DIFFRACTION 2008
International Workshop on Diffraction in High-Energy Physics
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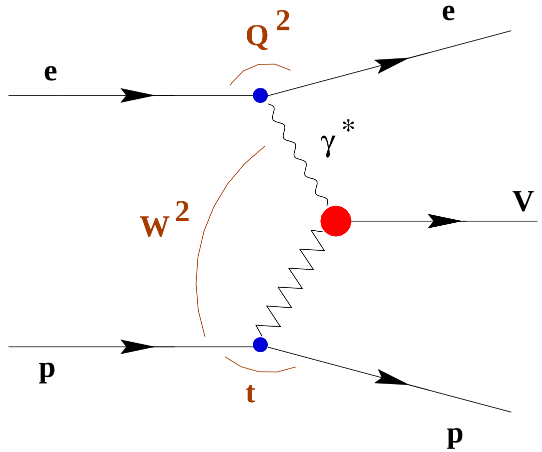
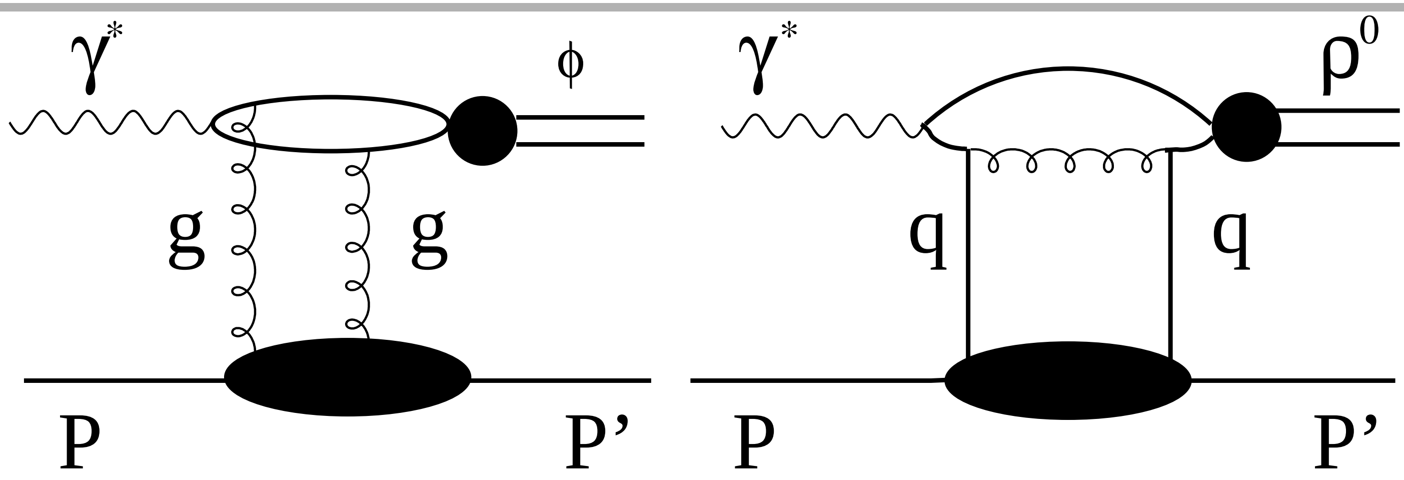
Spin Density Matrix Elements from ρ^0 and ϕ Meson Electroproduction at hermes

- Objectives: Generalized Parton Distributions
- Total and Longitudinal Cross Sections of ρ^0 and ϕ
- ρ^0 and ϕ Meson Spin Density Matrix Elements
  - Longitudinal-to-Transverse Cross-Section Ratios
  - Kinematic Dependences
  - Hierarchy of Helicity Amplitudes
  - Unnatural Parity Exchange
- Summary and Outlook



Alexander Borissov, DESY, on behalf of HERMES Collaboration

Test of GPDs via Exclusive Vector Meson Production



Properties of ρ^0 and ϕ meson data:

- different pQCD production mechanisms:
  - only two-gluon exchange for ϕ,
  - both two-gluon and quark exchanges for ρ^0

  $\longrightarrow$ GPDs as a flavor filter

- quark exchange mediated by
  - vector or scalar meson: ρ^0, ω, a_2 (natural parity: $J^P = 0^+, 1^-$) $\longrightarrow$ GPDs: H, E
  - pseudoscalar or axial meson: π, a_1, b_1 (unnatural parity $J^P = 0^-, 1^+$) $\longrightarrow$ GPDs: $\tilde{H}$, $\tilde{E}$

Experimental observables:

- total and logitudinal cross sections σ_{tot}, σ_L
- Spin Density Matrix Elements (SDMEs):
  $r^{\alpha}_{\lambda_\rho \lambda'_\rho} \sim \rho(V) = \frac{1}{2} T \rho(\gamma) T^+$
  vector meson spin-density matrix $\rho(V)$ via photon matrix $\rho(\gamma)$ and helicity amplitude $T_{\lambda_V \lambda_\gamma}$
  - *s-channel helicity conservation (SCHC)?* i.e. helicity of γ^* = helicity of ρ^0
  - Extracted from SDMEs natural and unnatural parity *helicity amplitudes and its ratios*
- Beam and target polarization asymmetries

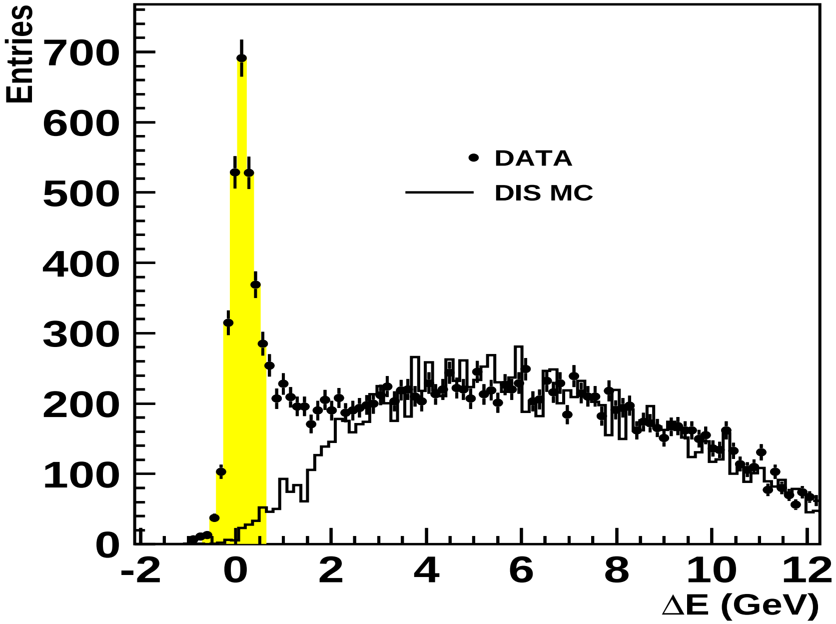
$\Longrightarrow$ **Comparison with GPD based calculations** S. V. Goloskokov, P. Kroll, Eur. Phys. J. C 53(2008) 367

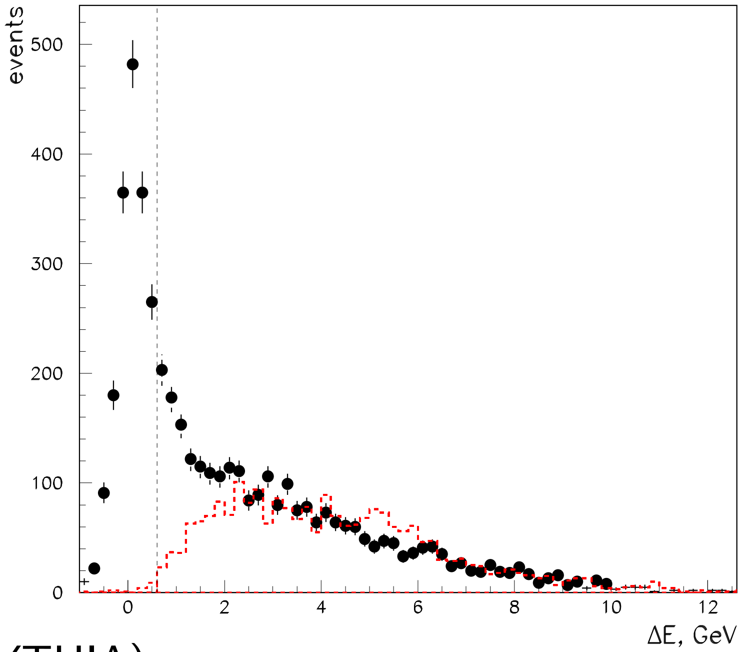
Exclusive ρ^0 and ϕ Meson Production

$$e + p \rightarrow e' + p' + \rho^0 \rightarrow \pi^+\pi^- \qquad e + p \rightarrow e' + p' + \phi \rightarrow K^+K^-$$

Clean exclusivity peaks of missing energy $\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$ for

ρ^0 ϕ





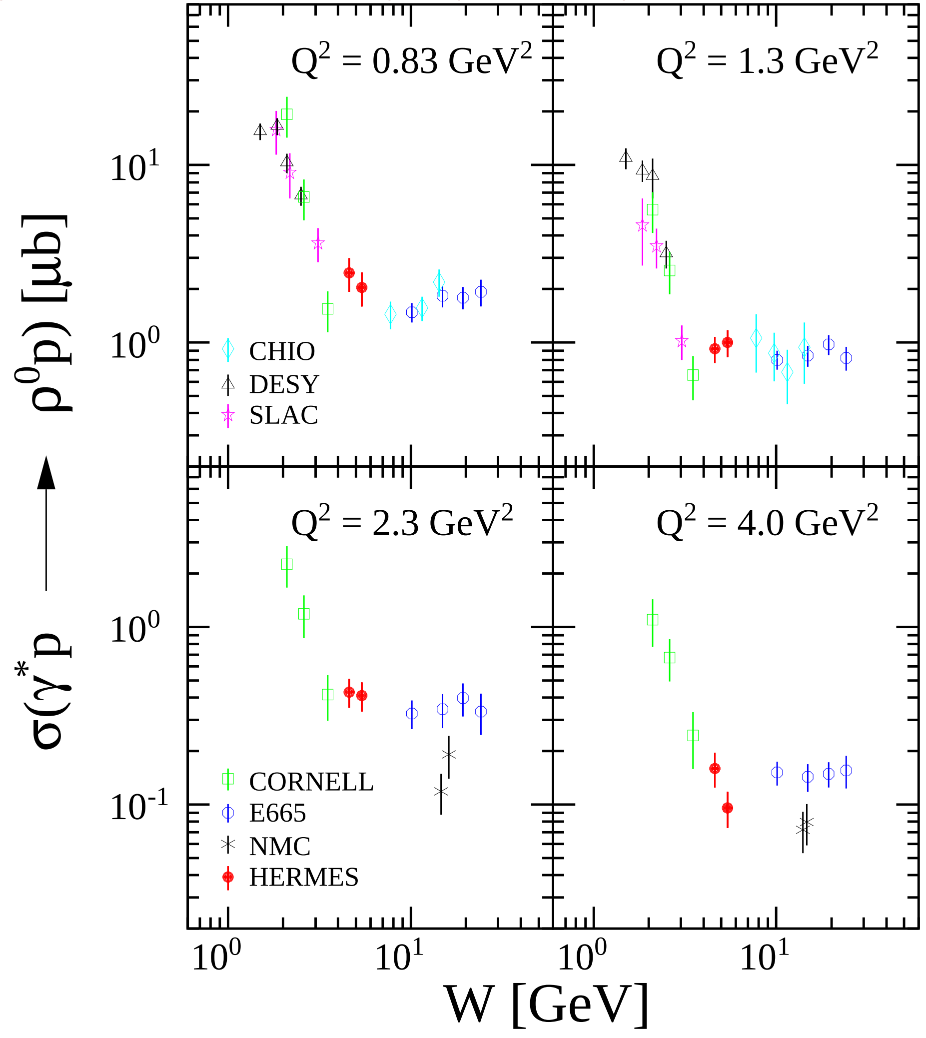
Background is subtracted using MC (PYTHIA)

Kinematics:

- $Q^2 = 0.5 \div 7.0$ GeV^2, $\langle Q^2 \rangle = 2.3$ GeV^2 $\qquad$ $W = 3.0 \div 6.5$ GeV, $\langle W \rangle = 4.9$ GeV,
- $x_{Bj} = 0.01 \div 0.35$, $\langle x_{Bj} \rangle = 0.07$ $\qquad$ $t' = 0 \div 0.4$ GeV^2, $\langle t' \rangle = 0.13$ GeV^2

ρ^0 Total and Longitudinal Cross Sections, application of GPDs

(HERMES collab. EPJ C 17 (2000) 3, 389-398).



