d^7\sigma^\downarrow \propto \left(1-y+\frac{y^2}{2}\right) f_1(x)\, D_1(z)\, \boxed{\mathcal{BW}(M^2;M_\Lambda^2)}$$

Extremely sharp Breit-Wigner
invariant mass distribution

We can perform the integration over four variables

$$\int dM^2 d\theta_R\, d\phi_R\, d\phi_S \left(d^7\sigma^\uparrow + d^7\sigma^\downarrow\right) \propto 2\pi\left(1-y+\frac{y^2}{2}\right) f_1(x)\, D_1(z)$$

Polarized Λ production

$$d^7\sigma^\uparrow - d^7\sigma^\downarrow \propto (1-y)\cos\left(\phi_h+\phi_R\right)\, \boxed{h_1(x)}\,\boxed{H_1(z)}\,\alpha\sin\theta_R\, \mathcal{BW}(M^2;M_\Lambda^2)$$

Transversity

Λ transversity fragmentation function

We can perform the integration over four variables

$$\int dM^2\, d\theta_R\, d\phi_R\, d\phi_S \cos\left(\phi_R+\phi_S\right) \left(d^7\sigma^\uparrow - d^7\sigma^\downarrow\right) \propto \alpha\, 2\pi\,(1-y)\, h_1(x)\, H_1(z)$$

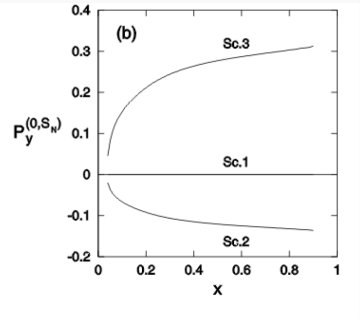
Asymmetry for Λ production

$$A_T \left\langle \cos\left(\phi_h + \phi_S\right)\right\rangle(x,y,z) = \frac{(1-y)}{\left(1-y+\frac{y^2}{2}\right)} \frac{h_1(x)\,\alpha\, H_1(z)}{f_1(x)\, D_1(z)}$$

$$\alpha = 0.642$$

Estimates of Λ asymmetry

$$\frac{1}{\alpha} A_T \left\langle \cos\left(\phi_h + \phi_S\right)\right\rangle \approx 1.5\, A_T \left\langle \cos\left(\phi_h + \phi_S\right)\right\rangle$$



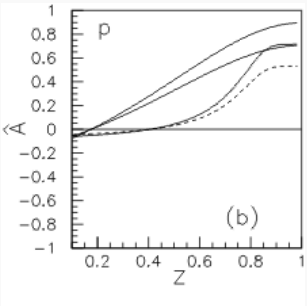
All light quarks contribute equally to the Λ spin

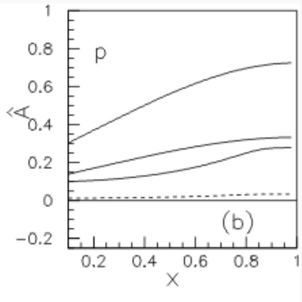
The whole Λ spin is carried by the *s* quark

M. Anselmino, M. Boglione, F. Murgia, PLB 481 (2000)

Estimates of Λ asymmetry

$$\approx 1.5\, A_T \left\langle \cos\left(\phi_h + \phi_S\right)\right\rangle$$





B.-Q. Ma, I. Schmidt, J.-J. Tang, PRD 64 (2001)

Λ production summary

- The Λ can easily be distinguished from the continuos background.
- The transversity fragmentation function is probably sizable, but is difficult to give a reliable estimate for the asymmetries.
- The evolution of the function is known.

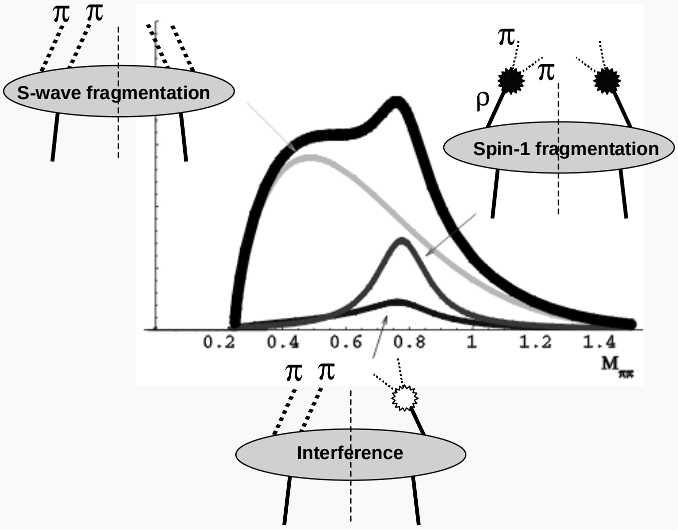
Two-pion fragmentation

$$d^7\sigma^{\uparrow} + d^7\sigma^{\downarrow} \propto \left(1 - y + \frac{y^2}{2}\right) f_1(x)\, D_1(z, \theta_R, \phi_R, M^2)$$

