Quark Correlations and $\perp$ Single-Spin Asymmetries

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Outline

- What are single spin asymmetries and why are they interesting
- Sivers effect in QCD
- Implications for nucleon structure
- Summary

What are Single Spin Asymmetries (SSA)?

- Target (or projectile) transversely polarized

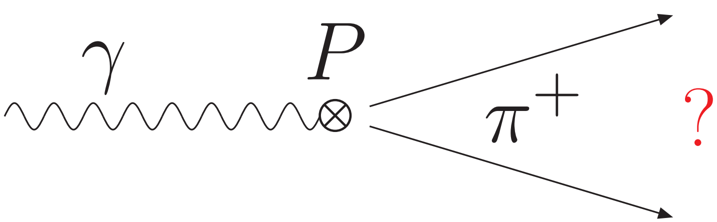
$\hookrightarrow$ left-right asymmetry of particles in the final state

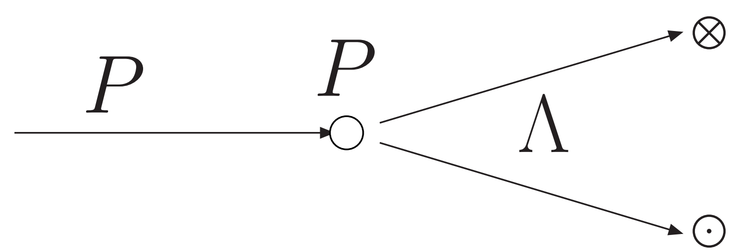
$$\gamma + p\uparrow \longrightarrow \pi^+ + X$$

- or target and projectile unpolarized

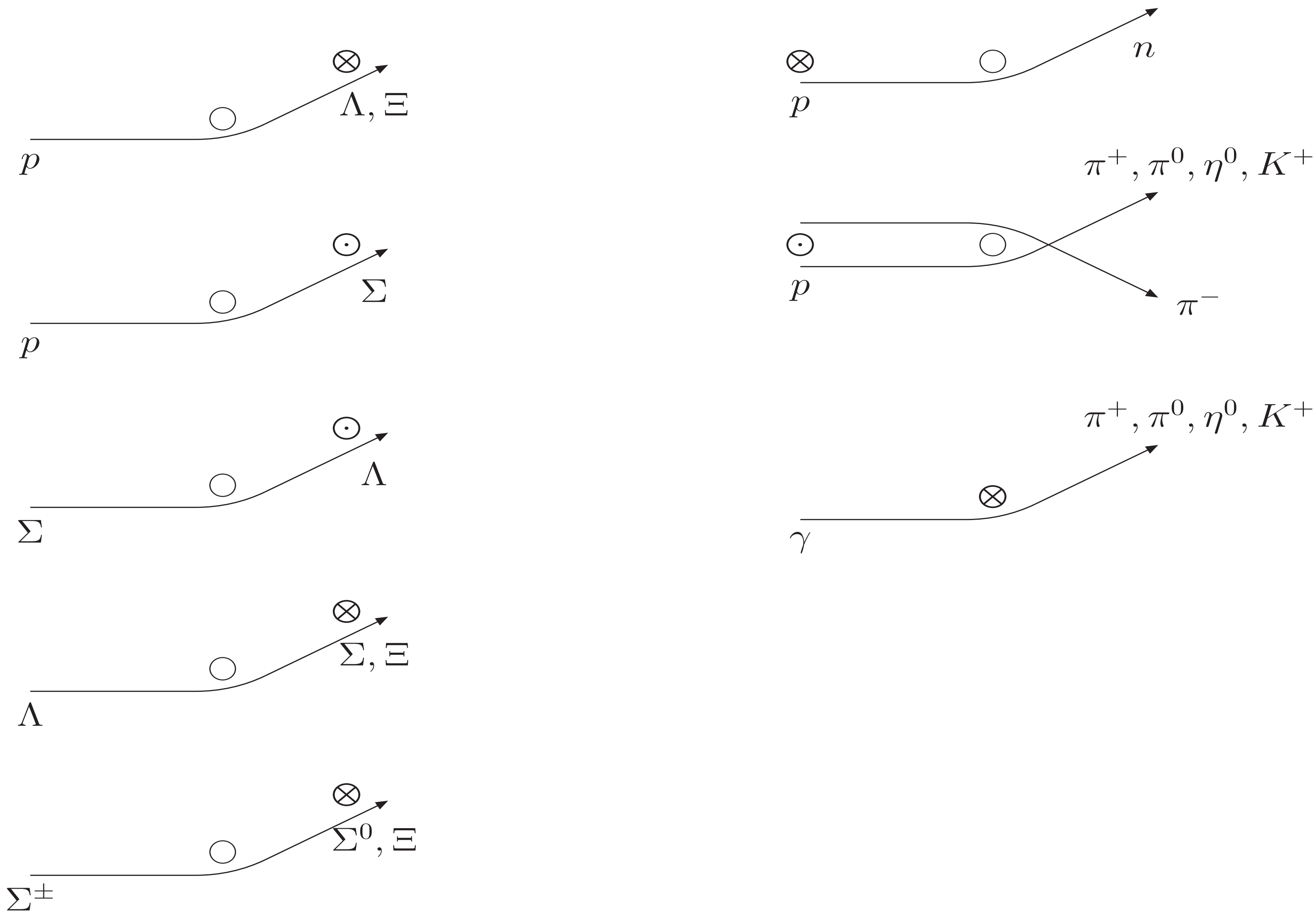
$\hookrightarrow$ transverse polarization ($\perp$ to scattering plane) is observed in final state

$$p + p \longrightarrow \Lambda\uparrow + X$$





Other Examples for $\perp$ SSA:



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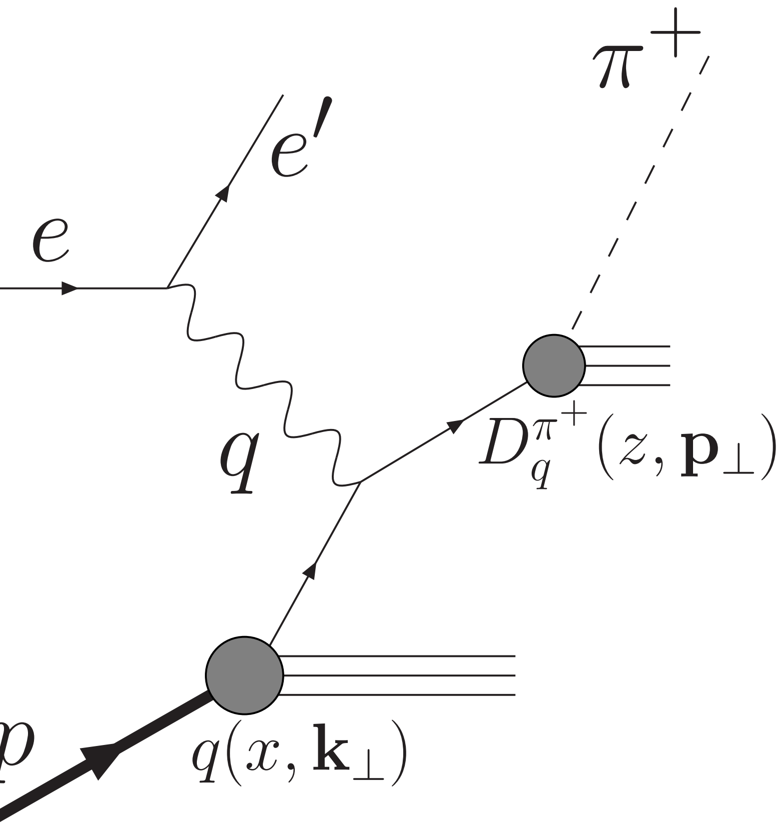
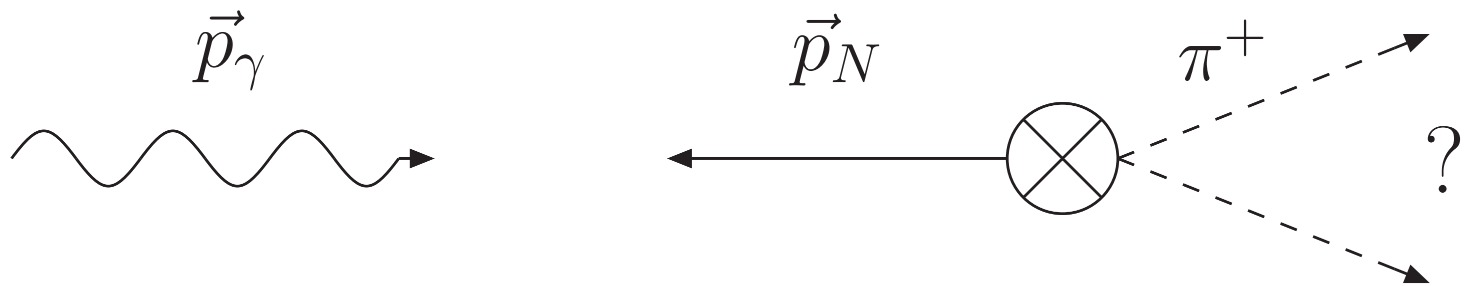
Theoretical Description ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon
  - $\hookrightarrow$ unintegrated parton density $q(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q
  - $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$

- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

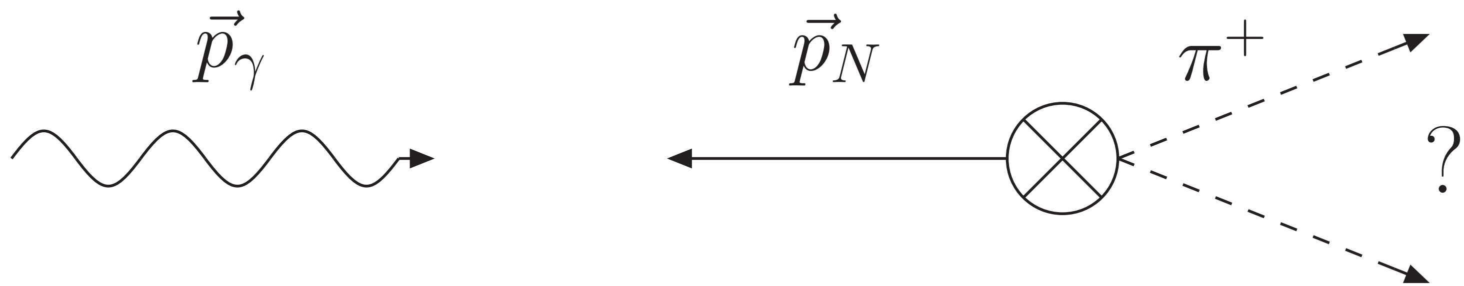
Theoretical Description ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



What is the sign/magnitude of the left-right asymmetry?

- Sivers effect: asymmetry of π^+ due to asymmetry in $\perp$ momentum distribution of quarks $q(x, \mathbf{k}_\perp)$ in target.
- Collins effect: asymmetry arises when transversely polarized quark fragments into π^+

Theoretical Description ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



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- Sivers effect: asymmetry of π^+ due to asymmetry in $\perp$ momentum distribution of quarks $q(x, \mathbf{k}_\perp)$ in target.
- Collins effect: asymmetry arises when transversely polarized quark fragments into π^+

Why is Sivers Interesting?