→HERMES data in the transition region

- The QCD factorization theorem is proven for the longitudinal part of the cross section J.Collins,L.L.Frankfurt,M.Strikman Phys.Rev.D**56**,2982 (1997);
- assuming SCHC:
  $\sigma_L = \frac{R}{1+\epsilon R}\sigma_{tot}$,
  where $R = \sigma_L/\sigma_T = \frac{r^{04}_{00}}{\epsilon(1-r^{04}_{00})}$
  - SDME r^{04}_{00} is measured from the fit of angular distributions (explained below)
  - longitudinal-to-transverse ratio of virtual photon fluxes

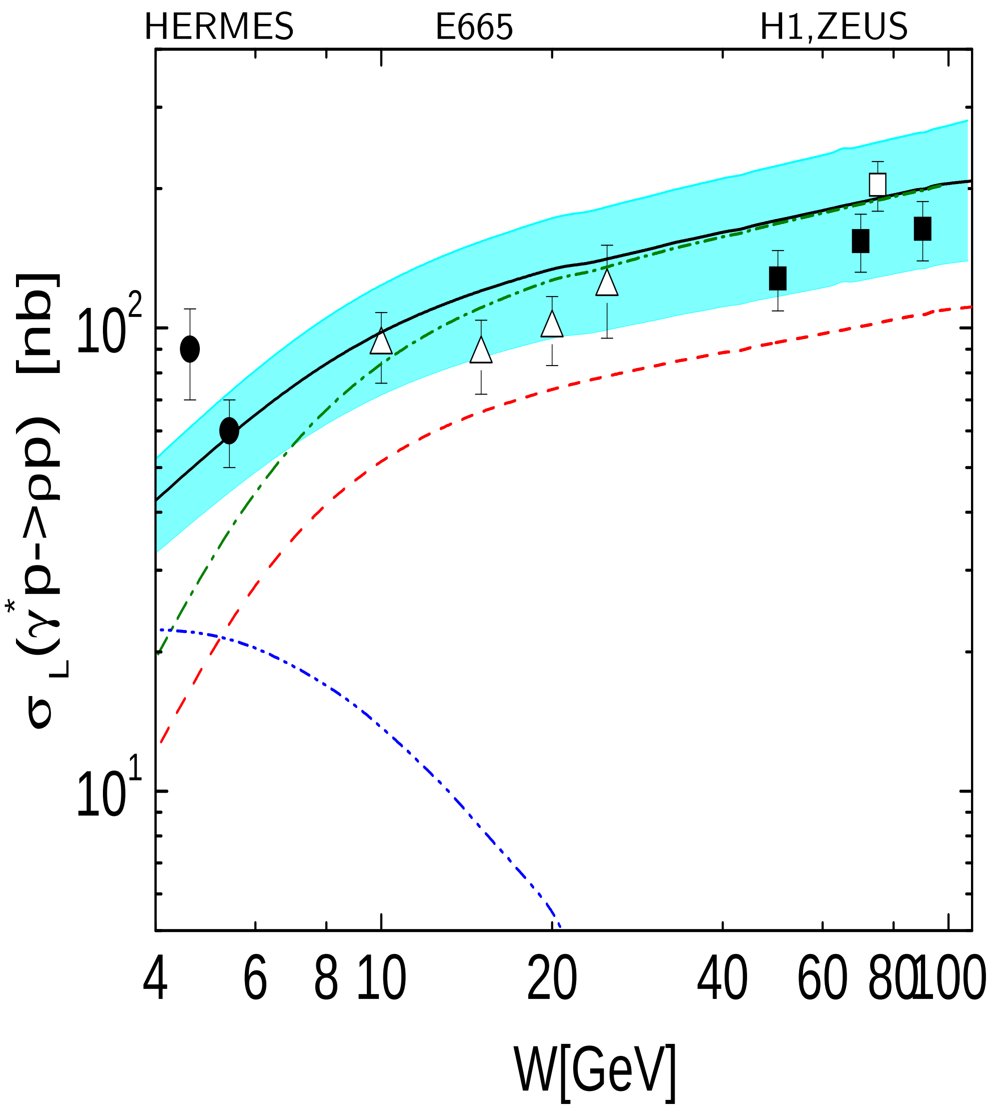
$$\epsilon = \frac{1-y-\frac{Q^2}{E^2}}{1-y+\frac{y^2}{2}+\frac{Q^2}{E^2}} \approx 0.8$$

$\Longrightarrow$ σ_L for the tests of spin-independent GPD function H

$\Longrightarrow$ **Which production mechanisms are involved?**

ρ^0 Total and Longitudinal Cross Sections, and GK Model

S.V.Goloskokov,P.Kroll,Eur.Phys.J. C 42,2005

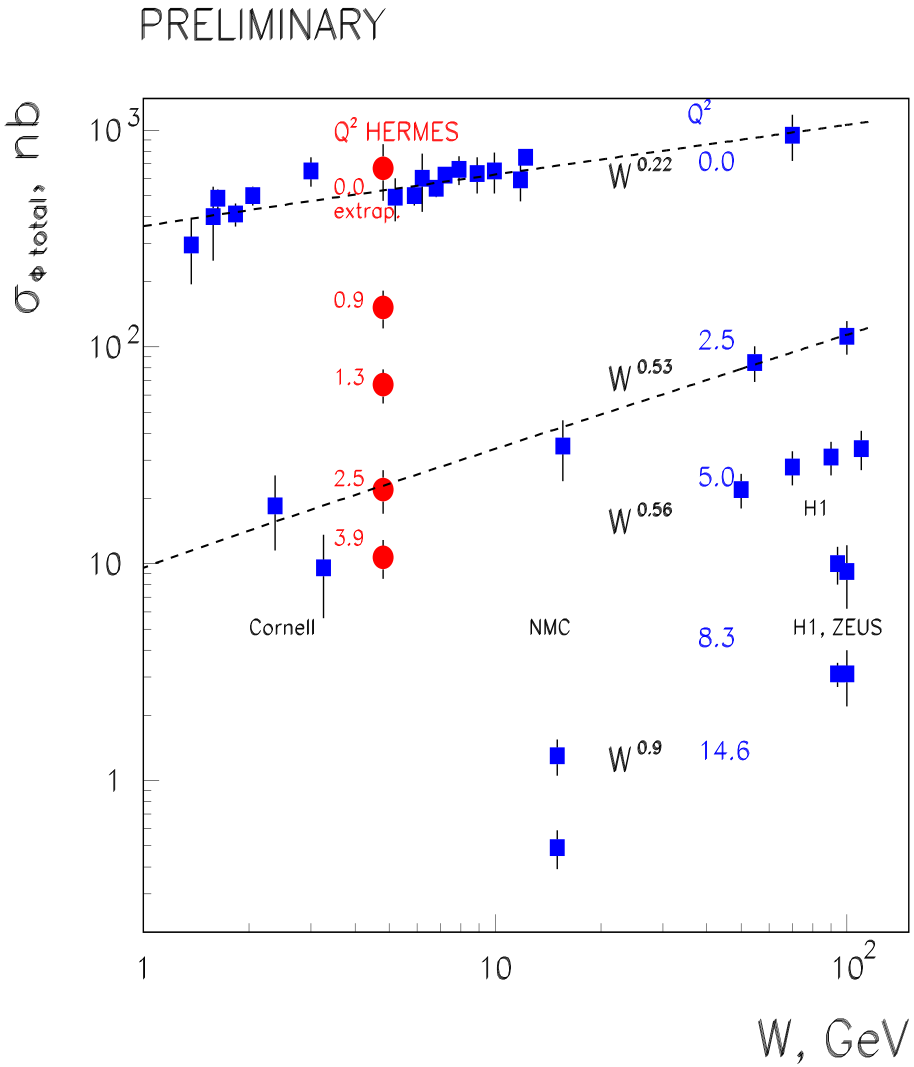


Which production mechanisms are involved?

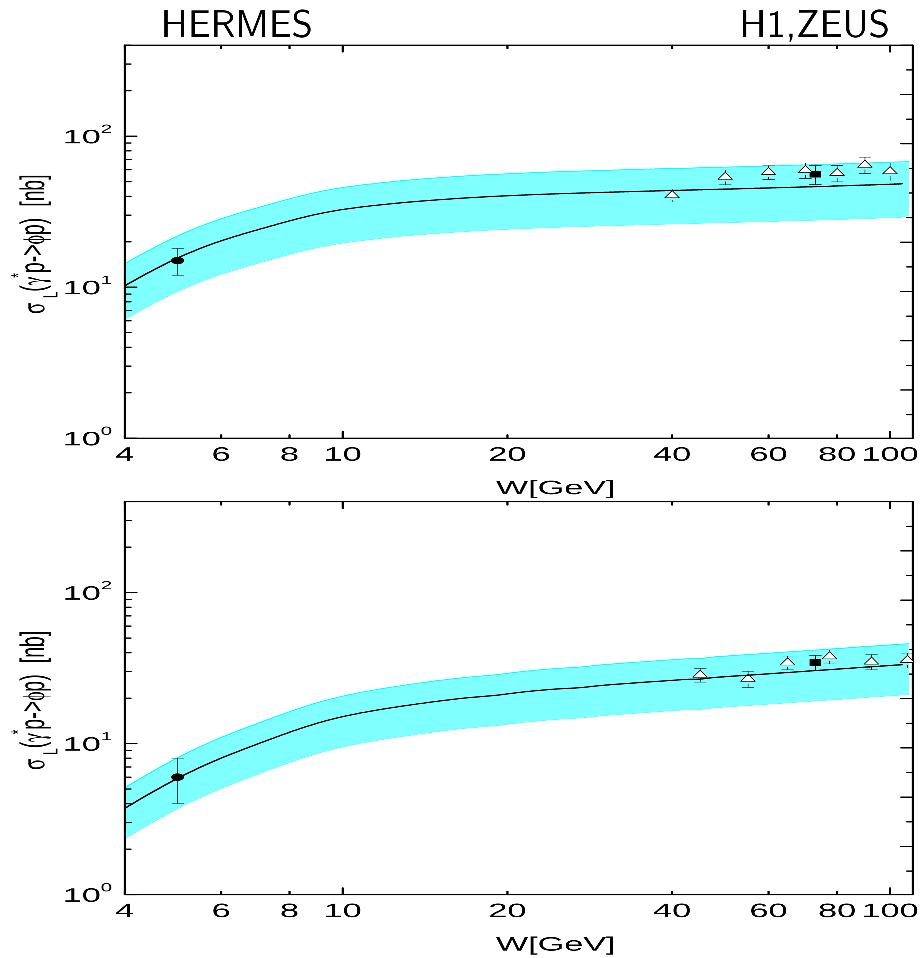
- two-gluon exchange
- two-gluon+sea interference
- quark exchange,
- sum, band represents uncertainties from Parton Distributions

$\Longrightarrow$ **Quark exchange is important for HERMES, i.e. at $W \leq 5$ GeV**

ϕ Total and Longitudinal Cross Sections, and GK model



S.V.Goloskokov,P.Kroll,Eur.Phys.J. C 42,2005



$\sigma_{L(\phi)}$: two-gluon exchange only

Band represents uncertainties in σ_L from Parton Distributions

$\longrightarrow$ $W^{\delta_\phi(Q^2)}$ dependence over all W

$\delta_\phi = 0.22$ at $Q^2 = 0$, $\delta_\phi = 0.53$ at $Q^2 = 2.5$ GeV^2

$\longrightarrow$ Two-gluon exchange is sufficient for σ^ϕ_{tot}

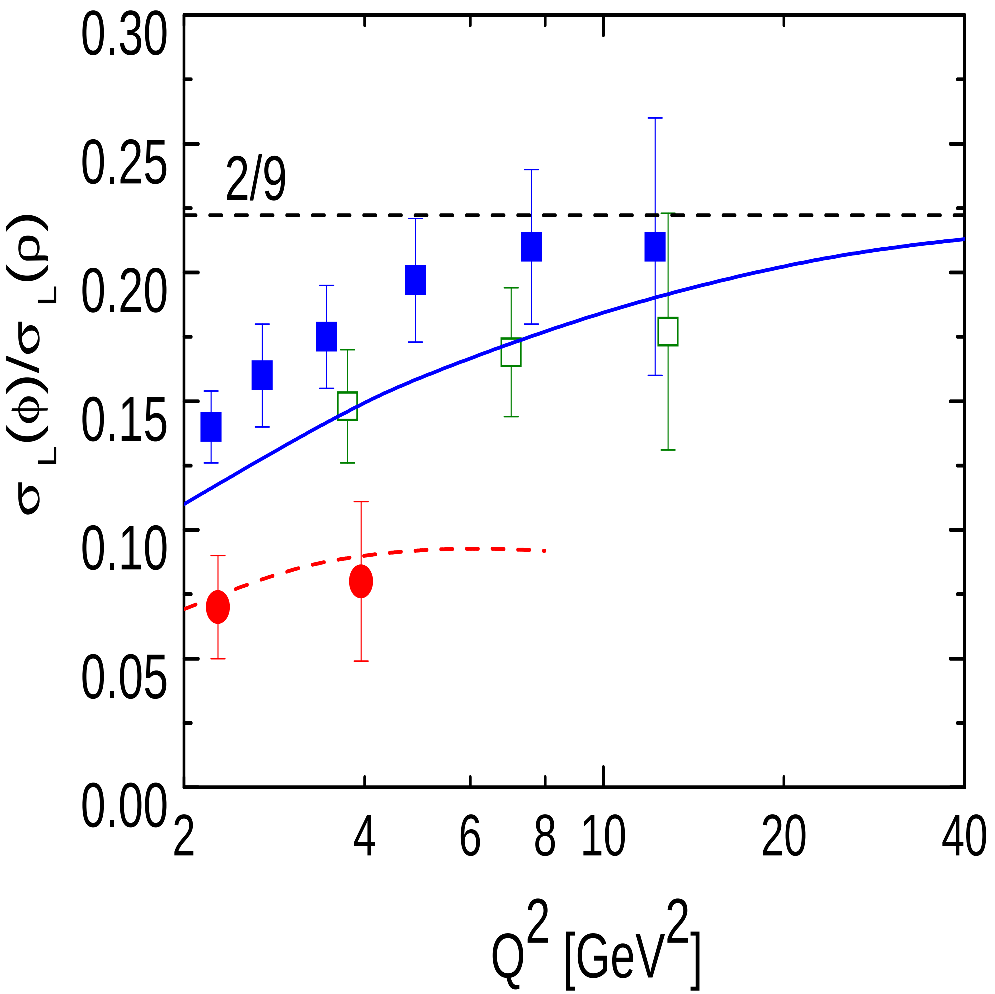
$\longrightarrow$ Good agreement of GK model calculations of $\sigma_L(W)$ at $Q^2 = 2.3, 3.8$ GeV^2.