We can perform the integration over three variables

$$\int d\theta_R\, d\phi_R\, d\phi_S\, \left(d^7\sigma^{\uparrow} + d^7\sigma^{\downarrow}\right) \propto 2\pi \left(1 - y + \frac{y^2}{2}\right) f_1(x)\, D_1(z, M^2)$$

Invariant mass spectrum



Two-pion fragmentation

Transversity

$$d^7\sigma^{\uparrow} - d^7\sigma^{\downarrow} \propto (1-y)\sin(\phi_h + \phi_R)\boxed{h_1(x)}\ \boxed{H_1^{\measuredangle}(z,\theta_R,M^2)}$$

Two-pion chiral-odd
fragmentation function

Interference fragmentation function

$$d^7\sigma^{\uparrow} - d^7\sigma^{\downarrow} \propto (1-y)\sin(\phi_h + \phi_R)h_1(x)\boxed{\sin\theta_R}H_1^{\measuredangle}(z,M^2)$$

Angular distribution typical
of an s-p interference

We can perform the integration over three variables

$$\int d\theta_R\, d\phi_R\, d\phi_S \sin(\phi_R + \phi_S)\left(d^7\sigma^{\uparrow} - d^7\sigma^{\downarrow}\right) \propto$$

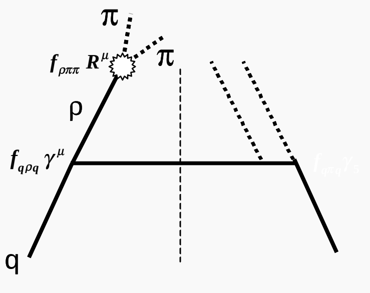
$$\alpha\, 2\pi\,(1-y)h_1(x)H_1^{\measuredangle}(z,M^2)$$

Asymmetry for interference fragmentation function

$$A_T \left\langle \sin\left(\phi_h + \phi_S\right) \right\rangle (x, y, z, M^2) = \frac{(1-y)}{\left(1 - y + \frac{y^2}{2}\right)} \frac{h_1(x)\, H_1^{\sphericalangle}(z, M^2)}{f_1(x)\, D_1(z, M^2)}$$

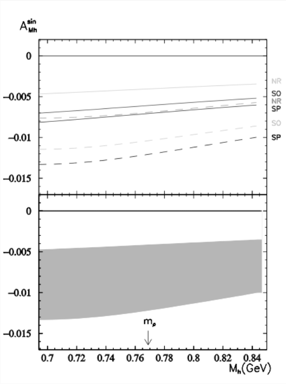
A model for interference fragmentation functions

