

Helicity Amplitude Ratios in Exclusive Electroproduction of the ρ^0 Meson at HERMES

B. Marianski

bohdan@fuw.edu.pl

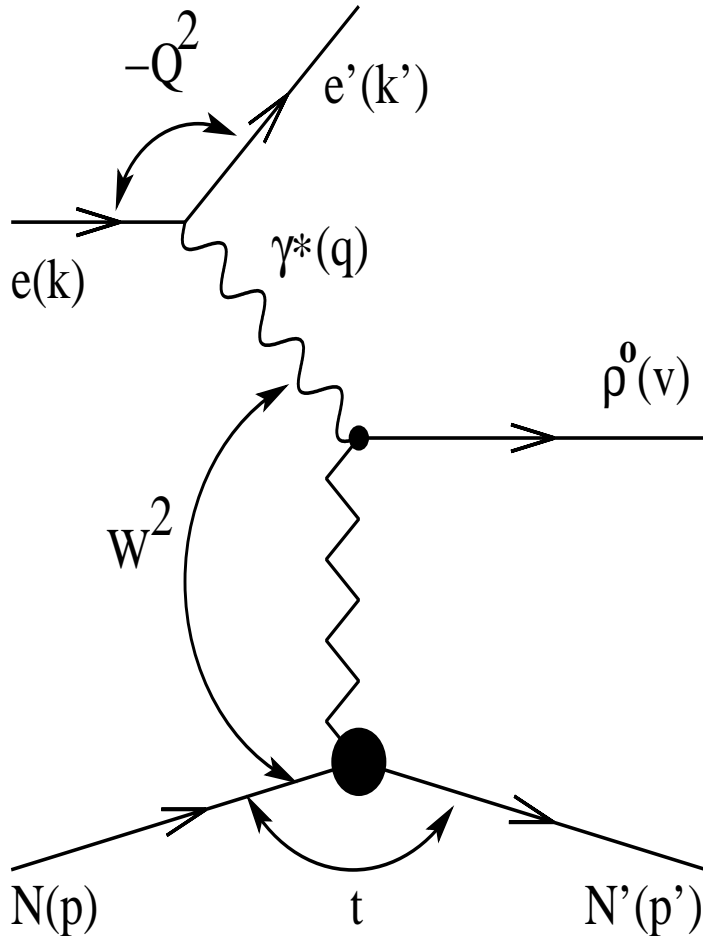
Andrzej Soltan Institute for Nuclear Studies, Warsaw, Poland

on behalf of HERMES Collaboration

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- Amplitudes and Spin Density Matrices.
- What can we learn from helicity amplitude ?
- HERMES Experiment and data processing.
- Kinematic dependences of ratios of helicity amplitudes.
- Comparison with H1 results.
- Summary.

Amplitudes and Spin Density Matrices in reaction $e + N \rightarrow e' + \rho^0 + N$



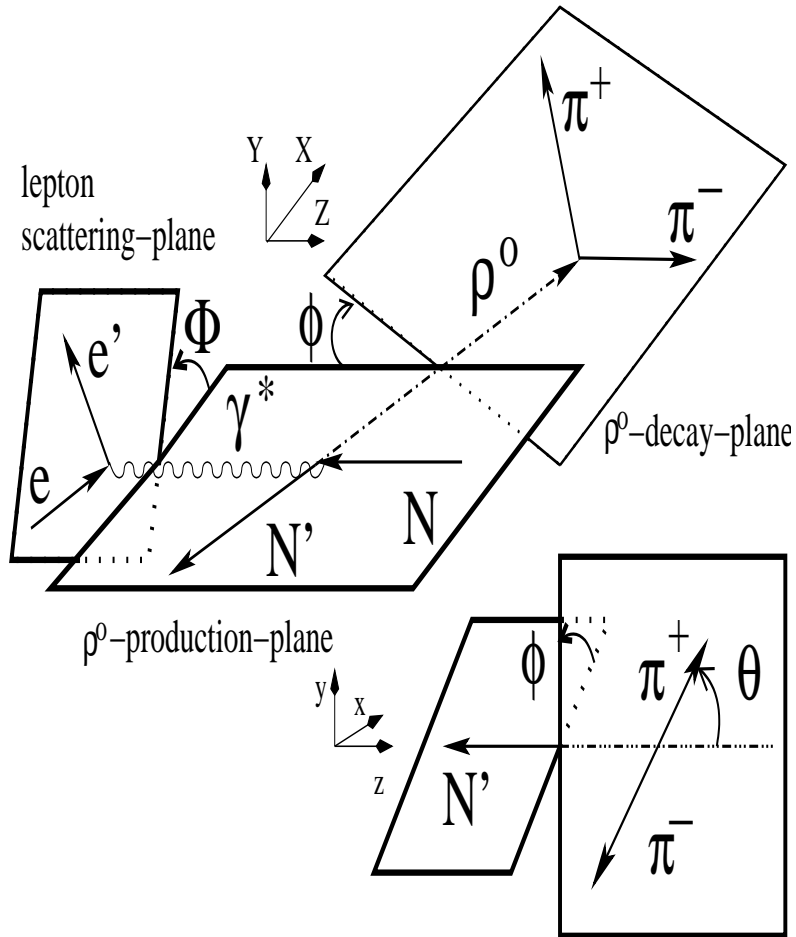
- $e \rightarrow e' + \gamma^*$ (QED). Spin-density matrix $\varrho_{\lambda_\gamma \lambda'_\gamma}^{U+L}(\epsilon, \Phi) = \varrho_{\lambda_\gamma \lambda'_\gamma}^U + P_{beam} \varrho_{\lambda_\gamma \lambda'_\gamma}^L$ of the virtual photon is known. U - unpolarized, L - polarized beam
- $\gamma^* + N \rightarrow \rho^0 + N \rightarrow \pi^+ + \pi^- + N$ (QCD). Vector-meson spin-density matrix $\rho_{\lambda_V \lambda'_V}$ is expressed by helicity amplitudes $F_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N}(W, Q^2, t')$. In CM frame of $\gamma^* N$ is given by the von Neumann formula:

$$\rho_{\lambda_V \lambda'_V} = \frac{1}{2N} \sum_{\lambda_\gamma \lambda'_\gamma \lambda_N \lambda'_N} F_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N} \varrho_{\lambda_\gamma \lambda'_\gamma}^{U+L} F_{\lambda'_V \lambda'_N; \lambda'_\gamma \lambda_N}^*$$

