

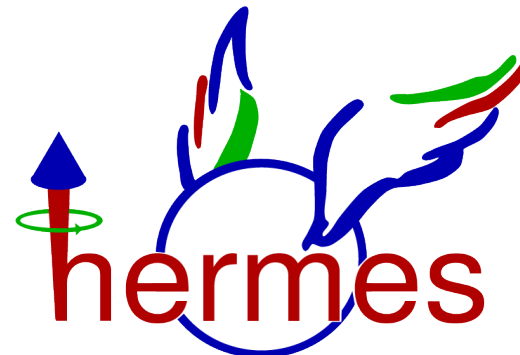
Results on the spin structure of the nucleon from HERMES

Charlotte Van Hulse,
University of the Basque Country



Universidad
del País Vasco

Euskal Herriko
Unibertsitatea



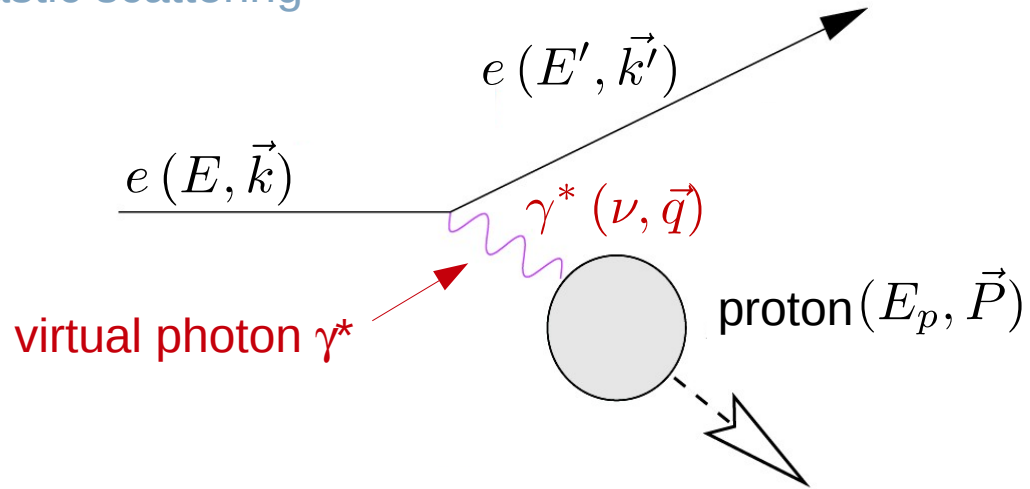
International Workshop on
Quark-gluon Structure of the Nucleon
November 8, 2012 – Tokyo Institute of Technology

Outline

- studying the proton: deep-inelastic scattering
- structure functions and parton model
- HERMES experiment
- quark-helicity distribution from deep-inelastic scattering
- semi-inclusive deep-inelastic scattering
 - quark-helicity, gluon helicity distribution and transversity
 - transverse-momentum distribution functions:
Sivers and Boer-Mulders
- orbital angular momentum: GPDs and DVCS

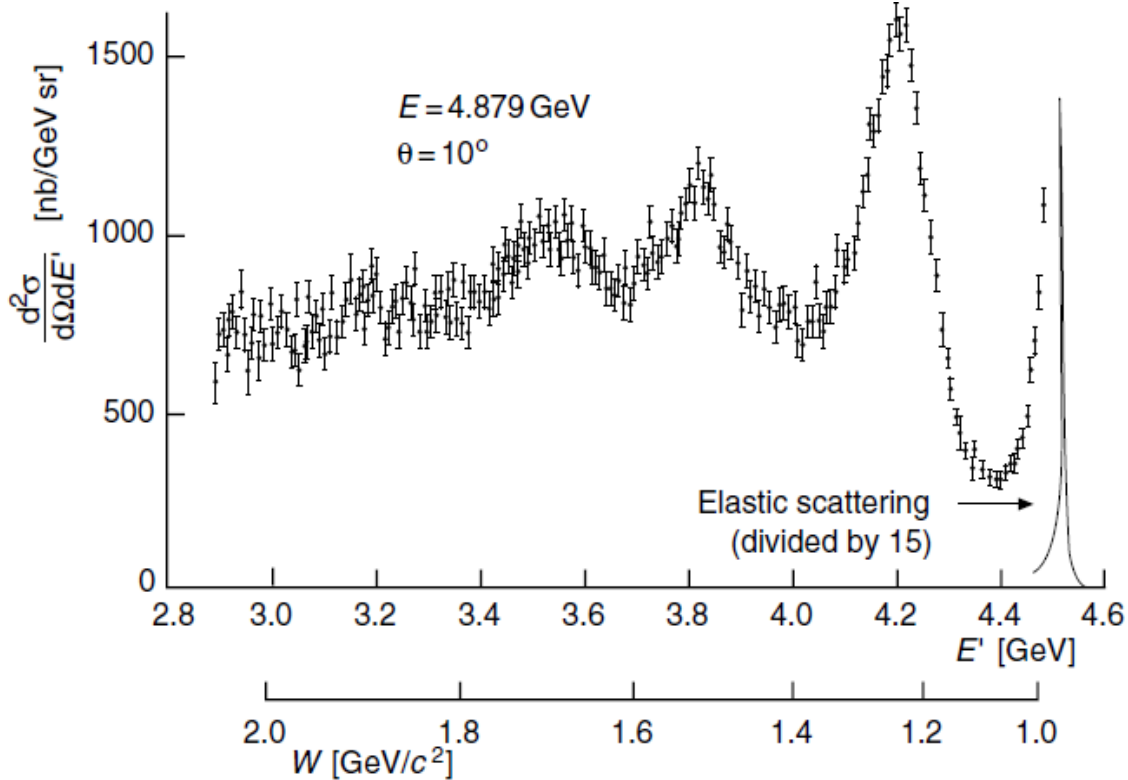
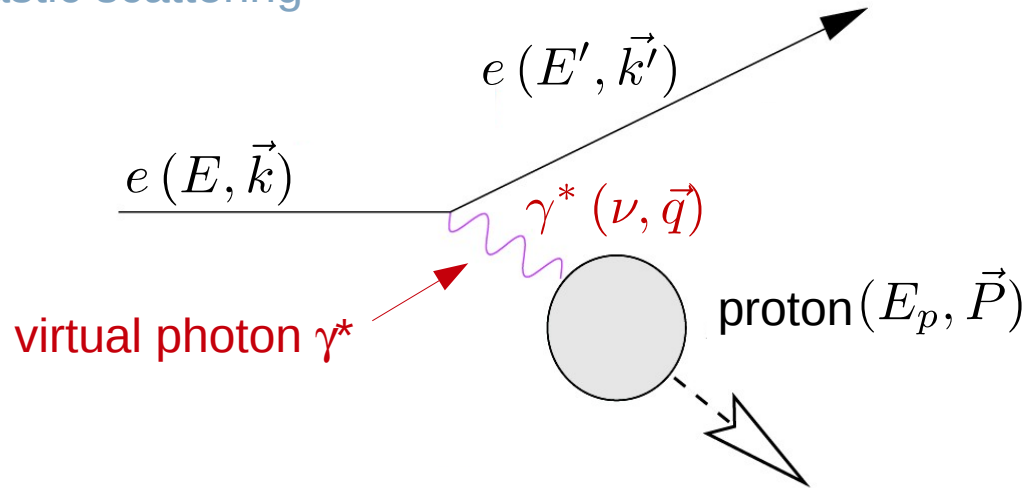
Studying the proton

elastic scattering



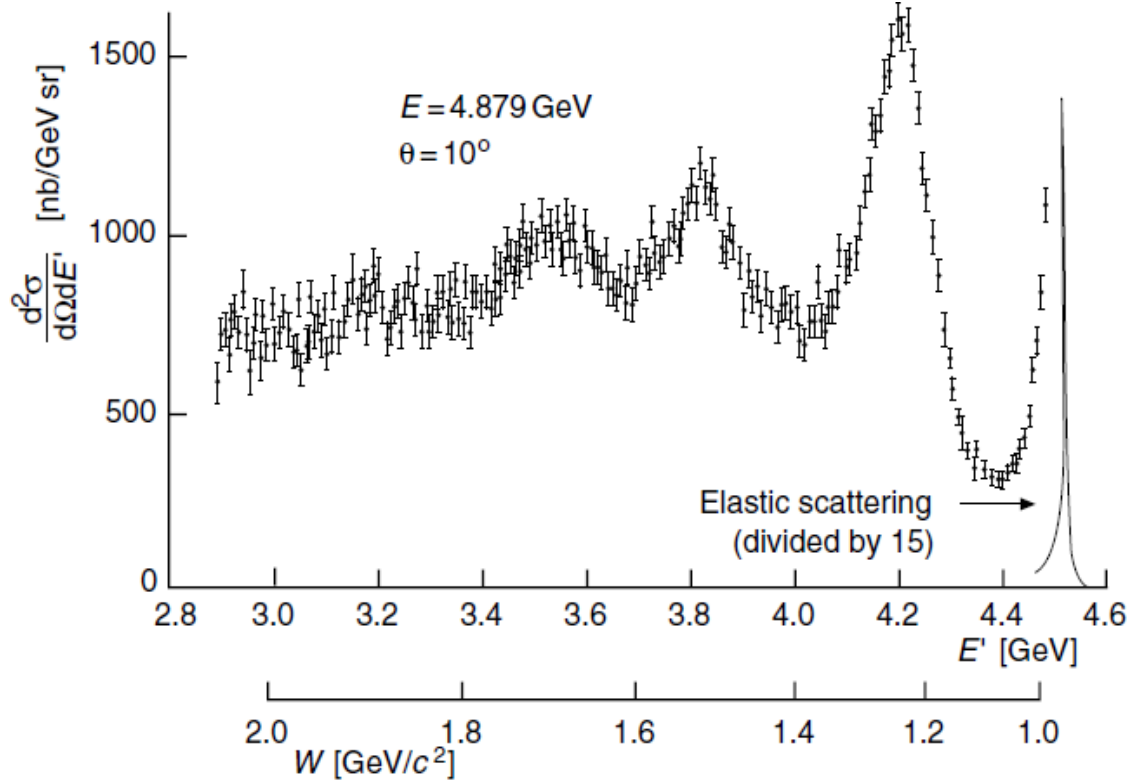
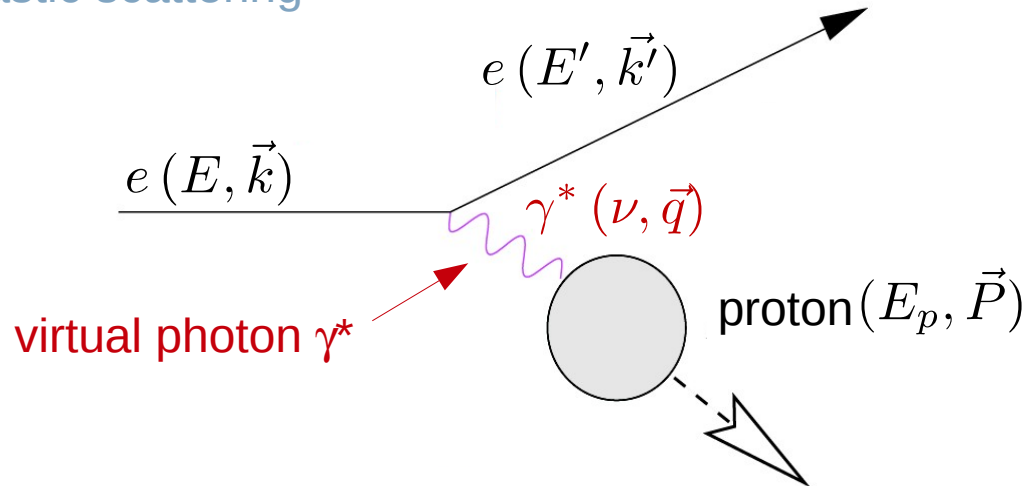
Studying the proton

elastic scattering



Studying the proton

elastic scattering



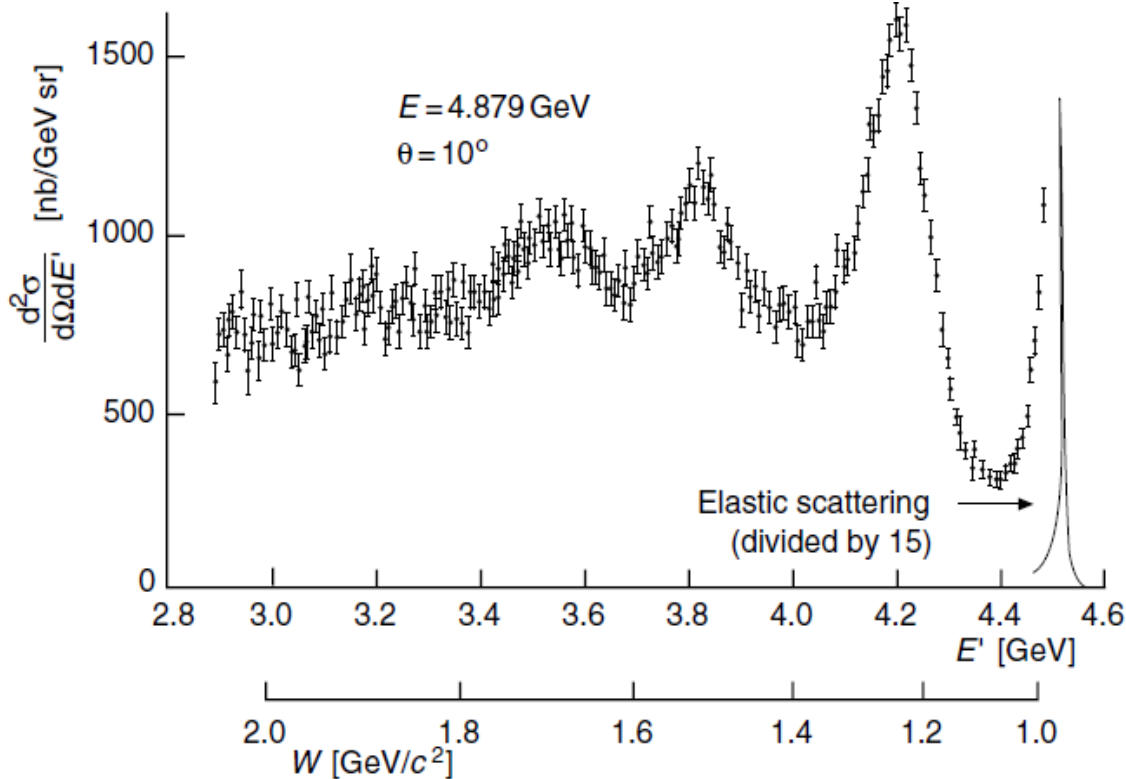
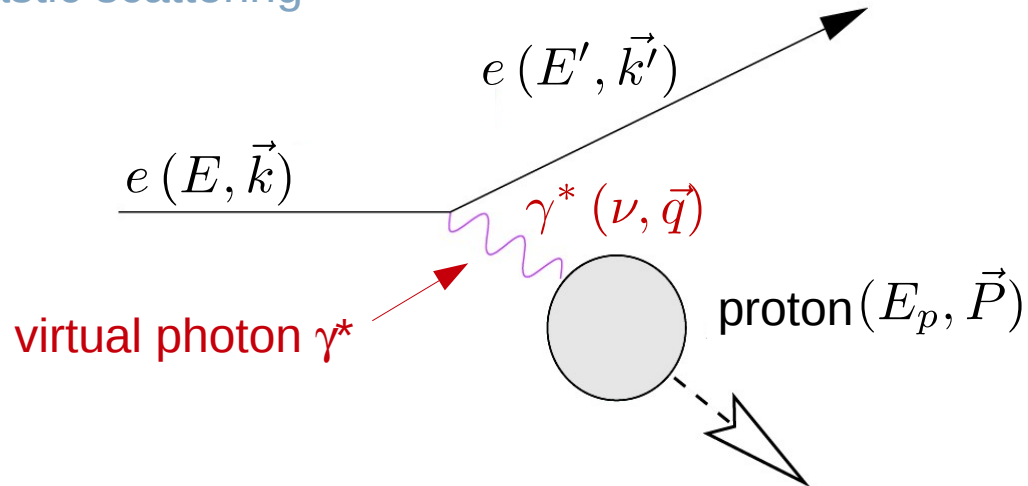
invariant mass W

