GPDs and SSAs

Matthias Burkardt

burkardt@nmsu.edu

New Mexico State University

Las Cruces, NM, 88003, U.S.A.

Outline

- Brief overview on generalized parton distributions (GPDs)
- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is transversely polarized

  $\hookrightarrow$ Sivers effect for single spin asymmetries
  - $2\tilde{H}_T + E_T \longrightarrow \perp$ distortion of $\perp$ polarized PDFs in unpolarized target

  $\hookrightarrow$ correlation between quark angular momentum and quark spin

  $\hookrightarrow$ Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$
- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,\xi,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t)$$

$$\int dx E_q(x,\xi,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- formal definition:

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

Generalized Parton Distributions (GPDs)

- measurement of the quark momentum fraction x singles out one space direction (the direction of the momentum)

$\hookrightarrow$ makes a difference whether the momentum transfer is parallel, or $\perp$ to this momentum

$\hookrightarrow$ GPDs must depend on an additional variable which characterizes the direction of the momentum transfer relative to the momentum of the active quark $\longrightarrow \xi$.

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

- GPDs are **form factor** for only those quarks in the nucleon carrying a certain **fixed momentum fraction** x

$\hookrightarrow$ t dependence of GPDs for fixed x, provides information on the **position space distribution** of quarks carrying a certain momentum fraction x

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define state that is localized in $\perp$ position:

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

GPDs $\longleftrightarrow$ $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ nucleon-helicity nonflip GPDs can be related to distribution of partons in $\perp$ plane

$$\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} H(x,0,-\mathbf{\Delta}_\perp^2), \\ \Delta q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2), \end{aligned}$$

- **no rel. corrections** to this result! (Galilean subgroup of $\perp$ boosts)
- $q(x, \mathbf{b}_\perp)$ has **probabilistic interpretation**, e.g.

$$\begin{aligned} q(x,\mathbf{b}_\perp) \geq |\Delta q(x,\mathbf{b}_\perp)| \geq 0 \quad &\text{for} \quad x > 0 \\ q(x,\mathbf{b}_\perp) \leq |\Delta q(x,\mathbf{b}_\perp)| \leq 0 \quad &\text{for} \quad x < 0 \end{aligned}$$

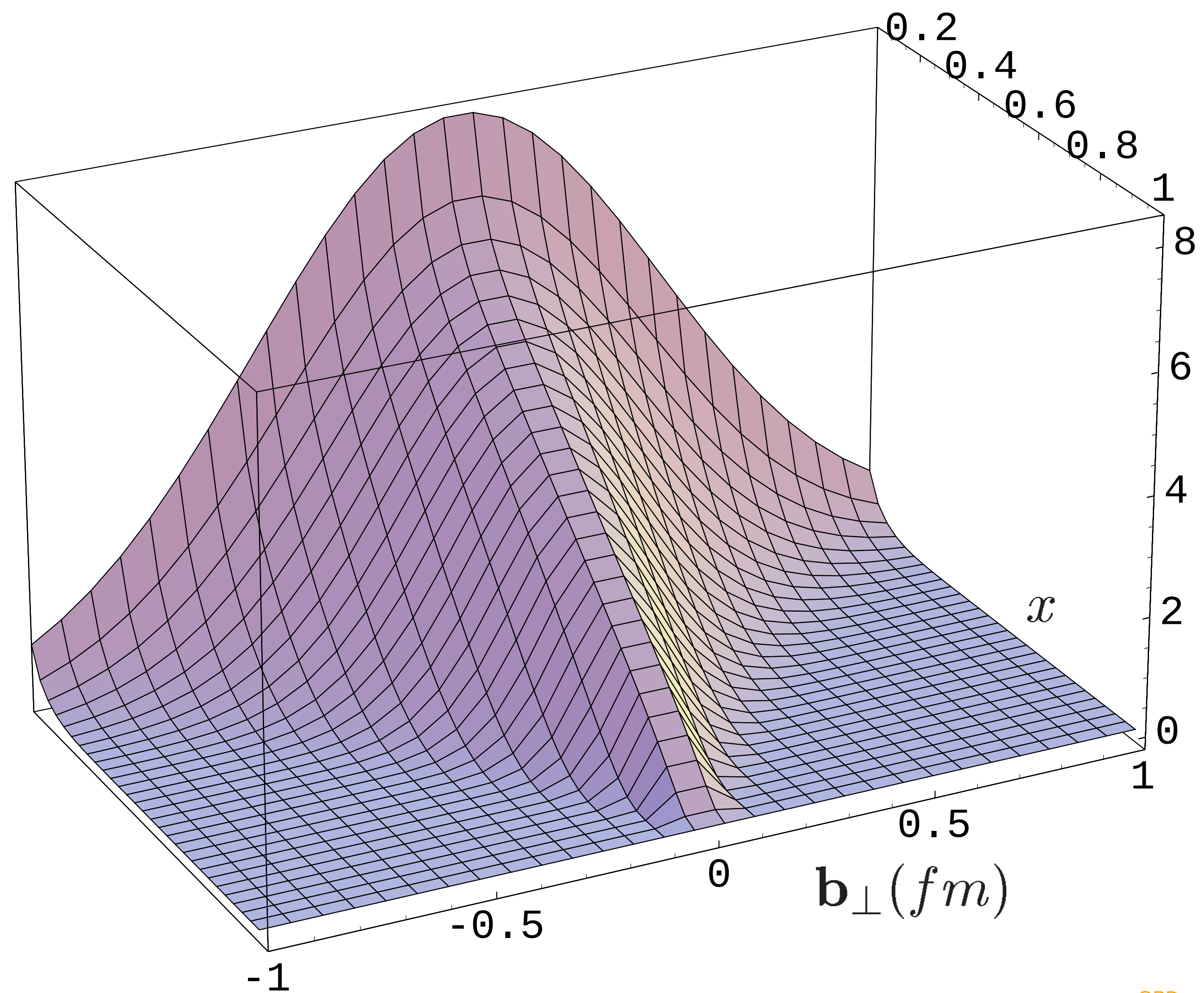
- Note that x already measures longitudinal momentum of quarks