- After decomposition of $\varrho_{\lambda_\gamma \lambda'_\gamma}^{L+U}$ into the set of nine hermitian matrices (3×3) Σ^α ($\alpha=0 \div 3$ - transv., 4 - long. 5 \div 8 - interf.), when we can not separate transverse and longitudinal photons, Spin Density Matrix Elements (SDMEs) are defined:

$$r_{\lambda_V \lambda'_V}^{04} = (\rho_{\lambda_V \lambda'_V}^0 + \epsilon R \rho_{\lambda_V \lambda'_V}^4) / (1 + \epsilon R),$$

$$r_{\lambda_V \lambda'_V}^\alpha = \begin{cases} \frac{\rho_{\lambda_V \lambda'_V}^\alpha}{(1 + \epsilon R)}, & \alpha = 1, 2, 3, \\ \frac{\sqrt{R} \rho_{\lambda_V \lambda'_V}^\alpha}{(1 + \epsilon R)}, & \alpha = 5, 6, 7, 8. \end{cases} \quad R = \sigma_L / \sigma_T$$



- $\rho^0 \Rightarrow \pi^+\pi^-$ (conservation of \vec{J})
 $|\rho^0; 1m\rangle \rightarrow |\pi^+\pi^-; 1m\rangle \Rightarrow Y_{1m}(\cos(\theta), \phi)$,
 $(m = \pm 1, 0)$. Angular distribution $\mathcal{W}(\Phi, \phi, \cos \Theta)$
depends linearly on $r_{\lambda_V \lambda'_V}^\alpha$ and beam polarization P_b .
- For longitudinally polarized beam and unpolarized target there are **23** SDMEs, which are determined from the fit of angular distribution of pions from decay $\rho^0 \Rightarrow \pi^+\pi^-$
- In turn, all SDMEs are bilinear combination of helicity amplitudes and in "principle" can also be determined from the fit of angular distribution.

- Total number of amplitudes:

 - Initial state 3 spin states of $\gamma^* \lambda_\gamma = (1, 0, -1)$ and 2 nucleon helicities $\lambda_N = (\frac{1}{2}, -\frac{1}{2})$
 - Final state 3 spin states of $\rho^0 \lambda'_V = (1, 0, -1)$ and nucleons $\lambda'_N = (\frac{1}{2}, -\frac{1}{2}) \rightarrow$ **36** amplitudes
 - Due to parity conservation:

$$F_{-\lambda_V - \lambda'_N; -\lambda_\gamma - \lambda_N} = (-1)^{(\lambda_V - \lambda'_N) - (\lambda_\gamma - \lambda_N)} F_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N} \rightarrow$$
 18 amplitudes.
- Helicity amplitude can be decomposed into a sum of an amplitude **T** for natural-parity exchange (NPE) ($P = (-1)^J$) and an amplitude **U** for unnatural-parity exchange (UPE) ($P = -(-1)^J$).

$$F_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N} = T_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N} + U_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N}$$
- The amplitudes obey the following symmetry relation:

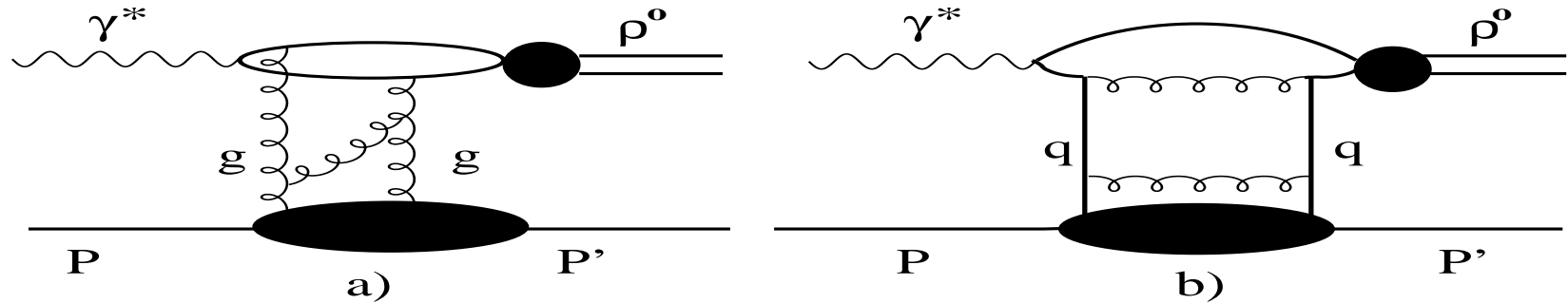
$$T_{\lambda_V \lambda_{N'}; \lambda_\gamma \lambda_N} = (-1)^{-\lambda_V + \lambda_\gamma} T_{-\lambda_V \lambda_{N'}; -\lambda_\gamma \lambda_N} = (-1)^{-\lambda'_N + \lambda_N} T_{\lambda_V - \lambda'_N; \lambda_\gamma - \lambda_N}$$

$$U_{\lambda_V \lambda_{N'}; \lambda_\gamma \lambda_N} = -(-1)^{-\lambda_V + \lambda_\gamma} U_{-\lambda_V \lambda_{N'}; -\lambda_\gamma \lambda_N} = -(-1)^{-\lambda'_N + \lambda_N} U_{\lambda_V - \lambda'_N; \lambda_\gamma - \lambda_N}$$
- Due to these symmetry relations production of vector meson is described by **10 NPE and 8 UPE amplitudes**.

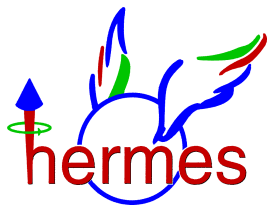
No UPE amplitude exists for the transition $\gamma_L \rightarrow \rho_L^0$. $T_{00} \equiv F_{00} \equiv F_{0\frac{1}{2}0\frac{1}{2}}$.

