# Probes of Orbital Angular Momentum

Matthias Burkardt

burkardt@nmsu.edu

New Mexico State University

Las Cruces, NM, 88003, U.S.A.

# Motivation

- Decomposition of nucleon spin

$$\frac{1}{2} = S_q + L_q + J_g \qquad \text{or} \qquad \frac{1}{2} = S_q + L_q + S_g + L_g$$

- Exp. (EMC, SLAC, HERMES): $S_q \ll \frac{1}{2}$

$\hookrightarrow$ spin crisis (i.e. nonrel. quark model picture for spin structure of nucleon too naive)

- What degrees of freedom carry the nucleon spin?

# Outline

- The anomalous magnetic moment
- Sivers effect for single spin asymmetries
- Generalized parton distributions (GPDs)
  - $\perp$ deformation of impact parameter distributions
  - $\hookrightarrow$ intuitive connection with Ji's sum rule for $J_q$
  - GPDs $\leftrightarrow$ $\perp$ correlations $\leftrightarrow$ SSA
- Summary

# Anomalous Magnetic Moment

- $$\langle p' | j^\mu(0) | p \rangle = \bar{u}(p') \left[ \gamma^\mu F_1(\Delta^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M} F_2(\Delta^2) \right] u(p)$$

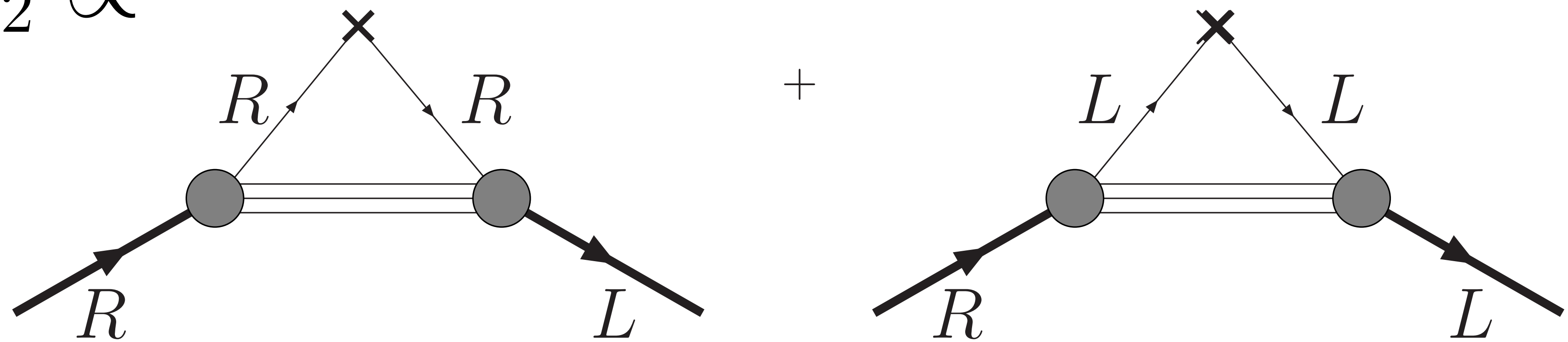
  with $\Delta = p' - p$.
- Drell-Yan West frame ($\Delta^+ = 0$)

$$\frac{1}{2p^+} \left\langle p', \uparrow \left| \bar{q}(0)\, \gamma^+ q(0) \right| p, \uparrow \right\rangle = F_1(-\boldsymbol{\Delta}_\perp^2)$$
$$\frac{1}{2p^+} \left\langle p', \uparrow \left| \bar{q}(0)\, \gamma^+ q(0) \right| p, \downarrow \right\rangle = -\frac{\Delta_x - i\Delta_y}{2M} F_2(-\boldsymbol{\Delta}_\perp^2).$$

# Anomalous Magnetic Moment and $\vec{L}$

- vector currect conserves quark helicity:

$$F_2 \propto$$




- total angular momentum conservation requires that initial and final state differ by one unit of orbital angular momentum

$\hookrightarrow$ Anomalous magnetic moment requires orbital angular momentum in nucleon wave function

# Anomalous Magnetic Moment and $\vec{L}_q$

- Anomalous magnetic moment requires quark OAM
- Let $q_{L_z \geq 1}(x)$ be the distribution of quarks with $L_z \geq 1$
- One can show (M.B. + G.Schnell, t.b.p.)

$$\left( \frac{E_q(x,0,0)}{4M} \right)^2 \leq q_{L_z \geq 1}(x) b_q^2(x)$$

where $b_q^2(x)$ is the $\mathbf{b}_\perp^2$-weighted distribution for quarks of flavor $q$ and $\int dx E_q(x,0,0) = \kappa_q = F_2^q(0)$

↪ As long as $b^2(x) < \infty$, a nonzero value of $E_q(x,0,0)$ provides a lower bound on the probability to find quarks with $L_z \geq 1$

- Note: nonrelativistically, no OAM needed to produce anomalous magnetic moment!
- physics: a relativistic particle that is confined must have OAM
- No statement about <u>net</u> OAM

# What are Single Spin Asymmetries (SSA)?

- Target (or projectile) transversely polarized

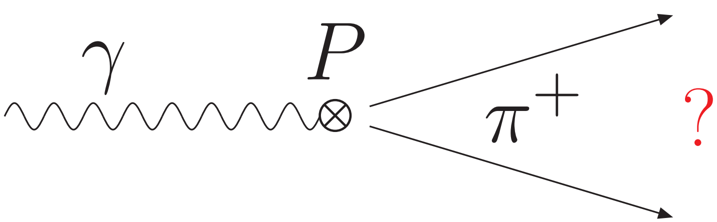
↪ left-right asymmetry of particles in the final state