$\hookrightarrow$ no simultaneous measurement of long. position of quarks

GPDs $\longleftrightarrow$ $q(x,\mathbf{b}_\perp)$

- $\mathbf{b}_\perp$ distribution measured w.r.t. $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{i,\perp}$
  $\hookrightarrow$ width of the $\mathbf{b}_\perp$ distribution should go to zero as $x \to 1$, since the active quark becomes the $\perp$ center of momentum in that limit!
  $\hookrightarrow$ $H(x,0,-\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp^2$-indep. as $x \to 1$. Confirmed by recent lattice studies (QCDSF, LHPC)
- Anticipated shape of $q(x,\mathbf{b}_\perp)$:
  large x: quarks from localized valence 'core',
  small x: contributions from larger ' meson cloud'
  $\hookrightarrow$ expect a gradual increase of the t-dependence ($\perp$ size) of $H(x,0,t)$ as x decreases

$q(x, \mathbf{b}_\perp)$ in a simple model



Transversely Distorted Distributions and $E(x,0,-\mathbf{\Delta}_\perp^2)$

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi=0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\uparrow\rangle \quad = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow|\bar{q}(0)\,\gamma^+ q(x^-)|\,P,\downarrow\rangle \quad = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF) $|X\rangle \equiv |p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\uparrow\rangle + |p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\downarrow\rangle$.

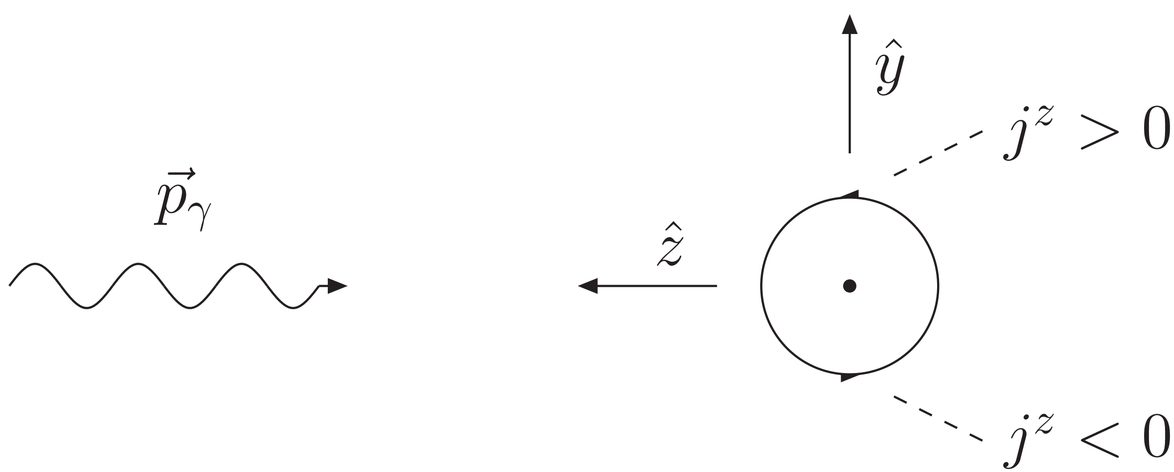
$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