For unpolarized target there is no interference between NPE and UPE amplitudes.
- The SDMEs are expressed by **18 complex helicity amplitudes, 36 parameters (real and imaginary parts)**.

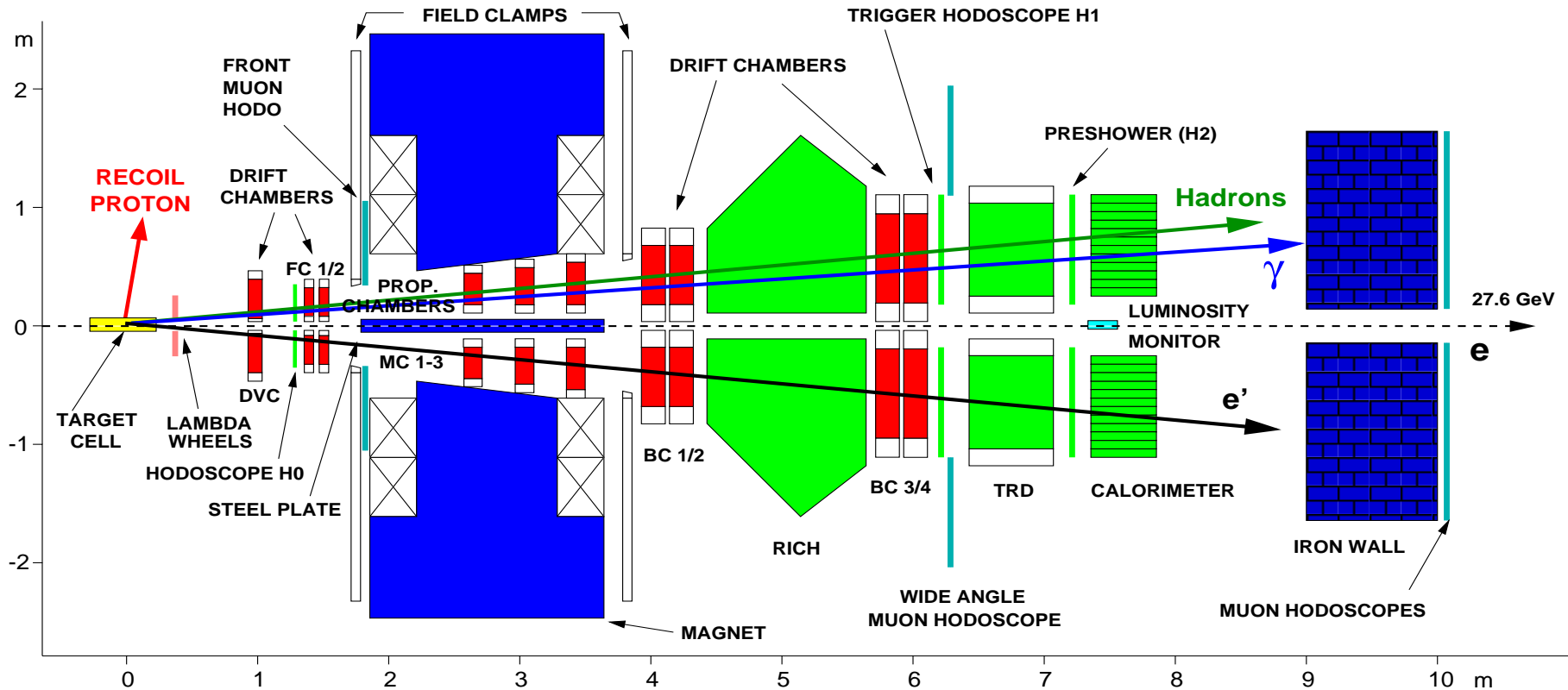
- On unpolarized target there is no linear contribution of nucleon-helicity-flip amplitudes to SDMEs (suppressed by factor $(\alpha)^2 = (\frac{\sqrt{-t'}}{M})^2$ ($t' = t - t_{min}$). This reduces the number of NPE amplitudes to five: Helicity conserving T_{00}, T_{11} , helicity non conserving T_{01}, T_{10}, T_{1-1} , where we used shorthand notation $T_{\lambda_V \lambda_\gamma} = T_{\lambda_V \frac{1}{2} \lambda_\gamma \frac{1}{2}}$. The dominance of diagonal transitions is called s-channel helicity conservation (SCHC).
- From the SDME analysis it has been found that for UPE transitions amplitudes obey the following hierarchy: $|U_{01}|^2, |U_{10}|^2, |U_{1-1}|^2 \ll |U_{11}|^2$, we keep only $|U_{11}| = \sqrt{|U_{1\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1-\frac{1}{2}1\frac{1}{2}}|^2}$. For UPE amplitudes it is not possible to prove the dominance of those without spin flip over those with spin flip.
- The hierarchy of amplitudes in the kinematic region of HERMES is:
 $|T_{00}|^2 \sim |T_{11}|^2 \gg |U_{11}|^2 > |T_{01}|^2 > |T_{10}|^2 \sim |T_{-1-1}|^2$,
- Since SDMEs depend rather on ratios of these complex amplitudes, the number of real parameters which determine all SDMEs is **9 (real and imaginary parts)**.
- Finally, we approximated the SDMEs through **9 real parameters**, namely: $Re\{T_{11}/T_{00}\}, Im\{T_{11}/T_{00}\}, Re\{T_{01}/T_{00}\}, Im\{T_{01}/T_{00}\}, Re\{T_{10}/T_{00}\}, Im\{T_{10}/T_{00}\}, Re\{T_{1-1}/T_{00}\}, Im\{T_{1-1}/T_{00}\}, |U_{11}/T_{00}|$ where $|U_{11}/T_{00}|$ is the module of U_{11}/T_{00} .