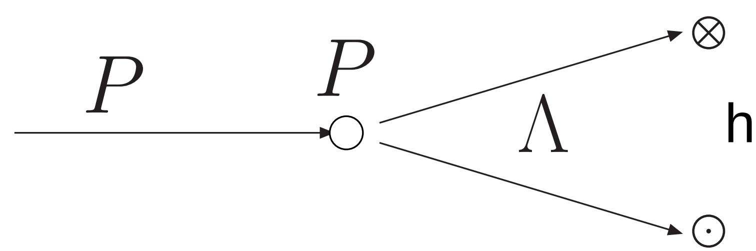
$$\gamma + p\!\uparrow \longrightarrow \pi^+ + X$$

- or target and projectile unpolarized

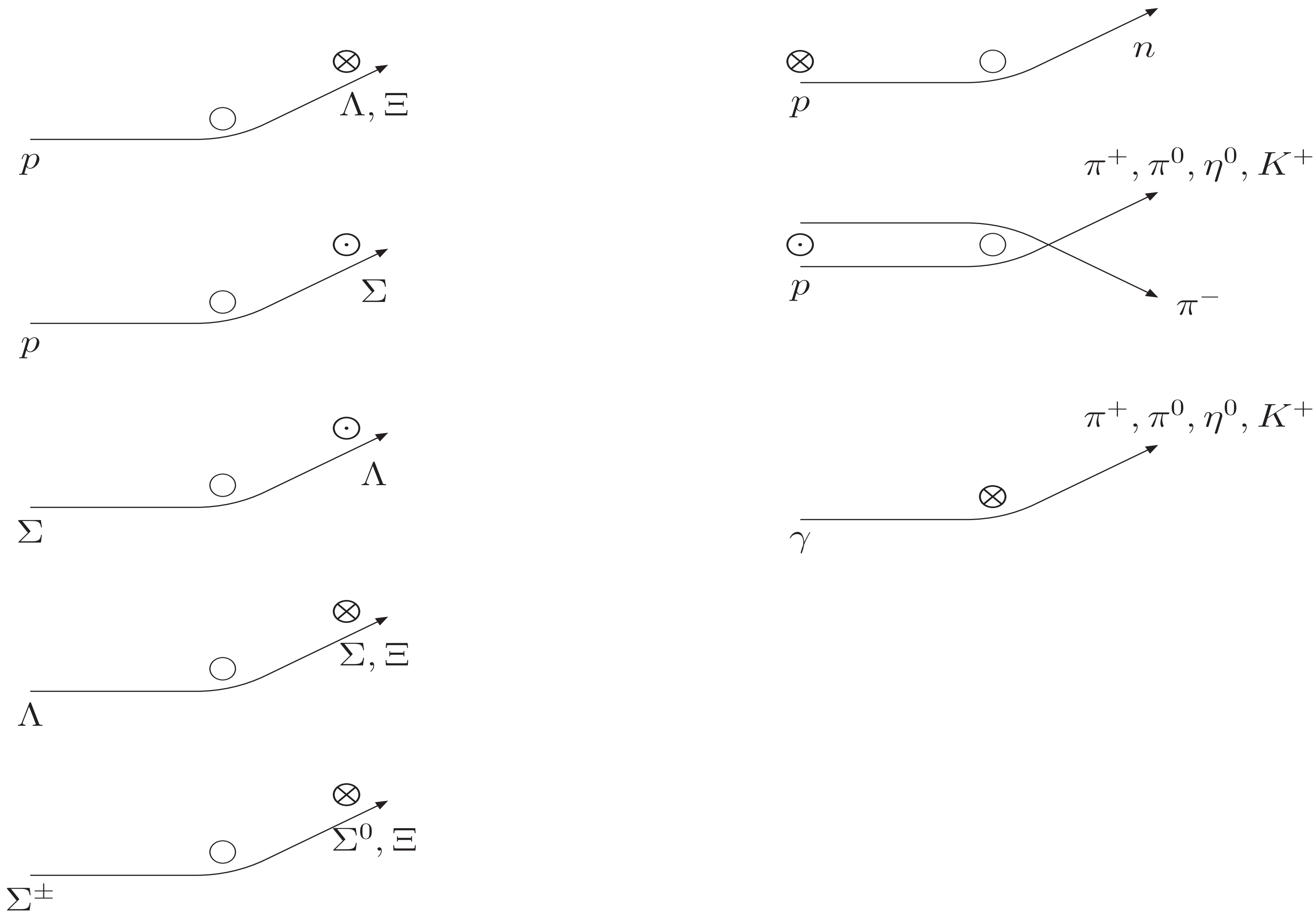
↪ transverse polarization ($\perp$ to scattering plane) is observed in final state