$\Longrightarrow$ Two-gluon exchange is sufficient to describe σ_L in ϕ-meson production

Longitudinal Cross Section Ratios: $\sigma_{L(\phi)}/\sigma_{L(\rho^o)}$

Asymptotic SU(4) pQCD predicts: $\rho^o : \omega : \phi : J/\Psi = 9 : 1 : 2 : 8$

S.V.Goloskokov,P.Kroll,Eur.Phys.J. C 42,2005



W=75 GeV, H1 (closed),ZEUS (open squares), W=5 GeV, HERMES PRELIMINARY (circules)

$\Longrightarrow$ **Remarkable agreement of calculations with W-dep. of $\sigma_{L(\phi)}/\sigma_{L(\rho^o)}$ ratio**

ρ^0 & ϕ-meson Spin Density Matrix Elements (SDMEs)

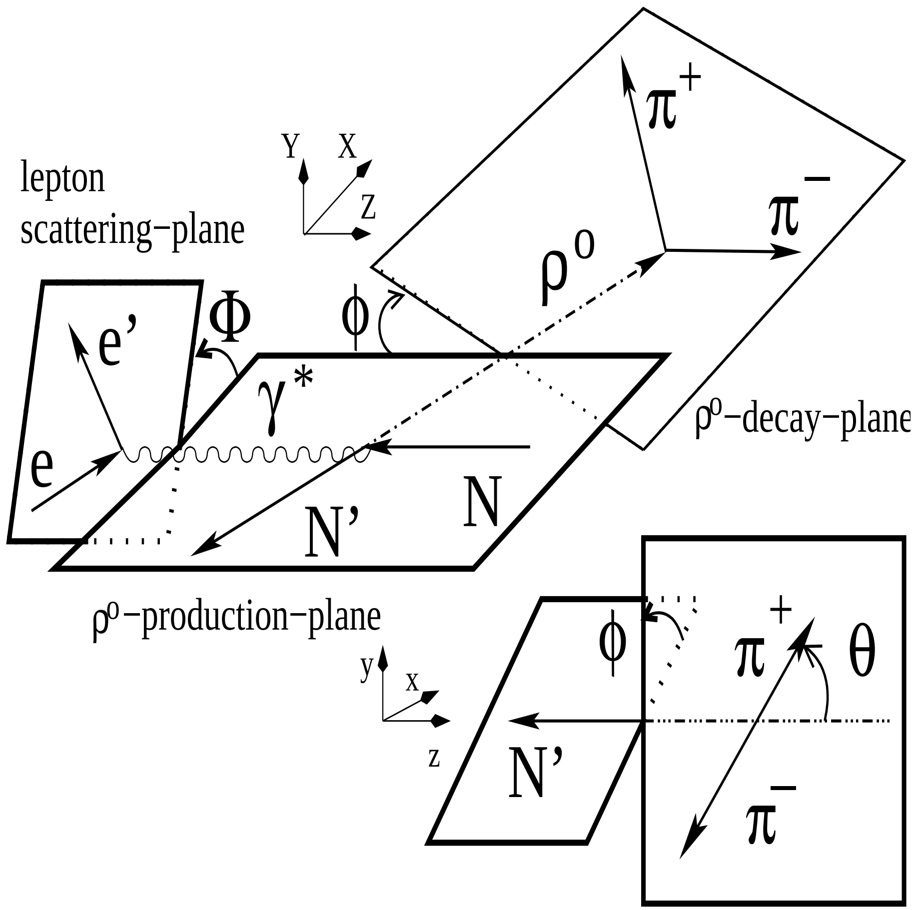
- $\gamma^* + N \to \rho^0(\phi) + N'$ is perfect to study the spin structure of production mechanism:
  - spin state of γ^* is known
  - $\rho^0 \to \pi^+\pi^-$ and $\phi \to K^+ + K^-$ decays are self-analysing

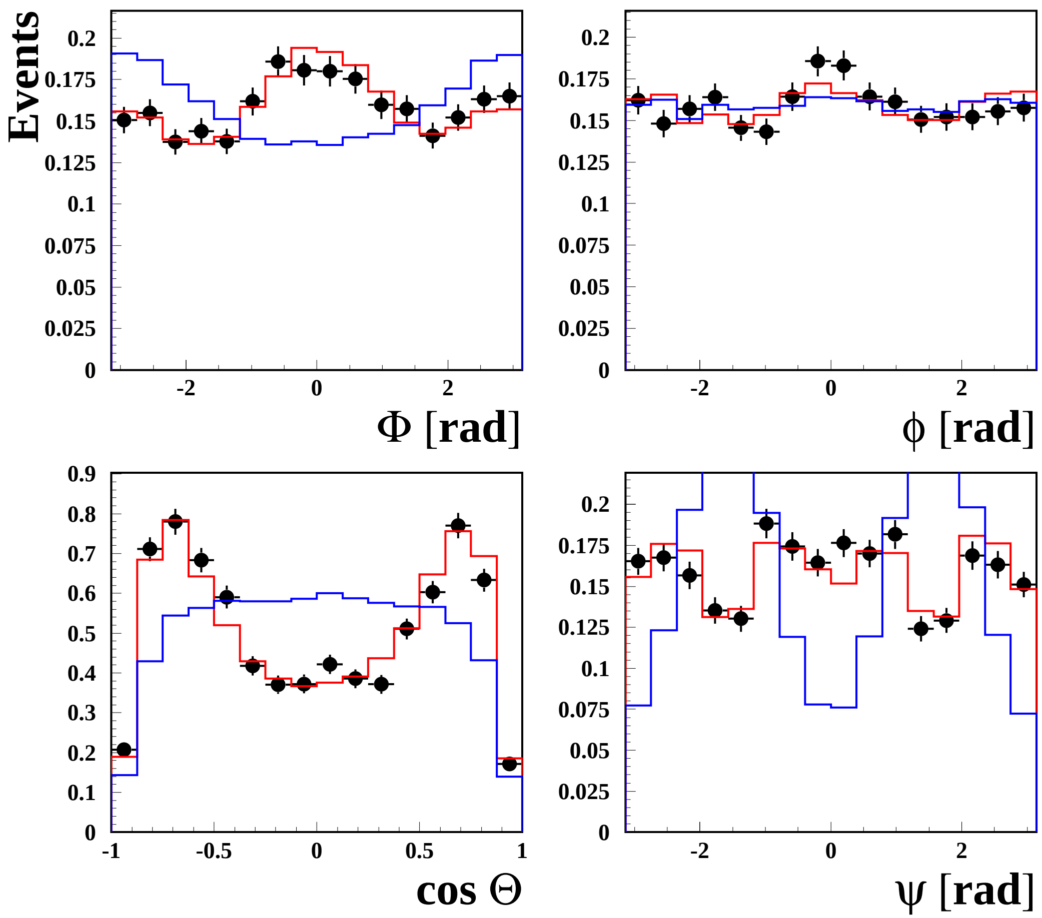
- SDMEs: $r^{\alpha}_{\lambda_\rho \lambda'_\rho} \sim \rho(V) = \frac{1}{2} T_{\lambda_V \lambda_\gamma} \rho(\gamma) T^+_{\lambda_V \lambda_\gamma}$
  spin-density matrix of the vector meson $\rho(V)$ in terms of the photon matrix $\rho(\gamma)$ and helicity amplitude $T_{\lambda_V \lambda_\gamma}$
  - presented according K.Schilling and G.Wolf (Nucl. Phys. B61 (1973) 381)
    $\alpha = 04, 1-3, 5-8$ long. or trans. photon, $\lambda_\rho = -1, 0, 1$ - polarization of $\rho^0(\phi)$
  - measured experimentally at $5 < W < 75$ GeV (HERMES,COMPASS,H1,ZEUS)
  - compared with ones calculated in GK GPD model at $W = 5$ GeV, $Q^2 = 3$ GeV^2

    S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **53** (2008) 367; Eur.Phys.J. C **50**,829 (2007); Eur.Phys.J. C **42**,281 (2005)
  - provide access to *helicity amplitudes* $T_{\lambda_V \lambda_\gamma}$, which are:
    * extracted experimentally from SDMEs
    * calculated from GPDs

$\Longrightarrow$ **Constraints and detailed tests of GPDs**

Fit of Angular Distributions Using Max. Likelihood Method in MINUIT





- Fit of 23 SDMEs after full detector simulation done at initial uniform angular distribution
- Binned Maximum Likelihood Method: $8 \times 8 \times 8$ bins of $\cos(\Theta), \phi, \Phi$. Simultaneous fit of 23 SDMEs $r^{\alpha}_{ij} = W(\Phi, \phi, \cos\Theta)$for data with negative and positive beam helicity ($\langle|P_b|\rangle = 53.5\%$, $\Psi = \Phi - \phi$)

$\Longrightarrow$ **Full agreement of fitted angular distributions with data**

Function for the Fit of 23 SDME r_{ij}^{α}

$$W(\cos\Theta,\phi,\Phi) = W^{unpol} + W^{long.pol},$$

$$W^{unpol}(\cos\Theta,\phi,\Phi) = \frac{3}{8\pi^2}\Big[\frac{1}{2}(1-r_{00}^{04}) + \frac{1}{2}(3r_{00}^{04}-1)\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r_{10}^{04}\}\sin 2\Theta\cos\phi - r_{1-1}^{04}\sin^2\Theta\cos 2\phi$$

$$-\ \epsilon\cos 2\Phi\Big(r_{11}^{1}\sin^2\Theta + r_{00}^{1}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r_{10}^{1}\}\sin 2\Theta\cos\phi - r_{1-1}^{1}\sin^2\Theta\cos 2\phi\Big)$$

$$-\ \epsilon\sin 2\Phi\Big(\sqrt{2}\mathrm{Im}\{r_{10}^{2}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r_{1-1}^{2}\}\sin^2\Theta\sin 2\phi\Big)$$

$$+\ \sqrt{2\epsilon(1+\epsilon)}\cos\Phi\Big(r_{11}^{5}\sin^2\Theta + r_{00}^{5}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r_{10}^{5}\}\sin 2\Theta\cos\phi - r_{1-1}^{5}\sin^2\Theta\cos 2\phi\Big)$$

$$+\ \sqrt{2\epsilon(1+\epsilon)}\sin\Phi\Big(\sqrt{2}\mathrm{Im}\{r_{10}^{6}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r_{1-1}^{6}\}\sin^2\Theta\sin 2\phi\Big)\Big],$$

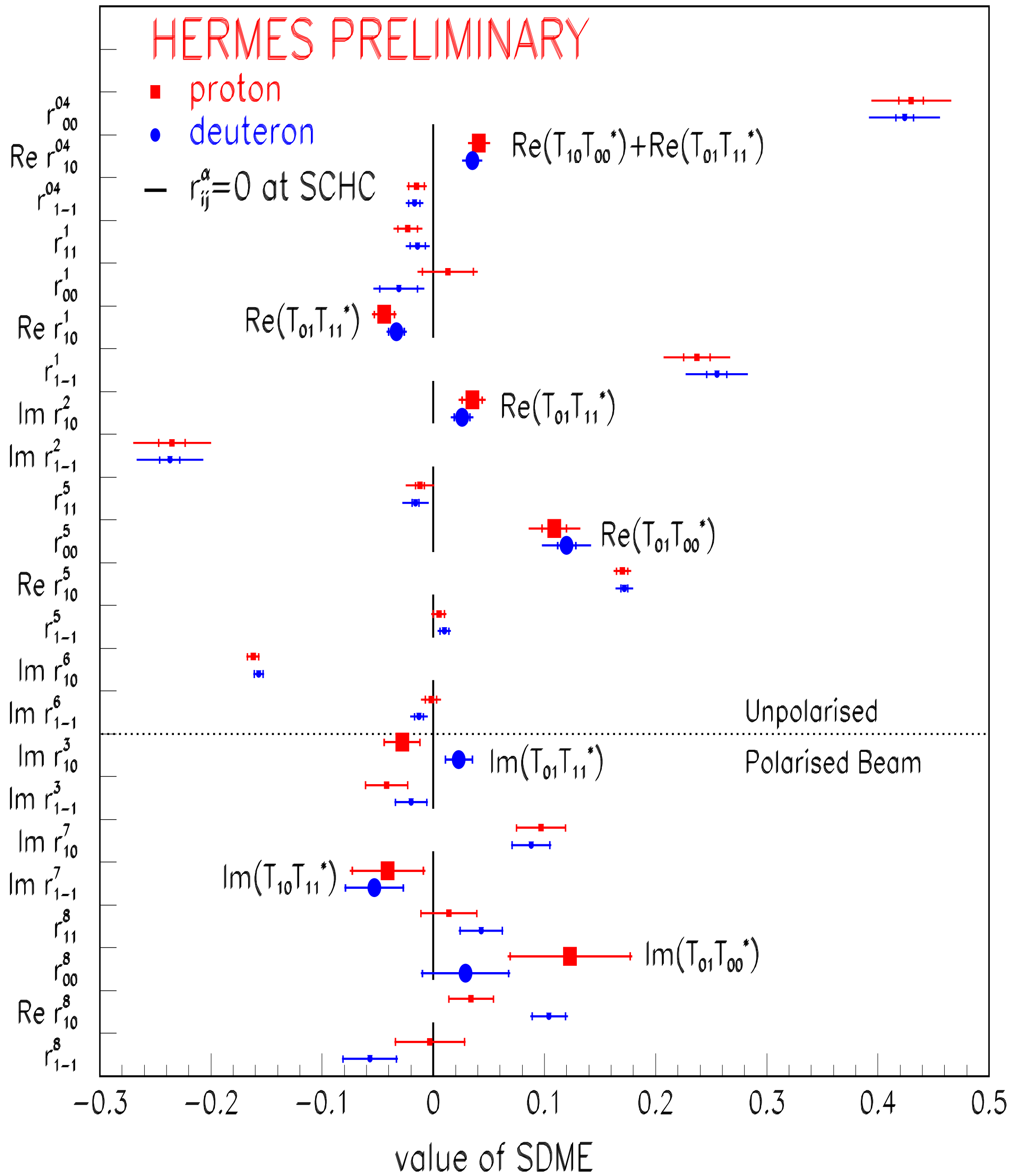
$$W^{long.pol.}(\cos\Theta,\phi,\Phi) = \frac{3}{8\pi^2}P_{beam}\Big[\sqrt{1-\epsilon^2}\Big(\sqrt{2}\mathrm{Im}\{r_{10}^{3}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r_{1-1}^{3}\}\sin^2\Theta\sin 2\phi\Big)$$