- $\mathbf{k}_\perp$-dependence of the nucleon wave function
- "naive" time-reversal invariance predicts vanishing effect: $(\vec{p} \times \vec{s}) \cdot \vec{k}$ is T-odd
- orbital angular momentum
- probe of space-time structure of the hadron wavefunction

Sivers Effect and $\vec{L}_q$

- optical theorem $\Rightarrow$ inclusive X-section $\leftrightarrow$ forward Compton amplitude
- $\perp$ asymmetry arises from amplitudes where helicity of initial and final state (in forward Compton amplitude) have opposite helicity

$$|x\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle + |L\rangle\right) \qquad |-x\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle - |L\rangle\right)$$

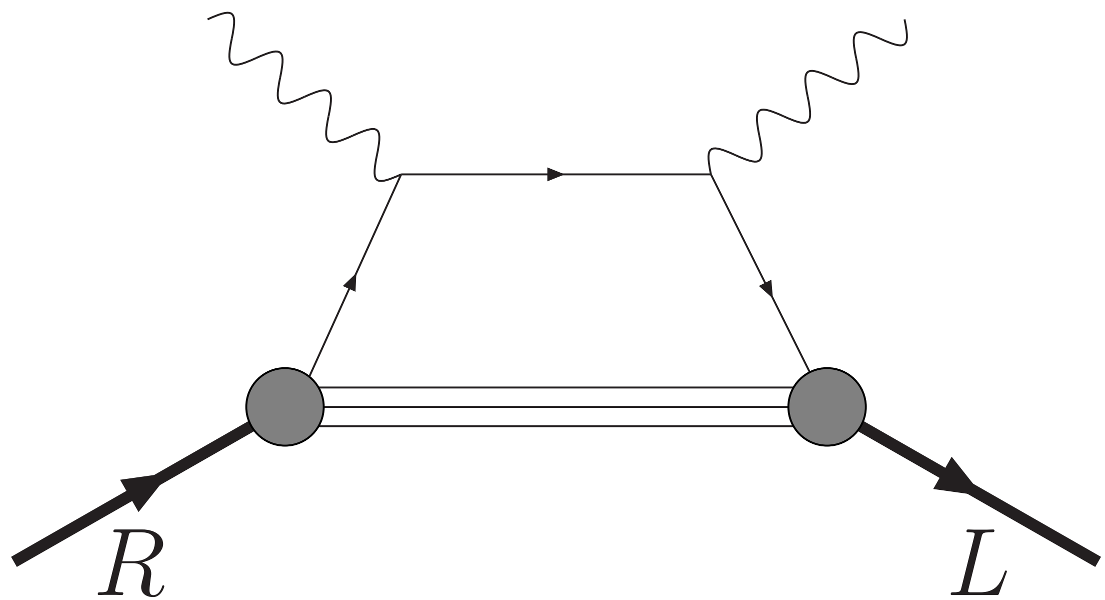
$\hookrightarrow$

$$\text{asymmetry} \propto \frac{d\sigma^{+x}}{d\Omega} - \frac{d\sigma^{-x}}{d\Omega} \propto \left\langle R \left| \hat{T} \right| L \right\rangle,$$

where $\hat{T}$ represents the operator that probes the forward Compton amplitude.

Sivers Effect and $\vec{L}_q$

asymmetry $\propto$



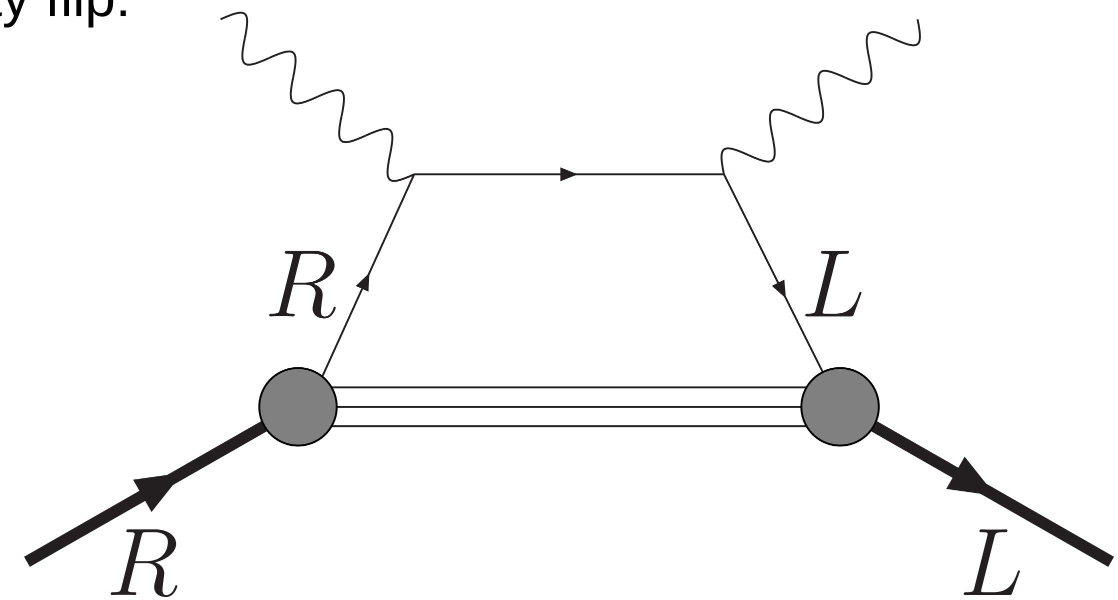
(with appropriate cuts on the momentum of the active quark ...)

- Here the quark helicity can either flip too (suppressed by chiral symmetry) or it can remain unchanged

Sivers Effect and $\vec{L}$

- quark helicity flip:

asymmetry $\propto$

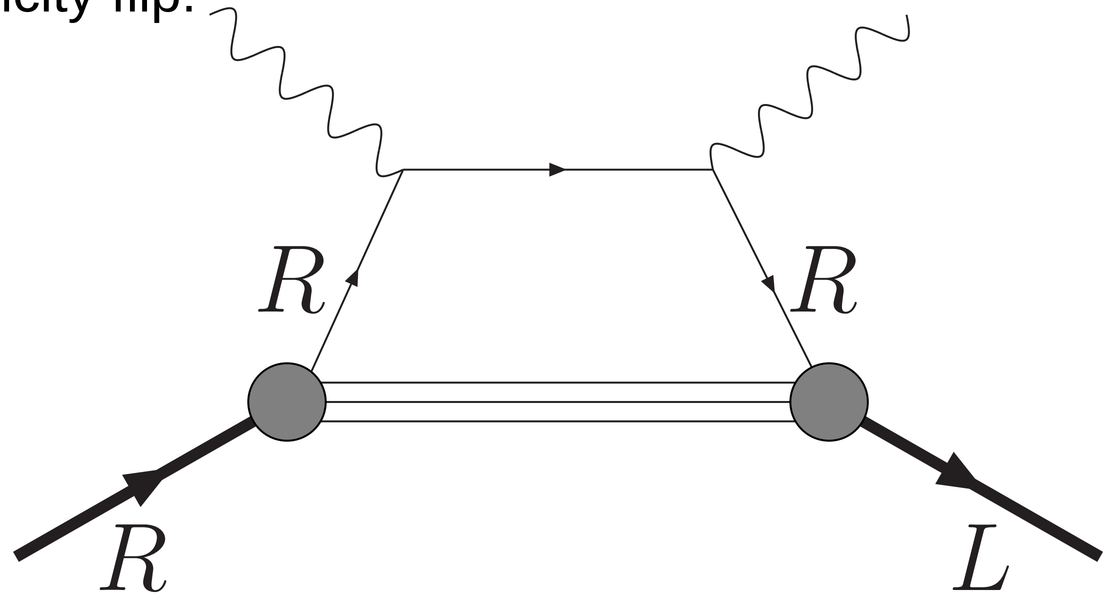


- helicity flip of quark supressed by chiral symmetry $m_q \approx 0$

Sivers Effect and $\vec{L}$

- no quark helicity flip:

asymmetry $\propto$



- total angular momentum conservation requires that initial and final state differ by one unit of orbital angular momentum

$\hookrightarrow$ Sivers effect requires orbital angular momentum in nucleon wave function

Why is Sivers interesting?

- $\mathbf{k}_\perp$-dependence of the nucleon wave function
- “naive” time-reversal invariance predicts vanishing effect: $(\vec{p} \times \vec{s}) \cdot \vec{k}$ is T-odd
- orbital angular momentum
- probe of space-time structure of the hadron wavefunction: final state interaction crucial for nonzero Sivers effect ...

$\perp$ Single Spin Asymmetry (Sivers)

- Naive definition of unintegrated parton density

$$P(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P, S \left| \bar{q}(0)\gamma^+ q(\xi) \right| P, S \right\rangle \Big|_{\xi^+=0} .$$

- Time-reversal invariance $\Rightarrow P(x, \mathbf{k}_\perp) = P(x, -\mathbf{k}_\perp)$

$\hookrightarrow$ Asymmetry $\int d^2\mathbf{k}_\perp P(x, \mathbf{k}_\perp)\, \mathbf{k}_\perp \; = 0$

- Same conclusion for gauge invariant definition with straight Wilson line

$$P(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P, S \left| \bar{q}(0)\gamma^+ U_{[0,\xi]} q(\xi) \right| P, S \right\rangle \Big|_{\xi^+=0} ,$$

where $U_{[0,\xi]} = P \exp\left(ig \int_0^1 ds \xi_\mu A^\mu(s\xi) \right)$.

$\perp$ Single Spin Asymmetry (Sivers)

- Naively (time-reversal invariance) $P(x, \mathbf{k}_\perp) = P(x, -\mathbf{k}_\perp)$
- However, including the final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)
- Gauge invariant definition requires quark to be connected by gauge link. Choice of path not arbitrary but must be chosen along path of outgoing quark to incorporate FSI

$$P(x, \mathbf{k}_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip\cdot\xi} \left\langle P, S \left| \bar{q}(0) U_{[0,\infty]} \gamma^+ U_{[\infty,\xi]} q(\xi) \right| P, S \right\rangle \Big|_{\xi^+=0}$$

with $U_{[0,\infty]} = P \exp\left(ig \int_0^\infty d\eta^- A^+(\eta)\right)$

- What is sign/magnitude of this result?
- What do we learn about the nucleon if we know this matrix element?

$\perp$ Single Spin Asymmetry (Sivers)

- Modulo gauge links this yields ... (Mankiewicz et al., Sterman, Qiu, Koike, Boer et al.,..)

$$\langle \mathbf{k}_\perp \rangle \sim \left\langle P,S \left| \bar{q}(0)\gamma^+ \int_0^\infty d\eta^- G^{+\perp}(\eta) q(0) \right| P,S \right\rangle$$

- physical (semi-classical) interpretation:
  - net transverse momentum is result of averaging over the transverse force from spectators on active quark
  - $\int_0^\infty d\eta^- G^{+\perp}(\eta)$ is $\perp$ impulse due to FSI

Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P \exp\left(ig \int_0^\infty d\eta^- A^+(\eta) \right) = 1$$

$\hookrightarrow$ Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

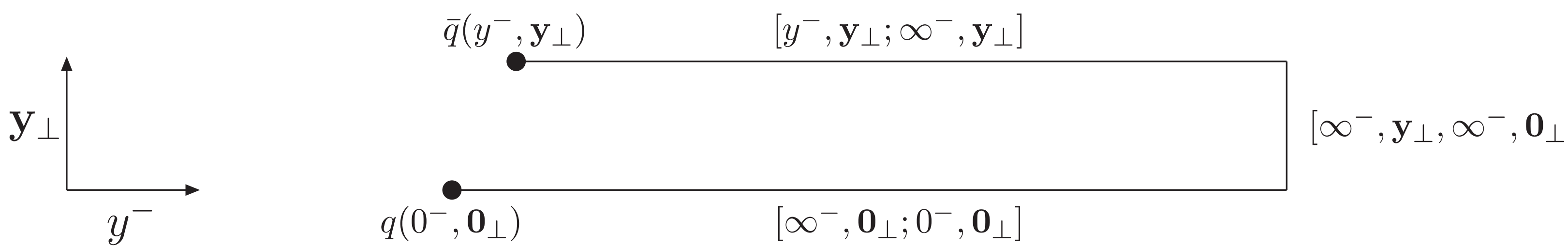
- X.Ji: fully gauge invariant definition for $P(x, \mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$\begin{aligned} P(x, \mathbf{k}_\perp) &= \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\ &\times \left\langle p, s \left| \bar{q}(y) \gamma^+ U_{[y^-, \mathbf{y}_\perp; \infty^-, \mathbf{y}_\perp]} U_{[\infty^-, \mathbf{y}_\perp, \infty^-, \mathbf{0}_\perp]} U_{[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]} q(0) \right| p, s \right\rangle \end{aligned}$$