$$p + p \longrightarrow \Lambda\!\uparrow + X$$



# Other Examples for ⊥ SSA:

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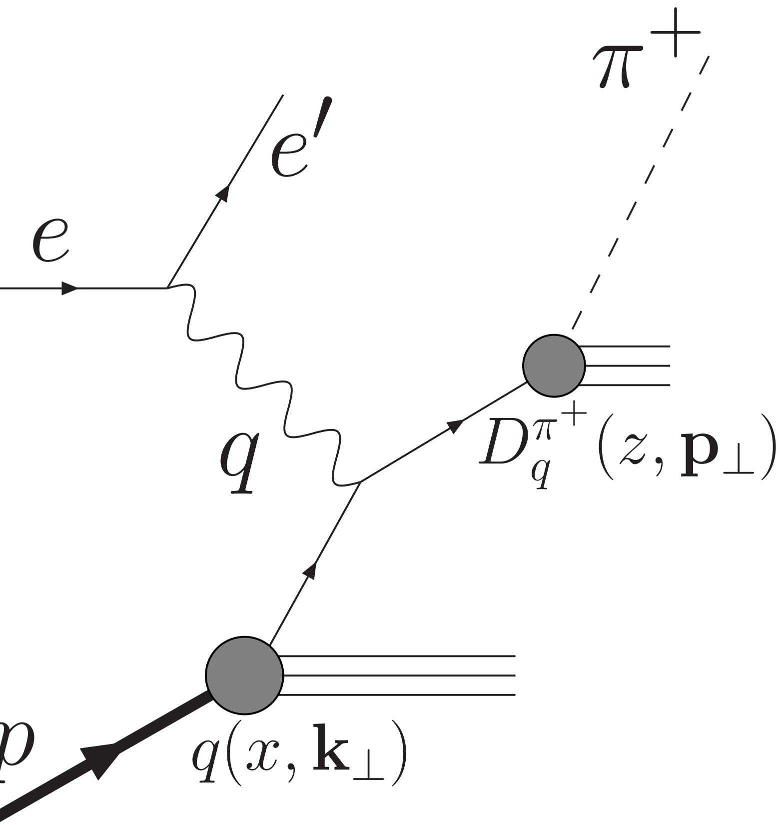
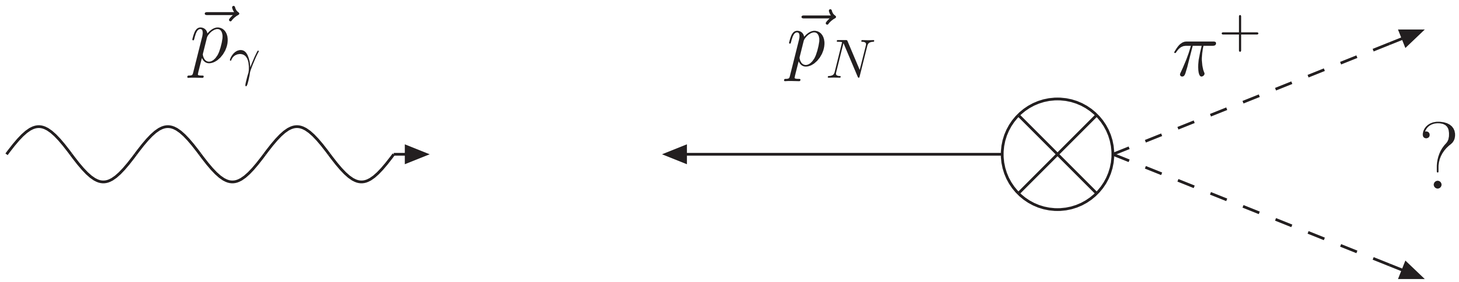
# Theoretical Description ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)




- use factorization (high energies) to express momentum distribution of outgoing $\pi^+$ as convolution of
  - momentum distribution of quarks in nucleon
  - ↪ unintegrated parton density $q(x, \mathbf{k}_\perp)$
  - momentum distribution of $\pi^+$ in jet created by leading quark $q$
  - ↪ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$
- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

# Theoretical Description ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)




What is the sign/magnitude of the left-right asymmetry?

- Sivers effect: asymmetry of $\pi^+$ due to asymmetry in $\perp$ momentum distribution of quarks $q(x, \mathbf{k}_\perp)$ in target.
- Collins effect: asymmetry arises when transversely polarized quark fragments into $\pi^+$

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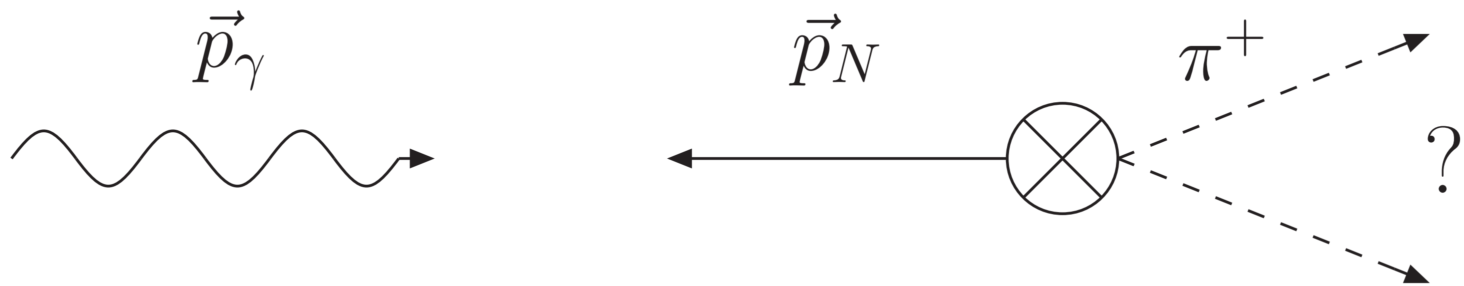