$$+\ \sqrt{2\epsilon(1-\epsilon)}\cos\Phi\Big(\sqrt{2}\mathrm{Im}\{r_{10}^{7}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r_{1-1}^{7}\}\sin^2\Theta\sin 2\phi\Big)$$

$$+\ \sqrt{2\epsilon(1-\epsilon)}\sin\Phi\Big(r_{11}^{8}\sin^2\Theta + r_{00}^{8}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r_{10}^{8}\}\sin 2\Theta\cos\phi - r_{1-1}^{8}\sin^2\Theta\cos 2\phi\Big)\Big]$$

ρ^0 23 Spin Density Matrix Elements

at $0 < t' < 0.4$ GeV^2 and $1 < Q^2 < 5$ GeV^2



- SDMEs: $r^{\alpha}_{\lambda_\rho \lambda'_\rho} \sim \rho(V) = \frac{1}{2} T \rho(\gamma) T^+$
  $\Longrightarrow$ Beam-polarization dependent SDMEs measured for the first time

- $q\bar{q}$-exchange with isospin 1 can be observed in case of difference between proton and deuteron data,
  $\Longrightarrow$ No significant difference between proton and deuteron, as well as for ϕ meson SDMEs

- SCHC?

  $\Longrightarrow$ **Enlarged** SDMEs are violating SCHC $(2 \div 5 \quad \sigma)$. Indication on hierarchy of non-zero spin-flip amplitudes: T_{01}, T_{10}, T_{1-1}

SDMEs According to Hierarchy of Amplitudes with(out) Helicity Flip: ρ^0 ϕ

- A, $\gamma^*_L \to \rho^0_L$ and $\gamma^*_T \to \rho^0_T$

  $|T_{11}|^2 \propto 1 - r^{04}_{00} \propto r^1_{1-1} \propto -Im\{r^2_{1-1}\}$

- B, Interference: γ^*_L, ρ^0_T

  $Re\{T_{00}T^*_{11}\} \propto Re\,\{r^5_{10}\} \propto -Im\{r^6_{10}\}$

  $Im\{T_{11}T^*_{00}\} \propto Im\{r^7_{10}\} \propto Re\{r^8_{10}\}$

- C, Spin Flip: $\gamma^*_T \to \rho^0_L$

  $Re\{T_{11}T^*_{01}\} \propto Re\{r^{04}_{10}\} \propto Re\{r^1_{10}\} \propto Im\{r^2_{10}\}$

  $Re\{T_{01}T^*_{00}\} \propto r^5_{00}$

  $|T_{01}|^2 \propto r^1_{00}$

  $Im\{T_{01}T^*_{11}\} \propto Im\{r^3_{10}\}$

  $Im\{T_{01}T^*_{00}\} \propto r^8_{00}$

- D, Spin Flip: $\gamma^*_L \to \rho^0_T$

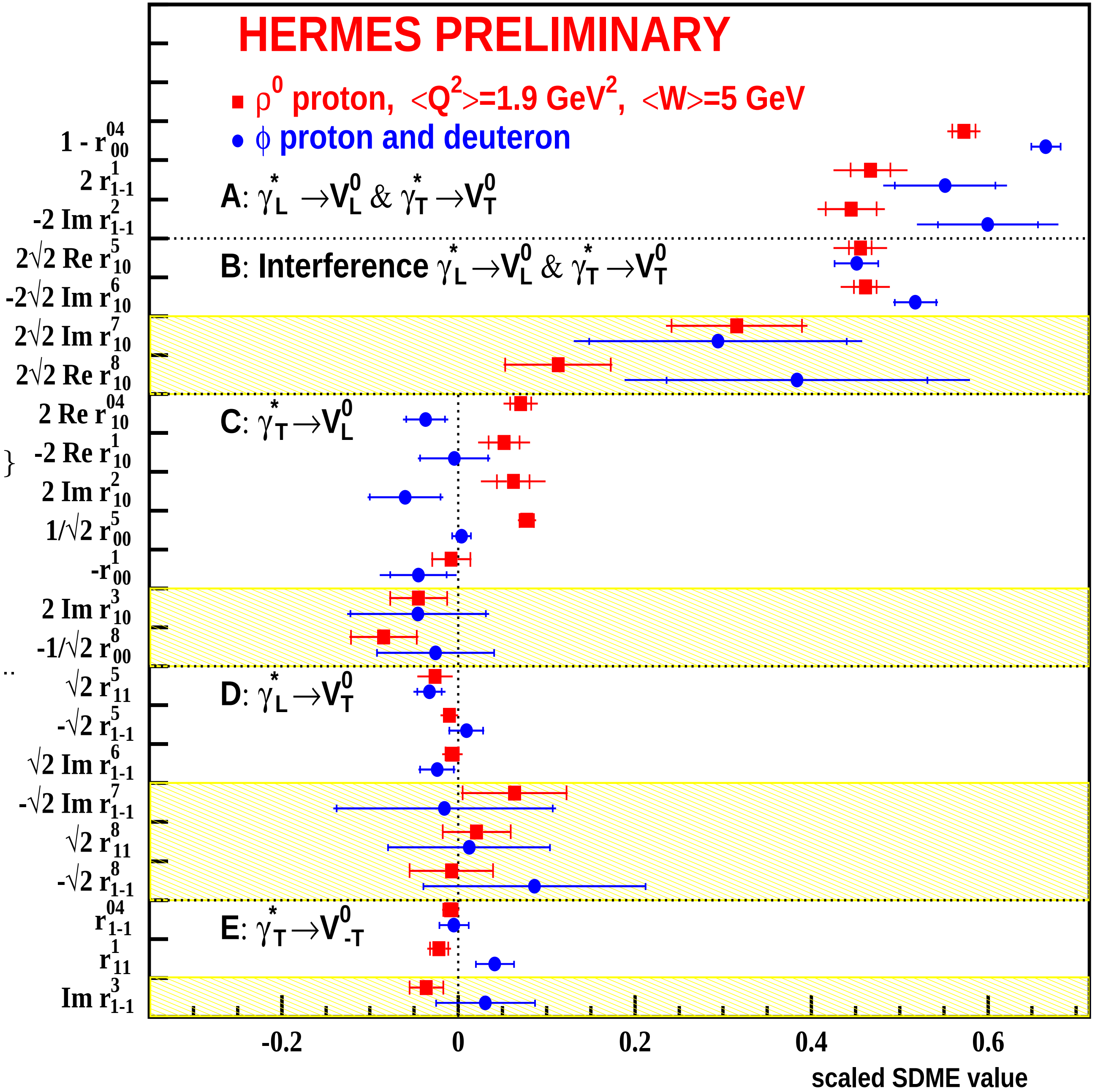
  $Re\{T_{10}T^*_{11}\} \propto r^5_{11} \propto r^5_{1-1} \propto Im\{r^6_{1-1}\}$

  $Im\{T_{10}T^*_{11}\} \propto Im\{r^7_{1-1}\} \propto r^8_{11} \propto r^8_{1-1}$

- E, Spin Flip: $\gamma^*_T \to \rho^0_{-T}$

  $Re\{T_{1-1}T^*_{11}\} \propto r^{04}_{1-1} \propto r^1_{11}$

  $Im\{T_{1-1}T^*_{11}\} \propto Im\{r^3_{1-1}\}$



$\Longrightarrow$ **Hierarchy of ρ^0 amplitudes:** $|T_{00}| \sim |T_{11}| \gg |T_{01}| > |T_{10}| \gtrsim |T_{1-1}|$, $(0 \to L, 1 \to T)$

$\Longrightarrow$ ϕ meson SDMEs are consistent with SCHC, $|T_{00}| \sim |T_{11}|$

ρ^0 Longitudinal-to-Transverse Cross-Section Ratio

Presented commonly measured $R^{04} = \frac{1}{\epsilon}\frac{r^{04}_{00}}{1-r^{04}_{00}}$,

$$r^{04}_{00} = \sum\{\epsilon|T_{00}|^2 + |T_{01}|^2 + |U_{01}|^2\}/\sigma_{tot}$$

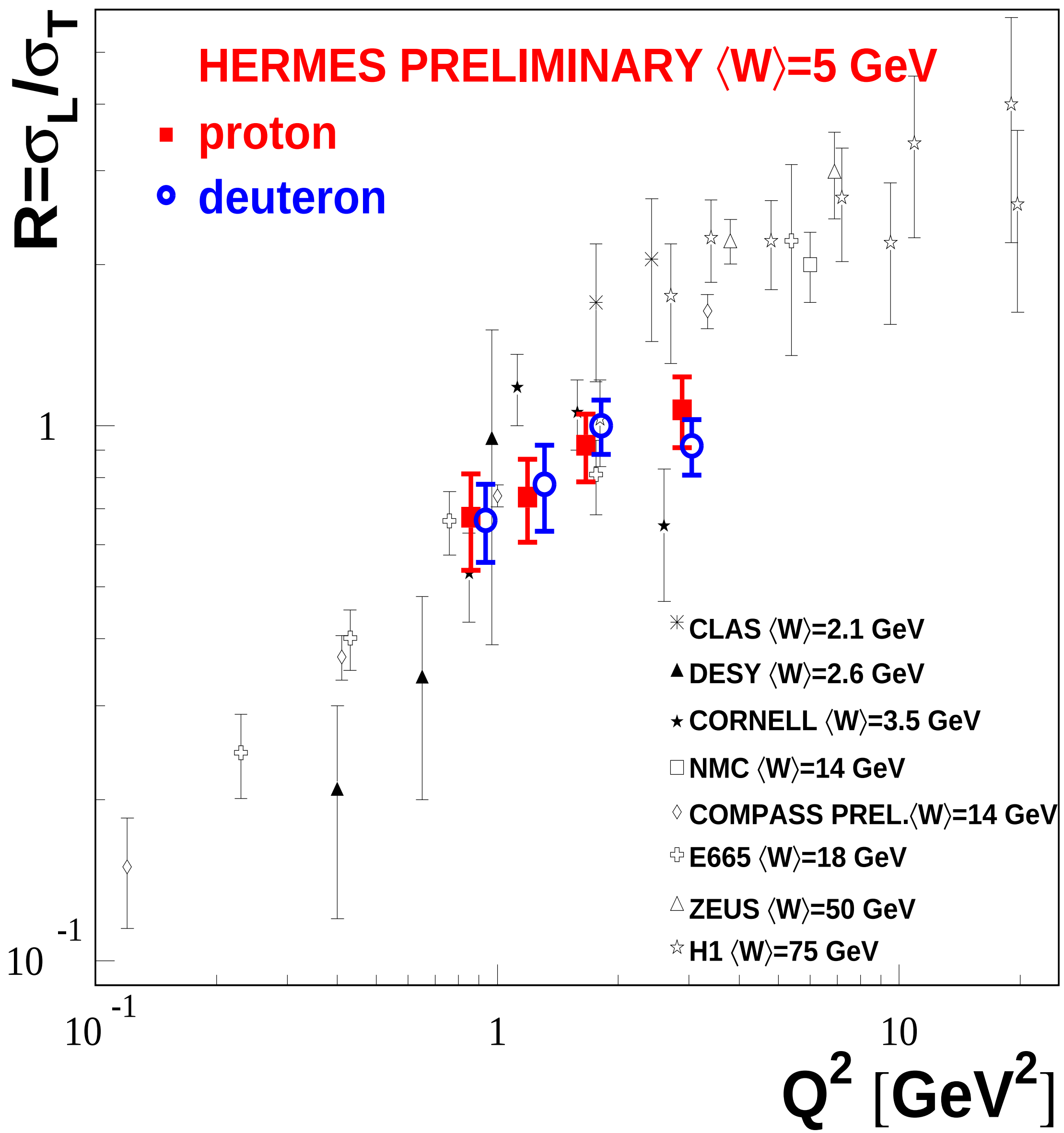
$$\sigma_{tot} = \epsilon\sigma_L + \sigma_T$$

$$\sigma_T = \sum\{|T_{11}|^2 + |T_{01}|^2 + |T_{1-1}|^2 + |U_{11}|^2\}$$

$$\sigma_L = \sum\{|T_{00}|^2 + 2|T_{10}|^2\}$$

Due to the helicity-flip and unnatural parity amplitudes R^{04} depends on kinematic conditions, and is not identical to $R \equiv |T_{00}|^2/|T_{11}|^2$ at SCHC and NPE dominance.