Sivers Mechanism in $A^+ = 0$ gauge

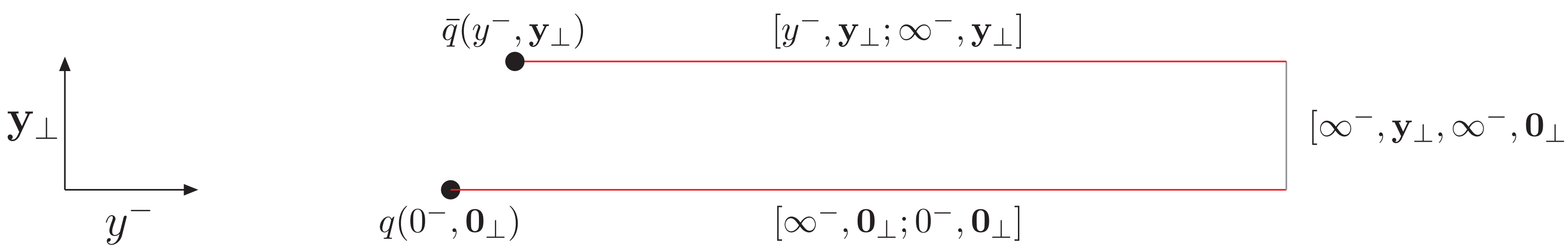
- fully gauge invariant definition for $P(x, \mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$
\begin{aligned}
P(x, \mathbf{k}_\perp) &= \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\
&\times \left\langle p, s \left| \bar{q}(y) \gamma^+ U_{[y^-,\mathbf{y}_\perp;\infty^-,\mathbf{y}_\perp]} U_{[\infty^-,\mathbf{y}_\perp,\infty^-,\mathbf{0}_\perp]} U_{[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]} q(0) \right| p, \right.
\end{aligned}
$$

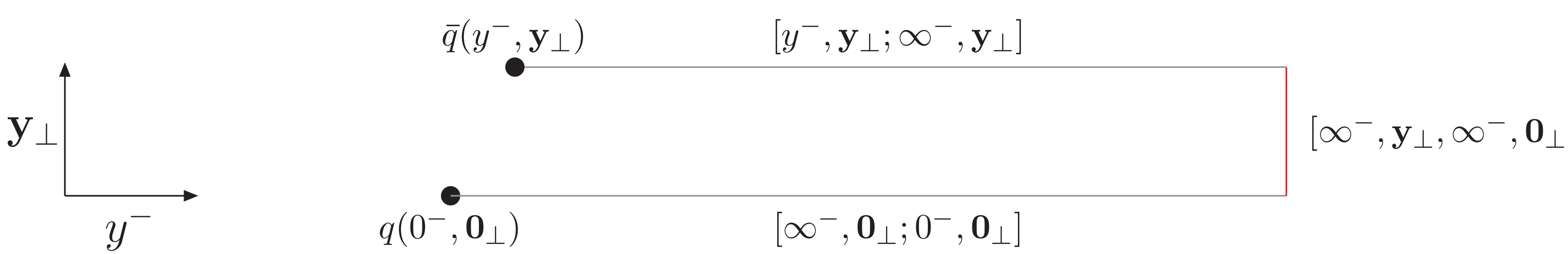


Sivers Mechanism in $A^+ = 0$ gauge

- Most gauges (e.g. Feynman gauge): link at $x^- = \infty$ yields no contribution $U_{[\infty^-,\mathbf{y}_\perp,\infty^-,\mathbf{0}_\perp]} \longrightarrow 1$



- LC-gauge: only link at $x^- = \infty$ nontrivial



Sivers Mechanism in $A^+ = 0$ gauge

↪ (LC gauge)

$$P(x, \mathbf{k}_\perp) = \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \times \left\langle p, s \left| \bar{q}(y)\gamma^+ U_{[\infty^-, \mathbf{y}_\perp, \infty^-, \mathbf{0}_\perp]} q(0) \right| p, s \right\rangle .$$

↪ ... $\longrightarrow$

$$\begin{aligned} \langle \mathbf{k}_\perp \rangle &\equiv \int dx \int d^2\mathbf{k}_\perp P(x, \mathbf{k}_\perp)\, \mathbf{k}_\perp \\ &= -\frac{g}{2p^+} \left\langle p, s \left| \bar{q}(0) \mathbf{A}_\perp(\infty^-, \mathbf{0}_\perp) \gamma^+ q(0) \right| p \right\rangle . \\ &= -\frac{g}{2p^+} \left\langle p, s \left| \bar{q}(0) \alpha_\perp(\mathbf{0}_\perp) \gamma^+ q(0) \right| p \right\rangle . \end{aligned}$$

with $\alpha_\perp(\mathbf{x}_\perp) \equiv \frac{1}{2}\left[\mathbf{A}_\perp(\infty^-, \mathbf{x}_\perp) - \mathbf{A}_\perp(-\infty^-, \mathbf{x}_\perp)\right]$

Sivers Mechanism in $A^+ = 0$ gauge

- naive treatment: $\mathbf{A}_\perp(\pm\infty^-, \mathbf{x}_\perp) = 0 \quad \Rightarrow \quad$ no SSA
- Proper treatment of $x^- = \pm\infty$ requires careful regularization (prescription) for gluon propagator in $k^+ = 0$ region! (Brodsky, Hoyer, Schmidt; Kovchegov; ...)
- Relation of SSA to ground state LC wave functions?
- What is correlation between quark field and the gauge field at $x^- = \pm\infty$?