- Physics: $j^+=j^0+j^3$, and left-right asymmetry from j^3 !

Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons "see" (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

$\hookrightarrow$ j^+ is distorted not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to j^+) "see" the quarks on the $+\hat{y}$ side better than on the $-\hat{y}$ side (for $L_x > 0$).

Transversely Distorted Distributions and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is distorted compared to longitudinally polarized nucleons !
- mean $\perp$ displacement of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

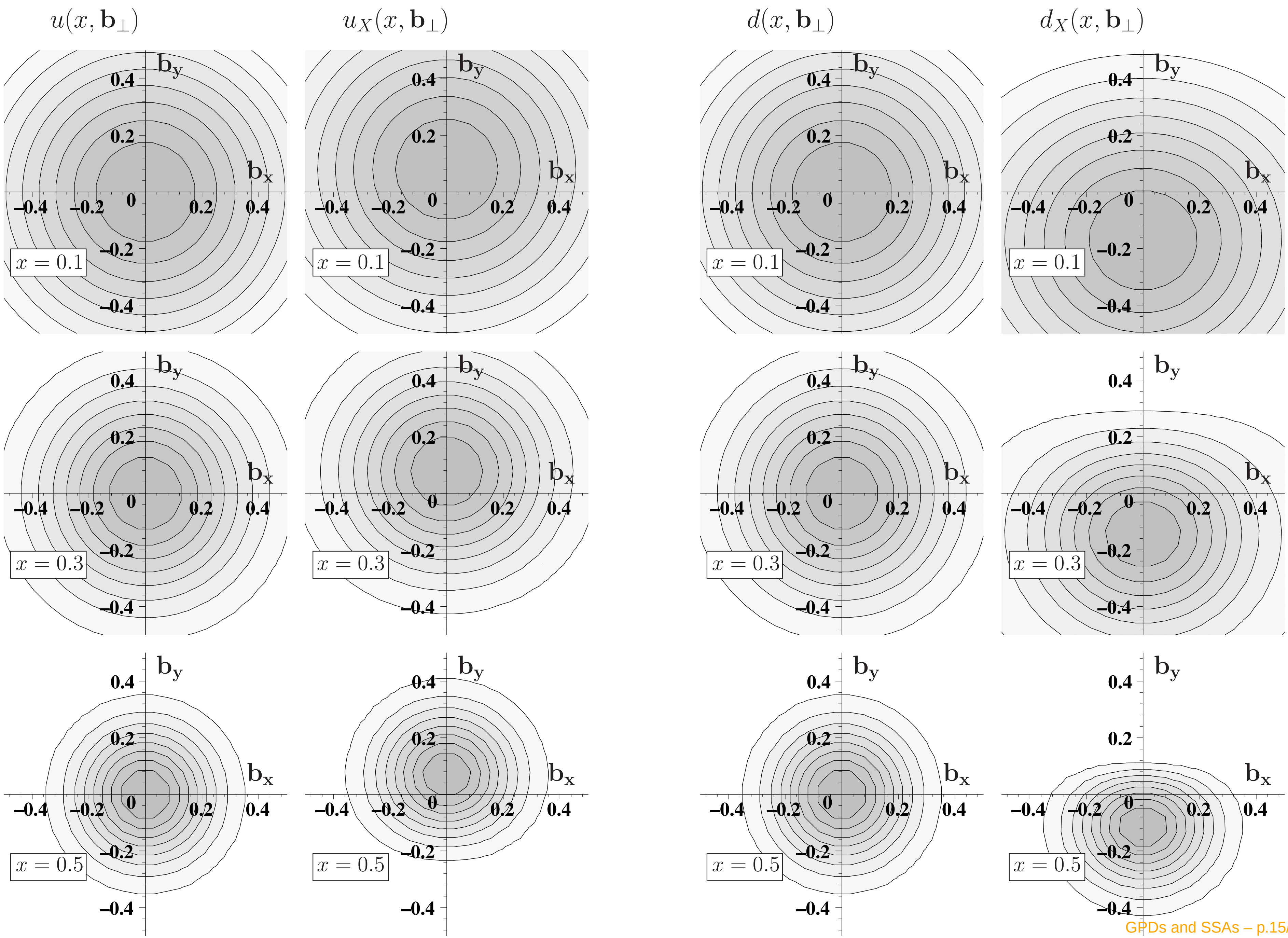
with $\kappa^p_{u/d} \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