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# Sivers Effect and $\vec{L}_q$

- optical theorem $\Rightarrow$ inclusive X-section $\leftrightarrow$ forward Compton amplitude
- $\perp$ asymmetry arises from amplitudes where helicity of initial and final state (in forward Compton amplitude) have opposite helicity

$$|x\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle + |L\rangle\right) \qquad |-x\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle - |L\rangle\right)$$

$\hookrightarrow$

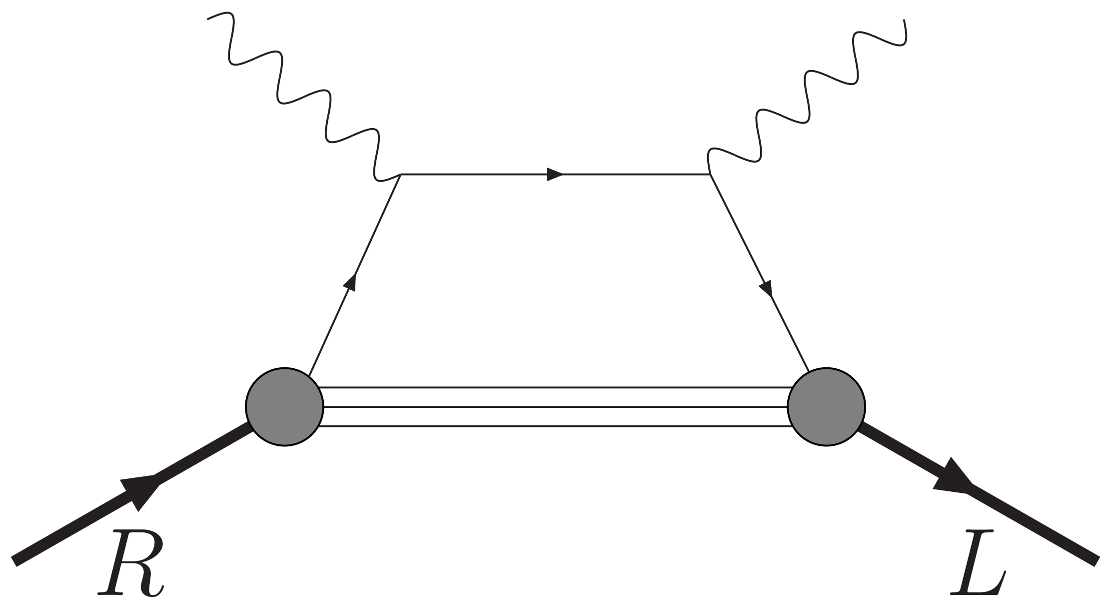
$$\text{asymmetry} \propto \frac{d\sigma^{+\hat{x}}}{d\Omega} - \frac{d\sigma^{-\hat{x}}}{d\Omega} \propto \left\langle R \middle| \hat{T} \middle| L \right\rangle,$$

where $\hat{T}$ represents the operator that probes the forward Compton amplitude.

$\hookrightarrow$ Sivers requires nonzero interference between $L$ and $R$ (nucleon) helicity amplitudes in SIDIS

# Sivers Effect and $\vec{L}_q$

asymmetry $\propto$



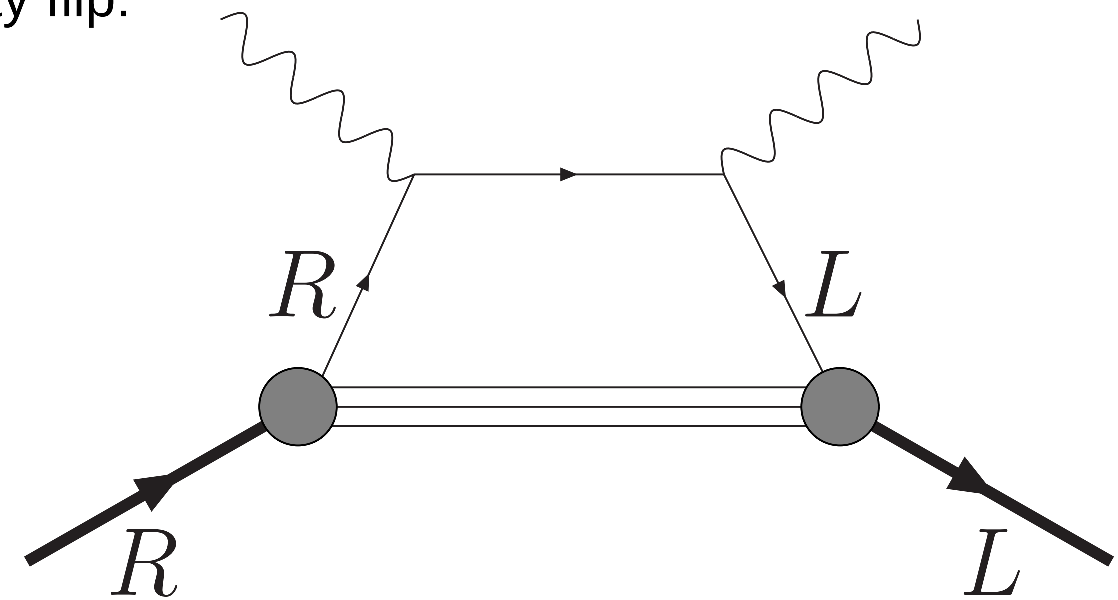

(with appropriate cuts on the momentum of the active quark ...)