$$W^2 \equiv (P + q)^2$$

$$= M^2 + 2Pq + q^2$$

Studying the proton

elastic scattering



invariant mass W

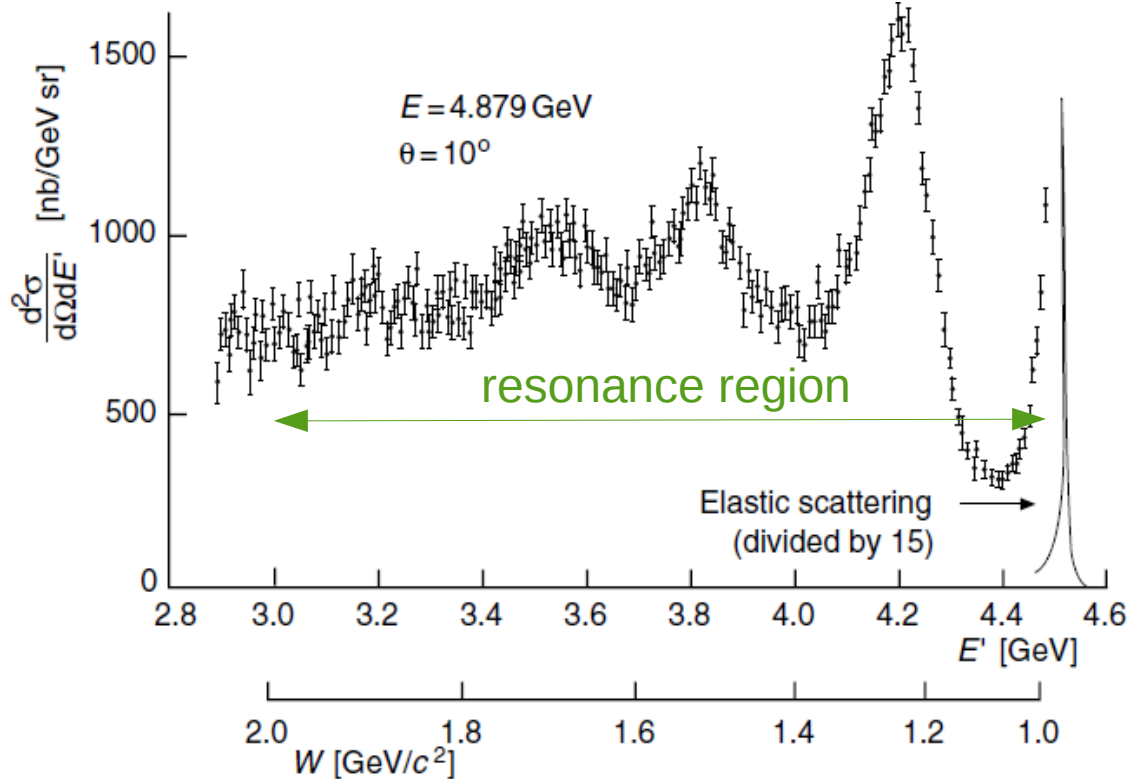
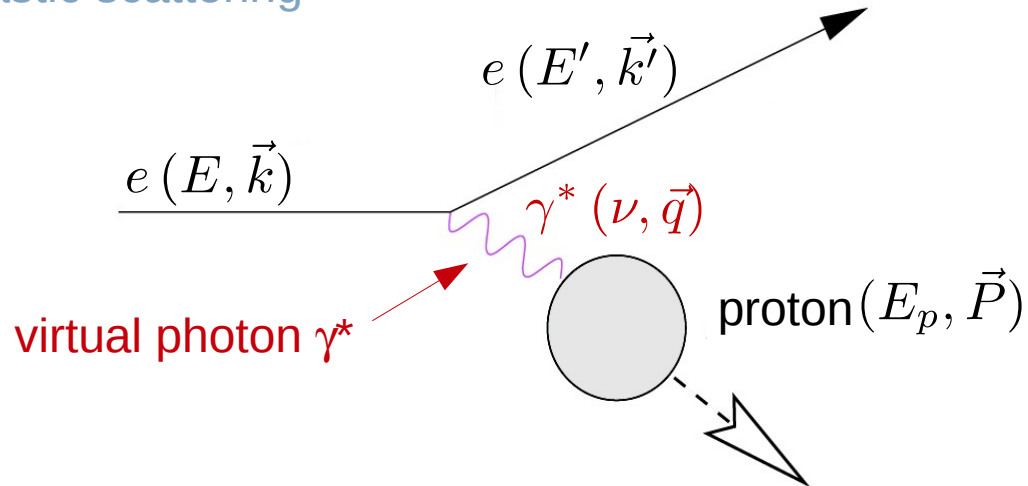
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$W^2 = M^2 \rightarrow$ elastic scattering

Studying the proton

elastic scattering



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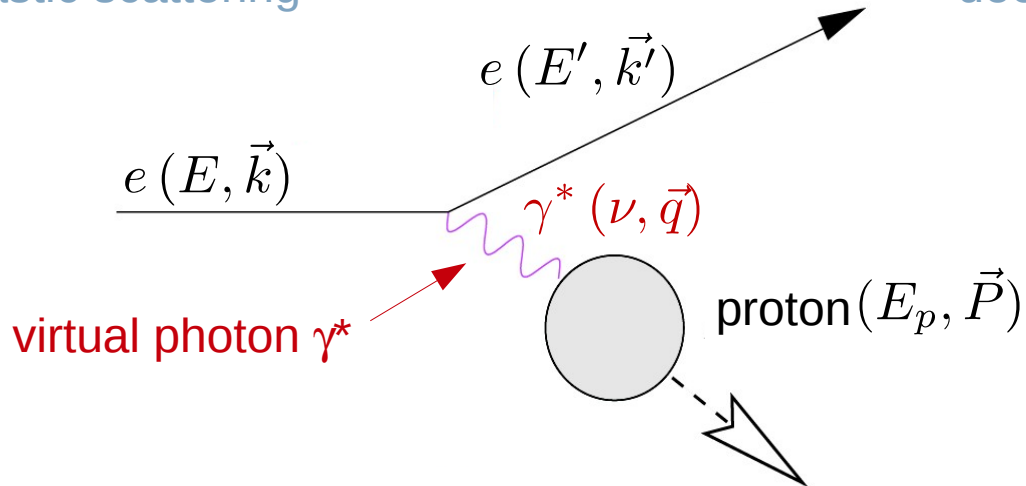
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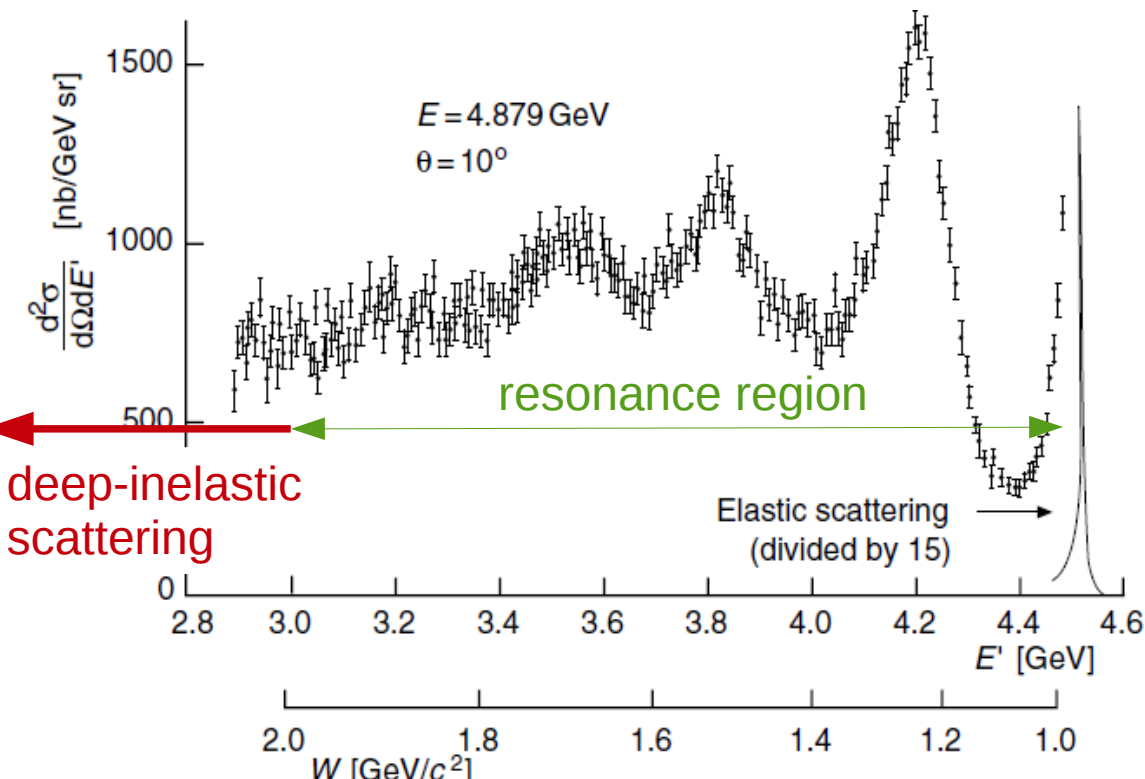
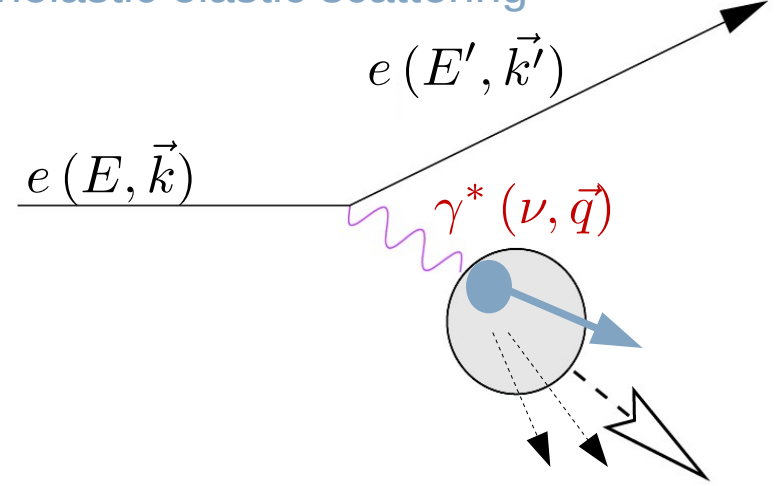
$W^2 > M^2 \rightarrow$ inelastic scattering

Studying the proton

elastic scattering



deep-inelastic elastic scattering



invariant mass W

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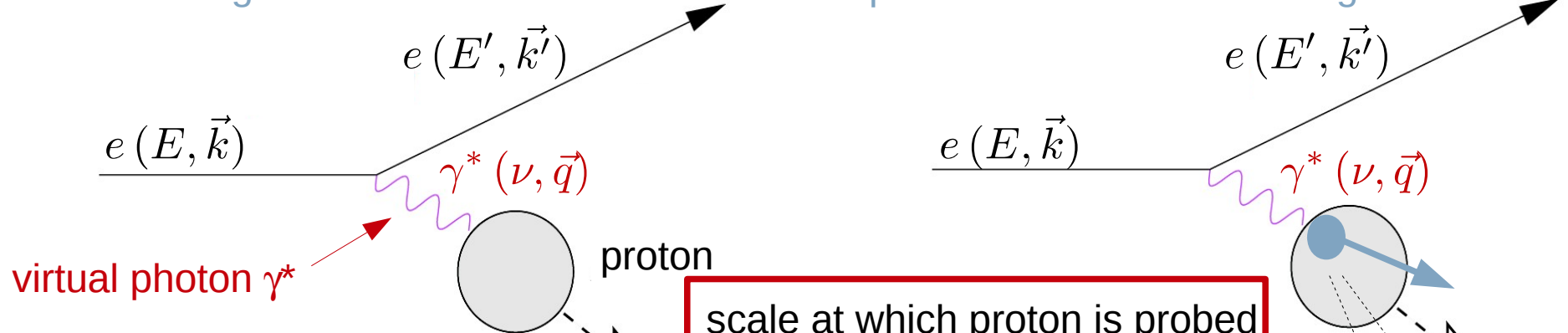
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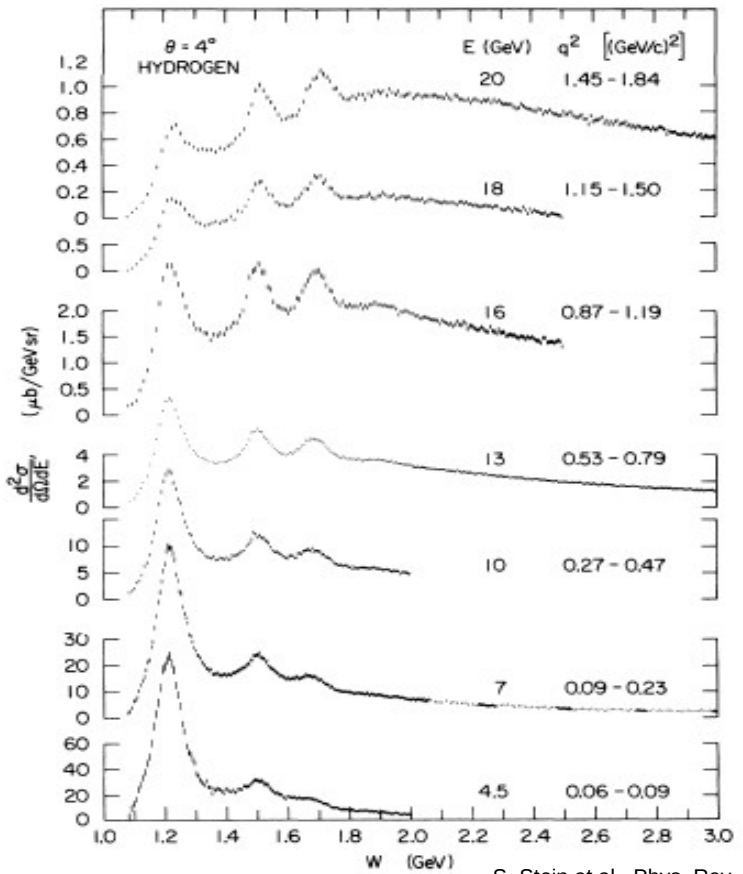
Studying the proton

elastic scattering

deep-inelastic elastic scattering



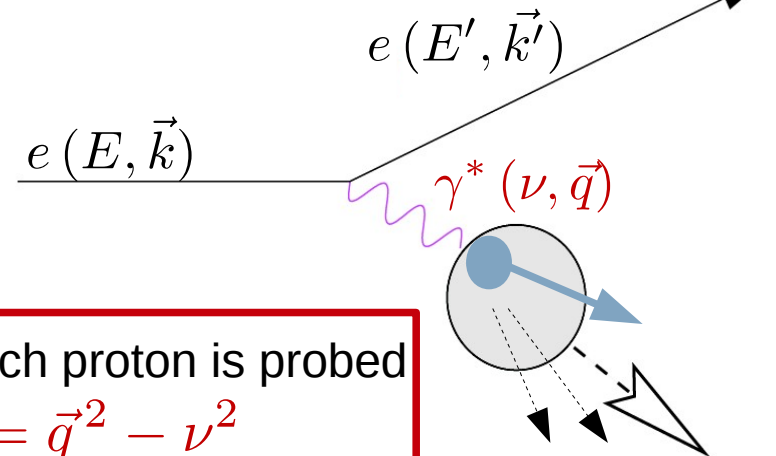
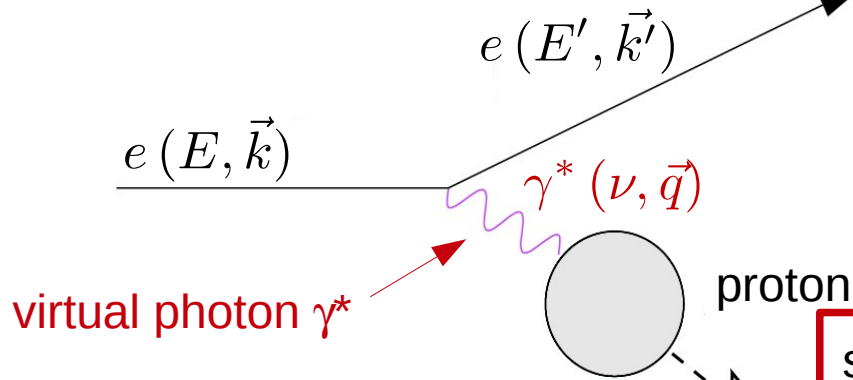
scale at which proton is probed