- NPE ($J^P = 0^+, 1^-, \dots$) amplitudes $T_{\lambda_V \lambda_\gamma}$ (Two-gluon exchange = pomeron, ρ , ω, a_2, \dots reggeons = $q\bar{q}$ exchange). UPE ($J^P = 0^-, 1^+, \dots$) amplitudes $U_{\lambda_V \lambda_\gamma}$ (π, a_1, b_1, \dots reggeons = $q\bar{q}$ exchange) In the GPD formalism, NPE amplitudes are described by H and E, UPE by \tilde{H} , \tilde{E} . The amplitude ratios can be used to distinguish between contribution of NPE and UPE processes. For this aim, **an amplitude ratio is more convenient than SDMEs** as any SDME depends on all amplitude ratios.
- Violation of s -channel helicity ($\lambda_V \neq \lambda_\gamma$) can be studied more reliably using amplitude ratios rather than SDMEs. The spin flip amplitudes T_{01}, T_{10} provide information on valence quark motion in vector meson. (They are to be zero in the absence of quark motion in meson). The double spin flip amplitudes T_{1-1} contain information on gluon distribution in nucleon.
- Difference between proton and deuteron results would point to a contribution of $q\bar{q}$ -exchange with isospin $I = 1$ and natural parity $P = (-1)^J$ (ρ, a_0, a_2 reggeons).
- The present work is a continuation of the Spin Density Matrix Elements (SDME) analysis



Hermes Detector was Two Identical Halves of Forward Spectrometer

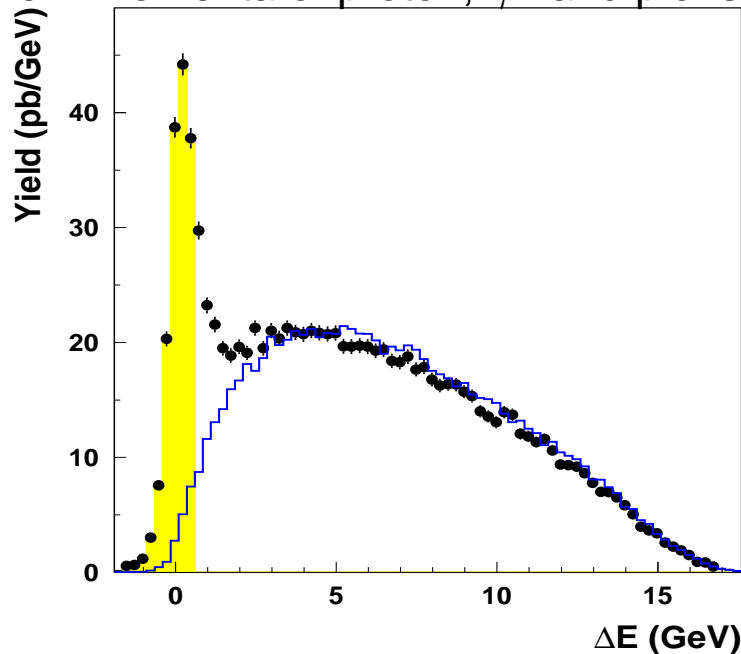


- Beam e^\pm , $P = 27.56$ GeV/c longitudinal polarization ~ 55 %.
- Target longitudinally, transversely polarized H or D or unpolarized gas target.
- Acceptance: $|\Theta_x| < 170$ mrad, $40 < |\Theta_y| < 140$ mrad.
- Resolution $\delta P/P \leq 1\%$, $\delta\Theta \leq 0.6$ mrad.
- PID: RICH, TRD, Preshower, Calorimeter.

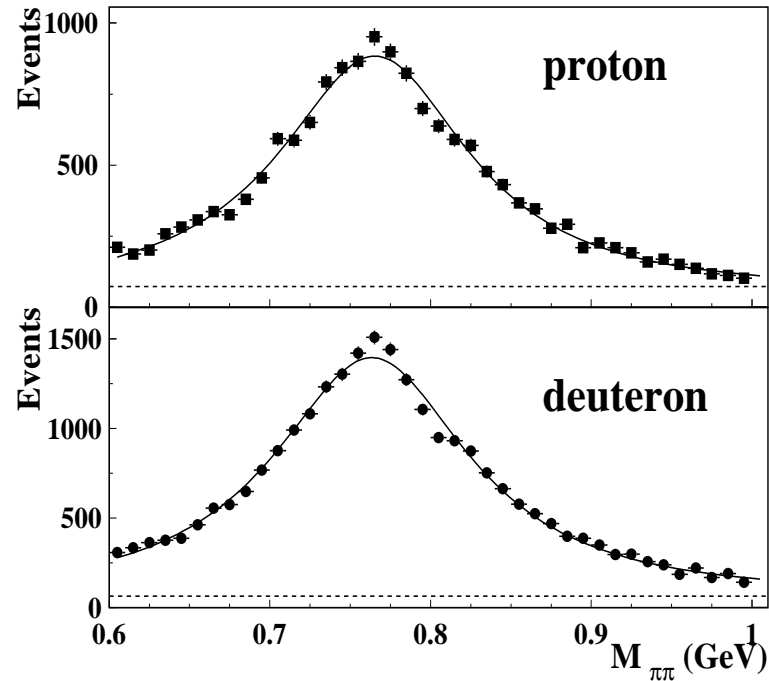
- $W = 3.0 \div 6.5 \text{ GeV}$, $\langle W \rangle = 4.9 \text{ GeV}$ total number of events (1996-2005) $W^2 = (q + p)^2$
- $Q^2 = 0.5 \div 7.0 \text{ GeV}^2$, $\langle Q^2 \rangle = 1.95 \text{ GeV}^2$ Deuteron: ρ^0 - 16388 $Q^2 = -(k - k')^2$
- $x_B = 0.01 \div 0.35$, $\langle x_B \rangle = 0.08$ Hydrogen: ρ^0 - 9860 $x_B = \frac{Q^2}{2pq}$
- $0 \leq -t' \leq 0.4 \text{ GeV}^2$, $\langle -t' \rangle = 0.13 \text{ GeV}^2$ with $t' = t - t_{min}$ $t = (q - v)^2$

$$\Delta E = \frac{M_X^2 - M_p^2}{2M_p} \text{ with } M_X^2 = (p + q - p_{\pi^+} - p_{\pi^-})^2 \text{ and } M_X \text{ being missing mass, } p, q, p_{\pi^+}, p_{\pi^-}$$

are 4-momenta of proton, γ^* and pions.