- Here the quark helicity can either flip too (suppressed by chiral symmetry) or it can remain unchanged

# Sivers Effect and $\vec{L}_q$

- quark helicity flip:

asymmetry $\propto$

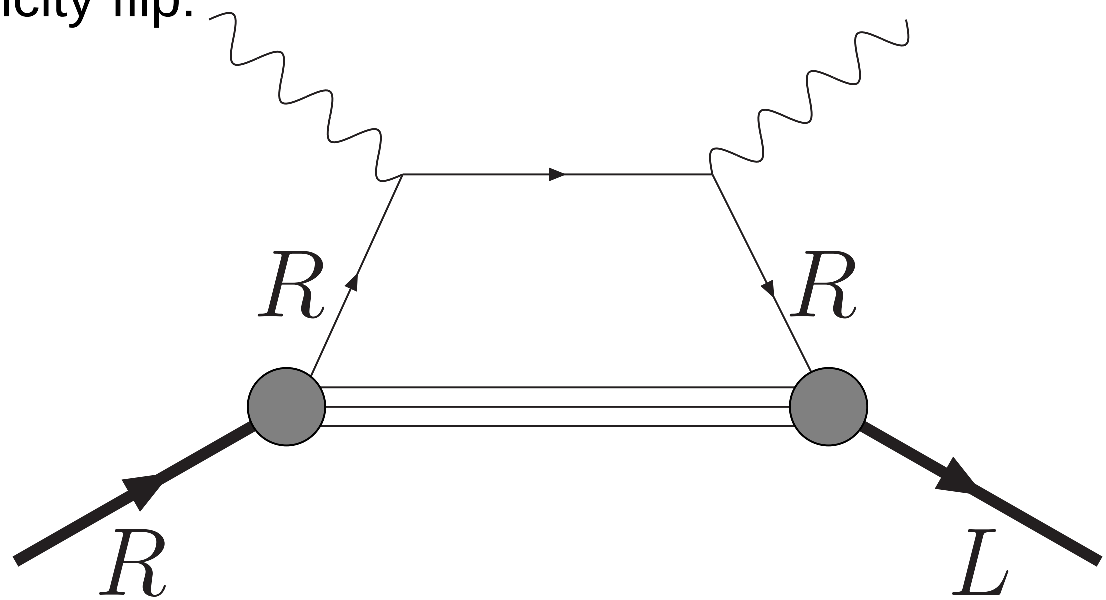



- helicity flip of quark supressed by chiral symmetry $m_q \approx 0$

# Sivers Effect and $\vec{L}$

- no quark helicity flip:

asymmetry $\propto$




- total angular momentum conservation requires that initial and final state differ by one unit of orbital angular momentum

$\hookrightarrow$ Sivers effect requires OAM in nucleon wave function

# Sivers and FSI

- average $\perp$ momentum due to FSI

$$\langle \mathbf{k}_\perp \rangle \sim \left\langle P,S \left| \bar{q}(0)\gamma^+ \int_0^\infty d\eta^- G^{+\perp}(\eta) q(0) \right| P,S \right\rangle$$

- (semi-classical) interpretation: net transverse momentum is result of averaging over the transverse force from spectators on active quark
- $\int_0^\infty d\eta^- G^{+\perp}(\eta)$ is $\perp$ impulse due to FSI
- FSI in $1^{st}$ order perturbation theory

$$\langle \mathbf{k}_q \rangle = -\frac{g}{4p^+} \int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{\mathbf{y}_\perp}{|\mathbf{y}_\perp|^2} \left\langle p,s \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0) \rho_a(\mathbf{y}_\perp) \right| p,s \right\rangle$$

$\hookrightarrow$

| Sivers effect $\longleftrightarrow$ color density-density correlations in $\perp$ plane |
|---|

# Sivers and FSI

$$\langle \mathbf{k}_q \rangle = -\frac{g}{4p^+} \int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{\mathbf{y}_\perp}{|\mathbf{y}_\perp|^2} \left\langle p,s \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0)\rho_a(\mathbf{y}_\perp) \right| p,s \right\rangle$$

- Sivers effect probes color density-density correlations in impact parameter space
- under rotations generated by $\mathbf{L}_q + \mathbf{L}_\mathbf{g}$, operator on r.h.s. transforms as a vector in plane

$\hookrightarrow$ changes OAM by one unit

$\hookrightarrow$ need OAM in wave function (quark and/or glue)

# Generalized Parton Distributions (GPDs)

- GPDs provide **decomposition of form factors** at a given value of $t$, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$
$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - $x_i$ and $x_f$ are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
  - $F_1^q(t)$, $F_2^q(t)$, $G_A^q(t)$, and $G_P^q(t)$ are the Dirac, Pauli, axial, and pseudoscalar formfactors, respectively.

# Generalized Parton Distributions (GPDs)

- measurement of the quark momentum fraction $x$ singles out one space direction (the direction of the momentum)

$\hookrightarrow$ makes a difference whether the momentum transfer is parallel, or $\perp$ to this momentum

$\hookrightarrow$ GPDs must depend on an additional variable which characterizes the direction of the momentum transfer relative to the momentum of the active quark $\longrightarrow \xi$.

