What is Orbital Angular Momentum?

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Motivation

- polarized DIS: only $\sim 30\%$ of the proton spin due to quark spins

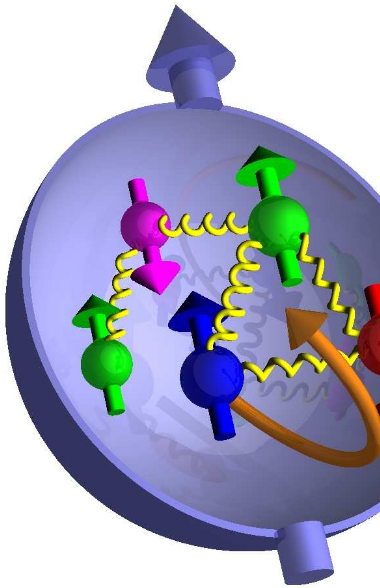
$\hookrightarrow$ 'spin crisis'$\longrightarrow$ 'spin puzzle', because $\Delta\Sigma$ much smaller than the quark model result $\Delta\Sigma = 1$

$\hookrightarrow$ quest for the remaining 70%

  - quark orbital angular momentum (OAM)
  - gluon spin
  - gluon OAM

$\hookrightarrow$ How are the above quantities defined?

$\hookrightarrow$ How can the above quantities be measured



example: angular momentum in QED

- consider, for simplicity, QED without electrons:

$$\vec{J} = \int d^3r\, \vec{x} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$

- integrate by parts

$$\vec{J} = \int d^3r \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \left(\vec{x} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- drop 2^{nd} term (eq. of motion $\vec{\nabla} \cdot \vec{E} = 0$), yielding $\vec{J} = \vec{L} + \vec{S}$ with

$$\vec{L} = \int d^3r\, E^j \left(\vec{x} \times \vec{\nabla}\right) A^j \qquad \vec{S} = \int d^3r\, \vec{E} \times \vec{A}$$

- note: $\vec{L}$ and $\vec{S}$ not separately gauge invariant

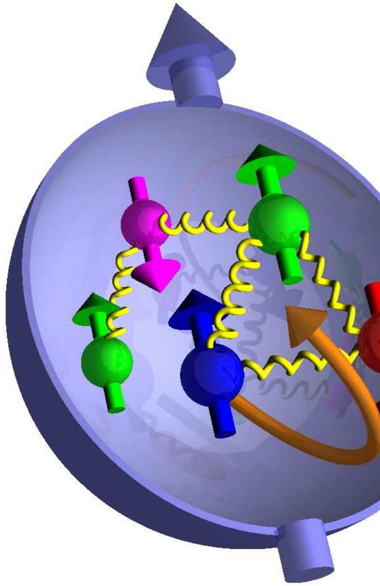
example (cont.)

- total angular momentum of isolated system uniquely defined
- ambiguities arise when decomposing $\vec{J}$ into contributions from different constituents
- gauge theories: changing gauge may also shift angular momentum between various degrees of freedom

$\hookrightarrow$ decomposition of angular momentum in general depends on 'scheme' (gauge & quantization scheme)