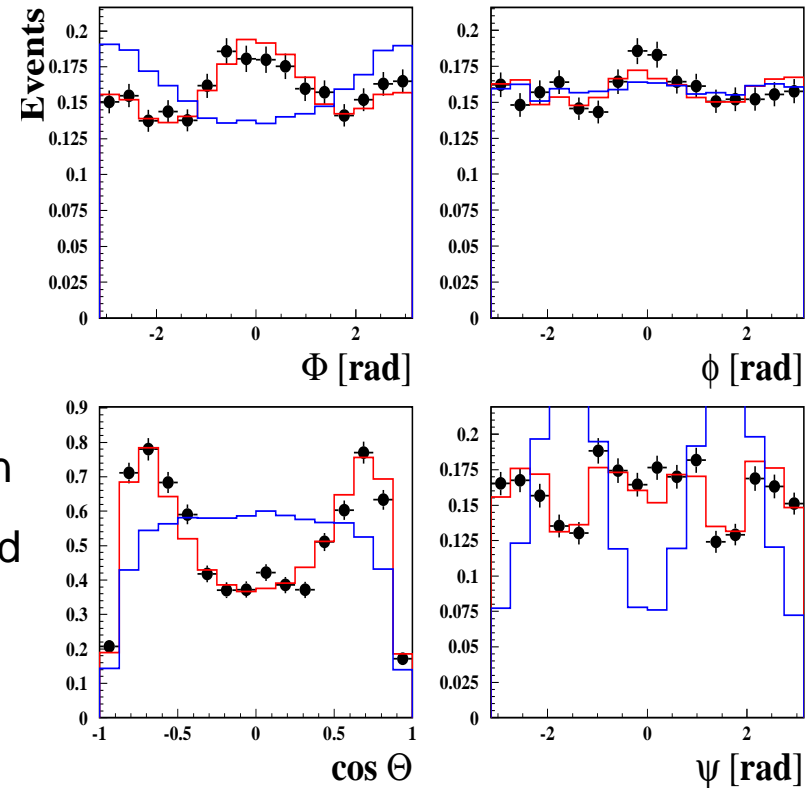


$$-1.0 < \Delta E < 0.6 \text{ GeV},$$

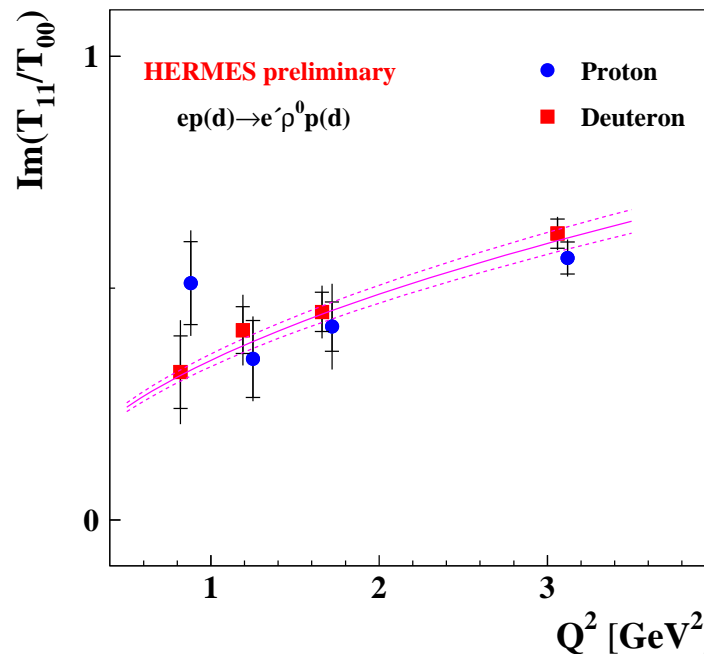
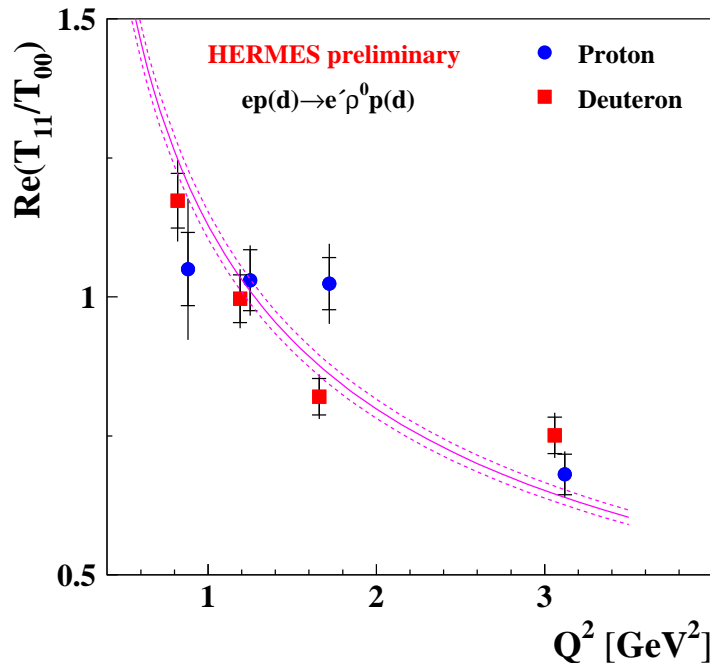