- in the limit of vanishing $t$ and $\xi$, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

- GPDs are **form factor** for only those quarks in the nucleon carrying a certain **fixed momentum fraction** $x$

$\hookrightarrow$ $t$ dependence of GPDs for fixed $x$, provides information on the **position space distribution** of quarks carrying a certain momentum fraction $x$

# GPDs $\longleftrightarrow$ $q(x,\mathbf{b}_\perp)$

$\hookrightarrow$ nucleon-helicity nonflip GPDs can be related to distribution of partons in $\perp$ plane

$$q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} H(x,0,-\mathbf{\Delta}_\perp^2),$$

$$\Delta q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2),$$

- Note that $x$ already measures longitudinal momentum of quarks

$\hookrightarrow$ no similtaneous measurement of long. position of quarks (Heisenberg)

# GPDs $\longleftrightarrow \quad q(x, \mathbf{b}_\perp)$

- Impact parameter space interpretation (consider nucleon polarized in $x$ direction)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$
  with $|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\rangle = \int d^2\mathbf{k}_\perp \, |p^+, \mathbf{k}_\perp, \lambda\rangle$ .

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q_X(x,\mathbf{b}_\perp) = q(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

where $q(x, \mathbf{b}_\perp)$ is the impact parameter space distribution in a longitudinally polarized nucleon

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from $j^3$ !

# The physics of $E(x, 0, -\Delta_\perp^2)$

- $q_X(x, \mathbf{b}_\perp)$ in transversely polarized nucleon is transversely distorted compared to longitudinally polarized nucleons !
- mean displacement of flavor $q$ ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

# simple model for $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

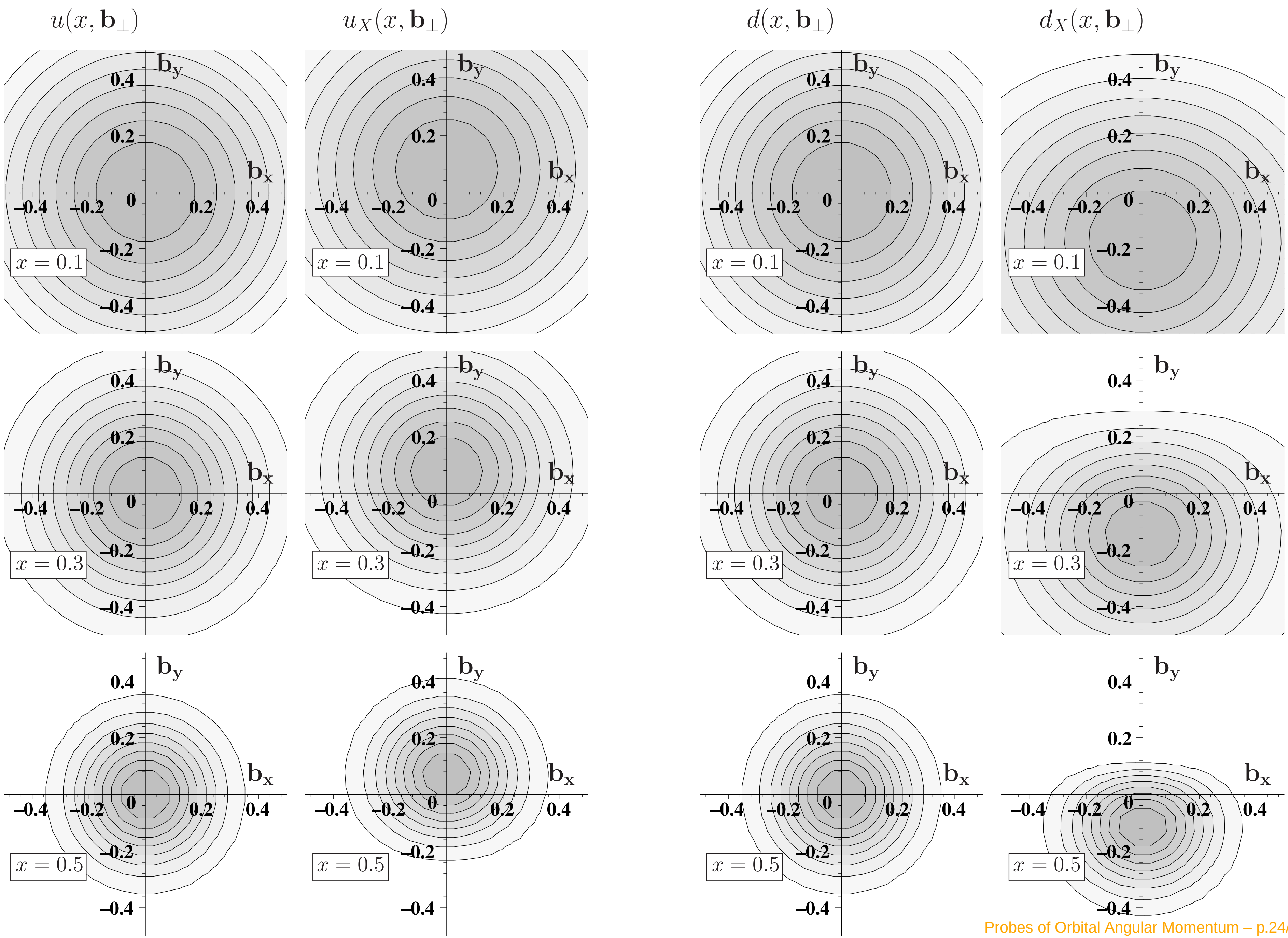
- For simplicity, make ansatz where $E_q \propto H_q$

$$
\begin{aligned}
E_u(x, 0, -\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\mathbf{\Delta}_\perp^2) \\
E_d(x, 0, -\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x, 0, -\mathbf{\Delta}_\perp^2)
\end{aligned}
$$

with

$$
\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.
$$

- Satisfies: $\int dx E_q(x, 0, 0) = \kappa_q^P$
- Model too simple but illustrates that anticipated distortion is very significant since $\kappa_u$ and $\kappa_d$ known to be large!