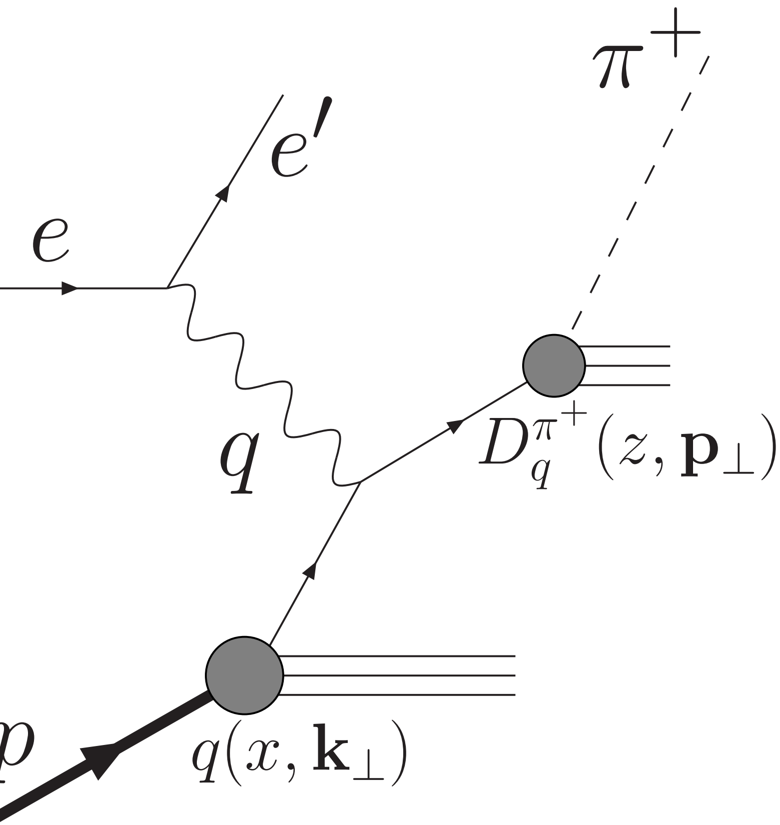
$$\begin{aligned} E_u(x, 0, -\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\mathbf{\Delta}_\perp^2) \\ E_d(x, 0, -\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x, 0, -\mathbf{\Delta}_\perp^2) \end{aligned}$$

with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated distortion is very significant since κ_u and κ_d known to be large!


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_x}$
$\mathbf{b_y}$
−0.4
−0.2
0
0.2
0.4

SSA ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- use factorization (high energies) to express momentum distribution of outgoing π^+ as convolution of
  - momentum distribution of quarks in nucleon
  - $\hookrightarrow$ unintegrated parton density $q(x, \mathbf{k}_\perp)$
  - momentum distribution of π^+ in jet created by leading quark q
  - $\hookrightarrow$ fragmentation function $D_q^{\pi^+}(z, \mathbf{p}_\perp)$

- average $\perp$ momentum of pions obtained as sum of
  - average $\mathbf{k}_\perp$ of quarks in nucleon (Sivers effect)
  - average $\mathbf{p}_\perp$ of pions in quark-jet (Collins effect)

Sivers distribution

- Sivers: Momentum distribution of **unpolarized** quarks in a $\perp$ polarized proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

GPD $\longleftrightarrow$ SSA (Sivers)

- without FSI, $\langle \mathbf{k}_\perp \rangle = 0$, i.e. $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) = 0$
- Brodsky, Hwang, Schmidt: simple model, which demonstrated that, including FSI, $\langle \mathbf{k}_\perp \rangle \neq 0$
- FSI formally included by appropriate choice of Wilson line gauge links (Boer et al; Collins; Ji et al;...) in gauge invariant def. of $q(x, \mathbf{k}_\perp)$