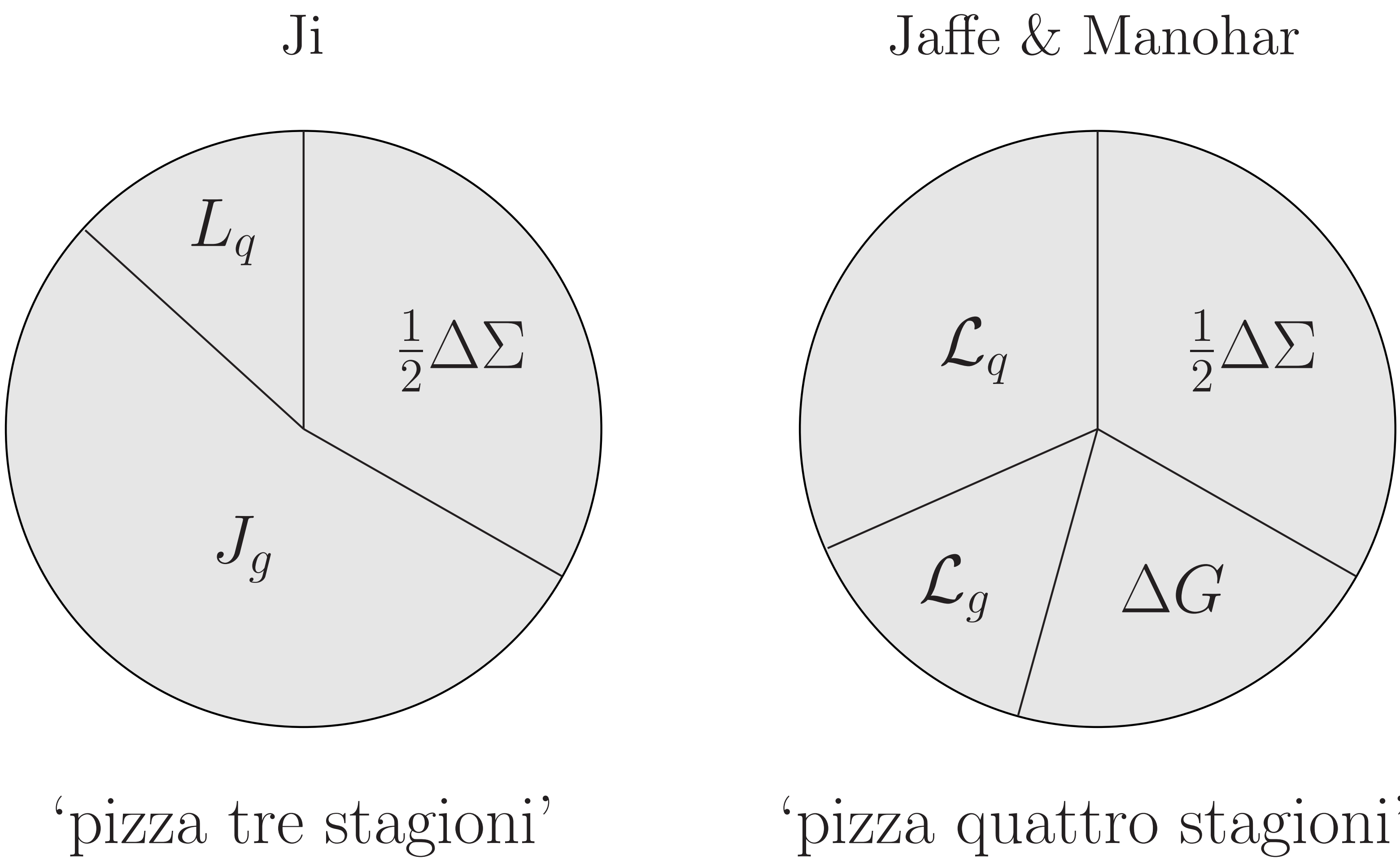
- does not mean that angular momentum decomposition is meaningless, but
- one needs to be aware of this 'scheme'-dependence in the physical interpretation of exp/lattice/model results in terms of spin vs. OAM
- and, for example, not mix 'schemes', e.t.c.

Outline



- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe

The nucleon spin pizza(s)



- only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\sum_q \Delta q$ common to both decompositions!

Angular Momentum Operator

- angular momentum tensor $M^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho}$
- $\partial_\rho M^{\mu\nu\rho} = 0$

$\hookrightarrow$ $\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk} \int d^3r M^{jk0}$ conserved

$$\frac{d}{dt}\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk} \int d^3x \partial_0 M^{jk0} = \frac{1}{2}\varepsilon^{ijk} \int d^3x \partial_l M^{jkl} = 0$$

- $M^{\mu\nu\rho}$ contains time derivatives (since $T^{\mu\nu}$ does)
  - use eq. of motion to get rid of these (as in T^{0i})
  - integrate total derivatives appearing in T^{0i} by parts
  - yields terms where derivative acts on x^i which then 'disappears'

  $\hookrightarrow$ J^i usally contains both
    - 'Extrinsic' terms, which have the structure '$\vec{x}\times$ Operator', and can be identified with 'OAM'
    - 'Intrinsic' terms, where the factor $\vec{x}\times$ does not appear, and can be identified with 'spin'

Angular Momentum in QCD (Ji)