Finiteness Conditions

- Demand absence of infrared divergences in LC Hamiltonian for $x^- \longrightarrow \pm\infty$

  $\hookrightarrow$ $G_{\mu\nu} = 0$ for $x^- = \pm\infty$

  $\hookrightarrow$ "finiteness conditions" on states:

  (1) $\partial^i \alpha_a^i(\mathbf{x}_\perp) \stackrel{!}{=} -\rho_a(\mathbf{x}_\perp)$, where $\rho_a(\mathbf{x}_\perp)$ is the total charge (quarks plus gluons) along a line with fixed $\mathbf{x}_\perp$

$$\rho_a(\mathbf{x}_\perp) = g \int dx^- \left[\sum_q \bar{q}\gamma^+ \frac{\lambda_a}{2} q - f_{abc} A_b^i \partial_- A_c^i \right]$$

  (2) $\alpha_\perp(\mathbf{x}_\perp)$ must be pure gauge

$$\alpha_i(\mathbf{x}_\perp) = \frac{i}{g} U^\dagger(\mathbf{x}_\perp) \partial_i U(\mathbf{x}_\perp)$$

Finiteness Conditions

- $\partial^i \alpha_a^i(\mathbf{x}_\perp) \stackrel{!}{=} -\rho_a(\mathbf{x}_\perp)$ another reminder that gauge field at $x^- = \pm\infty$ cannot be set to zero in LC-gauge

$\hookrightarrow$ shows again the need for a careful prescription of the k^+-singularity in gauge field propagator

- Conditions on $\alpha_a^i(\mathbf{x}_\perp)$ very similar to Eqs. derived by McLerran, Venugopalan et al. in context of gluon distributions at small x

$\hookrightarrow$ $\alpha_a^i(\mathbf{x}_\perp)$ nonzero and potentially large, but what is the net effect for Sivers asymmetry?

Quark Correlations $\longleftrightarrow$ SSA

- lowest order (small g) solution to finiteness conditions

$$\alpha_a^i(\mathbf{x}_\perp) = -\int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{x^i - y^i}{|\mathbf{x}_\perp - \mathbf{y}_\perp|^2} \rho_a(\mathbf{y}_\perp)$$

(equivalent to treating FSI in lowest order perturbation theory).

- Insert into expression for Sivers asymmetry in LC-gauge

$\hookrightarrow$

$$\langle k_q^i \rangle = -\frac{g}{4p^+} \int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{y^i}{|\mathbf{y}_\perp|^2} \left\langle p, s \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0) \rho_a(\mathbf{y}_\perp) \right| p, s \right\rangle$$

$\hookrightarrow$ Physics: $\perp$ impulse from Lorentz-contracted color-Coulomb field due to “spectators”

Sivers effect $\longleftrightarrow$ color density-density correlations in $\perp$ plane

Quark Correlations $\longleftrightarrow$ SSA

- Original expression (LC gauge) for net Sivers effect contained

$$\left\langle p, s \left| \bar{q}(0) \frac{\lambda^a}{2} q(0) A_\perp^a(\infty^-, \mathbf{0}_\perp) \right| p, s \right\rangle$$

$\hookrightarrow$ difficult to evaluate from LC wave functions

- replaced by (color) density-density correlations in $\perp$ plane

$$\left\langle p, s \left| \bar{q}(0) \gamma^+ \frac{\lambda_a}{2} q(0) \rho_a(\mathbf{y}_\perp) \right| p, s \right\rangle$$

$\hookrightarrow$ straightforward to evaluate from LC wave functions!

Modeling SSAs

- $\left\langle p,s\left|\bar{q}(0)\gamma^{+}\frac{\lambda_a}{2}q(0)\rho_a(\mathbf{y}_\perp)\right|p,s\right\rangle$ much easier to evaluate/interpret in terms of LC wave functions
- Example: (valence) quark model wave functions: color part of wave function factorizes ($\sim\epsilon^{ijk}$) and therefore

$$\left\langle\bar{q}(0)\gamma^{+}\frac{\lambda_a}{2}q(0)\rho_a(\mathbf{y}_\perp)\right\rangle=-\frac{2}{3}\left\langle\bar{q}(0)\gamma^{+}q(0)\rho(\mathbf{y}_\perp)\right\rangle$$

$\hookrightarrow$ relate SSA to (color neutral) density-density correlations in impact parameter space

$$\left\langle k_q^i\right\rangle=\frac{g}{6p^{+}}\int\frac{d^2\mathbf{y}_\perp}{2\pi}\frac{y^i}{\left|\mathbf{y}_\perp\right|^2}\left\langle p,s\left|\bar{q}(0)\gamma^{+}q(0)\rho(\mathbf{y}_\perp)\right|p,s\right\rangle$$