$$Q^2 = \vec{q}^2 - \nu^2$$


Studying the proton

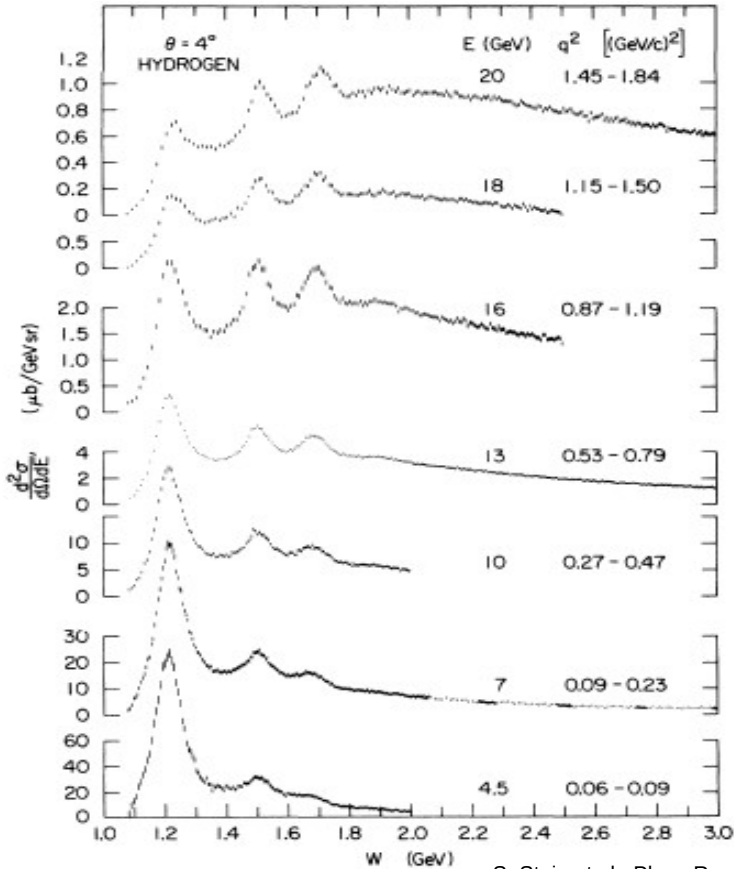
elastic scattering

deep-inelastic elastic scattering

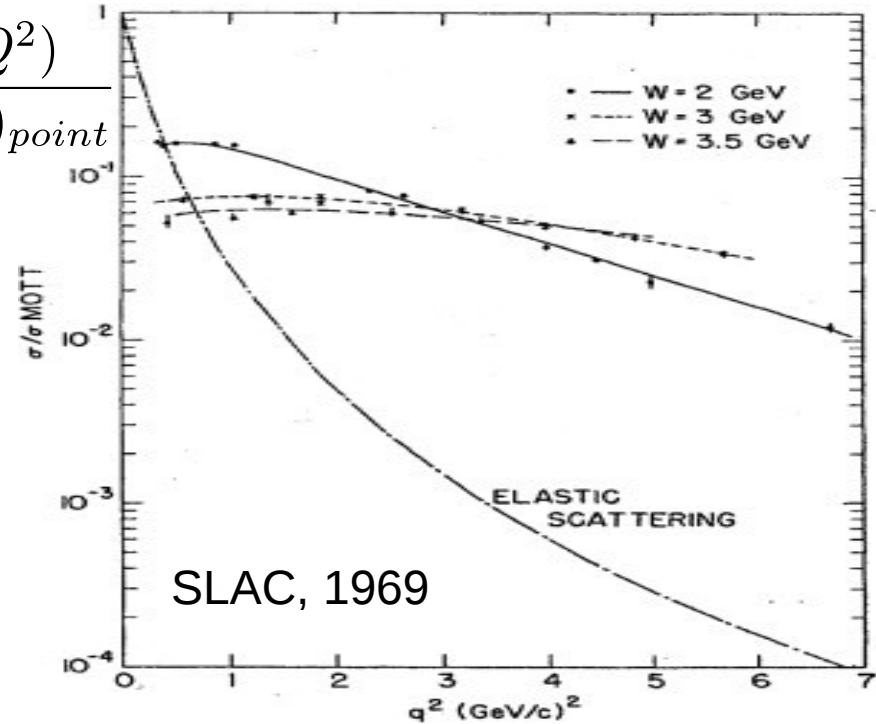


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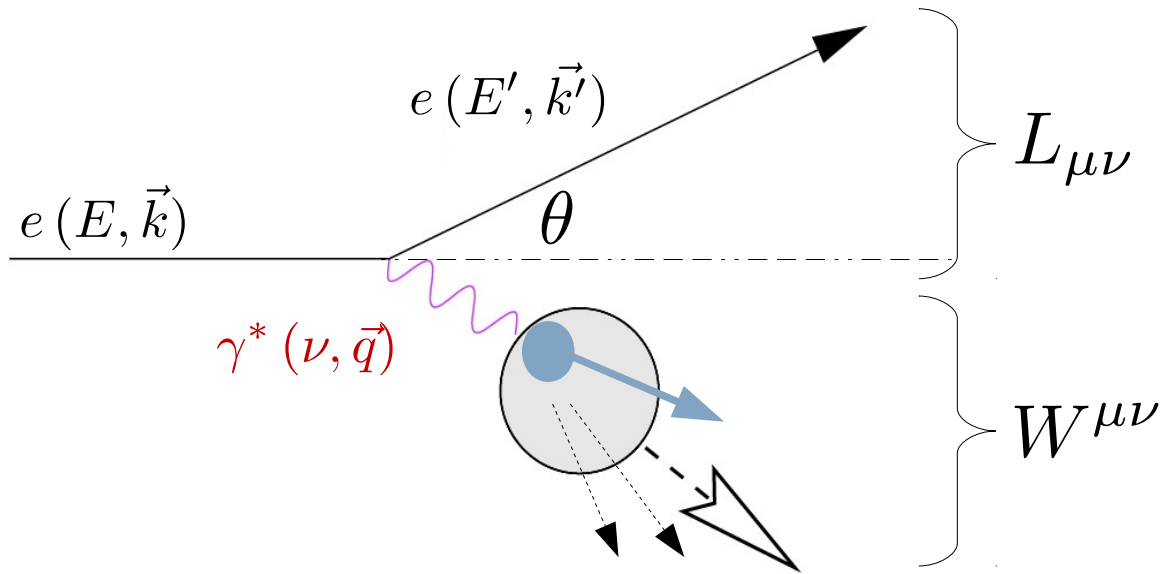
$$Q^2 = \vec{q}^2 - \nu^2$$



$$\frac{\sigma(Q^2)}{\sigma(Q^2)_{point}}$$



Deep-inelastic scattering cross section

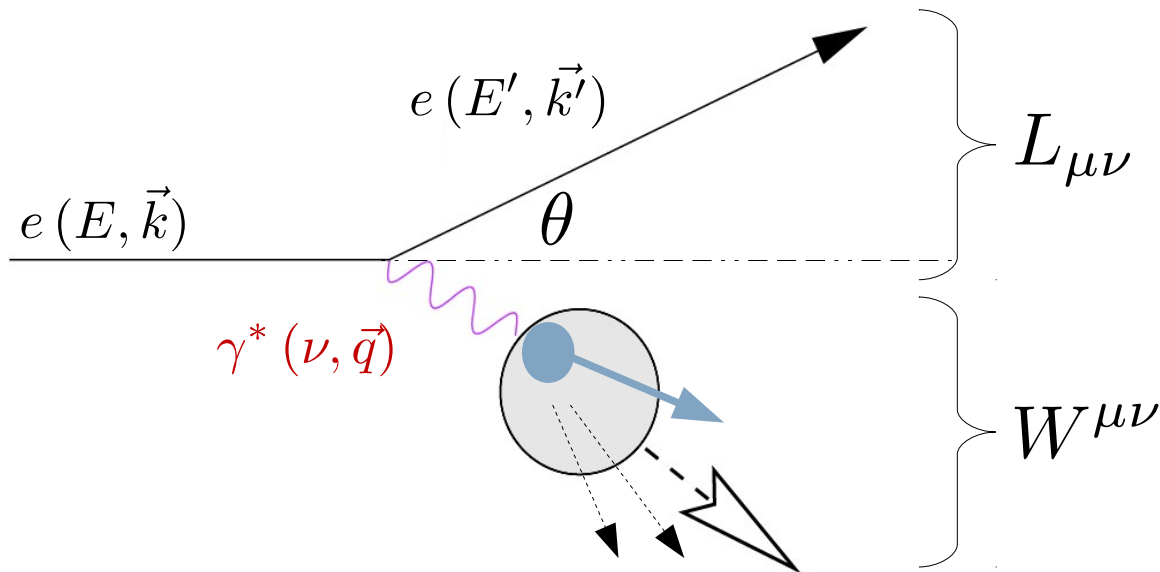


$$\frac{d^2\sigma}{dx_B dQ^2} \propto L_{\mu\nu} W^{\mu\nu}$$

$$Q^2 \equiv -q^2 \stackrel{lab}{=} 4EE' \sin^2\left(\frac{\theta}{2}\right)$$

$$x_B \equiv \frac{Q^2}{2Pq} = \frac{1}{1 + \frac{W^2 - M^2}{Q^2}}$$

Deep-inelastic scattering cross section



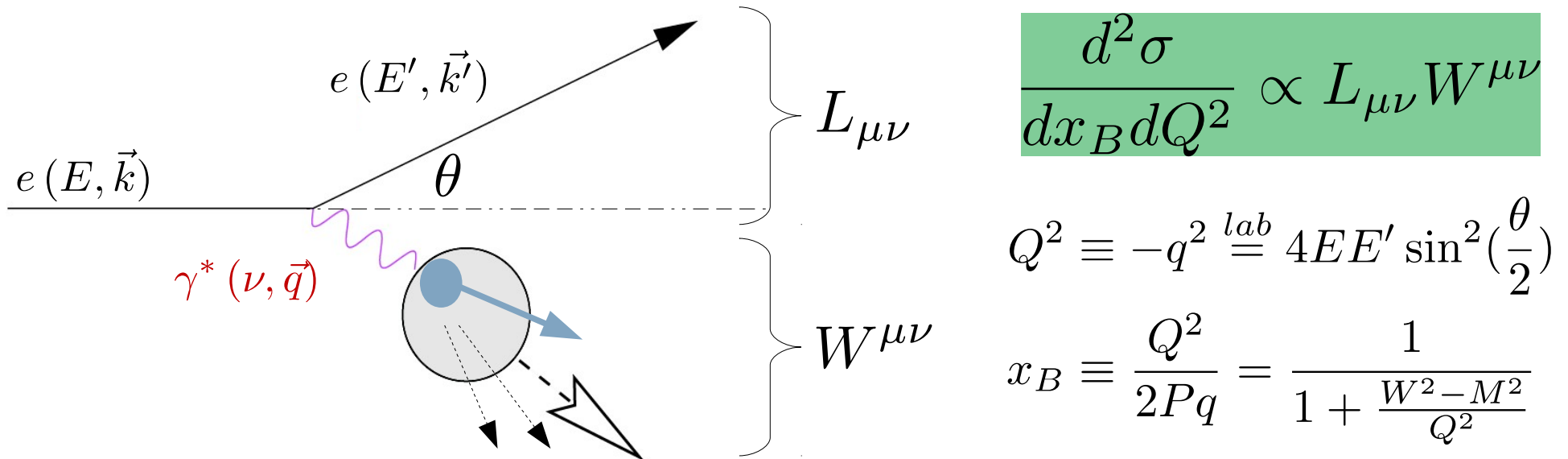
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elastic scattering : $W = M \rightarrow x_B = 1$
 inelastic scattering : $W > M \rightarrow 0 < x_B < 1$

Deep-inelastic scattering cross section



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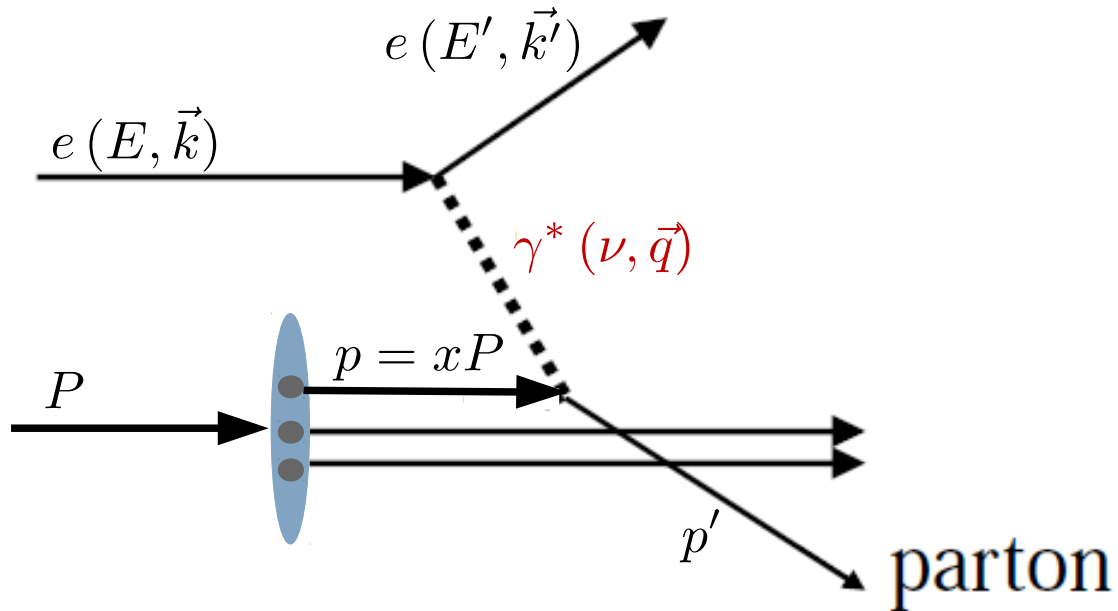
$L_{\mu\nu}$: calculable in QED

$$W_{\mu\nu} = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}\right) F_1 + \left(P_\mu + \frac{Pq}{Q^2} q_\mu\right) \left(P_\nu + \frac{Pq}{Q^2} q_\nu\right) \frac{F_2}{Pq} \\ + i\epsilon_{\mu\nu\alpha\beta} q^\alpha \frac{M}{Pq} \left[S^\beta g_1 + \left(S^\beta - \frac{Sq}{Pq} P^\beta \right) g_2 \right]$$

unpolarized structure functions F_1 and F_2

polarized structure functions g_1 and g_2 only contribute if both target and beam polarized

The parton model



proton in infinite-momentum frame
neglect masses and
transverse momenta of quarks

$$p = xP$$

$$p'^2 = (p + q)^2$$

$$= p^2 + 2pq - Q^2$$


$$0 = 0 + 2xPq - Q^2$$



$$x = \frac{Q^2}{2Pq} = x_B$$

The parton model

$$F_1(x) = \frac{1}{2} \sum_q e_q^2 q(x) = \frac{1}{2} \sum_q e_q^2 \left(\overset{\Rightarrow}{\vec{q}}(x) + \overset{\Rightarrow}{\vec{q}}(x) \right)$$



 proton spin
 (anti-)quark spin

$$F_2(x) = 2xF_1$$

spin-independent parton distribution function (PDF)

$$q(x) = \text{[yellow circle with blue dot and right arrow]} + \text{[yellow circle with blue dot and left arrow]}$$