$\Longrightarrow$ Contribution of spin-flip amplitudes to R^{04} is observed



$\Longrightarrow$ **HERMES ρ^0 data on R^{04} are suggestive to R(W)-dependence**

ϕ Longitudinal-to-Transverse Cross-Section Ratio

Presented commonly measured $R^{04} = \frac{1}{\epsilon}\frac{r^{04}_{00}}{1-r^{04}_{00}}$,

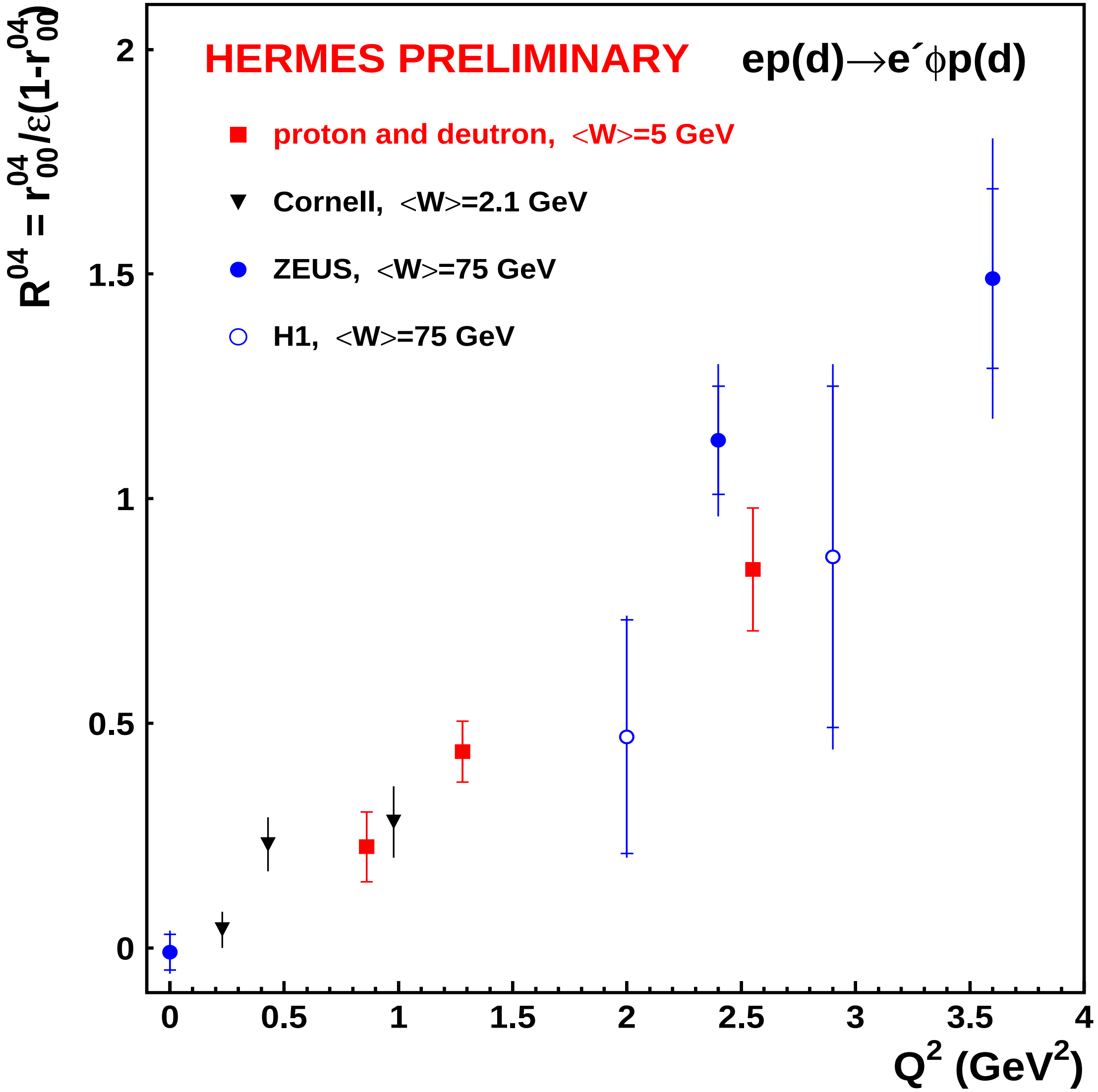
from the SDMEs analysis:

$$r^{04}_{00} = \sum\{\epsilon|T_{00}|^2\}/\sigma_{tot}$$

$$\sigma_{tot} = \epsilon\sigma_L + \sigma_T$$

$$\sigma_T = \sum\{|T_{11}|^2\}$$

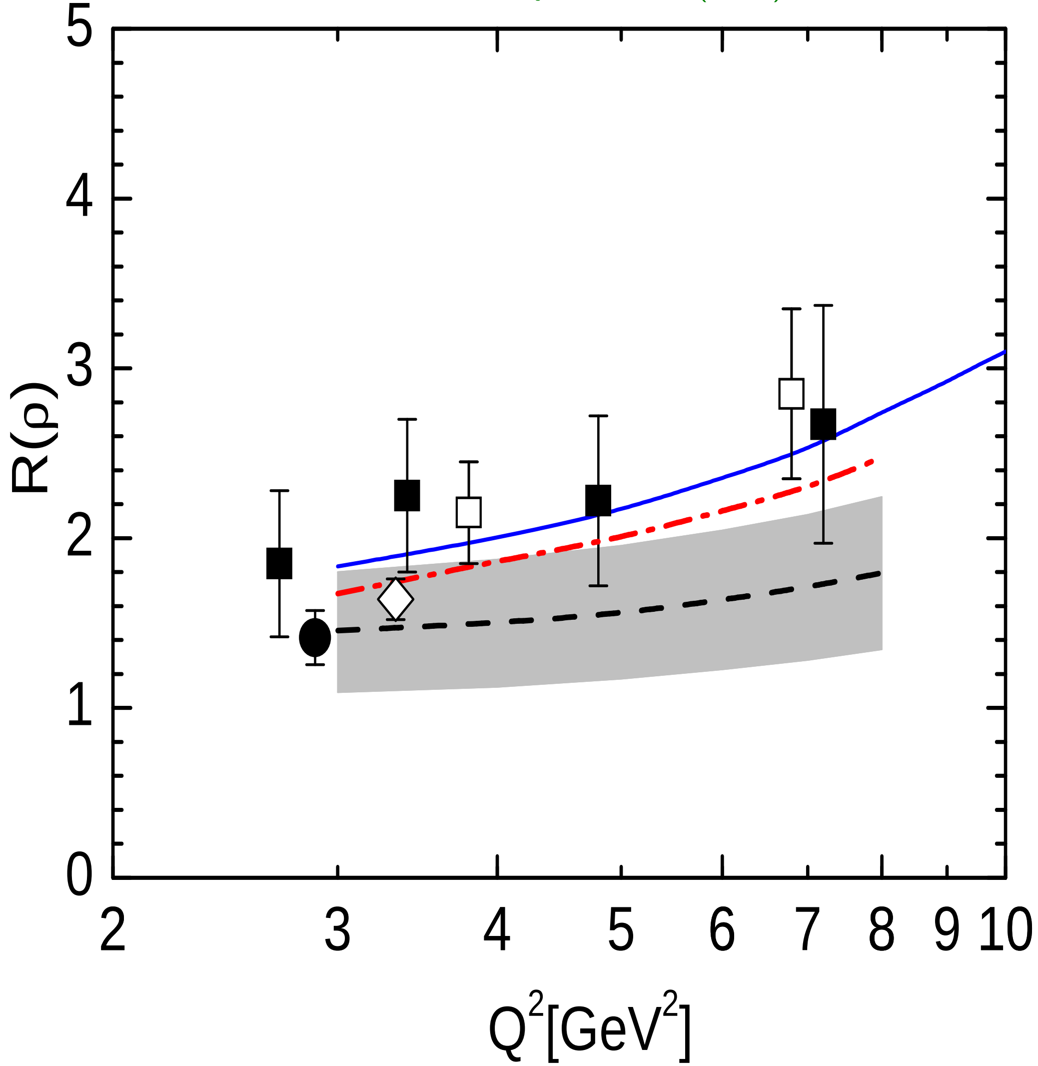
$$\sigma_L = \sum\{|T_{00}|^2\}$$

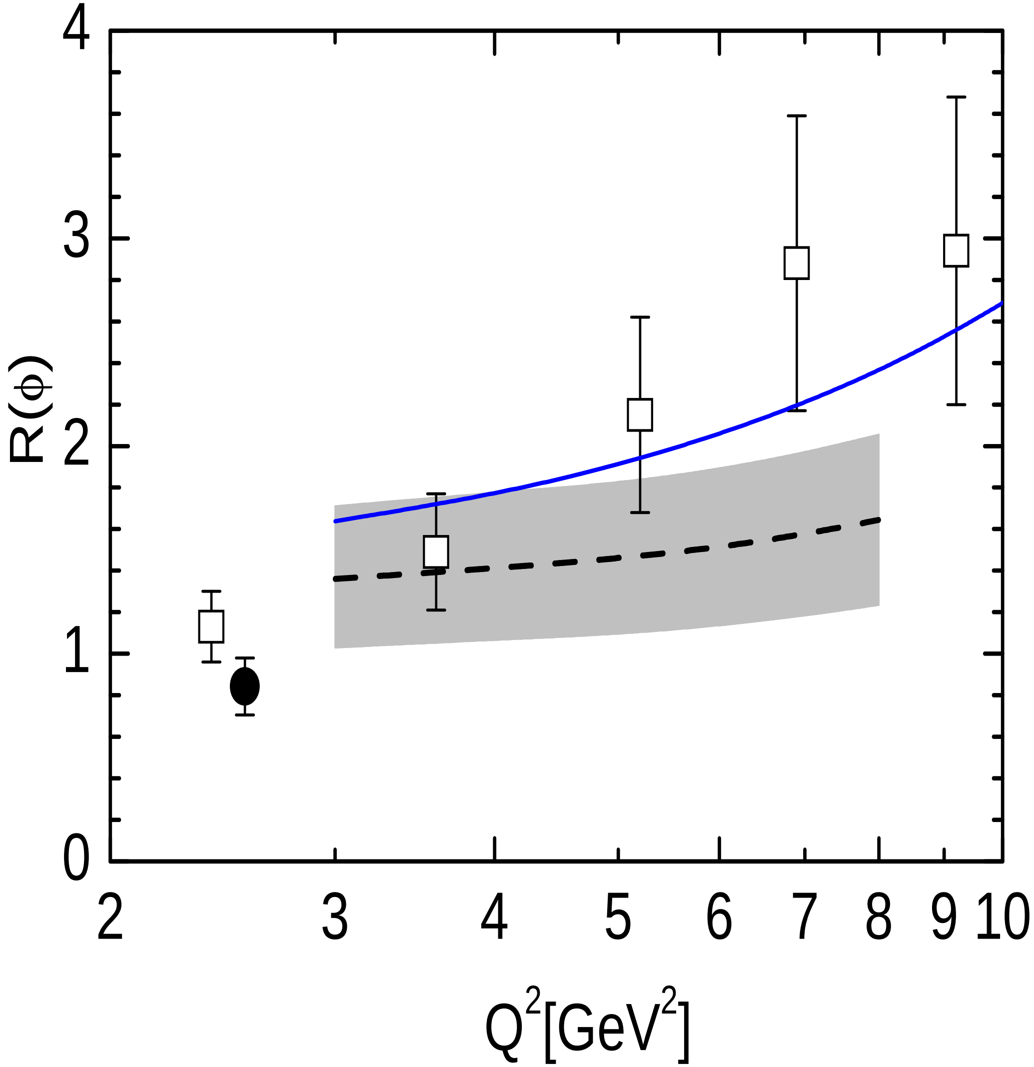


$\Longrightarrow$ R^{04} for ϕ meson at HERMES is in good agreement with world data

R^{04} of ρ^0 and ϕ-meson Compared with GK Model Calculations

S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **53** (2008) 367;