- following this general procedure, one finds in QCD

$$\vec{J} = \int d^3x \left[\psi^\dagger \vec{\Sigma} \psi + \psi^\dagger \vec{x} \times \left(i\vec{\partial} - g\vec{A}\right)\psi + \vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]$$

  with $\Sigma^i = \frac{i}{2}\varepsilon^{ijk}\gamma^j\gamma^k$

- Ji does <u>not</u> integrate gluon term by parts, <u>nor</u> identify gluon spin/OAM separately
- Ji-decomposition valid for all three components of $\vec{J}$, but usually only applied to $\hat{z}$ component, where the quark spin term has a partonic interpretation

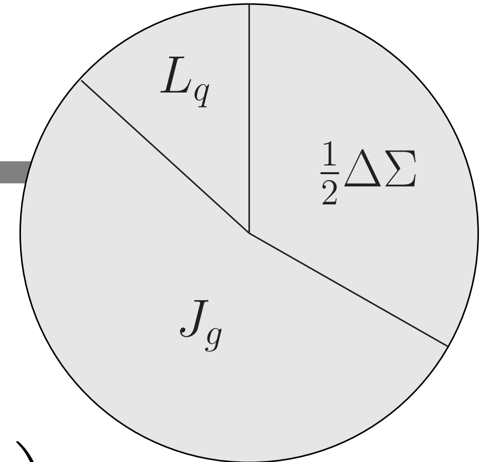
(+) all three terms manifestly gauge invariant

(+) DVCS can be used to probe $\vec{J}_q = \vec{S}_q + \vec{L}_q$

(-) quark OAM contains interactions

(-) only quark spin has partonic interpretation as a single particle density

Ji-decomposition



- Ji (1997)

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q \right) + J_g$$

with ($P^\mu = (M, 0, 0, 1)$, $S^\mu = (0, 0, 0, 1)$)

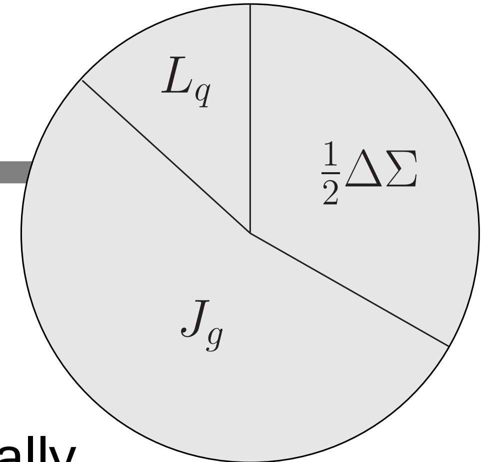
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P, S | \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, | P, S \rangle \qquad \Sigma^3 = i\gamma^1\gamma^2$$

$$L_q = \int d^3x \, \langle P, S | \, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D} \right)^3 q(\vec{x}) \, | P, S \rangle$$

$$J_g = \int d^3x \, \langle P, S | \left[\vec{x} \times \left(\vec{E} \times \vec{B} \right) \right]^3 | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Ji-decomposition



- $\vec{J} = \sum_q \frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D}\right) q + \vec{r} \times \left(\vec{E} \times \vec{B}\right)$ applies to each vector component of nucleon angular momentum, but Ji-decomposition usually applied only to $\hat{z}$ component where at least quark spin has parton interpretation as difference between number densities
- Δq from polarized DIS
- $J_q = \frac{1}{2}\Delta q + L_q$ from exp/lattice (GPDs)
- L_q in principle independently defined as matrix elements of $q^\dagger \left(\vec{r} \times i\vec{D}\right) q$, but in practice easier by subtraction $L_q = J_q - \frac{1}{2}\Delta q$
- J_g in principle accessible through gluon GPDs, but in practice easier by subtraction $J_g = \frac{1}{2} - J_q$
- further decomposition of J_g into intrinsic (spin) and extrinsic (OAM) that is local and manifestly gauge invariant has not been found

L_q for proton from Ji-relation (lattice)