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_y}$
$\mathbf{b_x}$
−0.4
−0.2
0
0.2
0.4

# Intuitive connection with $\vec{L}_q$

- (some) DIS-kinematics (target rest frame & momentum transfer in $-\hat{z}$ direction):

$$\mathbf{p}_{e\perp} = \mathbf{p}'_{e\perp} = \mathbf{0}_\perp \qquad p_e^z \to -\infty \qquad p_e^{z\prime} \to -\infty$$

$\hookrightarrow$ only "$-$" component of all momenta and momentum transfers large, e.g.

$$p_e^- \equiv \frac{1}{\sqrt{2}}\left(p_e^0 - p_e^z\right) \to \infty \qquad p_e^+ \equiv \frac{1}{\sqrt{2}}\left(p_e^0 + p_e^z\right) = \frac{m_e^2 + \mathbf{p}_{e\perp}^2}{2p_e^-} \to 0$$

$\hookrightarrow$ only "$-$" component of the electron current $j_e^\mu = \bar{u}(p')\gamma^\mu u(p)$ large

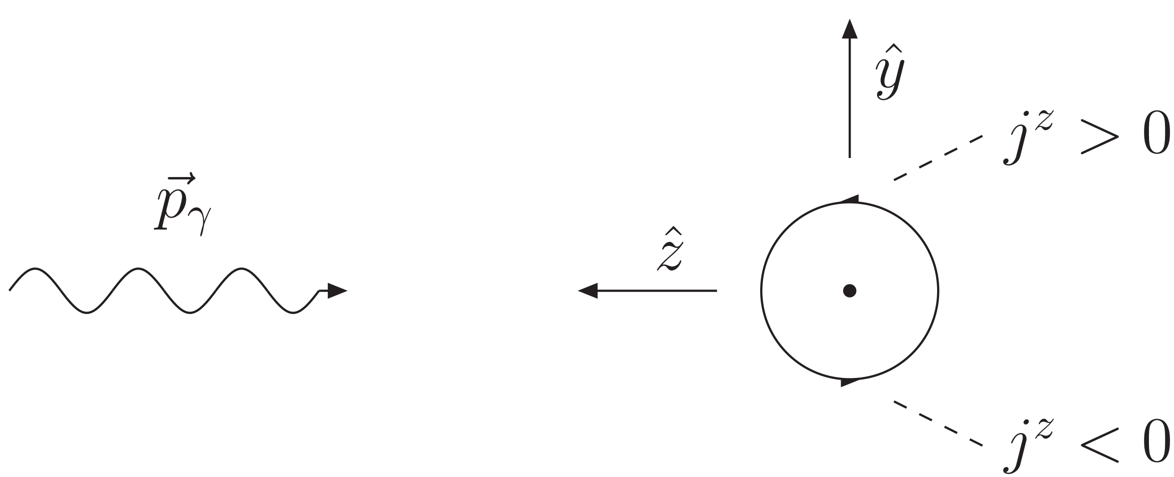
- vector-vector interaction

$$\mathcal{M} \propto j_e^\mu j_q^\nu g_{\mu\nu} = j_e^- j_q^+ + j_e^+ j_q^- - j_e^\perp j_q^\perp$$

$\hookrightarrow$ electron "sees" (for $p_e^z \to -\infty$) only $j_q^+$

# Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons "see" (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then $j^z$ is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



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- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then $j^z$ is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

$\hookrightarrow$ $j^+$ is distorted not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to $j^+$) "see" the quarks on the $+\hat{y}$ side better than on the $-\hat{y}$ side (for $L_x > 0$).

# Ji's sum rule (intuitive approach)

- Fourier transform of $H(x,0,-\boldsymbol{\Delta}_\perp^2)$ and $E(x,0,-\boldsymbol{\Delta}_\perp^2)$ describes distribution of (unpolarized) quarks in $\perp$ plane
-
$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) + \frac{\varepsilon^{ij}S_i}{2M}\frac{\partial}{\partial b_j}\mathcal{E}(x,\mathbf{b}_\perp)$$
- with $\mathcal{H}(x,\mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp^2}{(2\pi)^2} e^{i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp} H(x,0,-\boldsymbol{\Delta}_\perp^2)$, and
$\mathcal{E}(x,\mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp^2}{(2\pi)^2} e^{i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp} E(x,0,-\boldsymbol{\Delta}_\perp^2)$
- calculate
$$\langle J_q^x \rangle = \int T_q^{++} b^y$$
for this state
- $\mathcal{E}(x,\mathbf{b}_\perp)$ describes deformation of distribution of partons in $\perp$ plane when target has $\perp$ polarization $S_j$

$$\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp) b_i = \frac{\varepsilon^{ij}S_j}{2M} E_q(x,0,0)$$

# Ji's sum rule (intuitive approach)

- $\langle J_q^x \rangle = \int dx^- d^2\mathbf{b}_\perp T_q^{++} b^y$
- distribution of $T^+ +$ in $\perp$ plane given by $\int dx^- T_q^{++}(x^-, \mathbf{b}_\perp) = \int dx q(x, \mathbf{b}_\perp) x$

$\hookrightarrow$ find for this state

$$\langle J_q^x \rangle = \frac{1}{2} \int dx \int d^2\mathbf{b}_\perp q(x, \mathbf{b}_\perp) b^y = \frac{1}{2} \int dx E(x, 0, 0) x$$

which is part of Ji's sum rule

- Other part due to fact that light-cone helicity eigenstates not same as spin eigenstates in rest frame (Wigner-Melosh rotation)

$\hookrightarrow$ need to take expectation value in state that has small $\perp$ momenta and which corresponds to state that is $\perp$ polarized in rest frame!