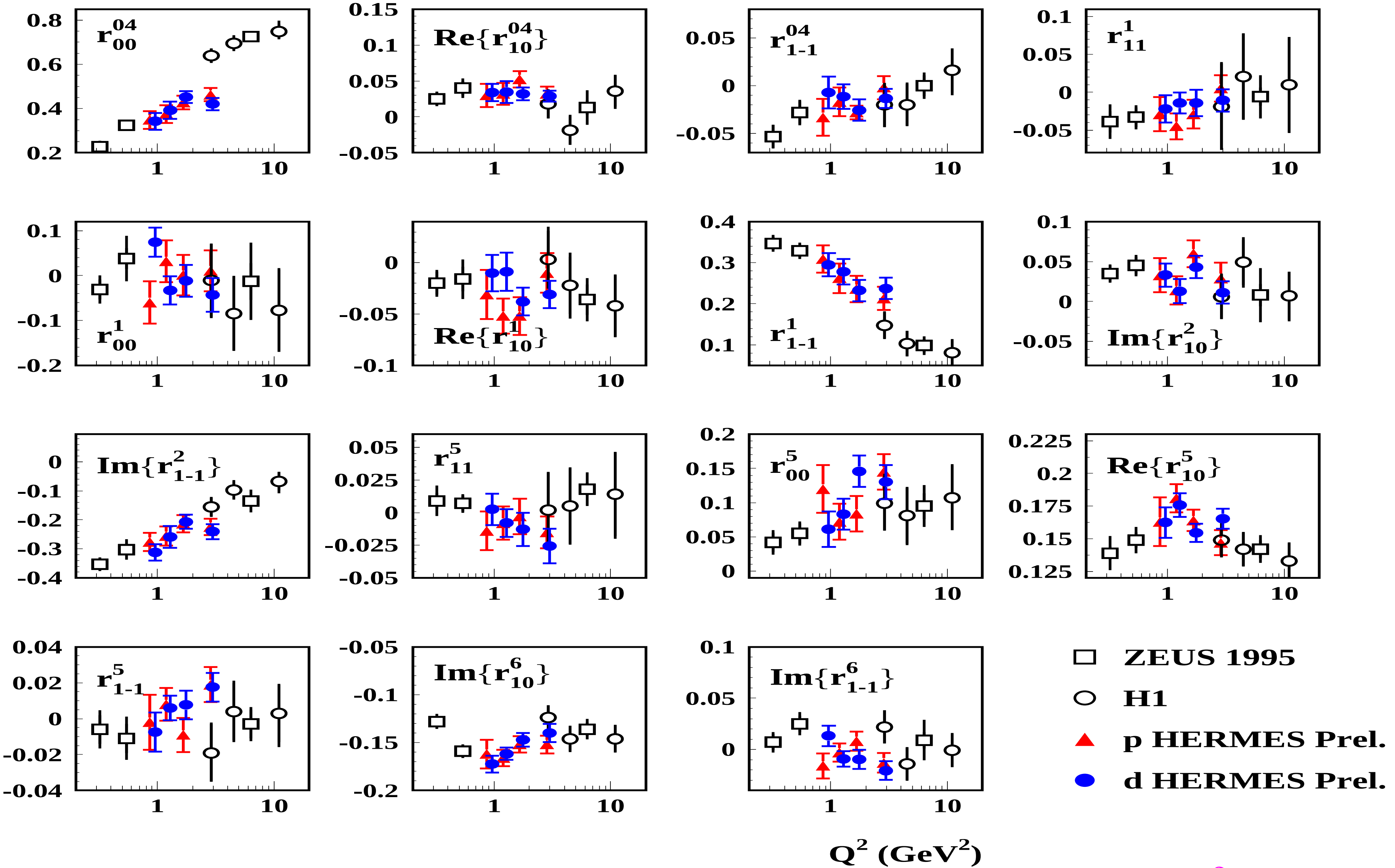




blue line W=90 GeV, squares: H1, ZEUS, red line W=10 GeV, diamond: COMPASS,
black line W=5 GeV, circle: HERMES PRELIMINARY, corrected to subtract UPE contribution for ρ^0

$\Longrightarrow$ **W-dependence of R^{04} is supported by calculations**

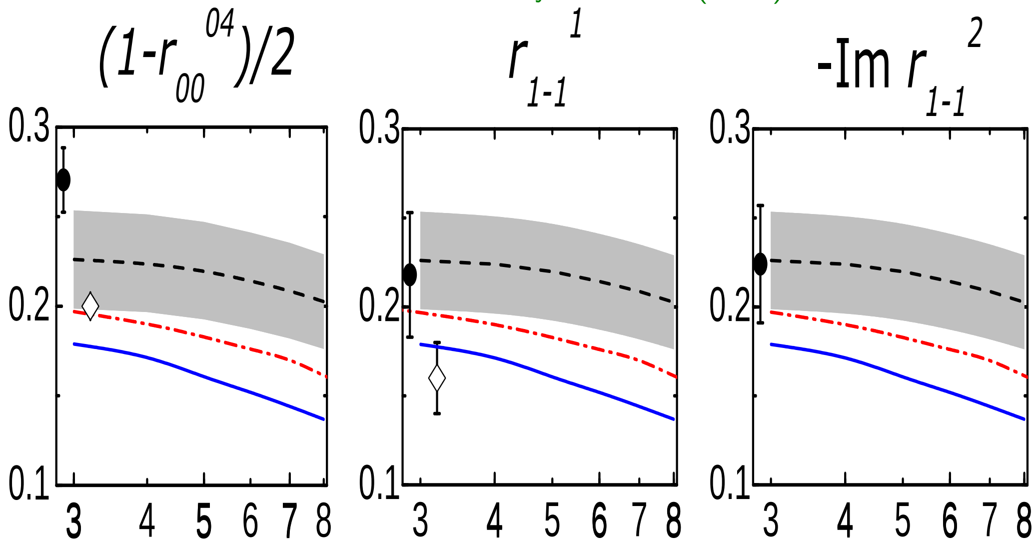
Q^2-dependence of HERMES ρ^0 SDMEs at W=5 GeV on proton and deuteron compared with H1 and ZEUS Data at W=75 GeV

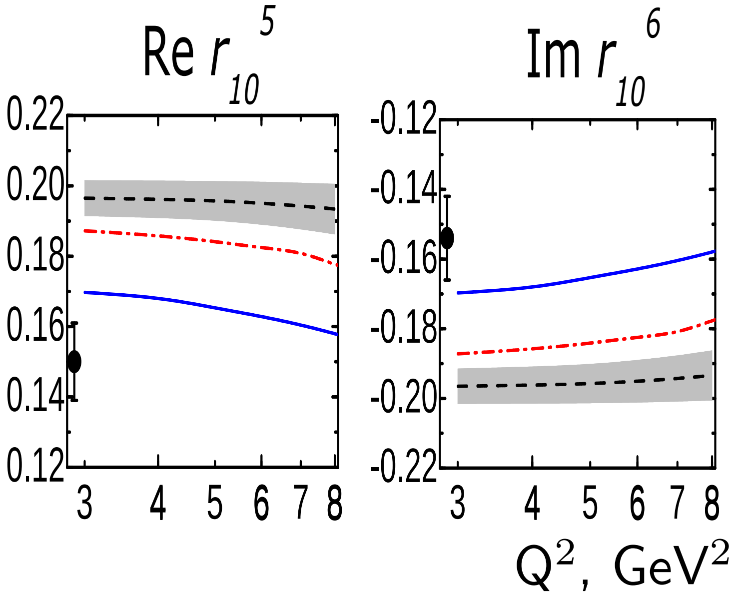


$\Longrightarrow$ Several SDMEs indicate possible W-dependence, in addition to Q^2-dependence

ρ^0 SDMEs Compared with GK Model Calculations

S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **53** (2008) 367;





$1 - r_{00}^{04} \propto r_{1-1}^{1} \propto -Im\{r_{1-1}^{2}\} \propto |T_{11}|^2$

i.e. amplitudes for $\gamma_L^* \to \rho_L^0$, $\gamma_T^* \to \rho_T^0$

– W=90 GeV

– W=10 GeV, diamond: COMPASS

– W=5 GeV, circle: HERMES PRELIMINARY

$\Longrightarrow$ Fair agreement with data

Re r_{10}^5 and Im r_{10}^6 correspond to interference of γ_L^*, ρ_T^0 amplitudes, phase difference between T_{11} and T_{00}:

$$\sin \delta_{LT} = \frac{2\sqrt{\epsilon}(\mathrm{Re}\{r_{10}^8\} + \mathrm{Im}\{r_{10}^7\})}{\sqrt{r_{00}^{04}(1 - r_{00}^{04} + r_{1-1}^1 - \mathrm{Im}\{r_{1-1}^2\})}}.$$

ρ^0 p: $\delta_{LT} = 28.1 \pm 2.8_{stat} \pm 3.7_{syst}$ deg

ρ^0 d: $\delta_{LT} = 30.2 \pm 2.0_{stat} \pm 3.7_{syst}$ deg

ϕ p+d: $\delta_{LT} = 33.0 \pm 7.4_{total}$ deg

But in GK model $\delta_{LT} = 3.1$ deg at W=5 GeV

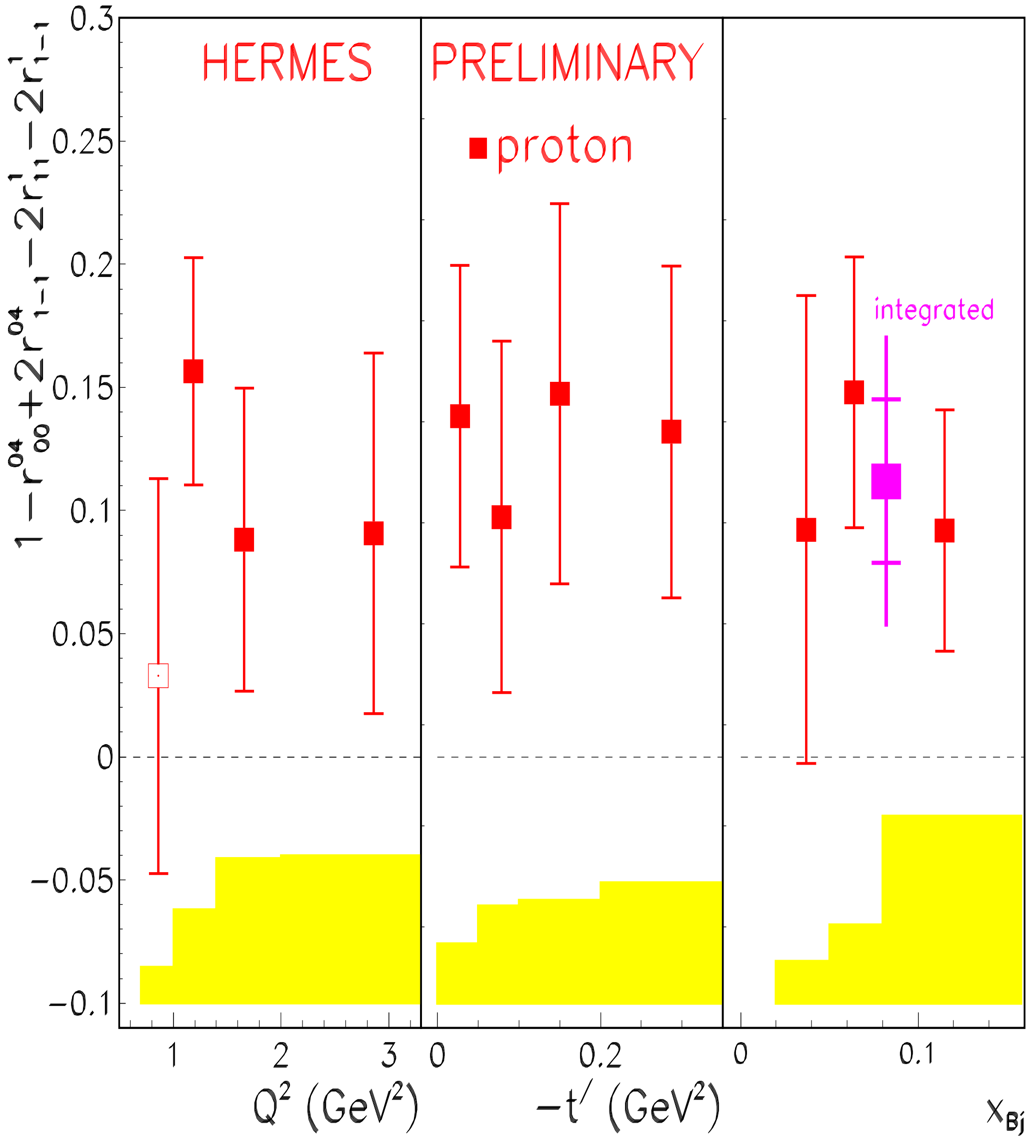
Observation of Unnatural Parity Exchange (UPE) in ρ^0 Leptoproduction

- Unnatural parity exchange is mediated by pseudoscalar or axial meson: $J^P = 0^-, 1^+$, e.g. π, a_1, b_1 $\longrightarrow$ only quark-exchange conribution
- No interference between NPE and UPE contributions on unpolarized target
- Extracted from SDMEs:
  - $U1 \propto \epsilon|U_{10}|^2 + 2|U_{11} + U_{1-1}|^2$
    $U1 = 1 - r^{04}_{00} + 2r^{04}_{1-1} - 2r^{1}_{11} - 2r^{1}_{1-1}$

    p: $U1 = 2|U_{11}|^2 \approx 0.13 \pm 0.06$
    d: $U1 \approx 0.09 \pm 0.05$
    p+d: $U1 \approx 0.11 \pm 0.04$
  - $U2 + iU3 \propto (U_{11} + U_{1-1}) * U_{10}$