$$0.6 < M_{\pi\pi} < 1 \text{ GeV},$$

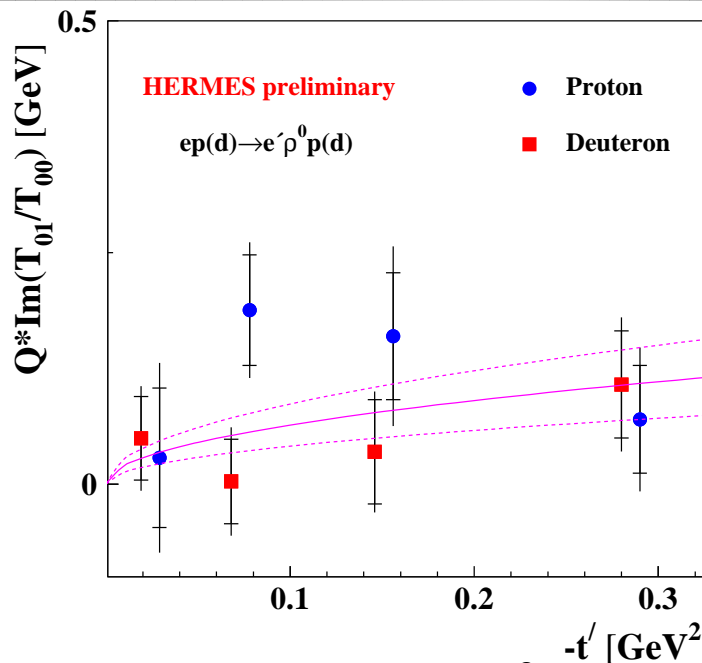
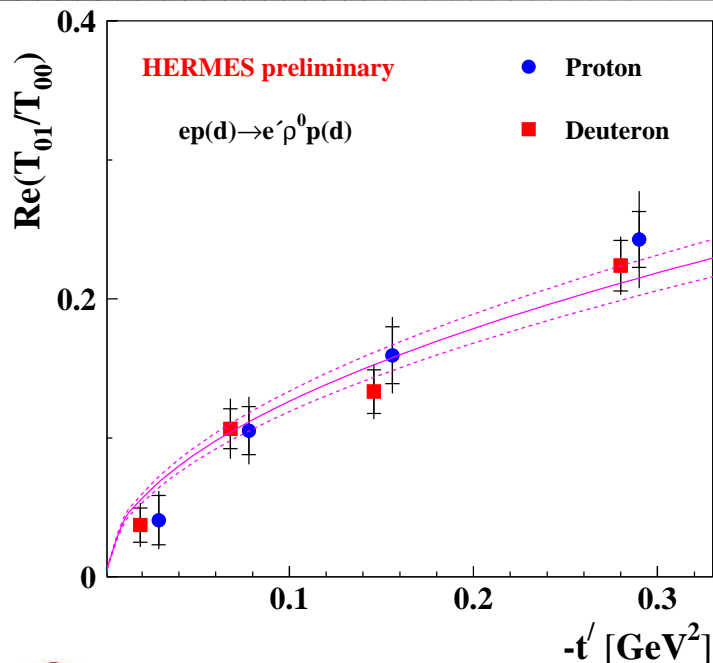
- Amplitude ratios are extracted **directly** from the measured angular distribution using **Binned Maximum Likelihood (BML)** method.
- 3-dimensional matrix $(\cos\Theta, \phi, \Phi)$ of data. $(8 \times 8 \times 8)$ cells.
- 3-dimensional matrix of fully reconstructed MC events generated with uniform angular distribution
- Minimizing the difference between data matrix and MC matrix reweighted by $\mathcal{W}(\Phi, \phi, \cos\Theta)$ which depends on 5 ratios of helicity amplitudes, i.e. 9 real fitted parameters. Simultaneous fit for data with negative and positive beam helicity ($\langle P_b \rangle = 47.0\%$)
- There is agreement of fitted angular distribution with the HERMES data.
- The amplitude ratios are extracted with the same **BML** method as SDMEs [EPJ C62\(2009\) 659](#).



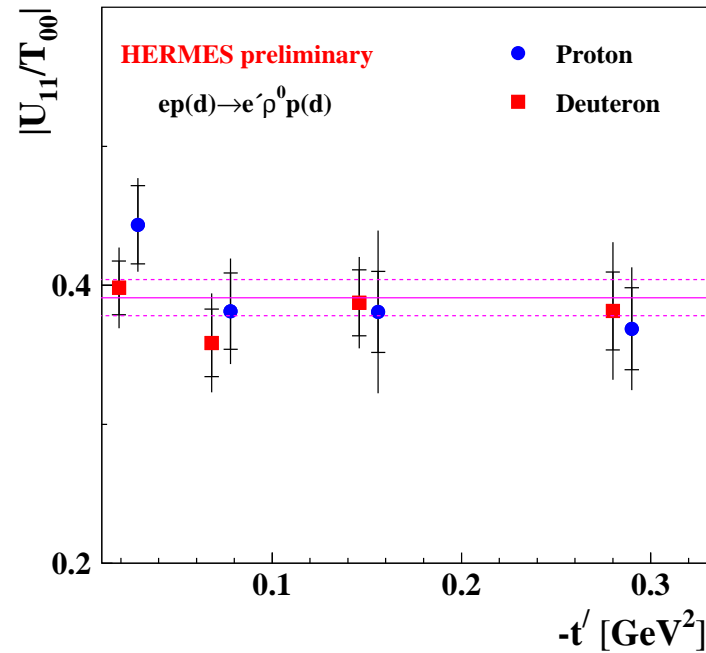
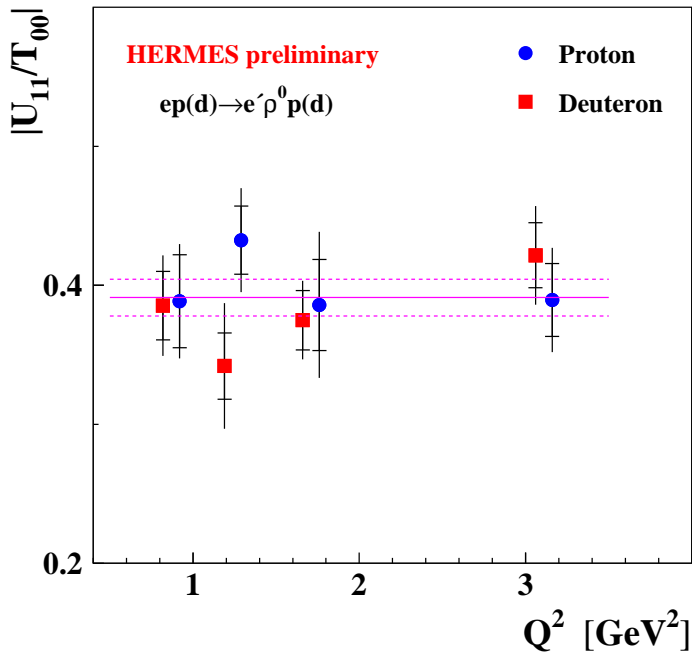
An example of fitted angular distribution. The blue lines represent isotropic input Monte Carlo distribution as modified by the HERMES acceptance, while the red lines are the results of the fit. $\Psi = \phi + \Phi$ (SCHC approximation)