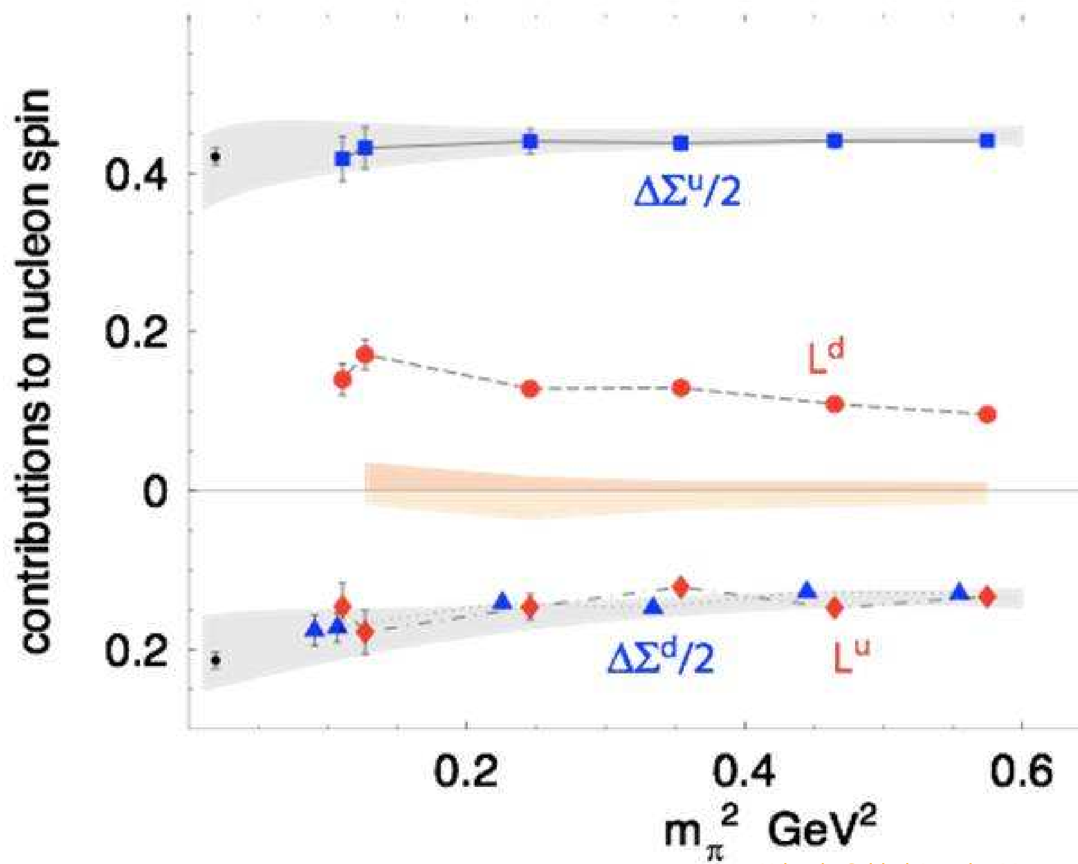
- lattice QCD $\Rightarrow$ moments of GPDs (LHPC; QCDSF)

$\hookrightarrow$ insert in Ji-relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0) + E_q(x,0) \right] x.$$

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- L_u, L_d both large!
- present calcs. show $L_u + L_d \approx 0$, but
  - disconnected diagrams ..?
  - m_π^2 extrapolation
  - parton interpret. of L_q...



Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypesurface rather than a space-like hypersurface

$$\tilde{J}^3 = \int d^2x_\perp \int dx^- M^{12+}$$

where $x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^-\right)$ and $M^{12+} = \frac{1}{\sqrt{2}}\left(M^{120} + M^{123}\right)$

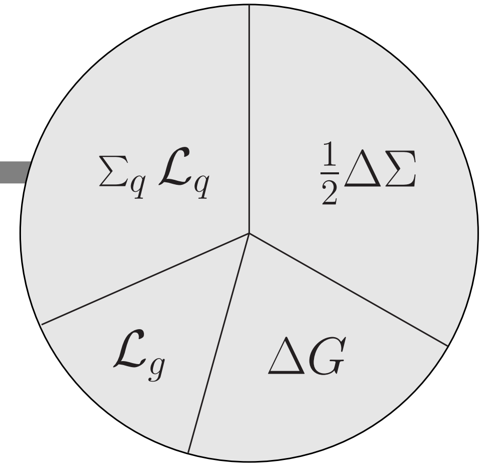
- Since $\partial_\mu M^{12\mu} = 0$

$$\int d^2\mathbf{x}_\perp \int dx^- M^{12+} = \int d^2\mathbf{x}_\perp \int dx^3 M^{120}$$

(compare electrodynamics: $\vec{\nabla}\cdot\vec{B} = 0 \quad \Rightarrow$ flux in = flux out)

- use eqs. of motion to get rid of ‘time’ (∂_+ derivatives) & integrate by parts whenever a total derivative appears in the T^{i+} part of M^{12+}