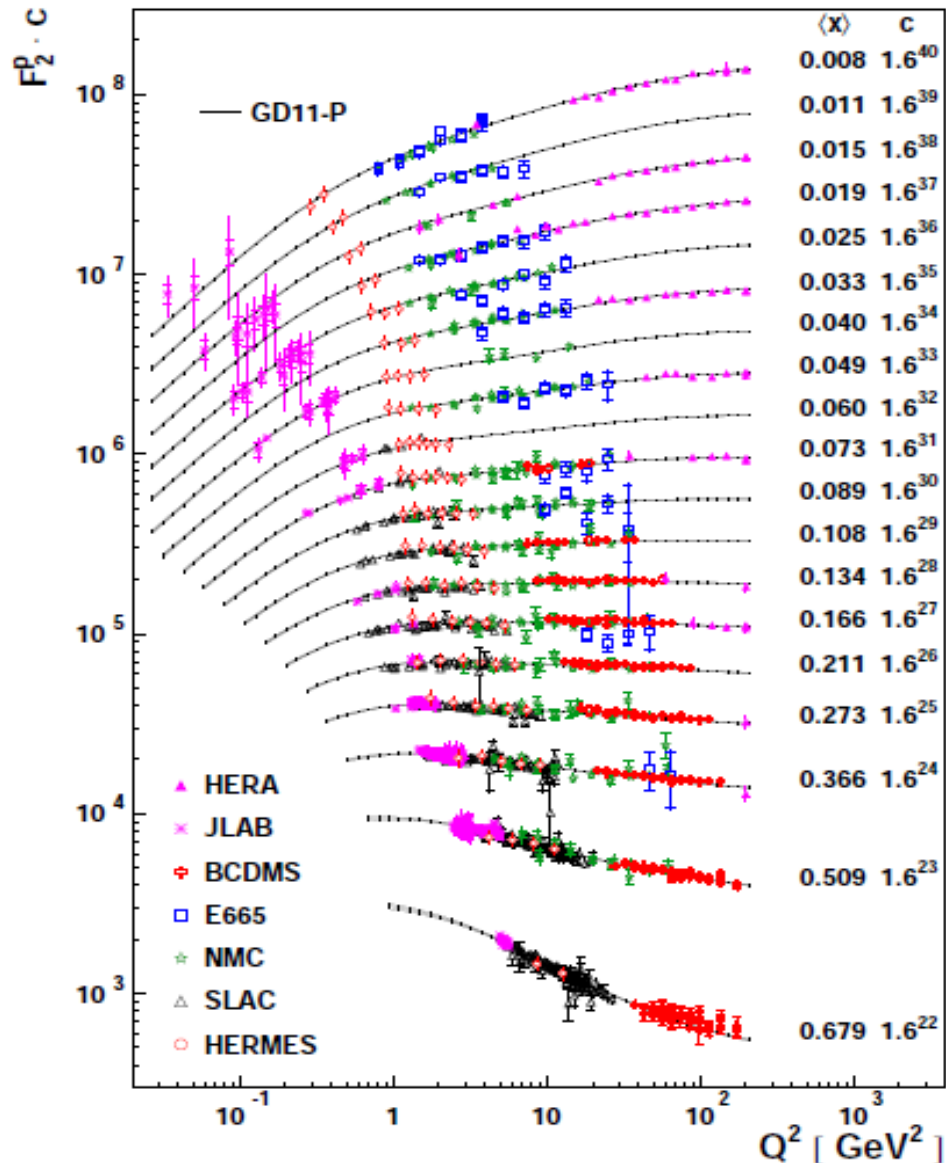
$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x) = \frac{1}{2} \sum_q e_q^2 \left(\overset{\Rightarrow}{\vec{q}}(x) - \overset{\Rightarrow}{\vec{q}}(x) \right)$$

$$g_2(x) = 0$$

helicity parton distribution function (PDF)

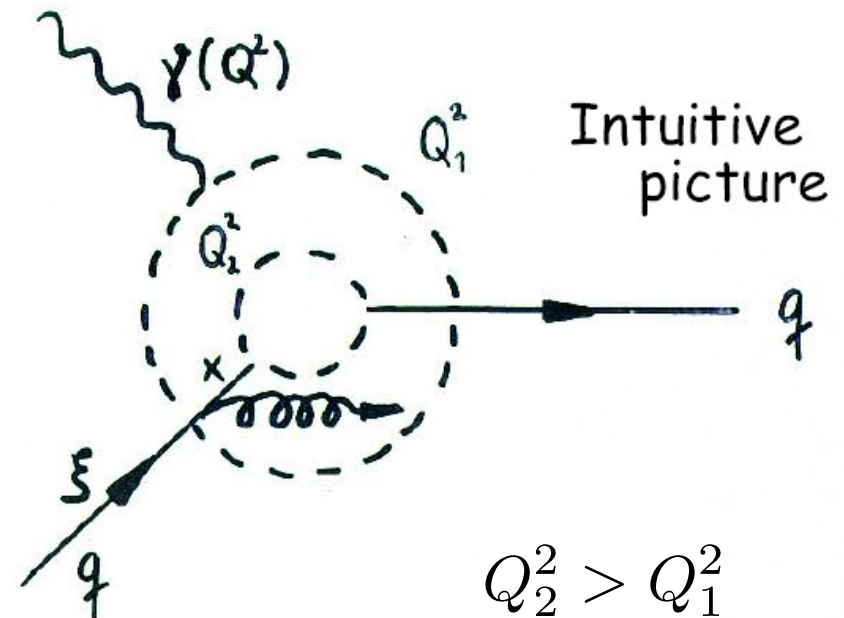
$$\Delta q(x) = \text{[yellow circle with blue dot and right arrow]} - \text{[yellow circle with blue dot and left arrow]}$$

Spin-independent structure functions

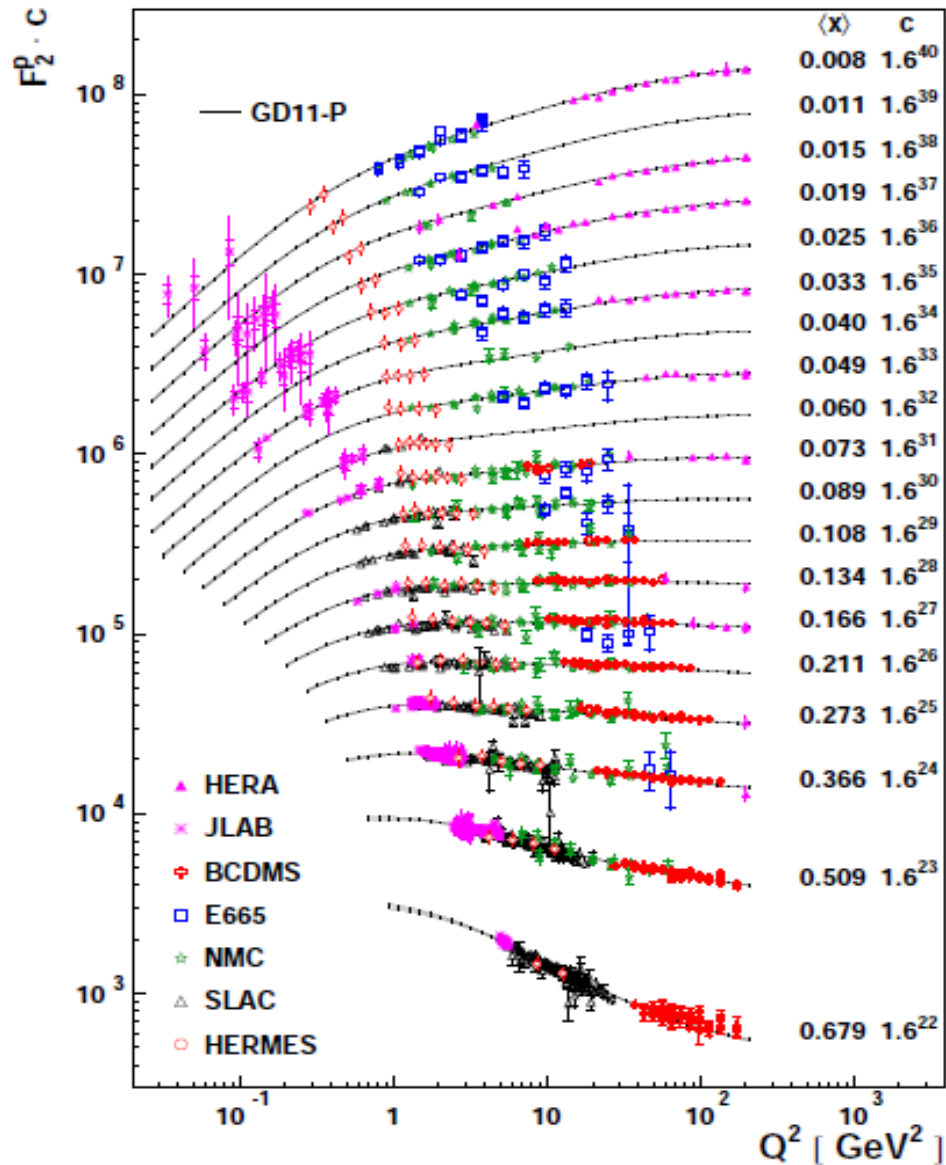


scaling violation:

structure functions and PDFs
depend on x_B and Q^2

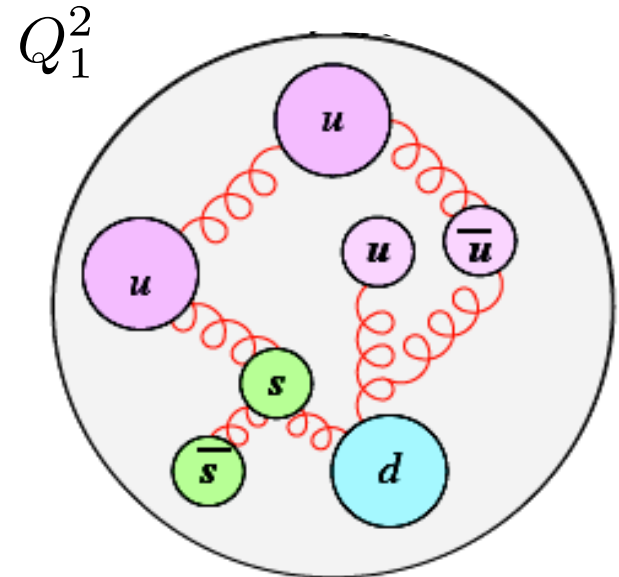


Spin-independent structure functions



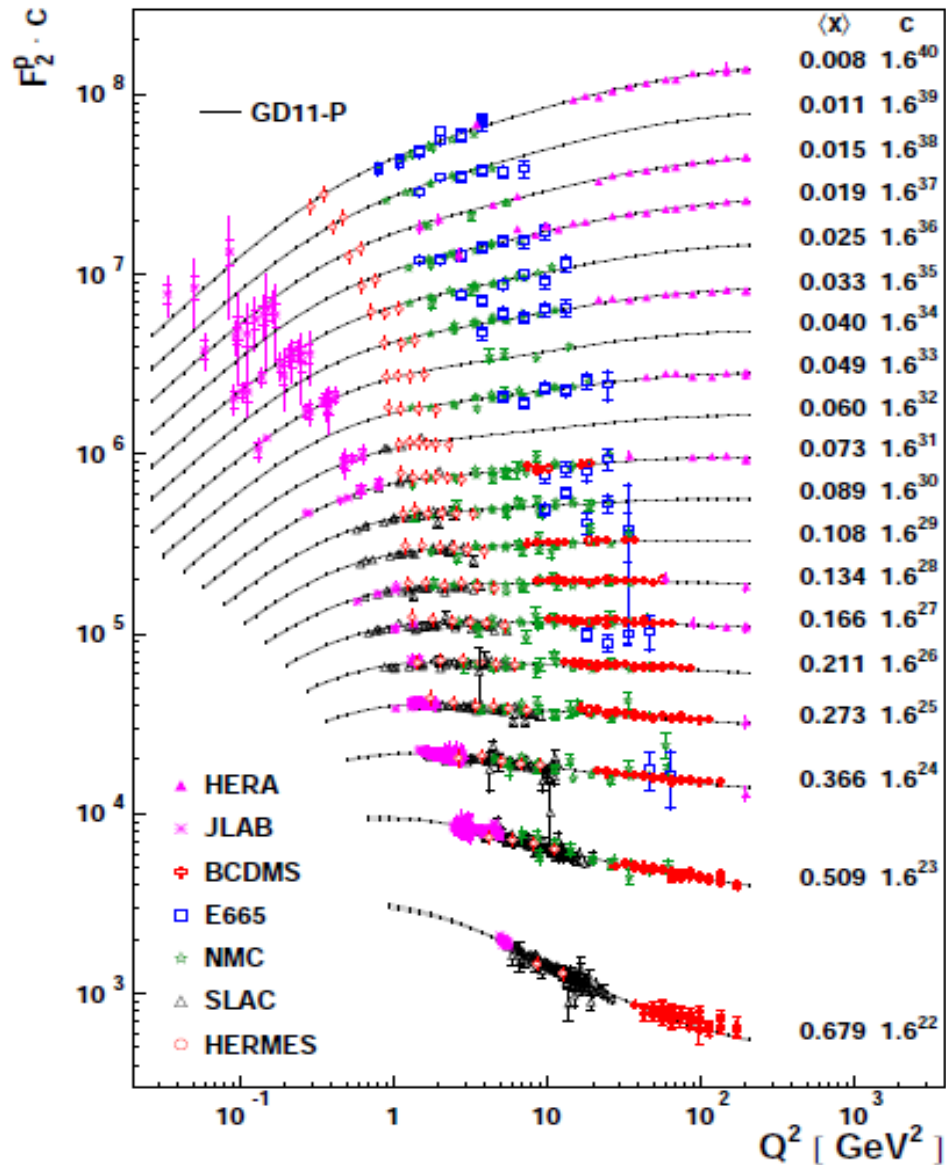
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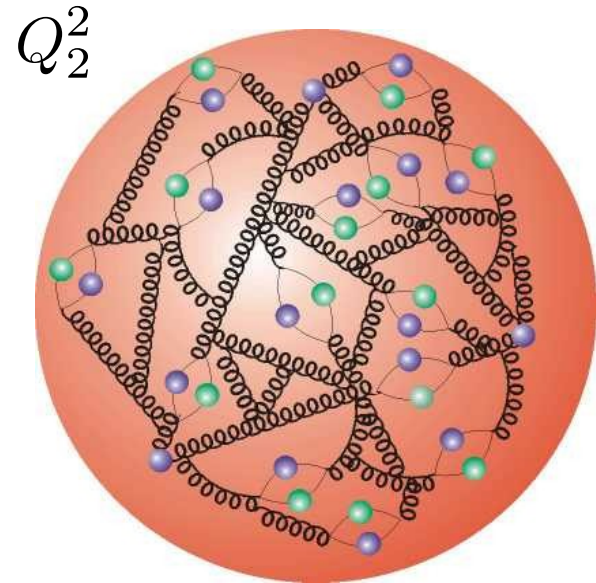
$$Q_2^2 > Q_1^2$$

Spin-independent structure functions



scaling violation:

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Helicity-dependent structure functions

measurement of

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 \Delta q(x) = \frac{1}{2} \sum_q e_q^2 \left(\overset{\Rightarrow}{\vec{q}}(x) - \overset{\Rightarrow}{\vec{q}}(x) \right)$$



$$\Delta\Sigma = \int_0^1 dx_B \Delta u(x) + \Delta d(x) + \Delta s(x) + \Delta\bar{u}(x) + \Delta\bar{d}(x) + \Delta\bar{s}(x)$$