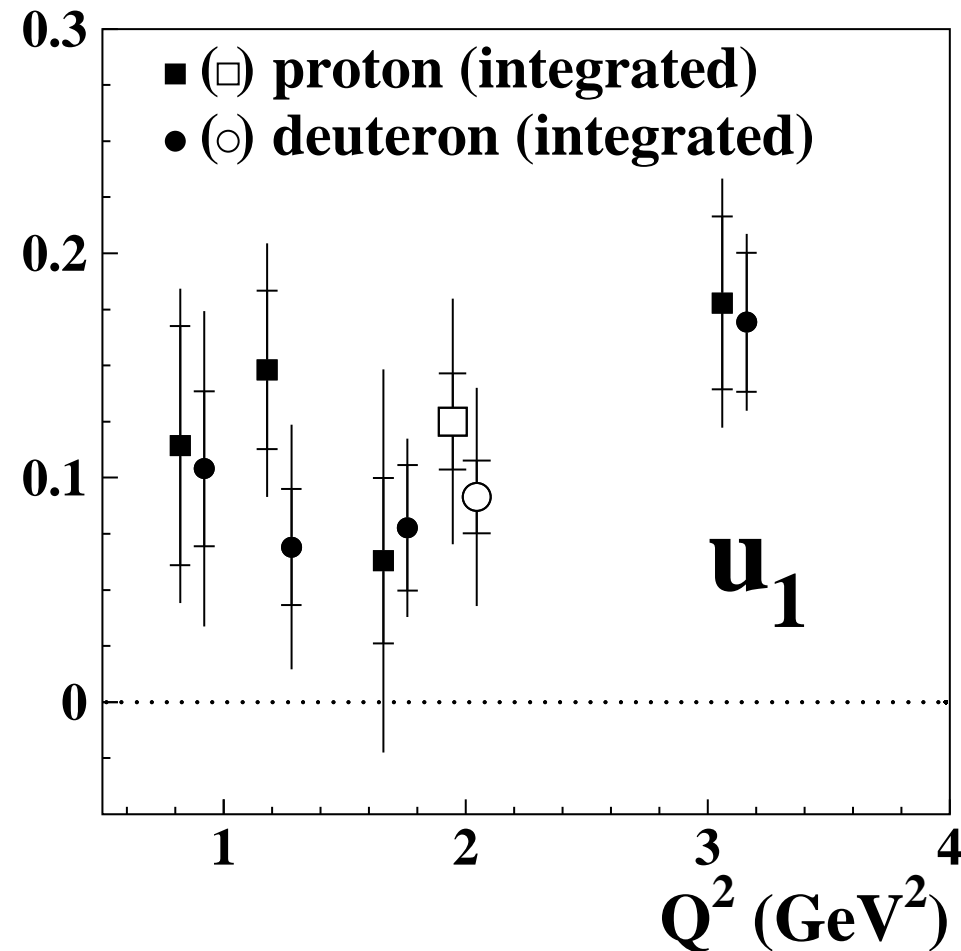
- No difference between proton and deuteron for amplitude ratio T_{11}/T_{00} .
- pQCD predicts the following dependence: $T_{11}/T_{00} \propto M_p/Q$.
- The Q dependence of T_{11}/T_{00} is fitted with $Re\{T_{11}/T_{00}\} = a/Q$, $Im\{T_{11}/T_{00}\} = b \cdot Q$.
Combined data on proton and deuteron: $a = 1.129 \pm 0.024 \text{ GeV}$, $\chi^2/N_{df} = 1.02$;
 $b = 0.344 \pm 0.014 \text{ GeV}^{-1}$, $\chi^2/N_{df} = 0.87$.
- Behaviour of $Im\{T_{11}/T_{00}\}$ is in **contradiction** with high- Q asymptotics in pQCD.
- The Q^2 dependence of the phase difference δ_{11} between the amplitudes T_{11} and T_{00} is given by $\tan \delta_{11} = Im\{T_{11}/T_{00}\}/Re\{T_{11}/T_{00}\} = bQ^2/a$.
Phase difference is $\delta_{11} \sim 30^\circ$ at $\langle Q^2 \rangle = 1.95 \text{ GeV}^2$ and grows with Q^2 in **disagreement** with pQCD calculation.



- The amplitude $T_{01} = T_{0\frac{1}{2}1\frac{1}{2}}$ describing the transition $\gamma_T^* \rightarrow \rho_L^0$ is the **largest** SCHC-violating amplitude.
- There is no difference between proton and deuteron for amplitude ratio T_{01}/T_{00} .
- pQCD predicts the following dependence: $\frac{T_{01}}{T_{00}} \propto \frac{\sqrt{-t'}}{Q}$.
- the t' dependence of T_{01}/T_{00} is fitted with
 $Re(T_{01}/T_{00}) = a\sqrt{-t'}$, $Im(T_{01}/T_{00}) = b\sqrt{-t'}/Q$.
 Combined proton and deuteron data: $a = 0.399 \pm 0.023 \text{ GeV}^{-1}$, $\chi^2/N_{df} = 0.72$;
 $b = 0.20 \pm 0.07$, $\chi^2/N_{df} = 1.09$.



- No difference between proton and deuteron for amplitude ratio $|U_{11}/T_{00}|$.
- pQCD predicts the following dependence: $U_{11}/T_{00} \propto M_p/Q$.
- We do not see either Q^2 or t' dependence: $|U_{11}|/|T_{00}| = a$, $a = 0.391 \pm 0.013$, $\chi^2/N_{df} = 0.44$
- **Contradiction** with both high-Q asymptotic and one-pion-exchange dominance.
- Unnatural Parity Exchange is **seen here much better** than in SDME method.