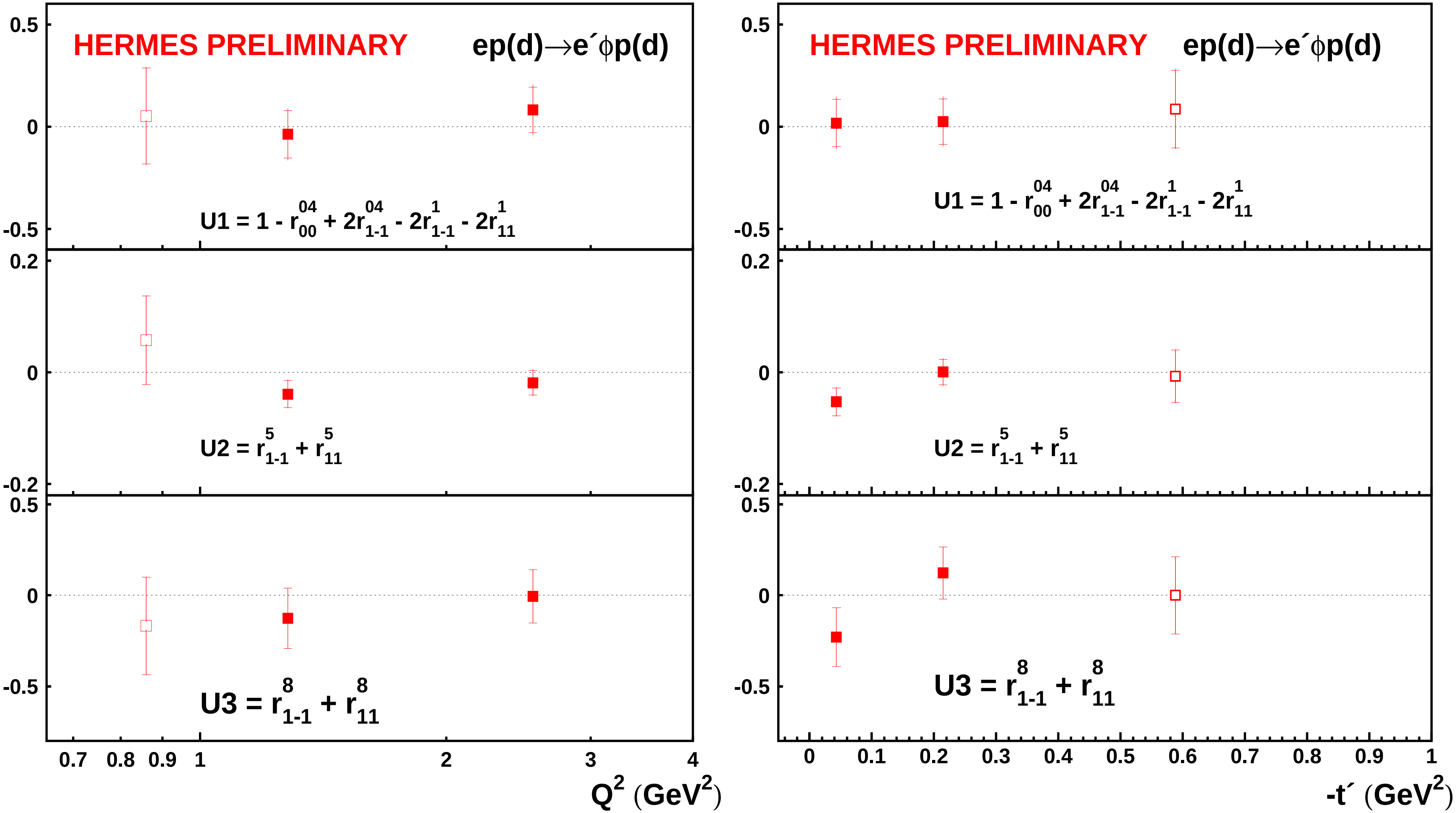
    $U2 = r^{5}_{11} + r^{5}_{1-1}$
    p: $U2 \approx -0.01 \pm 0.013$
    d: $U2 \approx -0.008 \pm 0.011$

    $U3 = r^{8}_{11} + r^{8}_{1-1}$
    p: $U3 \approx -0.02 \pm 0.05$
    d: $U3 \approx -0.02 \pm 0.04$



$\Longrightarrow$ **Indication on hierarchy of ρ^0 UPE amplitudes: $|U_{11}| \gg |U_{10}| \sim |U_{01}|$**

...Only Natural Parity Exchange in ϕ Meson Leptoproduction

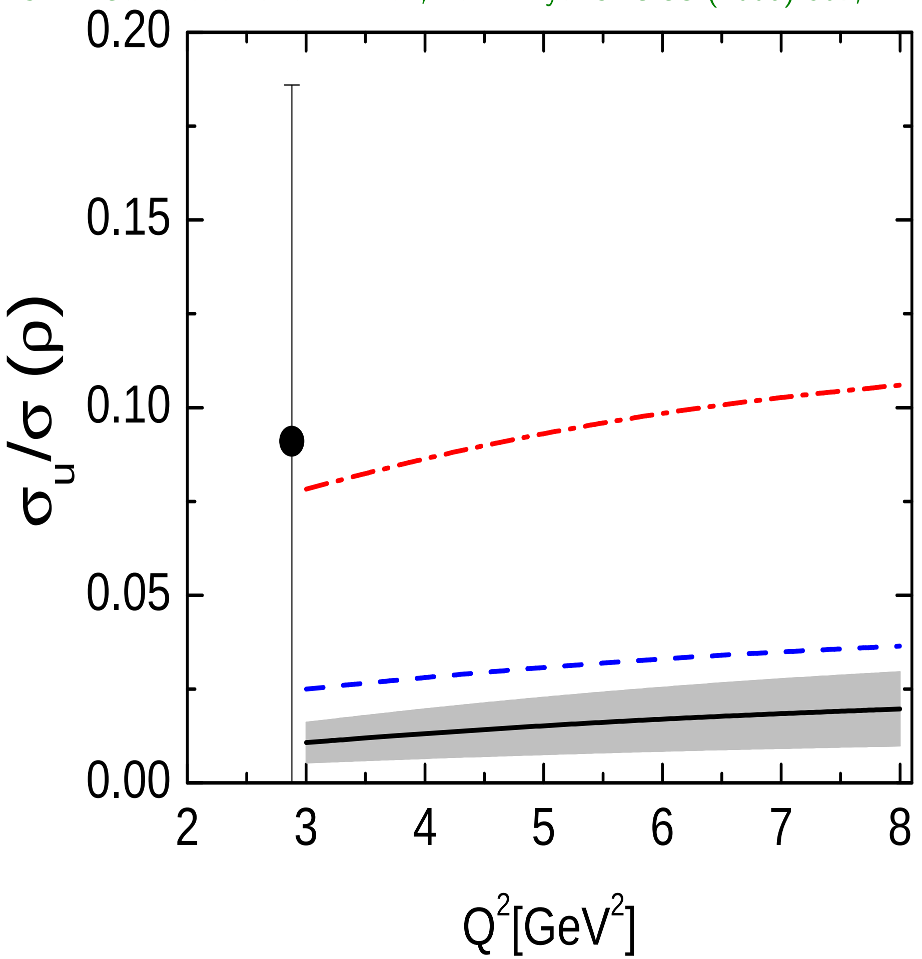


$$U1 \approx 0.02 \pm 0.17, \quad U2 \approx -0.03 \pm 0.04, \quad U3 \approx -0.05 \pm 0.13$$

$\Longrightarrow$ **no UPE for ϕ meson production, as expected**

Unnatural Parity Exchange contribution in GK model

S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **53** (2008) 367;



HERMES PRELIMINARY, W=5 GeV

- In GK model UPE requies $\tilde{H}$ GPD:

  $\sigma_U \propto e_u \tilde{H}^u_{val} - e_d \tilde{H}^d_{val}$ for ρ^0 production

  $\sigma_U / \sigma(\rho^0) = 2|U_{11}|^2 / \sigma(\rho^0)$

  Lines:

  – extreme assumption for valence quarks: $\tilde{H}^u_{val} = H^u_{val}$ and $\tilde{H}^d_{val} = H^d_{val}$

  – extreme assumption for valence quarks: $\tilde{H}^u_{val} = H^u_{val}$ and $\tilde{H}^d_{val} = -H^d_{val}$

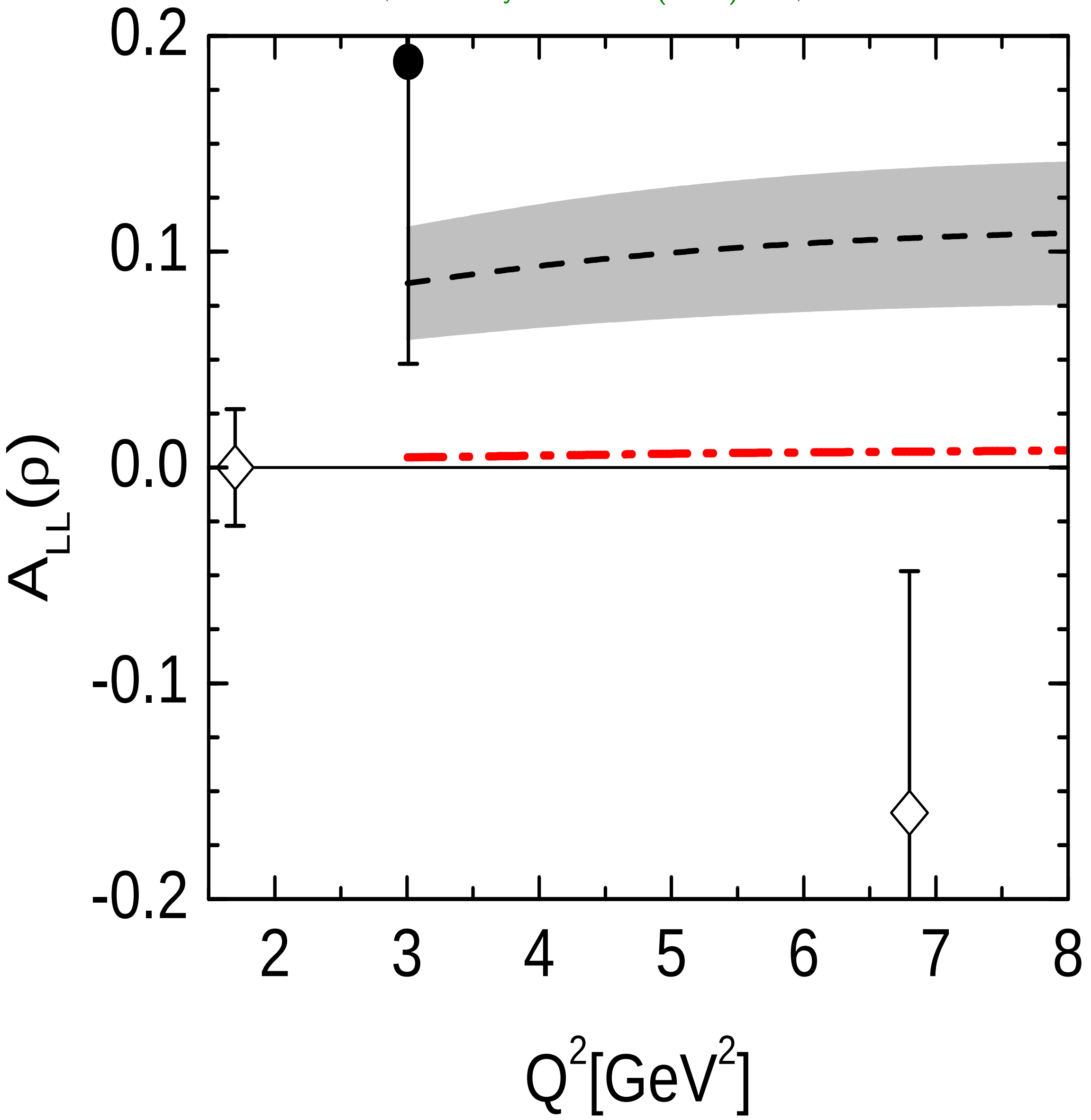
  – $\sigma_U \approx 0.013$ for gluons and sea contribution

- σ_U small for H1 and ZEUS ρ^0 data as gluon and sea contribution dominate
- σ_U small for ϕ at HERMES as gluon contribution dominate

...Better precision of $|U_{11}|^2$ measurement at $Q^2 \approx 3$ GeV^2 is needed, and planned

ρ^o Double Spin Asymmetry in GK model

S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **53** (2008) 367;



HERMES PRELIMINARY, W=5 GeV

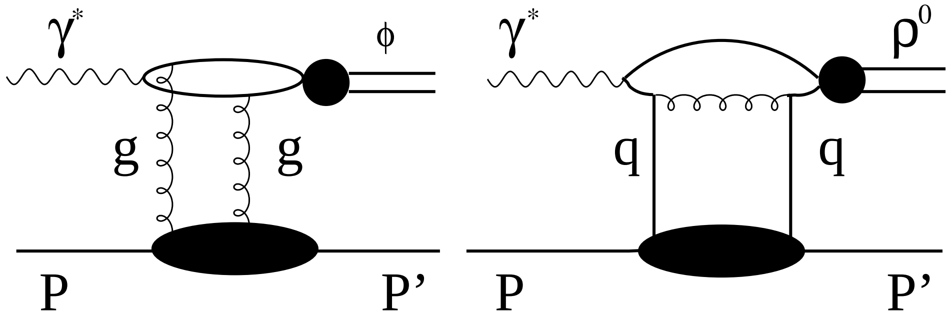
- Interference between leading NPE and UPE amplitudes on longitudinally polarized target results in A_{LL}
- $A_{LL} = 4\sqrt{1-\epsilon^2}\frac{Re(T_{11}U_{11}^*)}{\sigma(\rho^0)}$
- Lines:
  - W=10 GeV, diamods: COMPASS
  - W=5 GeV, circle: HERMES