Helicity-dependent structure functions

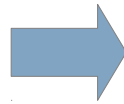
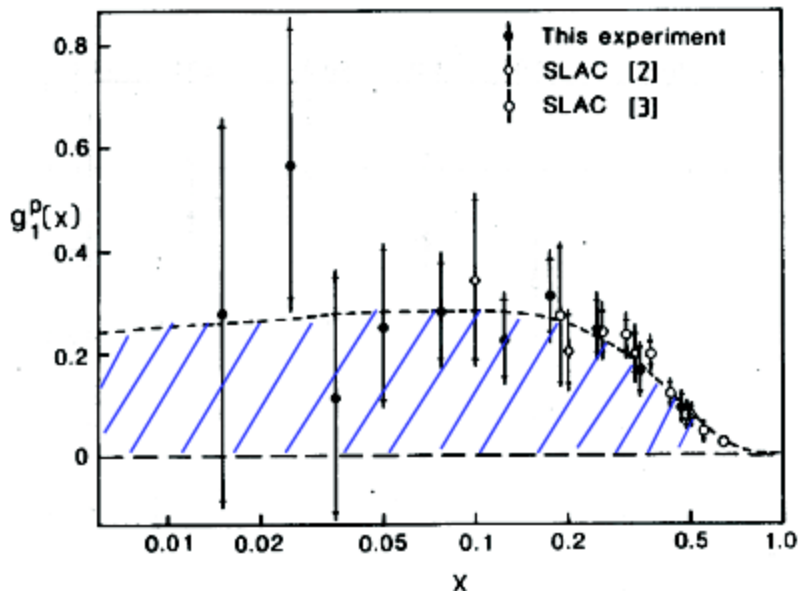
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first measurement: EMC, 1988



$$\Delta\Sigma = 0.14 \pm 0.09 \pm 0.21$$

Helicity-dependent structure functions

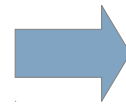
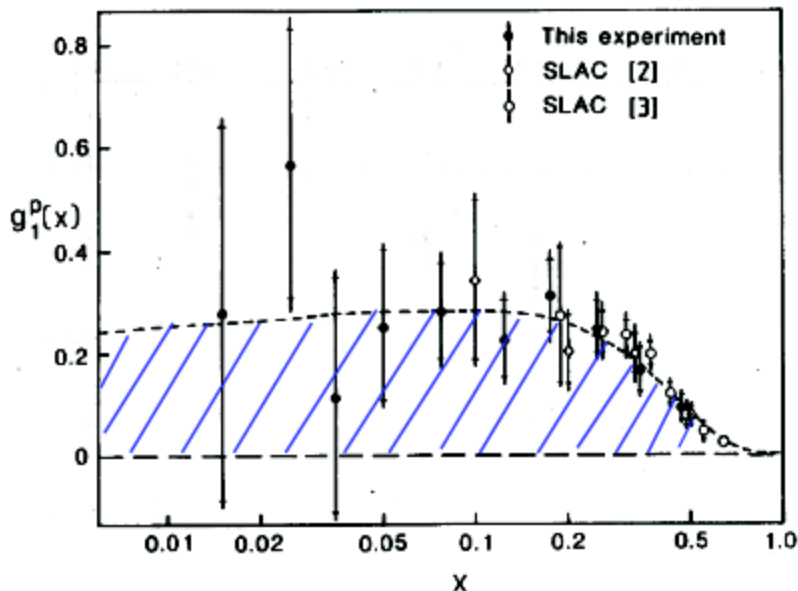
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Spin Puzzle

Helicity-dependent structure functions

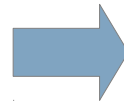
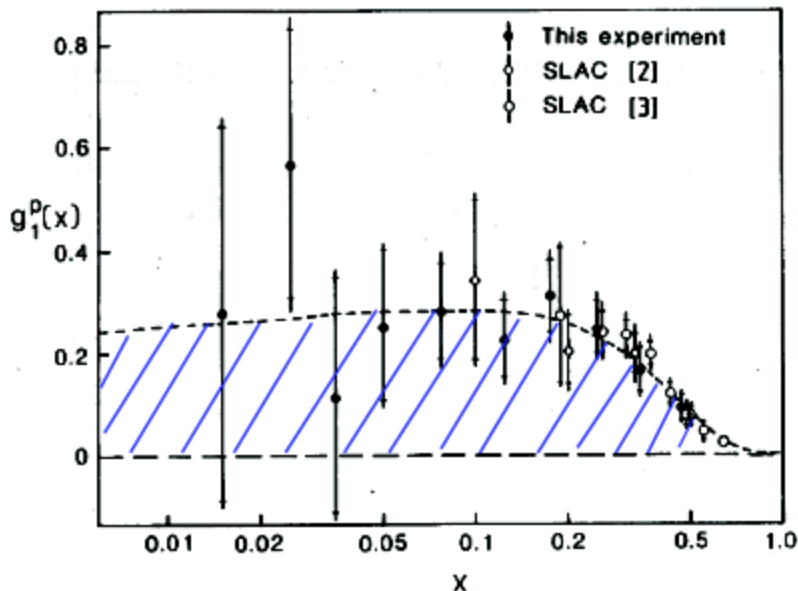
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Spin Puzzle

Spin Crisis!

Helicity-dependent structure functions

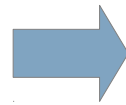
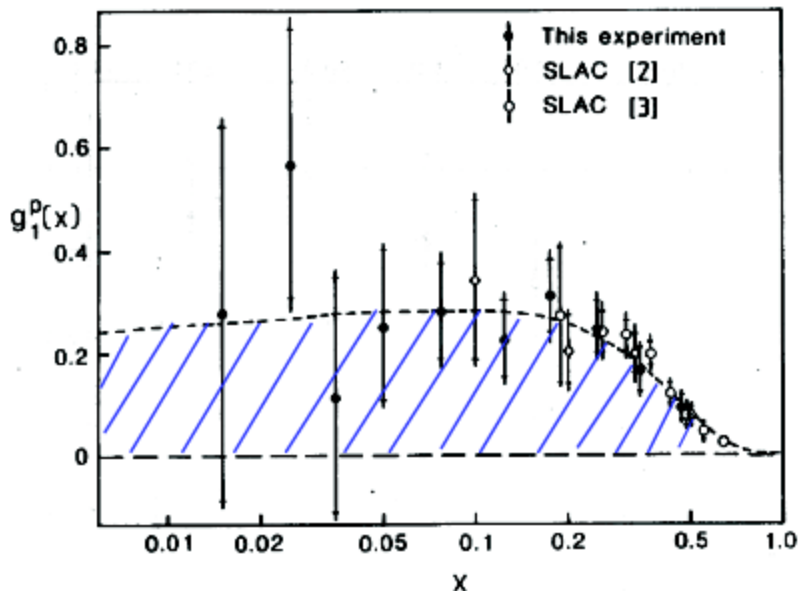
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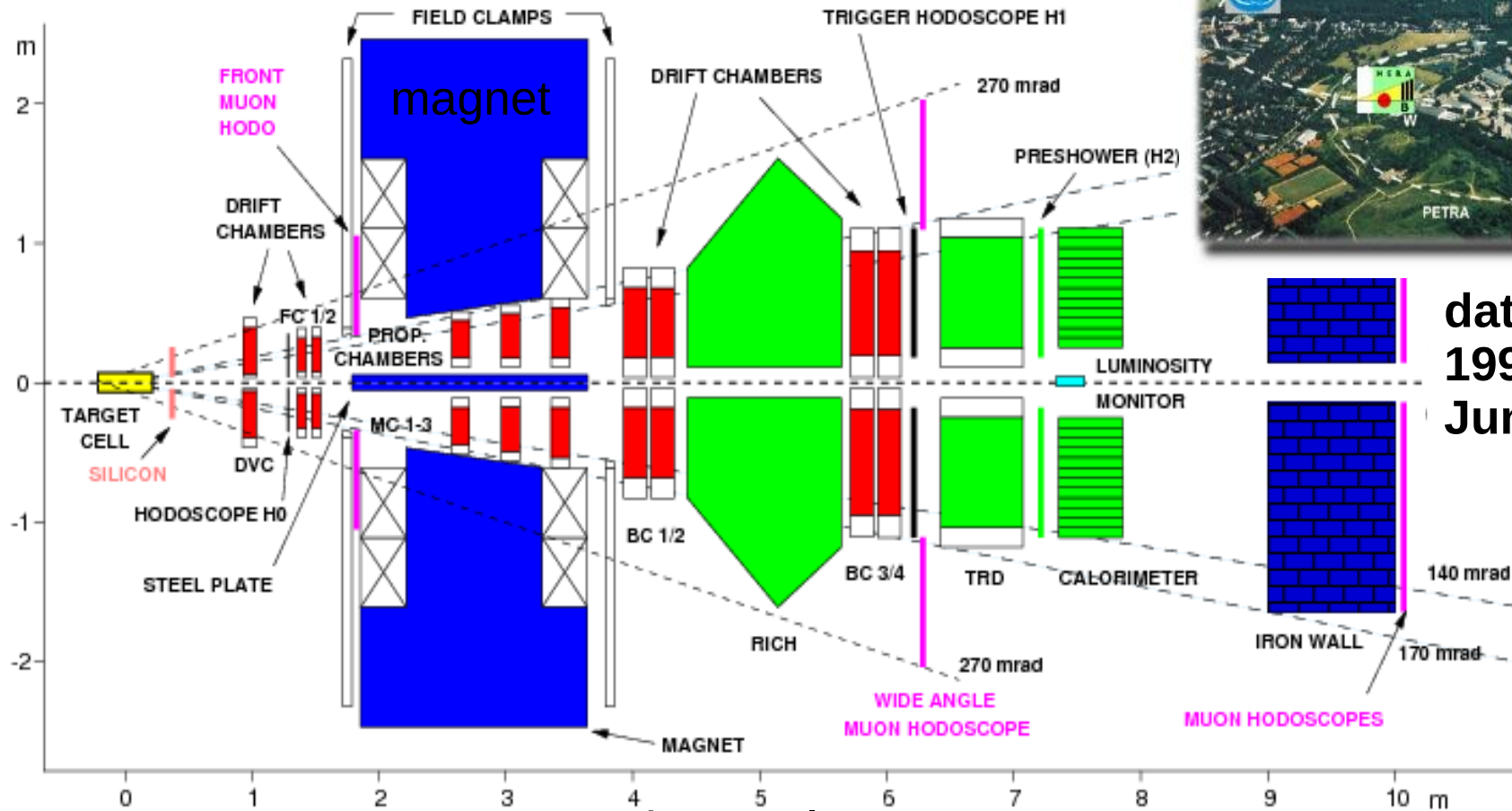
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Spin Puzzle

Spin Crisis!

new experiments saw the light

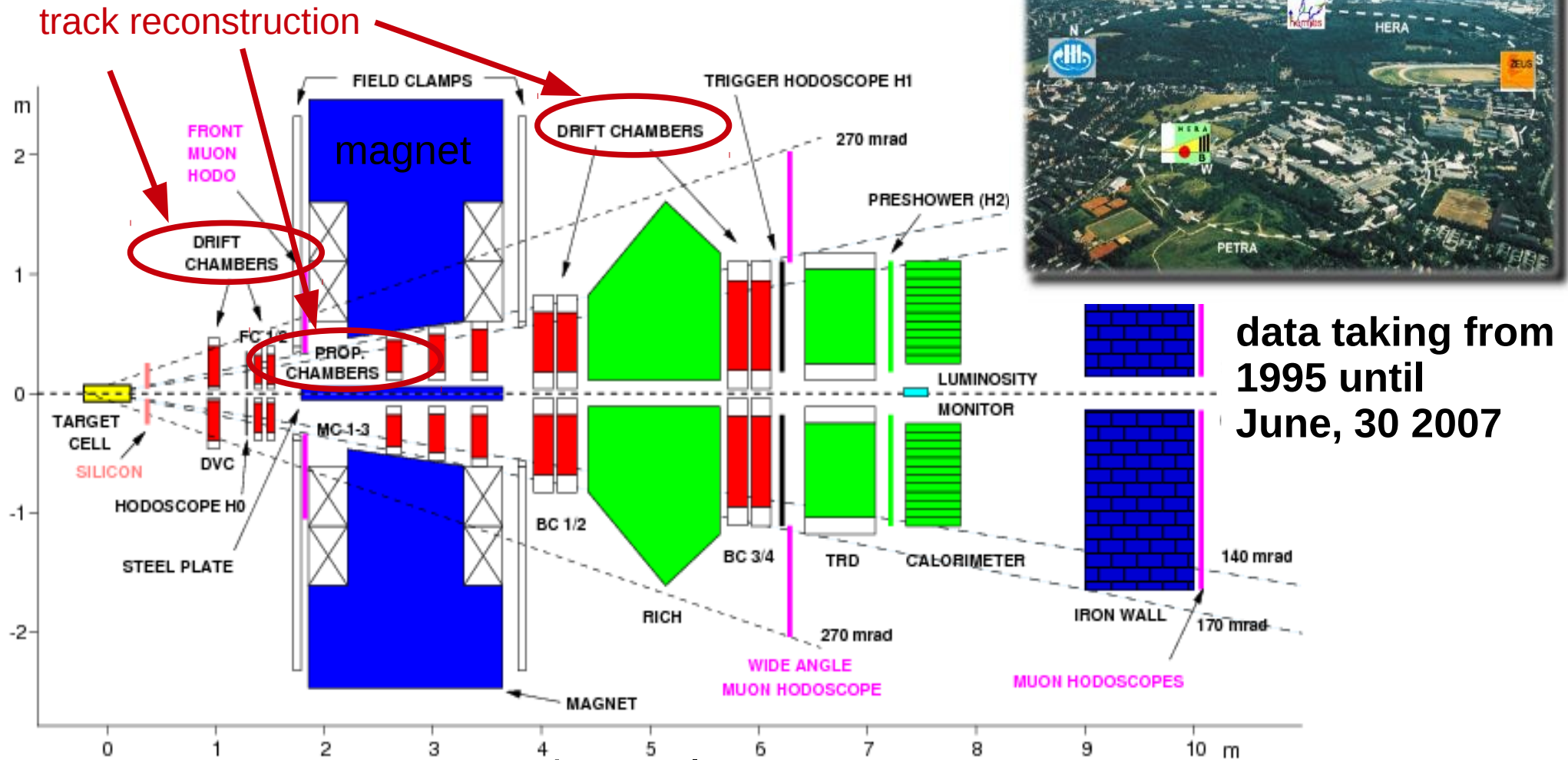
HERMES: HERA MEasurement of Spin



data taking from
1995 until
June, 30 2007

<p><u>Beam</u> longitudinally pol. e^+ & e^- $E = 27.6 \text{ GeV}$</p>	<p><u>Gaseous internal target</u> transversely pol. H (~75%) unpol. H, D, He, Ne, Kr, Xe longitudinally pol. H, D, He (~85%)</p>
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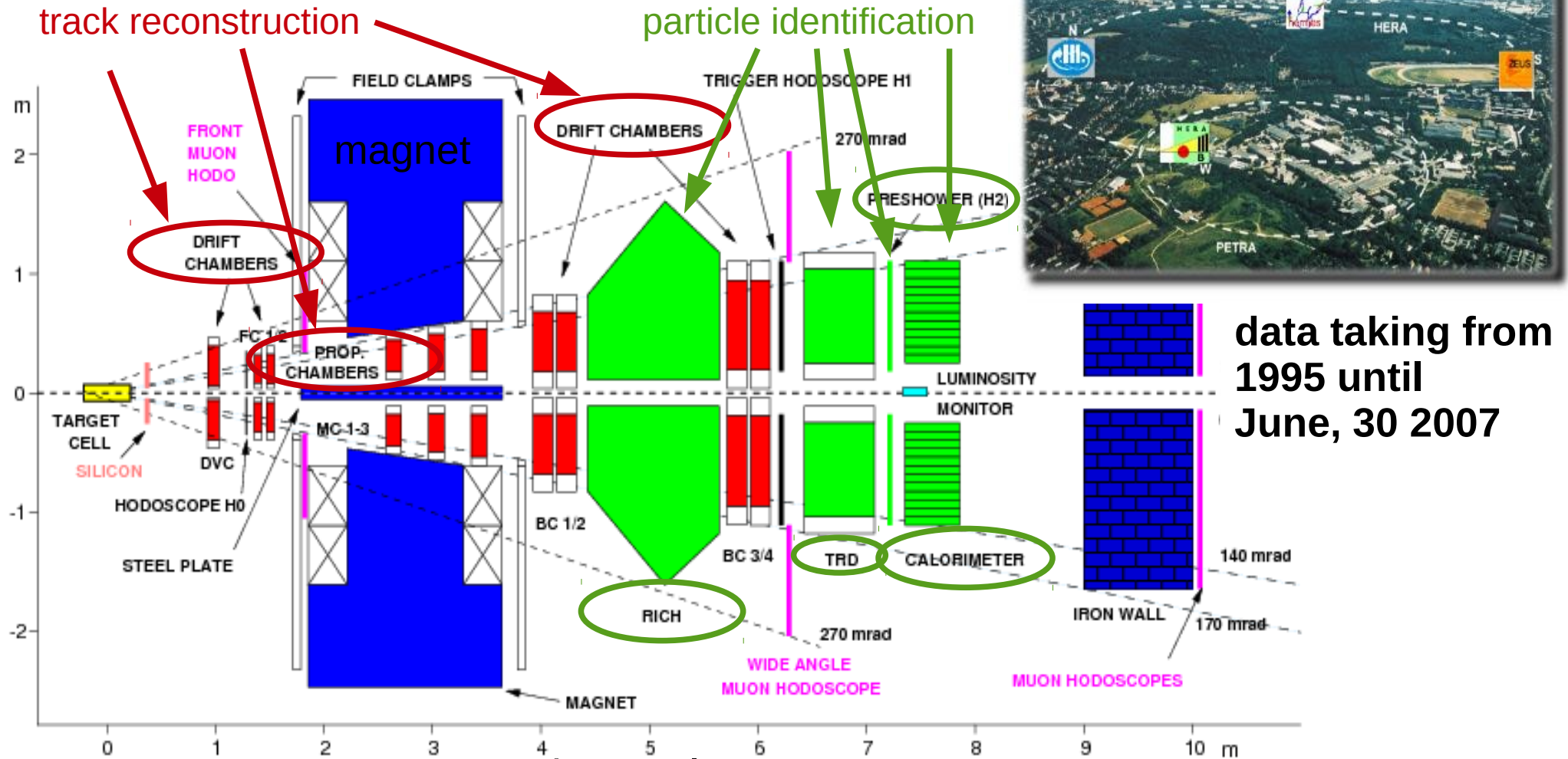
HERMES: HERA MEasurement of Spin