Jaffe/Manohar decomposition



- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dx^- d^2\mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

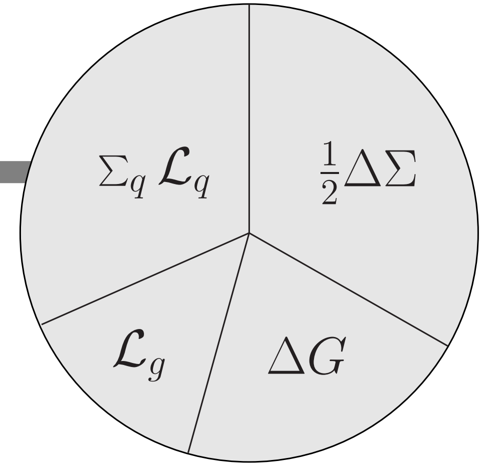
where $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\mathcal{L}_q = \int d^3r \, \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2 \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle$$

Jaffe/Manohar decomposition



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\overrightarrow{p}\overleftarrow{p}$ or polarized DIS (evolution)

$\hookrightarrow$ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to ‘mix’ Ji and JM decompositions, e.g. $J_g - \Delta G$ has no fundamental connection to OAM

$L_q \neq \mathcal{L}_q$

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- For nucleon at rest, matrix element of L_q same as that of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(xgA^y - ygA^x\right) q\Big|_{A^+=0}$

Summary part 1:

- Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- ΔG can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or $\vec{p}\overleftarrow{p}$

$\hookrightarrow$ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with 'spin' only in that gauge

- in general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$, but
- how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

OAM in scalar diquark model

[M.B. + Hikmat Budhathoki Chhetri (BC), PRD **79**, 071501 (2009)]

- toy model for nucleon where nucleon (mass M) splits into quark (mass m) and scalar 'diquark' (mass λ)

$\hookrightarrow$ light-cone wave function for quark-diquark Fock component

$$\psi^{\uparrow}_{+\frac{1}{2}}(x,\mathbf{k}_\perp) = \left(M + \frac{m}{x}\right)\phi \qquad\qquad \psi^{\uparrow}_{-\frac{1}{2}} = -\frac{k^1 + ik^2}{x}\phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x}}$.

- quark OAM according to JM: $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$
- quark OAM according to Ji: $L_q = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

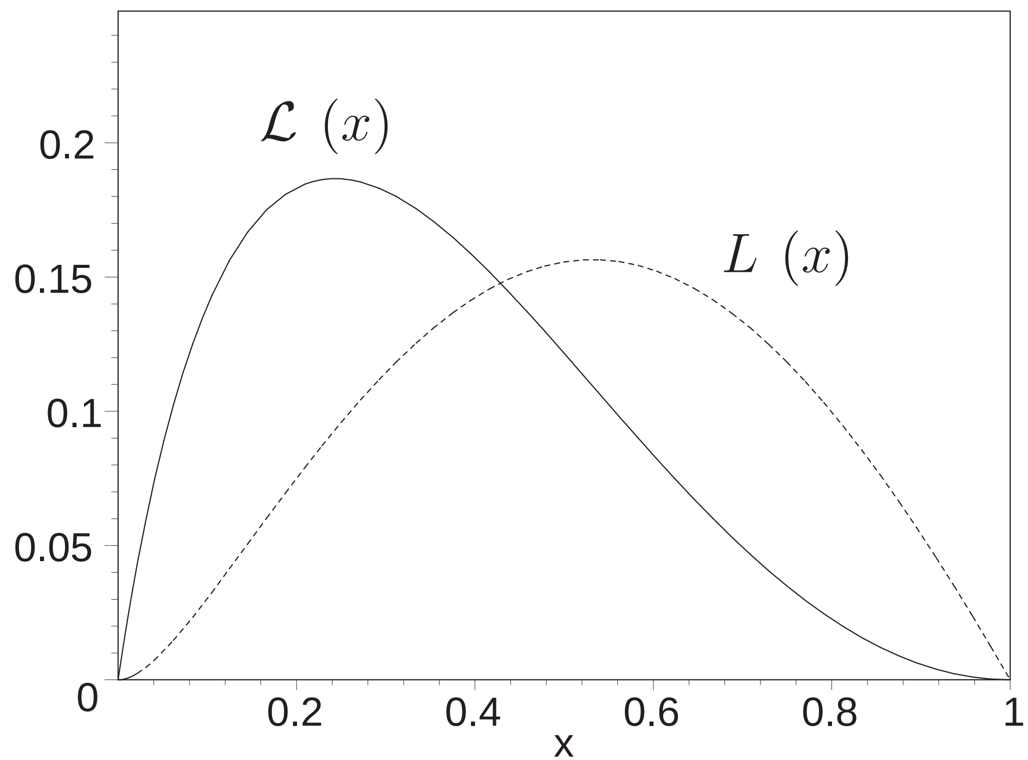
$\rightsquigarrow$ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is <u>not</u> a gauge theory