    HERMES collab., Phys.Lett.B 513 (2001) 301-310, and

    Eur.Phys.J. C 29, 171 - 179 (2003)

Summary

- HERMES data are unique due to the sensitivity to *both quark and two-gluon exchange processes* at sufficiently large W and Q^2 for the comparison with GPD handbag diagram based calculations:



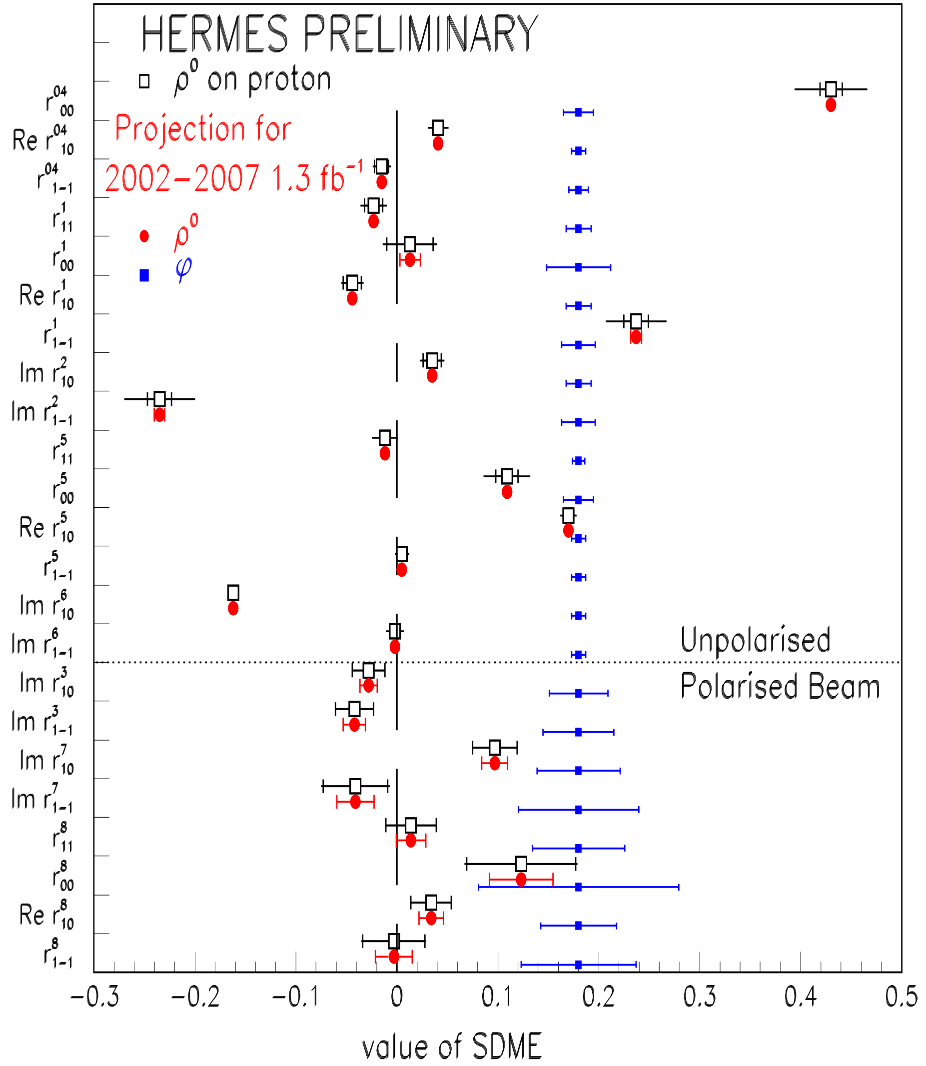
- *First comprehensive comparision* of data on vector meson production with GK model calculations is in fair agreement for:
  - longitudinal and total cross sections of ρ^0 and ϕ mesons
  - values of SDMEs and hierarchy of corresponding amplitudes
  - violation of SCHC in ρ^0 production
  - W-dependence of ρ^0 and ϕ SDMEs and σ_L/σ_T ratios

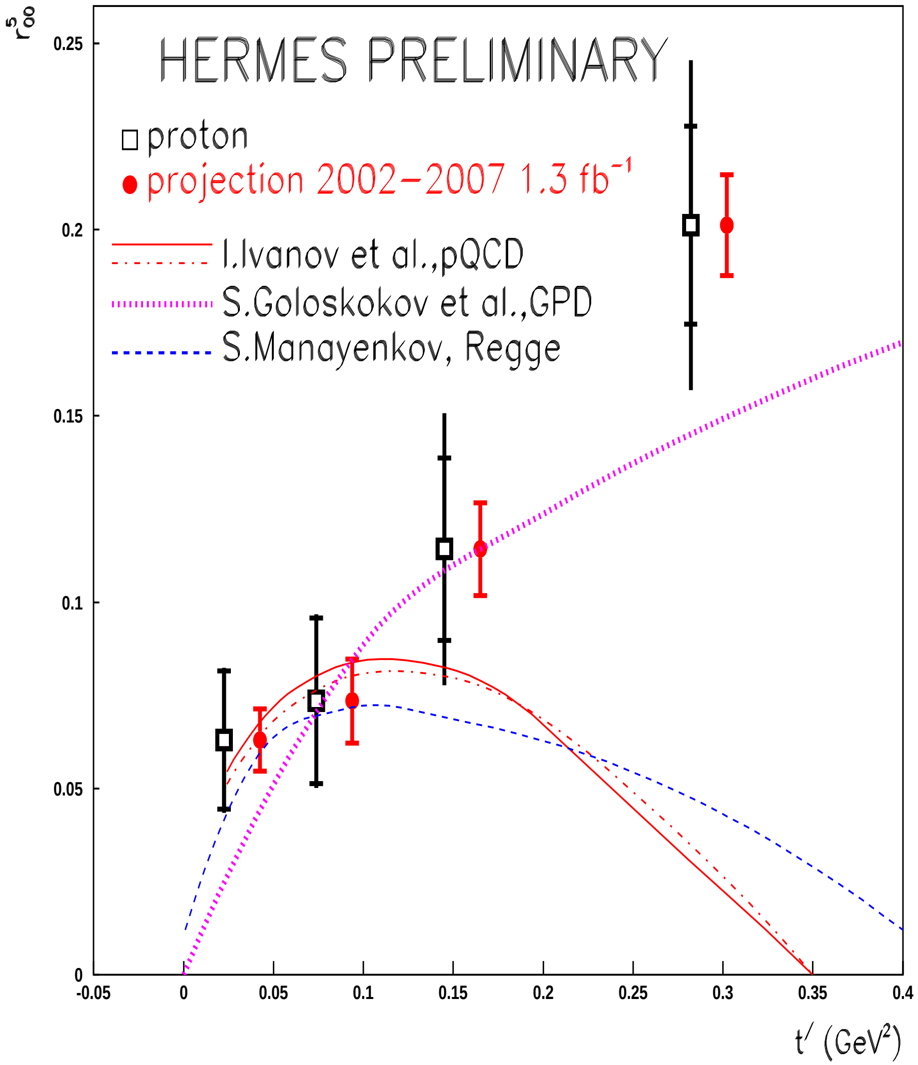
- Constraints of HERMES data in GPDs are for:
  - *phase difference* in the interference of $\gamma_L^* \to \rho_L^0 \& \gamma_T^* \to \rho_T^0$ transitions
  - $\tilde{H}_{val}^{u,d}$ contribution in Unnatural Parity Exchange amplitude and A_{LL}^{ρ}

Outlook

- Target-polarization dependent SDMEs are under analysis in M.Diehl representation
  (JHEP (2007) 0709:064, DESY-07-049)

- 2006-2007 data at $\mathcal{L} \sim 1.3$ fb^{-1} with detected recoil proton are available





Equations for SDMEs Ordered According Helicity Transfer Amplitudes

A: $\gamma^*_L \longrightarrow \rho^0_L$ and $\gamma^*_T \longrightarrow \rho^0_T$

$r^{04}_{00} = \widetilde{\sum}\{\epsilon|T_{00}|^2 + |T_{01}|^2 + |U_{01}|^2/\}/N_{full}$,

$r^{1}_{1-1} = \frac{1}{2}\widetilde{\sum}\{|T_{11}|^2 + |T_{1-1}|^2 - |U_{11}|^2 - |U_{1-1}|^2\}/N_{full}$,

$Im\{r^{2}_{1-1}\} = \frac{1}{2}\widetilde{\sum}\{-|T_{11}|^2 + |T_{1-1}|^2 + |U_{11}|^2 - |U_{1-1}|^2\}/N_{full}$,

B :interferenceof $\gamma^*_L \longrightarrow \rho^0_L$ and $\gamma^*_T \longrightarrow \rho^0_T$

$Re\{r^{5}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}Re\{2T_{10}T^*_{01} + (T_{11} - T_{1-1})T^*_{00}\}/N_{full}$,

$Im\{r^{6}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}Re\{2U_{10}U^*_{01} - (T_{11} + T_{1-1})T^*_{00}\}/N_{full}$,

$Im\{r^{7}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}Im\{2U_{10}U^*_{01} + (T_{11} + T_{1-1})T^*_{00}\}/N_{full}$,

$Re\{r^{8}_{10}\} = \frac{1}{\sqrt{8}}\widetilde{\sum}Im\{-2T_{10}T^*_{01} + (T_{11} - T_{1-1})T^*_{00}\}/N_{full}$,

C : $\gamma^*_T \longrightarrow \rho^0_L$

$Re\{r^{04}_{10}\} = \widetilde{\sum}Re\{\epsilon T_{10}T^*_{00} + \frac{1}{2}T_{01}(T_{11} - T_{1-1})^* + \frac{1}{2}U_{01}(U_{11} + U_{1-1})^*\}/N_{full}$,

$Re\{r^{1}_{10}\} = \frac{1}{2}\widetilde{\sum}Re\{-T_{01}(T_{11} - T_{1-1})^* + U_{01}(U_{11} + U_{1-1})^*\}/N_{full}$,

$Im\{r^{2}_{10}\} = \frac{1}{2}\widetilde{\sum}Re\{T_{01}(T_{11} + T_{1-1})^* - U_{01}(U_{11} - U_{1-1})^*\}/N_{full}$,

$r^{5}_{00} = \sqrt{2}\widetilde{\sum}Re\{T_{01}T^*_{00}\}/N_{full}$,

$r^{1}_{00} = \widetilde{\sum}\{-|T_{01}|^2 + |U_{01}|^2\}/N_{full}$,

$Im\{r^{3}_{10}\} = -\frac{1}{2}\widetilde{\sum}Im\{T_{01}(T_{11} + T_{1-1})^* + U_{01}(U_{11} - U_{1-1})^*\}/N_{full}$,

$r^{8}_{00} = \sqrt{2}\widetilde{\sum}Im\{T_{01}T^*_{00}\}/N_{full}$,

D : $\gamma^*_L \longrightarrow \rho^0_T$

$r^{5}_{11} = \frac{1}{\sqrt{2}}\widetilde{\sum}Re\{T_{10}(T_{11} - T_{1-1})^* + U_{10}(U_{11} - U_{1-1})^*\}/N_{full}$,

$r^{5}_{1-1} = \frac{1}{\sqrt{2}}\widetilde{\sum}Re\{-T_{10}(T_{11} - T_{1-1})^* + U_{10}(U_{11} - U_{1-1})^*\}/N_{full}$,

$Im\{r^{6}_{1-1}\} = \frac{1}{\sqrt{2}}\widetilde{\sum}Re\{T_{10}(T_{11} + T_{1-1})^* - U_{10}(U_{11} + U_{1-1})^*\}/N_{full}$,

$Im\{r^{7}_{1-1}\} = \frac{1}{\sqrt{2}}\widetilde{\sum}Im\{T_{10}(T_{11} + T_{1-1})^* - U_{10}(U_{11} + U_{1-1})^*\}/N_{full}$,

$r^{8}_{11} = -\frac{1}{\sqrt{2}}\widetilde{\sum}Im\{T_{10}(T_{11} - T_{1-1})^* + U_{10}(U_{11} - U_{1-1})^*\}/N_{full}$,

$r^{8}_{1-1} = \frac{1}{\sqrt{2}}\widetilde{\sum}Im\{T_{10}(T_{11} - T_{1-1})^* - U_{10}(U_{11} - U_{1-1})^*\}/N_{full}$,

E : $\gamma^*_T \longrightarrow \rho^0_{-T}$

$r^{04}_{1-1} = \widetilde{\sum}Re\{-\epsilon|T_{10}|^2 + \epsilon|U_{10}|^2 + T_{1-1}T^*_{11} - U_{1-1}U^*_{11}\}/N_{full}$,

$r^{1}_{11} = \widetilde{\sum}Re\{T_{1-1}T^*_{11} + U_{1-1}U^*_{11}\}/N_{full}$,

$Im\{r^{3}_{1-1}\} = -\widetilde{\sum}Im\{T_{1-1}T^*_{11} - U_{1-1}U^*_{11}\}/N_{full}$,

where N_{full} is normalized total ρ^0 production cross section