Beam
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 e^+ & e^-
 $E = 27.6 \text{ GeV}$

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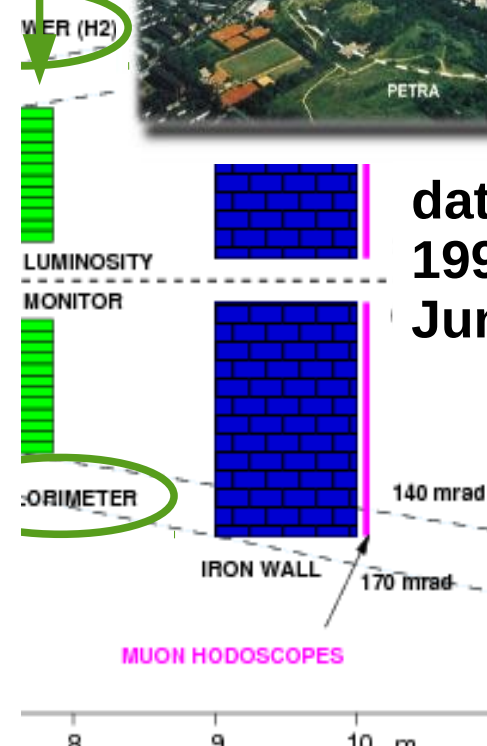
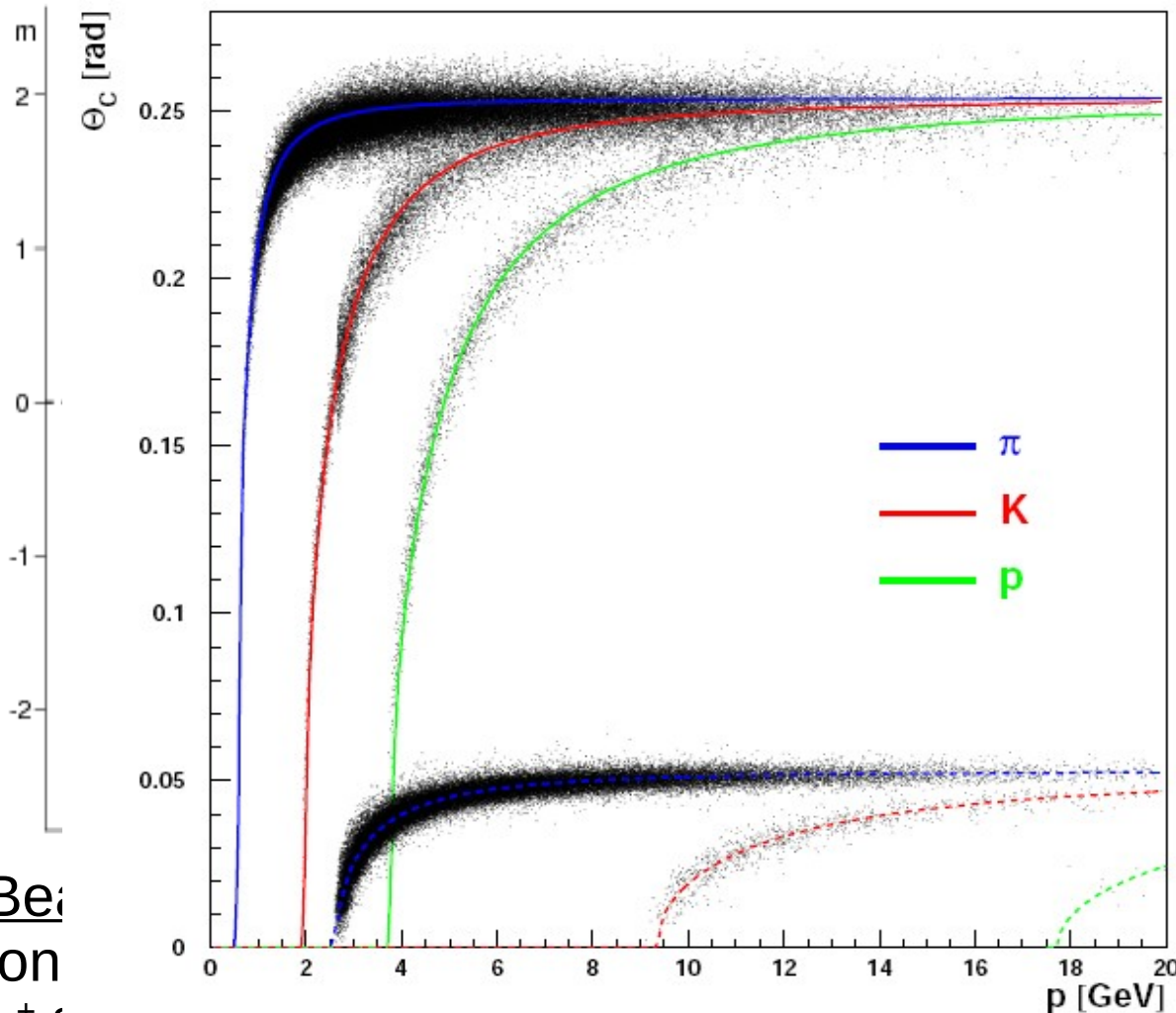
Gaseous internal target
 transversely pol. H (~75%)
 unpol. H, D, He, Ne, Kr, Xe
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- **lepton-hadron PID:** high efficiency (>98%) & low contamination (<1%)
- **hadron PID:** RICH 2-15 GeV

HERMES: HERA MEasurement of Spin

track reconstruction

particle identification



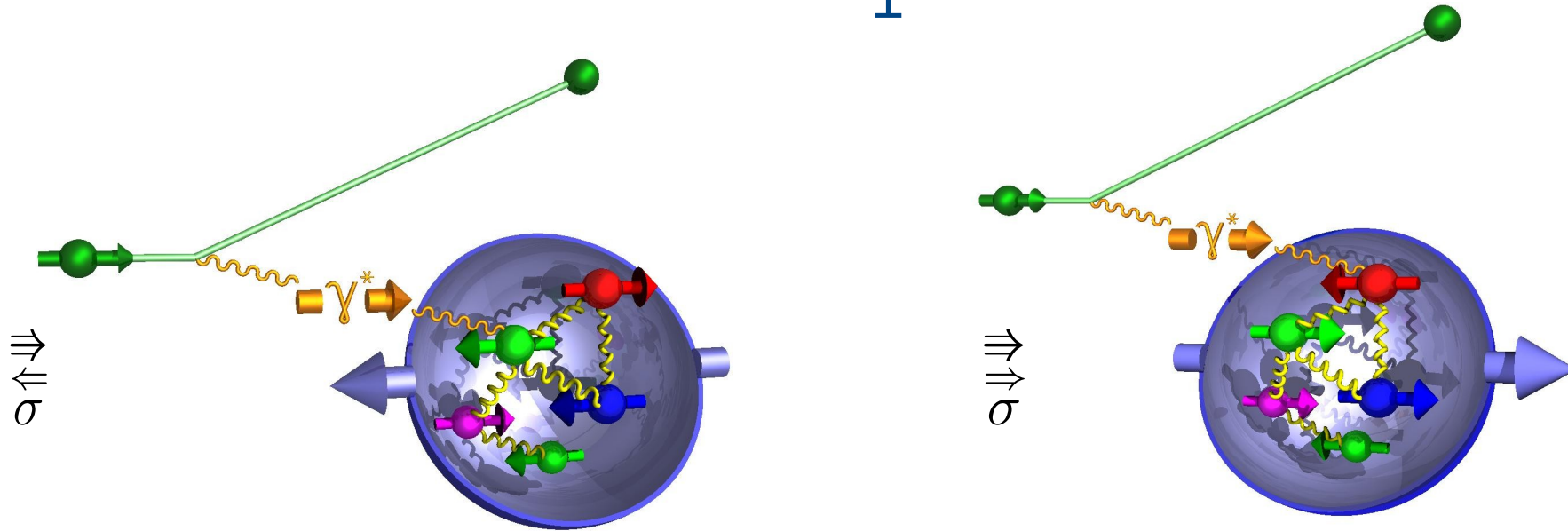
data taking from 1995 until June, 30 2007

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Be: Ion e⁺ & e⁻ E= 27.6 GeV

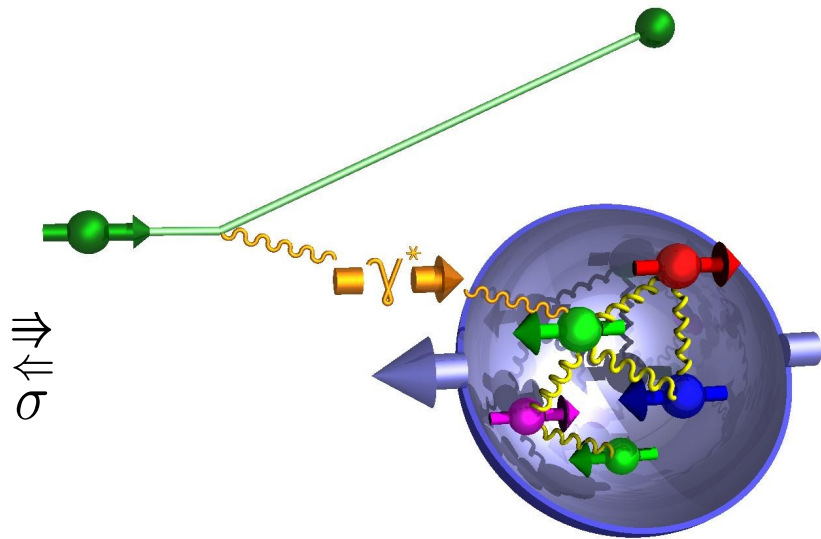
unpol. H,D,He (~85%) longitudinally pol. H,D,He (~85%)

Accessing g_1 at HERMES



$$\begin{aligned}
 A_1 &\approx \frac{\begin{array}{c} \Rightarrow \\ \sigma \end{array} - \begin{array}{c} \Rightarrow \\ \sigma \end{array}}{\begin{array}{c} \Rightarrow \\ \sigma \end{array} + \begin{array}{c} \Rightarrow \\ \sigma \end{array}} \quad \begin{array}{l} \longrightarrow \text{photon spin} \\ \longrightarrow \text{proton spin} \end{array} \\
 &\approx \frac{\sum_q e_q^2 \Delta q(x_B, Q^2)}{\sum_q e_q^2 q(x_B, Q^2)} \\
 &\approx \frac{g_1(x_B, Q^2)}{F_1(x_B, Q^2)}
 \end{aligned}$$