- after some work one finds that there is some additional $\perp$ displacement, which eventually yields

$$\langle J_q^x \rangle = \frac{1}{2} \int dx \int d^2\mathbf{b}_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2} \int dx \left[ H(x, 0, 0) + E(x, 0, 0) \right] x$$

# GPD $\leftrightarrow$ Quark Correlations $\leftrightarrow$ SSA

- Know from study of generalized parton distributions (GPDs) that distribution of partons in $\perp$ plane $q(x, \mathbf{b}_\perp)$ is significantly deformed for a transversely polarized target
- mean displacement of flavor $q$ ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

# GPD $\leftrightarrow$ Quark Correlations $\longleftrightarrow$ SSA

- $$\left\langle k_q^i \right\rangle = \frac{g}{6p^+} \int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{y^i}{\left|\mathbf{y}_\perp\right|^2} \left\langle p,s \left| \bar{q}(0)\gamma^+ q(0) \rho(\mathbf{y}_\perp) \right| p,s \right\rangle$$

$\hookrightarrow$ expect:

$$\langle k_u^y \rangle < 0 \qquad \text{and} \qquad \langle k_d^y \rangle > 0$$

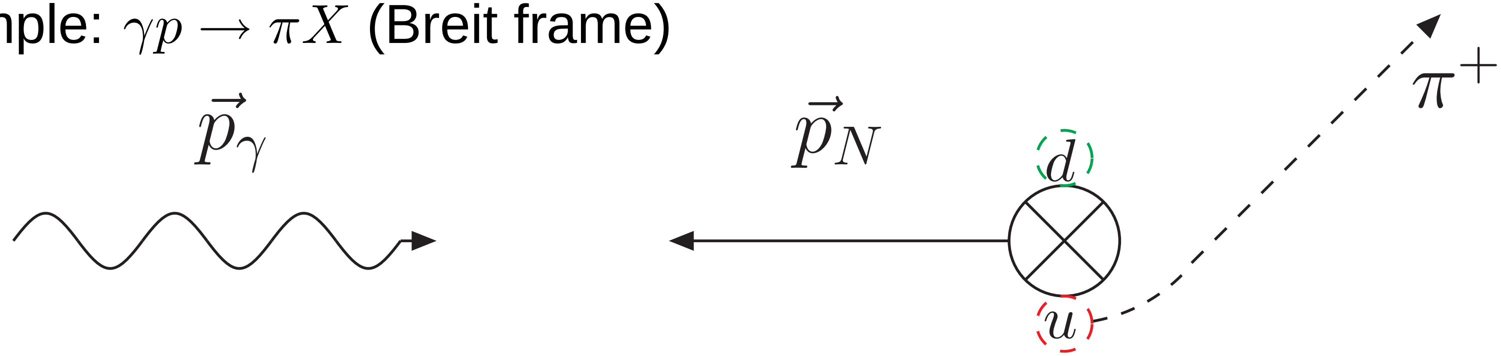
for proton polarized in $+\hat{x}$ direction

- Physics: FSI is attractive

$\hookrightarrow$ translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

# GPD $\leftrightarrow$ Quark Correlations $\longleftrightarrow$ SSA

- example: $\gamma p \rightarrow \pi X$ (Breit frame)




- $u, d$ distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by $\kappa_u$ & $\kappa_d$
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI converts left-right position space asymmetry of leading quark into right-left asymmetry in momentum

- compare: convex lens that is illuminated asymmetrically

$\hookrightarrow$ “chromodynamic lensing”

- naturally leads to correlation between sign of $\kappa_q/L_q$ and sign of SSA

# Summary

- anmalous magnetic moment of nucleon requires nonzero quark OAM — but not much — and no statement about net OAM
- left-right asymmetry of $\pi^+$ produced in $\gamma + p \longrightarrow \pi^+ + X$ on transversely polarized target can have two sources:
  - Sivers: unintegrated parton density $q(x, \mathbf{k}_\perp)$ for target polarized in $\hat{x}$ direction is not symmetric under $k_y \rightarrow -k_y$
  - Collins: distribution of $\pi^+$ in jet from quark polarized in $\hat{x}$ direction is not symmetric under $k_y \rightarrow -k_y$
- Sivers effect nonzero due to final state interactions (vanishes under naive time reversal)
- Sivers interesting because it probes
  - $\mathbf{k}_\perp$-dependence of the nucleon wave function
  - requires OAM
- can explain observed signs for Sivers from distortions of PDF in impact parameter space

# Summary

- “unintegrated Ji sum rule”

$$\frac{1}{2}\left[H_q(x,0,0)+E_q(x,0,0)\right]x$$

can be interpreted as decomposition of $\mathbf{J}_\perp^q$ w.r.t. the momentum of the quarks