OAM in scalar diquark model

- But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2 \neq \frac{1}{2}\left\{x\left[q(x)+E(x,0,0)\right]-\Delta q(x)\right\} \equiv L_q(x)$$



$\hookrightarrow$ 'unintegrated Ji-relation' does <u>not</u> yield x-distribution of OAM

OAM in QED

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1 - ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1+ik^2}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x} - m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar
$\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp \left[(1-x)\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$
- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\,[q(x) + E(x,0,0)] - \frac{1}{2}\Delta q$

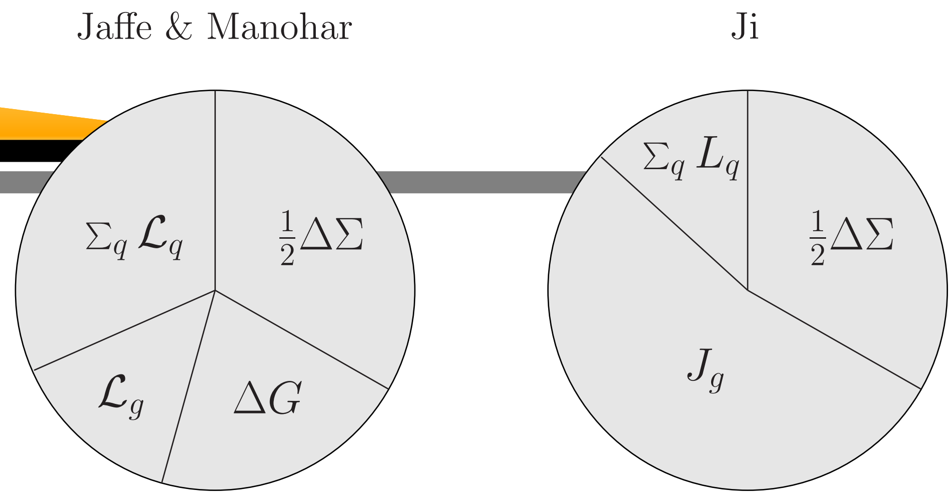
$\rightsquigarrow$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and defining $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields
$\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$
- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$

OAM in QCD

- $\hookrightarrow$ 1-loop QCD: $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$
- recall (lattice QCD): $L_u \approx -.15$; $L_d \approx +.15$
- QCD evolution yields negative correction to L_u and positive correction to L_d
- $\hookrightarrow$ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low Q^2) and lattice results ($Q^2 \sim 4GeV^2$)
- above result suggests that $\mathcal{L}_u > L_u$ and $\mathcal{L}_d > L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- $\hookrightarrow$ possible that lattice result consistent with $\mathcal{L}_u > \mathcal{L}_d$

Summary



- inclusive $\overrightarrow{e}\overleftarrow{p}$/$\overrightarrow{p}\overleftarrow{p}$ provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - gluon spin ΔG
  - parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - J_q & $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$ does <u>not</u> yield gluon OAM $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$ for O (α_s) dressed quark

Announcement:

- workshop on Orbital Angular Momentum of Partons in Hadrons
- ECT* 9-13 November 2009
- organizers: M.B. & Gunar Schnell
- confirmed participants: M.Anselmino, H.Avakian, A.Bacchetta, L.Bland, D.Boer, S.J.Brodsky, M.Diehl, D.Fields, L.Gamberg, G.Goldstein, M.Grosse-Perdekamp, P.Hägler, X.Ji, R.Kaiser, E.Leader, N.Makins, A.Miller, D.Müller, P.Mulders, A.Schäfer, G.Schierholz, O.Teryaev, W.Vogelsang, F.Yuan