M. Radici, R. Jakob, A. Bianconi, hep-ph/0110252

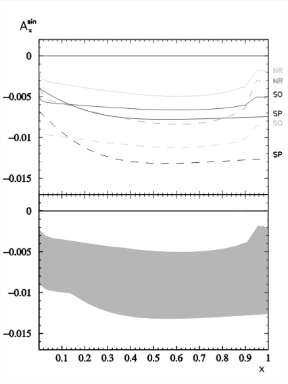


Results for the asymmetry

$$A_T \left\langle \sin\left(\phi_h + \phi_S\right)\right\rangle$$

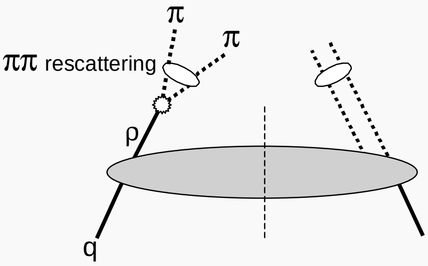


M

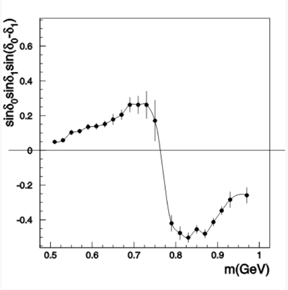


x

A different model



R. Jaffe,X. Jin, J. Tang, PRL 80 (1997)

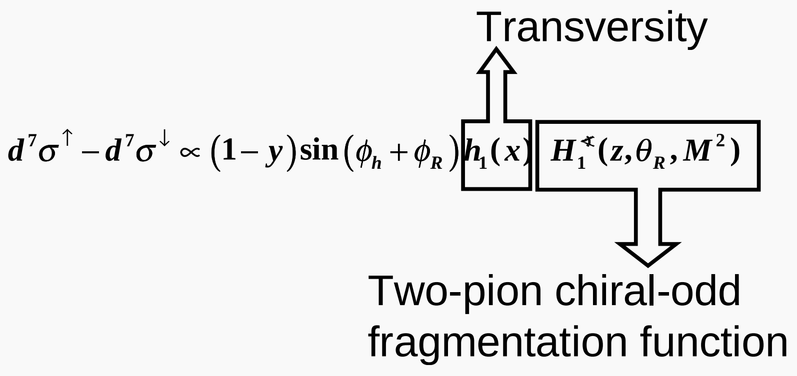


The model of Jaffe et al. suggests a different behavior with the invariant mass

Summary of interference fragmentation function

- The interference fragmentation function does not require the measurement of the angle $\theta_{R.}$
- The specific dependence on the invariant mass is not well known.
- The interesting part should anyway be around the ρ mass.
- The only estimate points to a few percent asymmetry.
- The evolution of the function is known.

Two-pion fragmentation



Transversity

$$d^7\sigma^{\uparrow} - d^7\sigma^{\downarrow} \propto (1-y)\sin(\phi_h+\phi_R)\boxed{h_1(x)}\,\boxed{H_1^{\sphericalangle}(z,\theta_R,M^2)}$$

Two-pion chiral-odd fragmentation function

A. Bianconi, S. Boffi, R. Jakob, M. Radici, PRD 62 (2000)

Spin-one fragmentation function

$$d^7\sigma^{\uparrow} - d^7\sigma^{\downarrow} \propto (1-y)\sin(\phi_h + \phi_R)$$

$$h_1(x)\boxed{\sin 2\theta_R}\, H_{1LT}^{\sphericalangle}(z)\boxed{\mathcal{BW}(M^2; M_\rho^2)}$$

Angular distribution typical of a spin-one system

Breit-Wigner invariant mass distribution typical of a resonance

We can perform the integration over two variables (we cannot integrate over θ_R)

$$\int d\phi_R\, d\phi_S \sin(\phi_R + \phi_S)\left(d^7\sigma^{\uparrow} - d^7\sigma^{\downarrow}\right) \propto$$

$$\pi (1-y) h_1(x) \sin 2\theta_R\, H_{1LT}(z)\, \mathcal{BW}(M^2; M_\rho^2)$$

Asymmetry for spin-one production

A. B., P. Mulders, PRD 62 (2000)

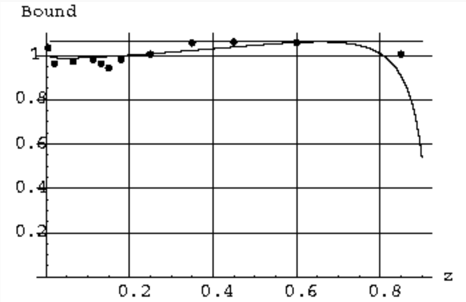
$$A_T \left\langle \sin(\phi_h + \phi_S) \right\rangle (x, y, z, \theta_R) = \frac{(1-y)}{\left(1 - y + \frac{y^2}{2}\right)} \frac{h_1(x) \sin 2\theta_R H_{1LT}(z)}{f_1(x)\, D_1(z)}$$

Positivity bound on H_{1LT}

A. B., P. Mulders, PLB 518 (2001)

$$H_{1LT}(z) \le \sqrt{\left(D_1(z)+\frac{2}{3}B_1(z)\right)\left(D_1(z)-\frac{1}{3}B_1(z)\right)} \le \frac{3}{2\sqrt{2}}D_1(z)$$

$$\frac{H_{1LT}(z)}{D_1(z)}$$



Example: positivity bound on H_{1LT} obtained from OPAL data for K*(892)

Summary for spin-one production

- The study of the dependence on the angle θ_R is necessary to isolate the spin-one contribution.
- At the moment there are no estimates of the asymmetry (but we are going to work at it!).
- The evolution of the function is known.

Conclusions

- Transversity is an interesting and significant object to measure.
- There are at least four different fragmentation mechanisms to probe it.
- The Collins function is more complex from the theoretical point of view, but there are promising indications on its size.
- Two-hadron fragmentation is theoretically more clear, but seems to be experimentally more challenging.