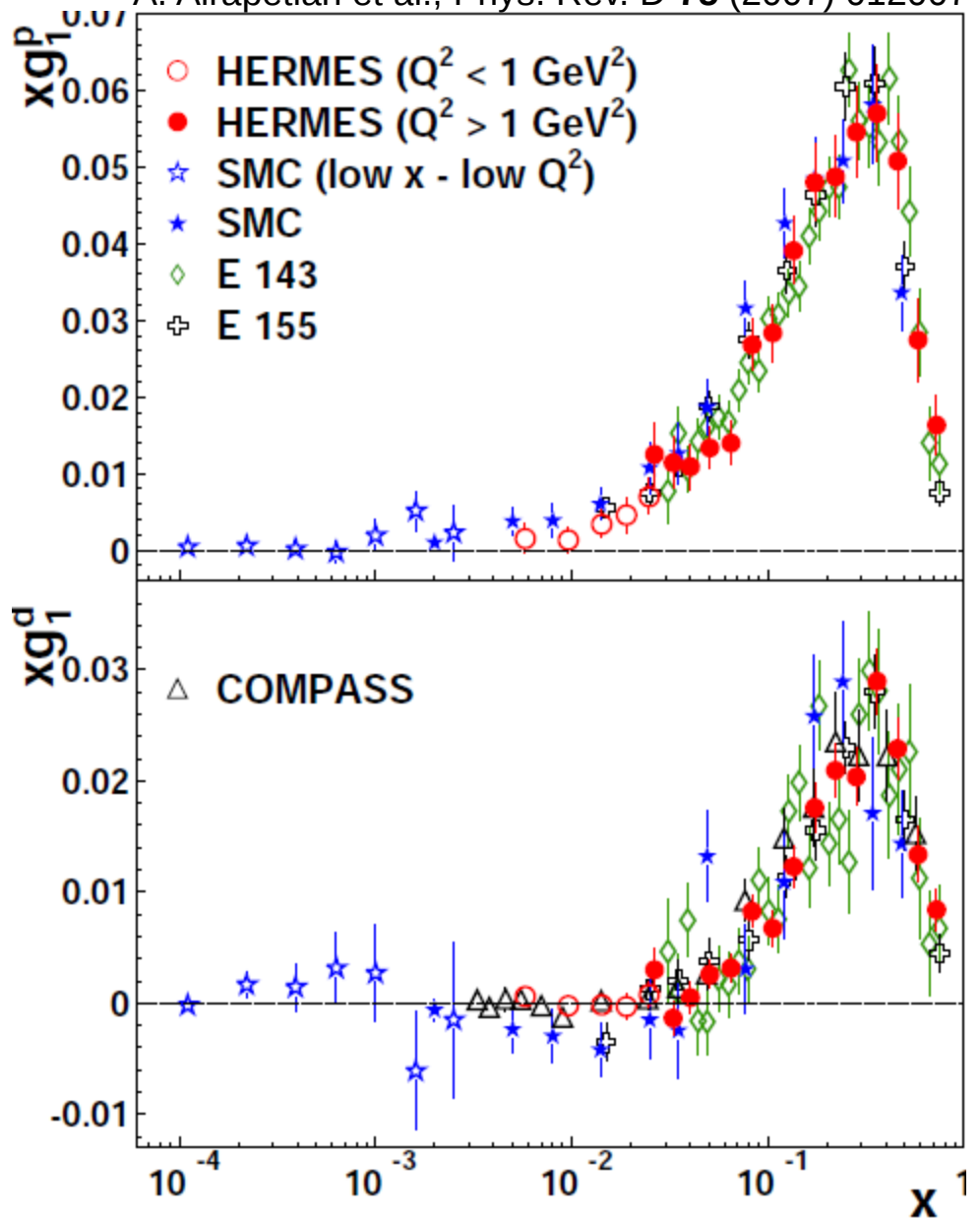
g_1 from HERMES



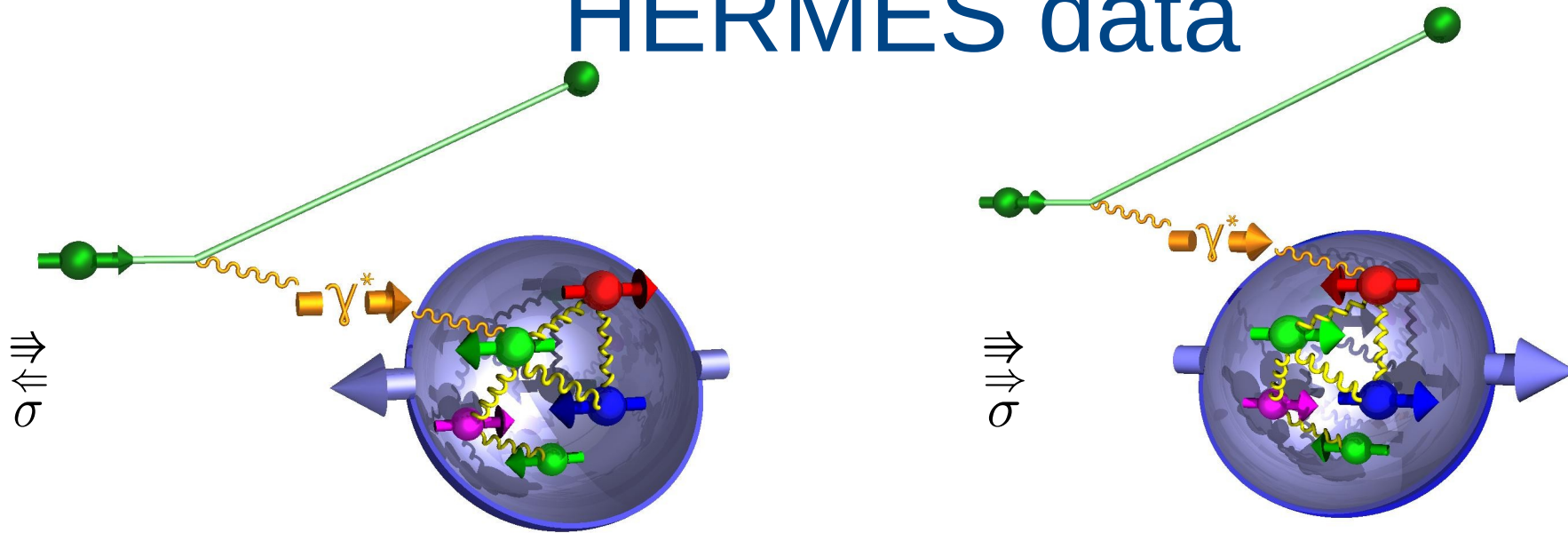
$$A_1 \approx \frac{\sigma_{\uparrow\uparrow} - \sigma_{\downarrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\downarrow\downarrow}} \approx \frac{\sum_q e_q^2 \Delta q(x_B, Q^2)}{\sum_q e_q^2 q(x_B, Q^2)} \approx \frac{g_1(x_B, Q^2)}{F_1(x_B, Q^2)}$$

$\begin{matrix} \uparrow \\ \uparrow \end{matrix}$ $\begin{matrix} \uparrow \\ \uparrow \end{matrix}$ \rightarrow photon spin
 $\begin{matrix} \uparrow \\ \uparrow \end{matrix}$ $\begin{matrix} \uparrow \\ \uparrow \end{matrix}$ \rightarrow proton spin

A. Airapetian et al., Phys. Rev. D **75** (2007) 012007



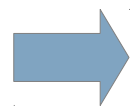
Quark spin contribution from HERMES data



$$A_1 \approx \frac{\begin{matrix} \Rightarrow & \Rightarrow \\ \sigma & - \sigma \end{matrix}}{\begin{matrix} \Rightarrow & \Rightarrow \\ \sigma & + \sigma \end{matrix}}$$

\Rightarrow photon spin
 \Rightarrow proton spin

$$\approx \frac{\sum_q e_q^2 \Delta q(x_B, Q^2)}{\sum_q e_q^2 q(x_B, Q^2)}$$

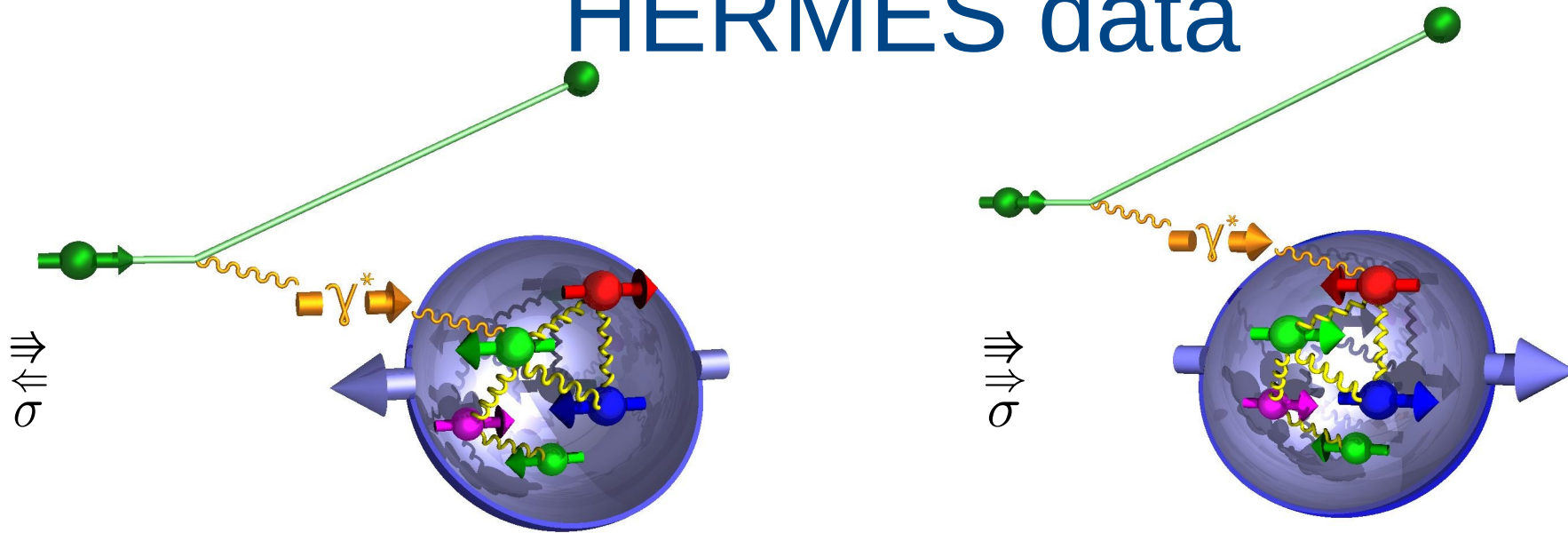


$$\Delta\Sigma = 0.330 \pm 0.025 \pm 0.011 \pm 0.028$$

(exp.)
(theor.)
(evol.)

$$\approx \frac{g_1(x_B, Q^2)}{F_1(x_B, Q^2)}$$

Quark spin contribution from HERMES data



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\Rightarrow photon spin
 \Rightarrow proton spin

$$\approx \frac{\sum_q e_q^2 \Delta q(x_B, Q^2)}{\sum_q e_q^2 q(x_B, Q^2)}$$

$$\approx \frac{g_1(x_B, Q^2)}{F_1(x_B, Q^2)}$$

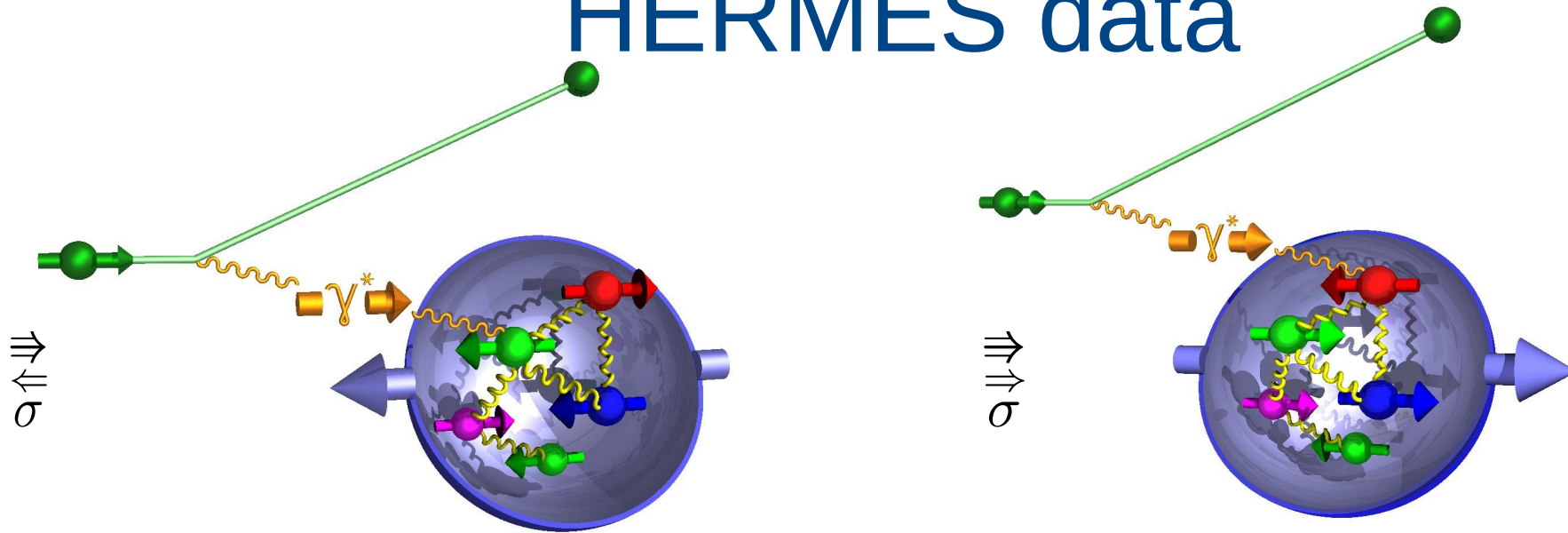
$$\Delta\Sigma = 0.330 \pm 0.025 \pm 0.011 \pm 0.028$$

(exp.) (theor.) (evol.)

Spin Puzzle

Where is the other 70%?

Quark spin contribution from HERMES data



$$A_1 \approx \frac{\begin{matrix} \Rightarrow & \Rightarrow \\ \sigma & - \sigma \end{matrix}}{\begin{matrix} \Rightarrow & \Rightarrow \\ \sigma & + \sigma \end{matrix}}$$

\Rightarrow photon spin
 \Rightarrow proton spin

$$\approx \frac{\sum_q e_q^2 \Delta q(x_B, Q^2)}{\sum_q e_q^2 q(x_B, Q^2)}$$

$$\approx \frac{g_1(x_B, Q^2)}{F_1(x_B, Q^2)}$$

$$\Delta\Sigma = 0.330 \pm 0.025 \pm 0.011 \pm 0.028$$

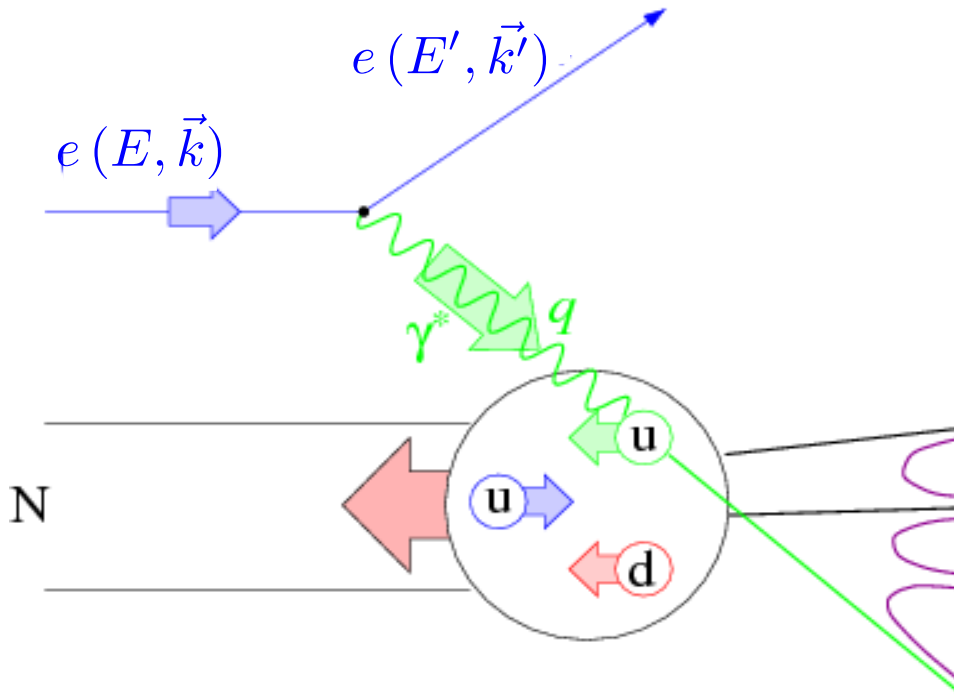
(exp.)
(theor.)
(evol.)

Spin Puzzle

gluon spin?
orbital angular momentum?

Tagging the quark flavor

inclusive deep-inelastic scattering



$$Q^2 \equiv -q^2$$

$$\nu \equiv \frac{Pq}{M} \stackrel{lab}{=} E - E'$$

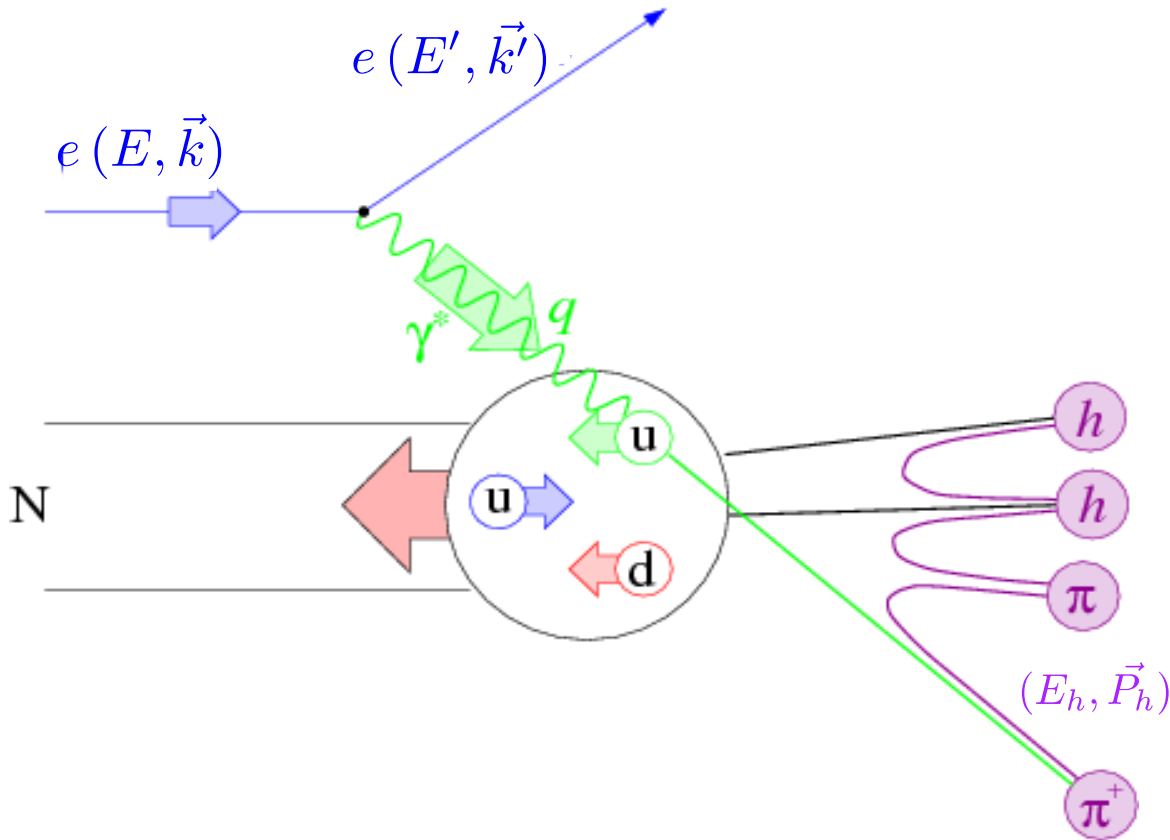
$$y \equiv \frac{Pq}{Pk} \stackrel{lab}{=} \frac{\nu}{E}$$