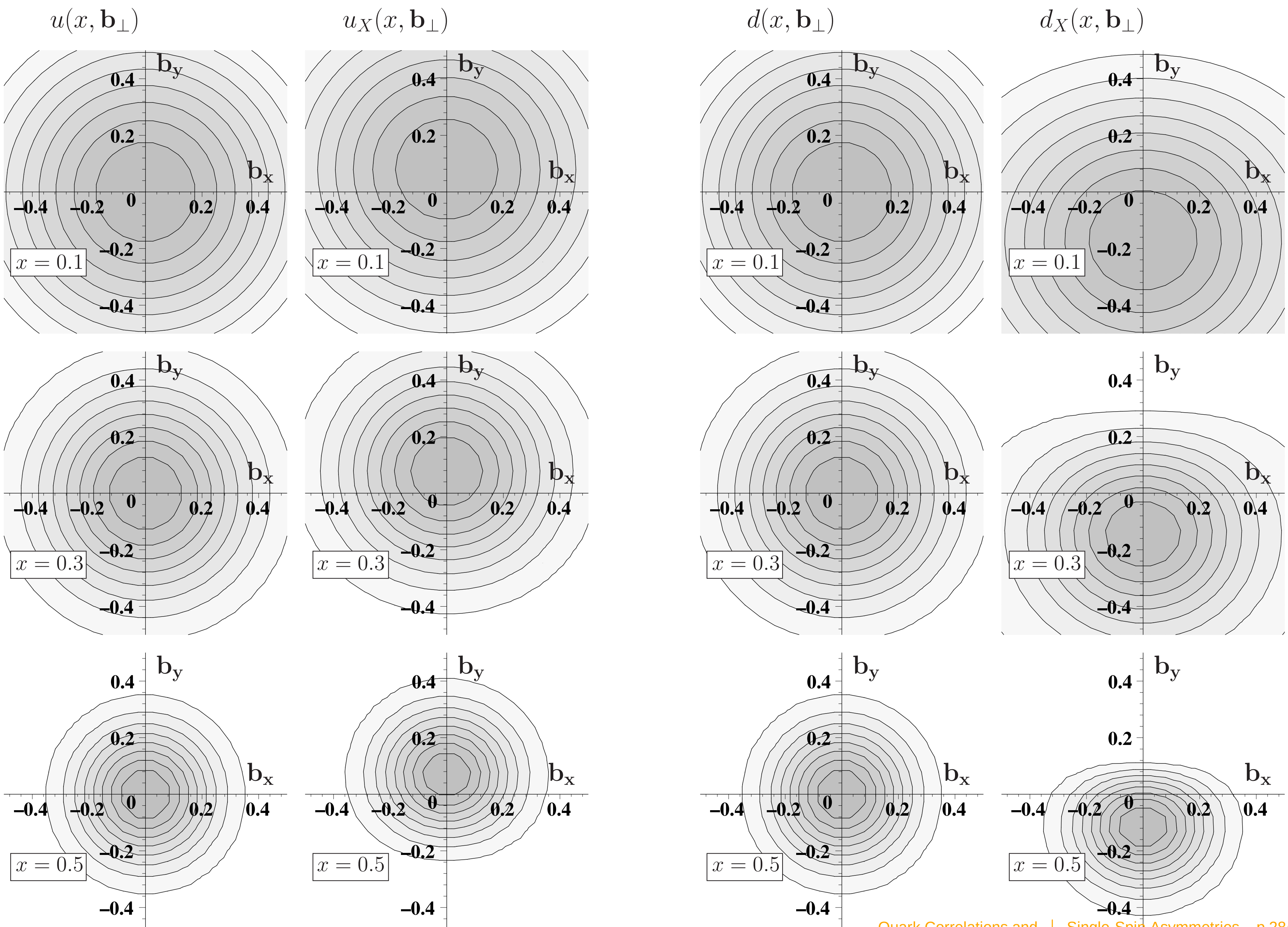
with $\rho(\mathbf{y}_\perp)=\sum_{q'}\int dy^{-}\bar{q}'(y^{-},\mathbf{y}_\perp)\gamma^{+}q'(y^{-},\mathbf{y}_\perp)$

GPD

- Know from study of generalized parton distributions (GPDs) that distribution of partons in $\perp$ plane $q(x, \mathbf{b}_\perp)$ is significantly deformed for a transversely polarized target
- mean displacement of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_y}$
$\mathbf{b_x}$

Quark Correlations $\longleftrightarrow$ SSA

- $$\langle k_q^i \rangle = \frac{g}{6p^+} \int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{y^i}{|\mathbf{y}_\perp|^2} \left\langle p, s \left| \bar{q}(0)\gamma^+ q(0)\rho(\mathbf{y}_\perp) \right| p, s \right\rangle$$

$\hookrightarrow$ expect:

$$\langle k_u^y \rangle < 0 \qquad \text{and} \qquad \langle k_d^y \rangle > 0$$

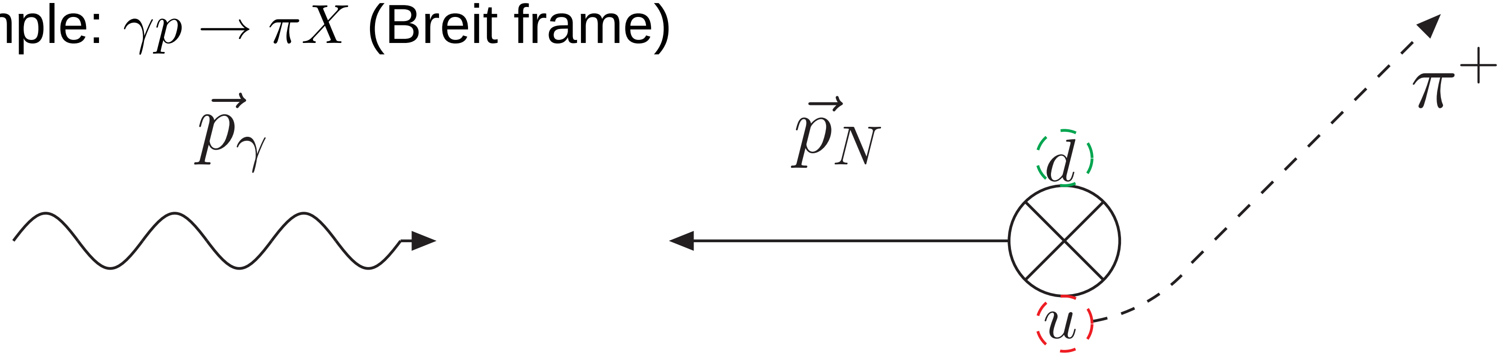
for proton polarized in $+\hat{x}$ direction

- Physics: FSI is attractive

$\hookrightarrow$ translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

Quark Correlations $\longleftrightarrow$ SSA

- example: $\gamma p \to \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI converts left-right position space asymmetry of leading quark into right-left asymmetry in momentum

- compare: convex lens that is illuminated asymmetrically

$\hookrightarrow$ “chromodynamic lensing”