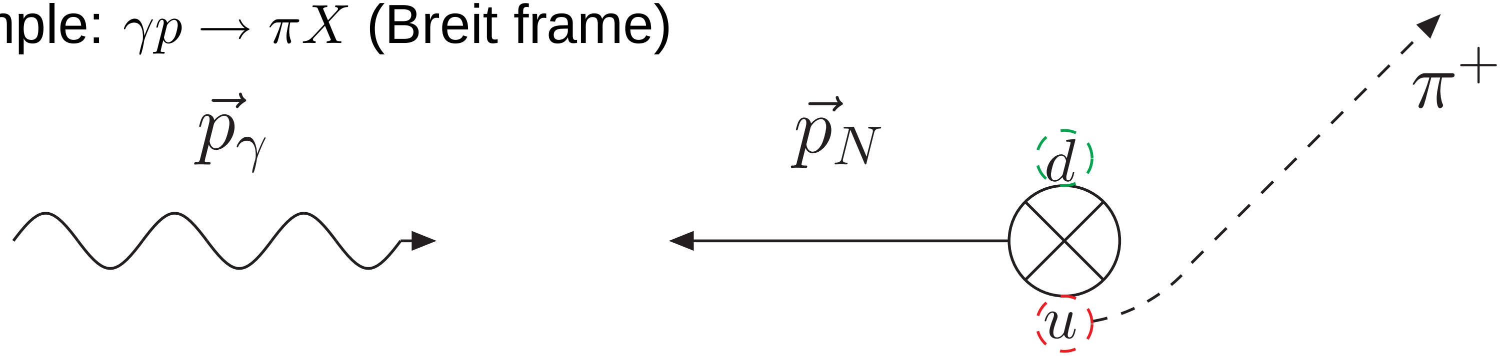
$\hookrightarrow$ Qiu, Sterman; Boer et al.;..

$$\langle \mathbf{k}_\perp \rangle \sim \left\langle P, S \left| \bar{q}(0)\gamma^+ \int_0^\infty d\eta^- G^{+\perp}(\eta) q(0) \right| P, S \right\rangle$$

  - $\int_0^\infty d\eta^- G^{+\perp}(\eta)$ is the $\perp$ impulse that the active quark acquires as it moves through color field of “spectators”
- What should we expect for Sivers effect in QCD ?
- What do we learn if we measure $\mathbf{k}_\perp$-asymmetry ?

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \rightarrow \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

GPD $\longleftrightarrow$ SSA (Sivers); formal argument

- treat FSI to lowest order in g

$\hookrightarrow$

$$\left\langle k_q^i \right\rangle = -\frac{g}{4p^+}\int \frac{d^2\mathbf{b}_\perp}{2\pi}\frac{b^i}{|\mathbf{b}_\perp|^2}\left\langle p,s\left|\bar{q}(0)\gamma^+\frac{\lambda_a}{2}q(0)\rho_a(\mathbf{b}_\perp)\right|p,s\right\rangle$$

with $\rho_a(\mathbf{b}_\perp) = \int dr^-\rho_a(r^-,\mathbf{b}_\perp)$ summed over all quarks and gluons

$\hookrightarrow$ SSA related to dipole moment of density-density correlations

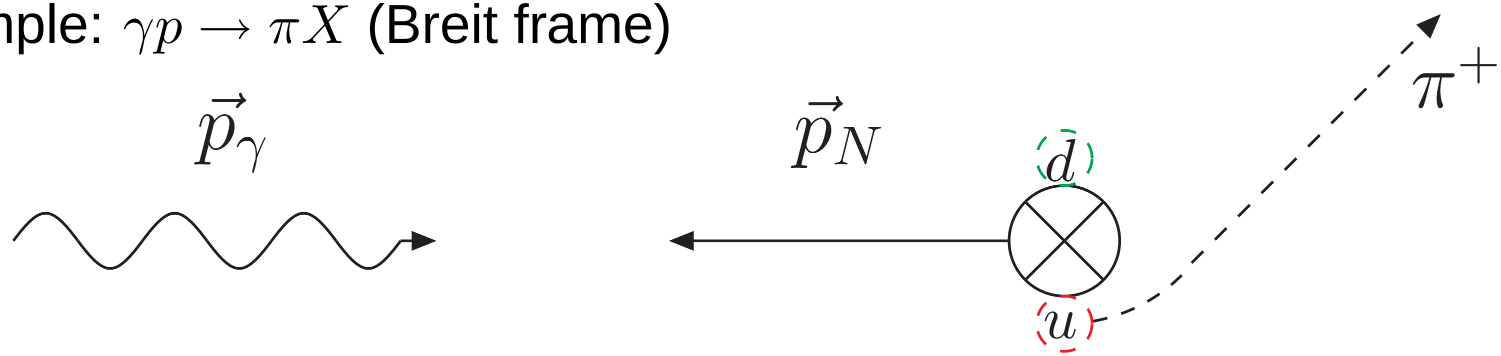
- GPDs (N polarized in $+\hat{x}$ direction): $u \longrightarrow +\hat{y}$ and $d \longrightarrow -\hat{y}$