- Natural and Unnatural Parity Exchanges in the t -channel
 NPE: GPD H, E ; $T_{\lambda\rho\lambda\gamma}$
 UPE: GPD \tilde{H}, \tilde{E} ; $U_{\lambda\rho\lambda\gamma}$
 NPE (Pomeron, $\rho, \omega, f_2, a_2, \dots$) dominate and
 UPE (π, a_1, b_1, \dots) are suppressed at high energies

- Signal of UPE in SDME method

$$u_1 = 1 - r_{00}^4 + 2r_{1-1}^4 - 2r_{11}^4 - 2r_{1-1}^4,$$

$$u_1 = \sum_{\lambda_N \lambda'_N} \frac{2\epsilon |U_{10}|^2 + |U_{11} + U_{-11}|^2}{N}$$

where $N = N_T + \epsilon N_L$,

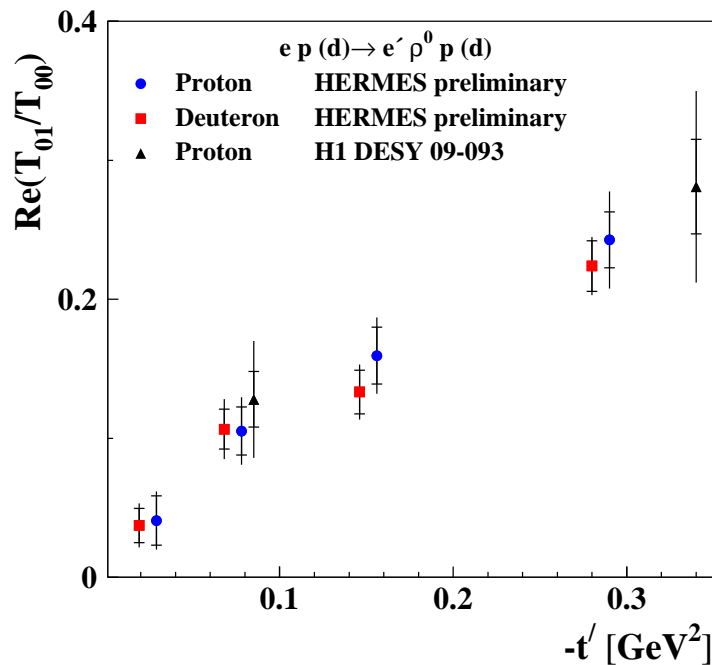
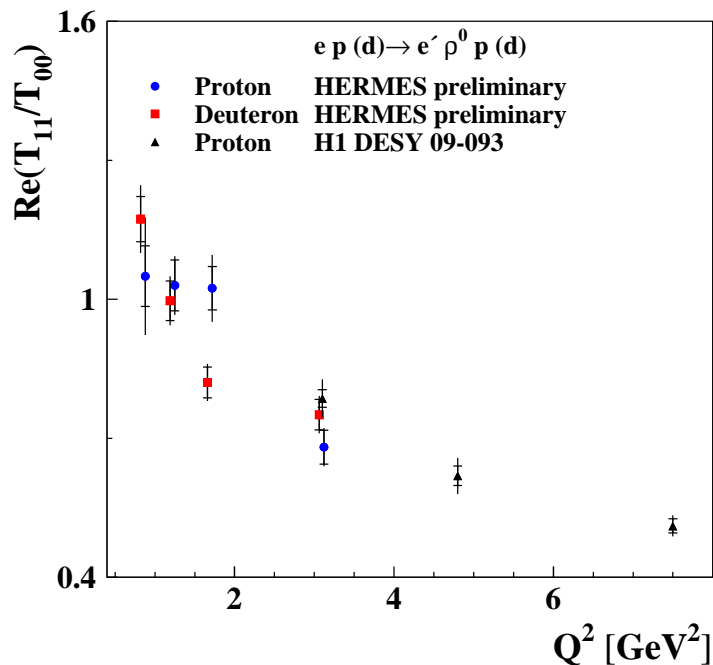
$$N_T = \sum_{\lambda_N \lambda'_N} (|T_{11}|^2 + |T_{01}|^2 + |T_{-11}|^2 + |U_{11}|^2 + |U_{01}|^2 + |U_{-11}|^2),$$

$$N_L = \sum_{\lambda_N \lambda'_N} (|T_{00}|^2 + |T_{10}|^2 + |T_{-10}|^2 + |U_{10}|^2 + |U_{-10}|^2).$$

$$u_1 = 0.125 \pm 0.021_{stat} \pm 0.050_{syst} \text{ (H)},$$

$$u_1 = 0.091 \pm 0.016_{stat} \pm 0.046_{syst} \text{ (D)}$$

$$u_1 = 0.106 \pm 0.036_{tot} \text{ (H+D)}$$



- H1: Unpolarized beam and unpolarized target (15 SDMEs), $\langle Q^2 \rangle = 3.3 \text{ GeV}^2$.
- Assumption: only NPE, all amplitudes are purely imaginary, all amplitude ratios are real.
- HERMES: Longitudinally polarized beam and unpolarized target (23 SDMEs).
- Both real and imaginary parts of ratios of helicity amplitudes are extracted.
- **Excellent agreement of amplitude ratios extracted by H1 and HERMES.**

- Study of electroproduction of ρ^0 vector meson on proton and deuteron enables to obtain ratios of helicity amplitudes, and to investigate their kinematic dependences.
- The kinematic dependences of $Im\{T_{11}/T_{00}\}$, $|U_{11}/T_{00}|$ are in contradiction with high-Q asymptotics behavior predicted in pQCD. The dependences of $Re\{T_{11}/T_{00}\}$ and $Im\{T_{01}/T_{00}\}$ are in agreement with pQCD prediction.
- The amplitude ratios for deuterons are compatible with those for protons.
- The UPE signal is seen here with very high significance for both proton and deuteron data and with higher precision than that obtained in SDME method.
- Violation of S-channel helicity conservation is determined with higher accuracy from studying amplitudes than from SMDEs.