- naturally leads to correlation between sign of κ_q/L_q and sign of SSA

Summary

- left-right asymmetry of π^+ produced in $\gamma + p \longrightarrow \pi^+ + X$ on transversely polarized target can have two sources:
  - Sivers: unintegrated parton density $q(x, \mathbf{k}_\perp)$ for target polarized in $\hat{x}$ direction is not symmetric under $k_y \rightarrow -k_y$
  - Collins: distribution of π^+ in jet from quark polarized in $\hat{x}$ direction is not symmetric under $k_y \rightarrow -k_y$
- Sivers effect nonzero due to final state interactions (vanishes under naive time reversal)
- Sivers interesting because it probes
  - $\mathbf{k}_\perp$-dependence of the nucleon wave function
  - requires orbital angular momentum

Summary

- Sivers asymmetry in $A^+ = 0$ gauge

$$\langle \mathbf{k}_{\perp,q} \rangle = -\frac{g}{2p^+} \left\langle p, s \left| \bar{q}(0) \alpha_\perp(\mathbf{x}_\perp) \gamma^+ q(0) \right| p \right\rangle$$

  with $\alpha_\perp(\mathbf{x}_\perp) \equiv \frac{1}{2} \left[\mathbf{A}_\perp(\infty^-, \mathbf{x}_\perp) - \mathbf{A}_\perp(-\infty^-, \mathbf{x}_\perp) \right]$

- finiteness conditions:

$$\partial^i \alpha_a^i(\mathbf{x}_\perp) \stackrel{!}{=} -\rho_a(\mathbf{x}_\perp),$$

  where $\rho_a(\mathbf{x}_\perp)$ is the total charge (quarks plus gluons) along x^- with fixed $\mathbf{x}_\perp$

- obviously $\alpha_\perp(\mathbf{x}_\perp) \neq 0$

Summary

- perturbative evaluation of $\alpha_\perp(\mathbf{x}_\perp)$ allows relating SSA to quark correlations in impact parameter space

$$\left\langle k_q^i \right\rangle = -\frac{g}{4p^+}\int \frac{d^2\mathbf{y}_\perp}{2\pi}\frac{y^i}{\left|\mathbf{y}_\perp\right|^2}\left\langle p,s\left|\bar{q}(0)\gamma^+\frac{\lambda_a}{2}q(0)\rho_a(\mathbf{y}_\perp)\right|p,s\right\rangle$$

$\hookrightarrow$ much easier to calculate/interpret in terms of LC wave functions than original expression involving $\mathbf{A}_\perp(\pm\infty^-,\mathbf{x}_\perp)$

- Quark models:

$$\left\langle k_q^i \right\rangle = \frac{g}{6p^+}\int \frac{d^2\mathbf{y}_\perp}{2\pi}\frac{y^i}{\left|\mathbf{y}_\perp\right|^2}\left\langle p,s\left|\bar{q}(0)\gamma^+q(0)\rho(\mathbf{y}_\perp)\right|p,s\right\rangle$$

$\hookrightarrow$ relate to asymmetry of parton distributions in impact parameter space, which can be simply related to GPDs/κ_q/L_q

Summary

- "explains" correlation between Sivers asymmetry and orbital angular momentum and/or magnetic moment that has been observed in many model calculations (e.g. Brodsky, Hwang, Schmidt)
- one can show [M.B. hep-ph/0402014 (to appear in PRD)]

$$\langle k_g^i \rangle + \sum_q \langle k_q^i \rangle = 0.$$

- M.B. PRD 69, 057501 (2004); connection to GPDs & magnetic moment M.B. NPA 735, 185 (2004); explicit example (scalar diquark model) M.B. & D.S.Hwang PRD 69, 074032 (2004).

$\perp$ Single Spin Asymmetry (Sivers)

- Modulo gauge links this yields ... (Mankiewicz et al., Sterman, Boer et al.,..)

$$\langle \mathbf{k}_\perp \rangle \sim \left\langle P,S \left| \bar{q}(0)\gamma^+ \int_0^\infty d\eta^- U_{[0,\eta]} G^{+\perp}(\eta) U_{[\eta,0]} q(0) \right| P,S \right\rangle$$

- physical (semi-classical) interpretation:
  - net transverse momentum is result of averaging over the transverse force from spectators on active quark
  - $\int_0^\infty \frac{d\eta^-}{2\pi} G^{+\perp}(\eta)$ is $\perp$ impulse due to FSI
- What is sign/magnitude of this result?
- What do we learn about the nucleon if we know this matrix element?

Finiteness Conditions

- LC energy divergent at $x^- = \pm\infty$ unless both $G^{+-}G^{+-}$ and $G^{12}G^{12}$ vanish at $x^- = \pm\infty$.
- $G^{12}G^{12} = 0 \quad \Rightarrow \quad A^j$ pure gauge

$$A^j(\infty^-, \mathbf{x}_\perp) = \frac{i}{g} U^\dagger(\mathbf{x}_\perp)\partial^j U(\mathbf{x}_\perp)$$

- $G_a^{+-} = \partial_- A_a^-$ in light cone gauge.
- Integrate constraint equation for A^- in LC gauge:

$$-\partial_-^2 A_a^- - \partial_- \partial^i A_a^i - g f_{abc} A_b^i G_c^{i+} = j_a^+$$

over x^-, using $\partial_- A_a^-(\pm\infty^-, \mathbf{x}_\perp) = 0$ yields

$$\partial^i \alpha_a^i(\mathbf{x}_\perp) = -\rho_a(\mathbf{x}_\perp)$$

Finiteness Conditions

with

$$\alpha^i(\mathbf{x}_\perp) = \frac{1}{2}\left[A_a^i(\infty^-, \mathbf{x}_\perp) - A_a^i(-\infty^-, \mathbf{x}_\perp)\right]$$

$$\rho_a(\mathbf{x}_\perp) = g\int dx^-\left[\bar{q}\gamma^+\frac{\lambda^a}{2}q + f_{abc}A_b^i G_c^{i+}\right]$$

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Impact parameter dependent PDFs

M.B., Int.J.Mod.Phys. A18, 173 (2003)

- define state that is localized in $\perp$ position:

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note:
  $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$
  this state has $\mathbf{R}_\perp \equiv \frac{1}{P^+}\int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \mathbf{0}_\perp$ (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{0}_\perp \right| \bar{\psi}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

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