$\hookrightarrow$ expect density density correlation to show same asymmetry $\langle b^y \bar{u}(0)\gamma^+\frac{\lambda_a}{2}u(0)\rho_a(\mathbf{b}_\perp)\rangle > 0$

$\hookrightarrow$ sign of SSA opposite to sign of distortion in position space

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q$

- signs of SSA confirmed at HERMES

Chirally Odd GPDs

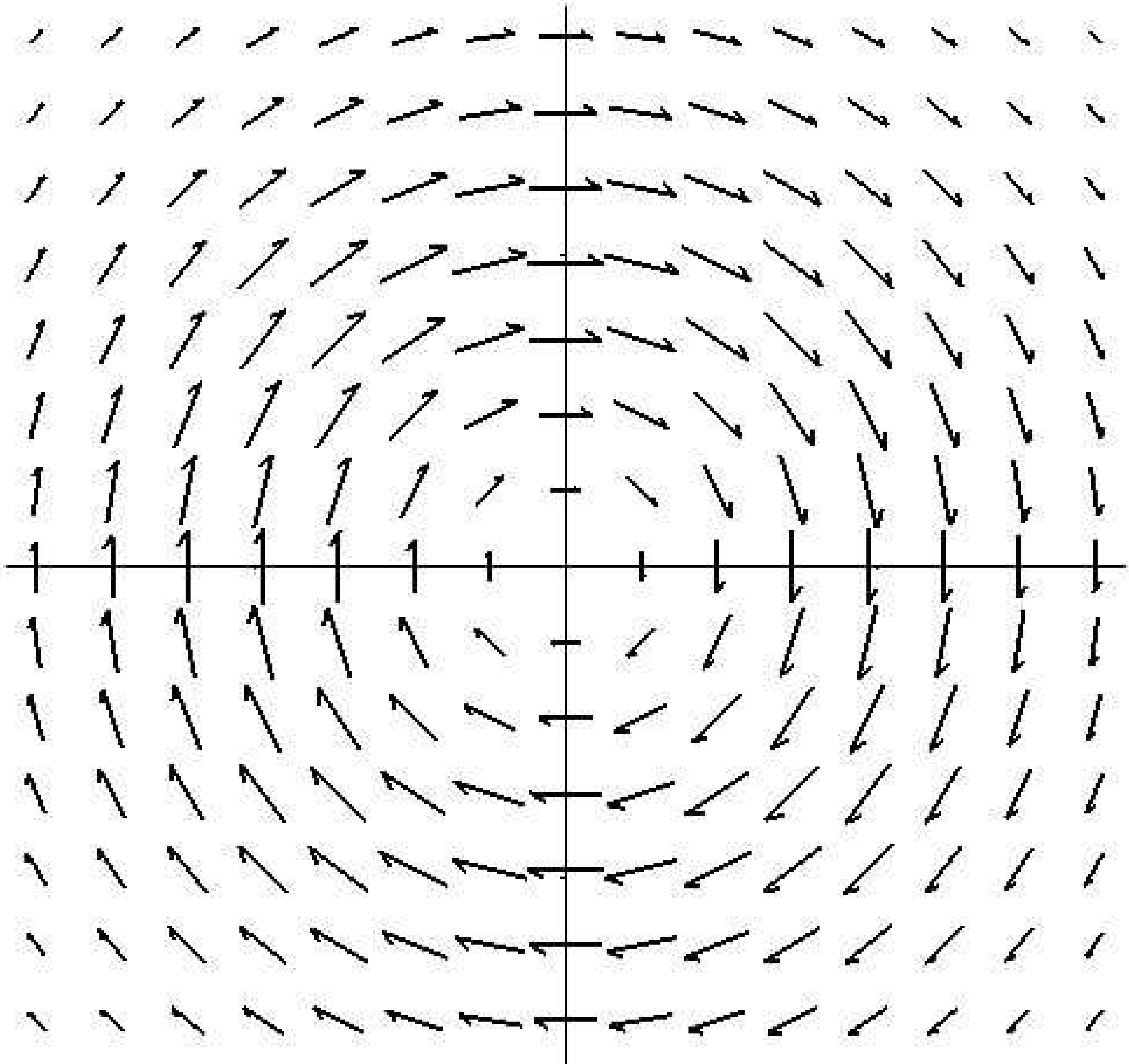
$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u + E_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha\gamma_\beta}{2M} u + \tilde{E}_T \bar{u}\frac{\varepsilon^{+j\alpha\beta}P_\alpha\gamma_\beta}{M}$$

- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of $2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity for unpolarized target in $\perp$ plane

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M}\frac{\partial}{\partial b_j}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \left[2\tilde{H}_T^q(x, 0, -\mathbf{\Delta}_\perp^2) + E_T^q(x, 0, -\mathbf{\Delta}_\perp^2)\right]$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



Transversity Distribution in Unpolarized Target

- attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $2\tilde{H}_T + E_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

Sivers vs. Boer-Mulders distribution

- Sivers: Momentum distribution of **unpolarized** quarks in a $\perp$ **polarized** proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- Boer-Mulders: Momentum distribution of $\perp$ **polarized** quarks in an **unpolarized** proton

$$f_{q^\uparrow/p}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - h_1^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S_q}{M}$$

Transversity Distribution in Unpolarized Target

- attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $2\tilde{H}_T + E_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

$\hookrightarrow$ qualitative predictions for $h_1^\perp(x, \mathbf{k}_\perp)$

  - sign of $h_1^\perp$ opposite sign of $2\tilde{H}_T + E_T$
  - "$\frac{h_1^\perp}{2\tilde{H}_T + E_T} \approx \frac{f_{1T}^\perp}{E}$"

- use measurement of $h_1^\perp$ to learn about spin-orbit correlation in nucleon wave function
- use LGT calcs. of $2\tilde{H}_T + E_T$ to make qualitative prediction for $h_1^\perp$

Summary

- GPDs provide decomposition of form factors w.r.t. the momentum of the active quark

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle$$

- GPDs resemble both PDFs and form factors: defined through matrix elements of light-cone correlator, but $\Delta \equiv p' - p \neq 0$.
- t-dependence of GPDs at $\xi = 0$ (purely $\perp$ momentum transfer) $\Rightarrow$ Fourier transform of impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ knowledge of GPDs for $\xi = 0$ provides novel information about nonperturbative parton structure of nucleons: distribution of partons in $\perp$ plane

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation, e.g. $q(x, \mathbf{b}_\perp) > 0$ for $x > 0$

Summary

- $\frac{\Delta_\perp}{2M}E(x,0,-\mathbf{\Delta}_\perp^2)$ describes how the momentum distribution of unpolarized partons in the $\perp$ plane gets transversely distorted when is nucleon polarized in $\perp$ direction.
- (attractive) final state interaction in semi-inclusive DIS converts $\perp$ position space asymmetry into $\perp$ momentum space asymmetry

$\hookrightarrow$ simple physical explanation for observed Sivers effect in $\gamma^* p \to \pi X$

- $2\tilde{H}_T + E_T$ measures correlation between $\perp$ spin and $\perp$ angular momentum
- physical explanation for Boer-Mulders effect; suggested relation between $h_1^\perp$ and the GPDs $2\tilde{H}_T + E_T$
- GPDs vs. $q(x,\mathbf{b}_\perp)$: M.B., PRD **62**, 71503 (2000), Int. J. Mod. Phys. **A18**, 173 (2003); see also D. Soper, PRD **15**, 1141 (1977).
- Connection to SSA in M.B., PRD **69**, 057501 (2004); NPA **735**, 185 (2004); PRD **66**, 114005 (2002).