$$W^2 \equiv M^2 + 2M\nu - Q^2$$

$$x_B \equiv \frac{Q^2}{2Pq}$$

Tagging the quark flavor

semi-inclusive deep-inelastic scattering



$$Q^2 \equiv -q^2$$

$$\nu \equiv \frac{Pq}{M} \stackrel{lab}{=} E - E'$$

$$y \equiv \frac{Pq}{Pk} \stackrel{lab}{=} \frac{\nu}{E}$$

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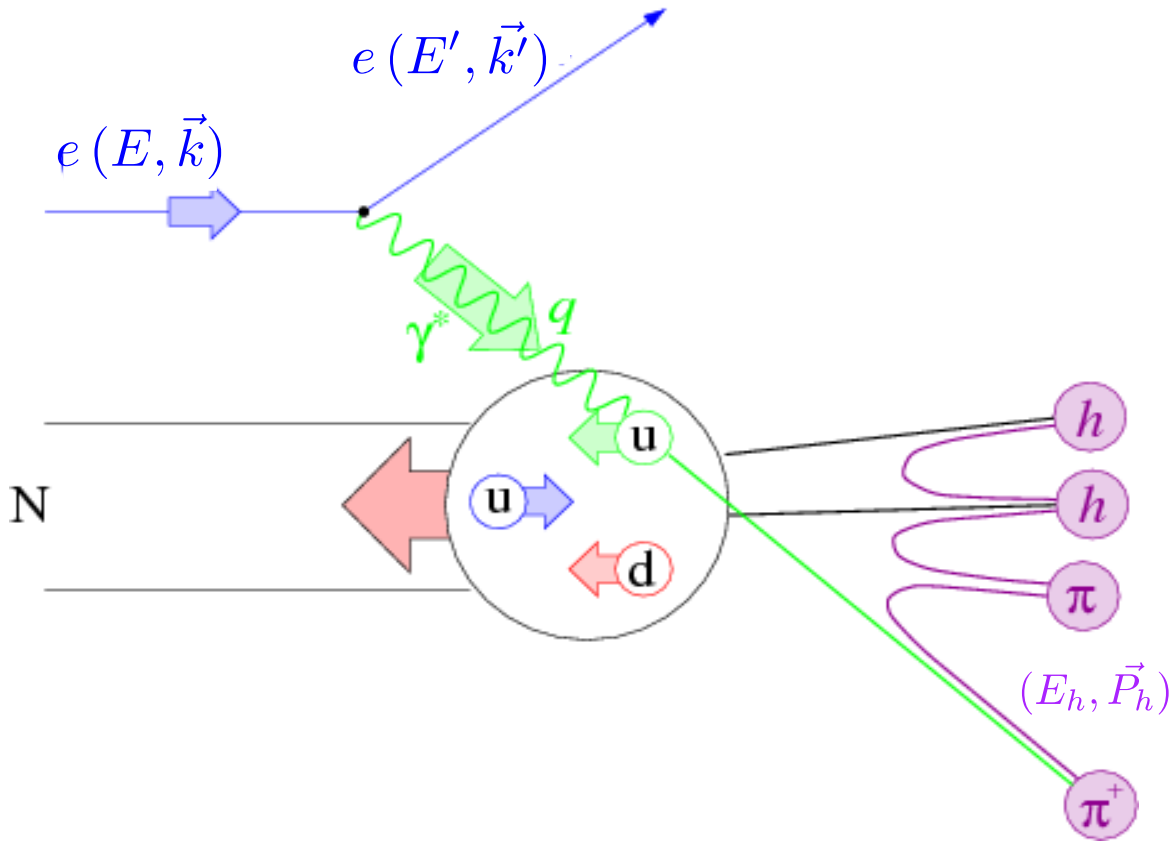
$$x_B \equiv \frac{Q^2}{2Pq}$$

$$z \equiv \frac{PP_h}{Pq} \stackrel{lab}{=} \frac{E_h}{\nu}$$

$$P_{h\perp} = \frac{|\vec{q} \times \vec{P}_h|}{|\vec{q}|}$$

Tagging the quark flavor

semi-inclusive deep-inelastic scattering



$$Q^2 \equiv -q^2$$

$$\nu \equiv \frac{Pq}{M} \stackrel{lab}{=} E - E'$$

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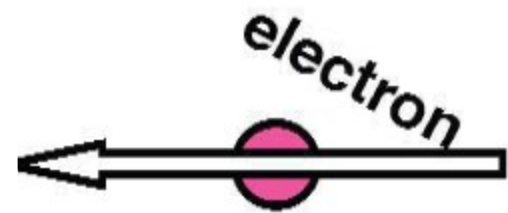
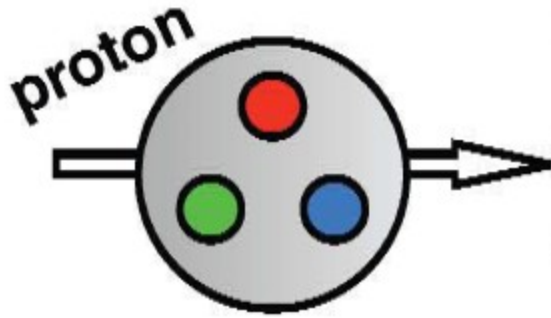
$$P_{h\perp} = \frac{|\vec{q} \times \vec{P}_h|}{|\vec{q}|}$$

$$\sigma^{ep \rightarrow eh} = \sum_q DF^{p \rightarrow q}(x_B, p_T^2, Q^2) \otimes \sigma^{eq \rightarrow eq} \otimes FF^{q \rightarrow h}(z, k_T^2, Q^2)$$

distribution function (DF):
distribution of quarks in nucleon
 p_T : transverse momentum of struck quark

fragmentation function (FF): fragmentation of
struck quark into final-state hadron
 k_T : transverse momentum of fragmenting quark

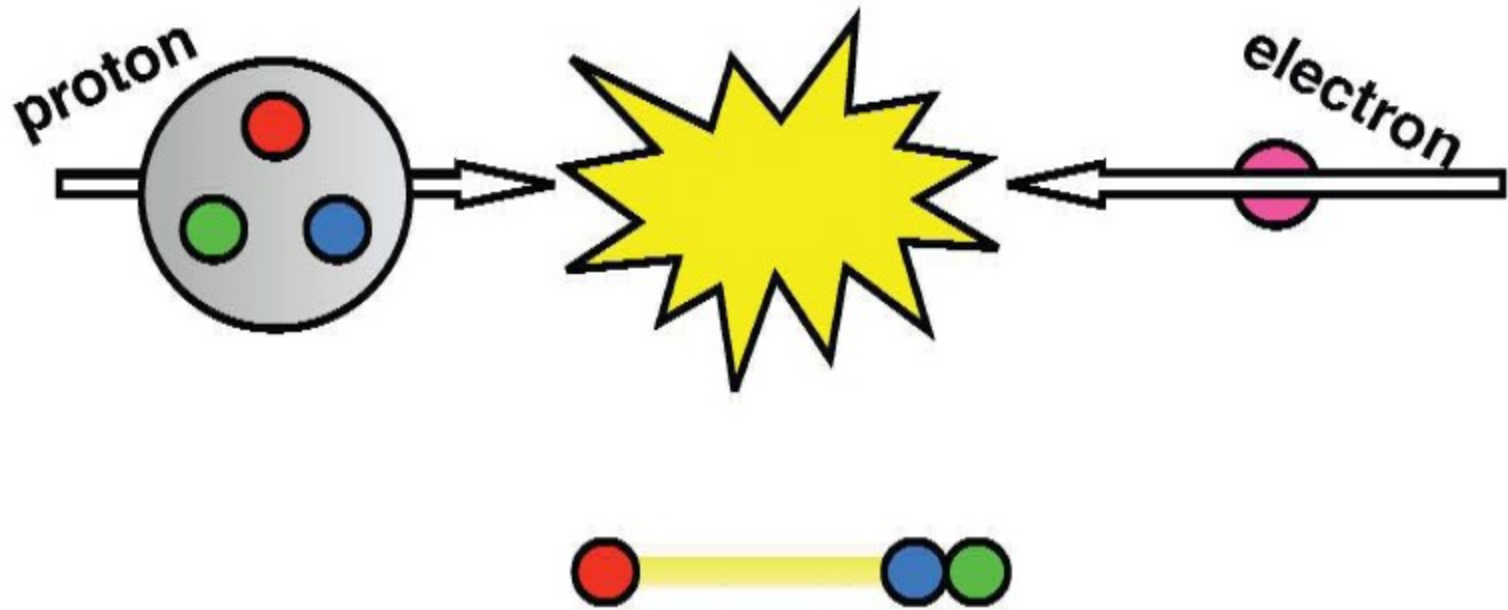
Fragmentation



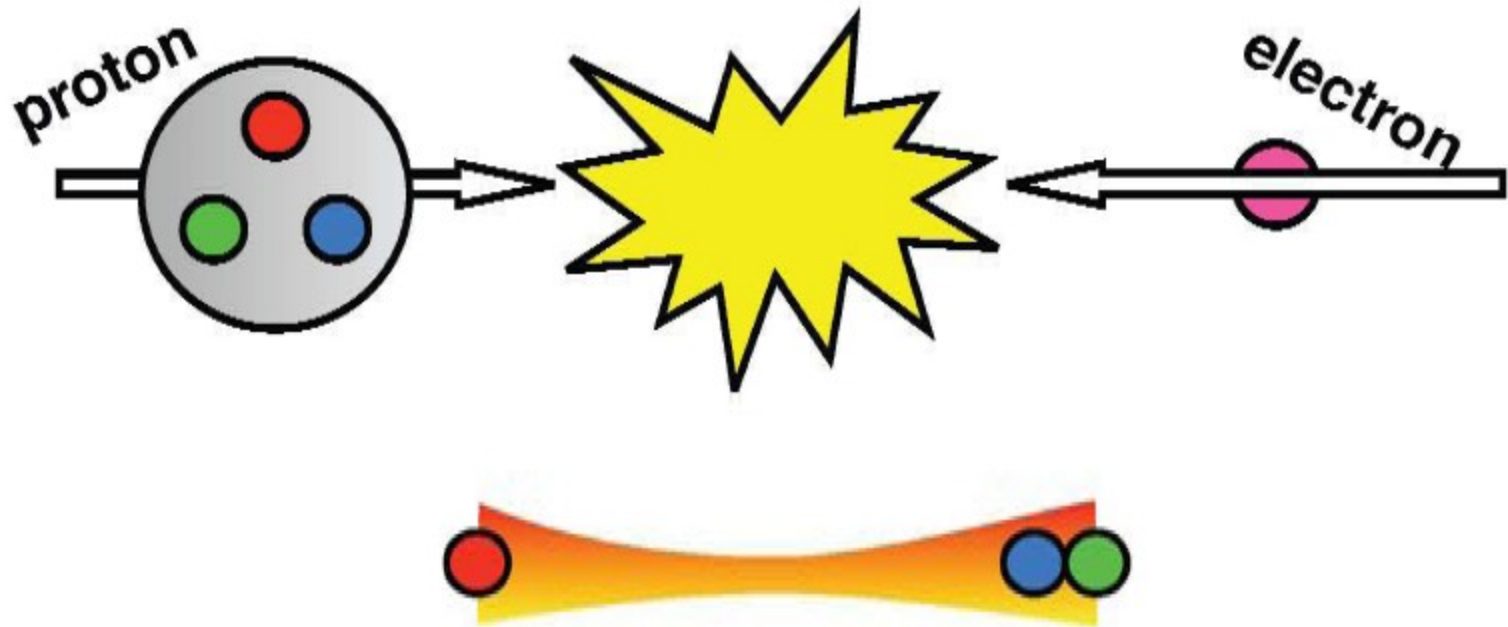
Fragmentation



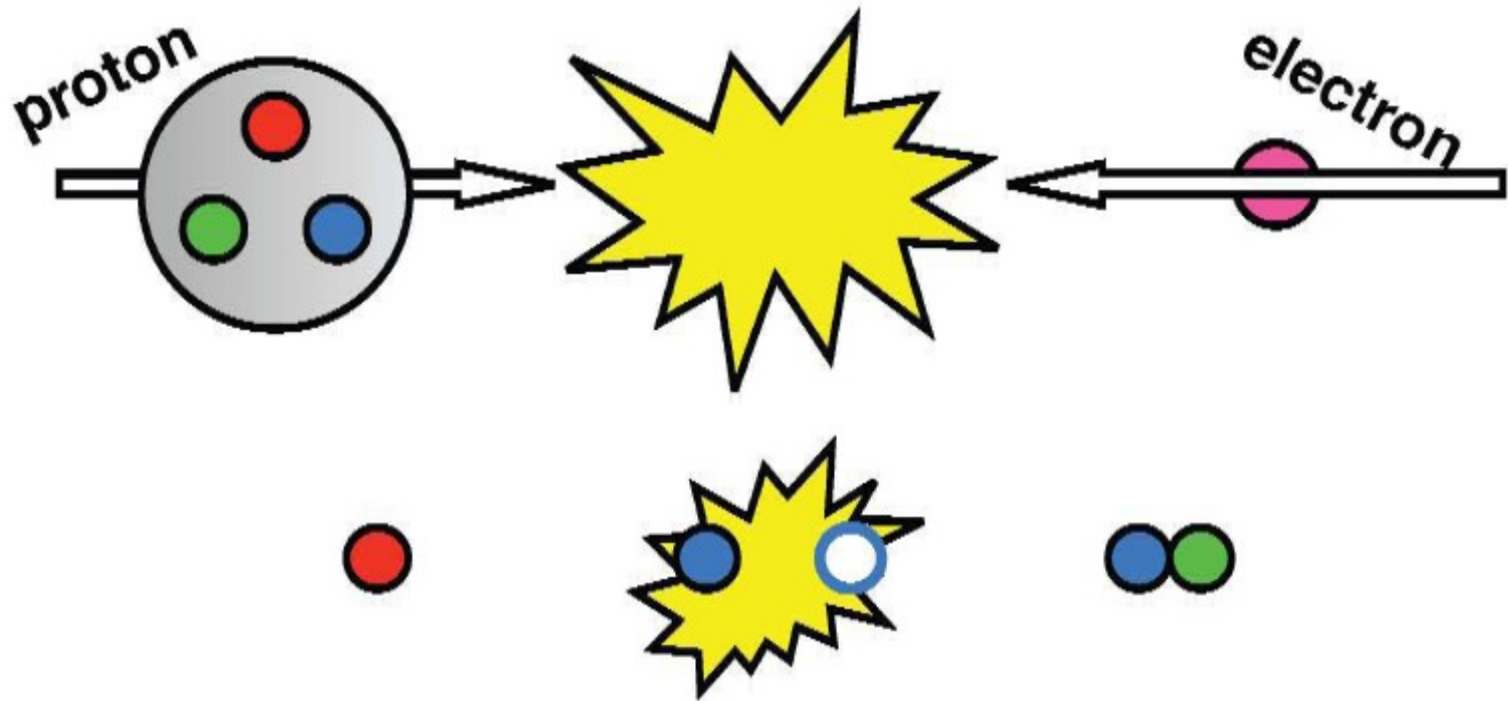
Fragmentation



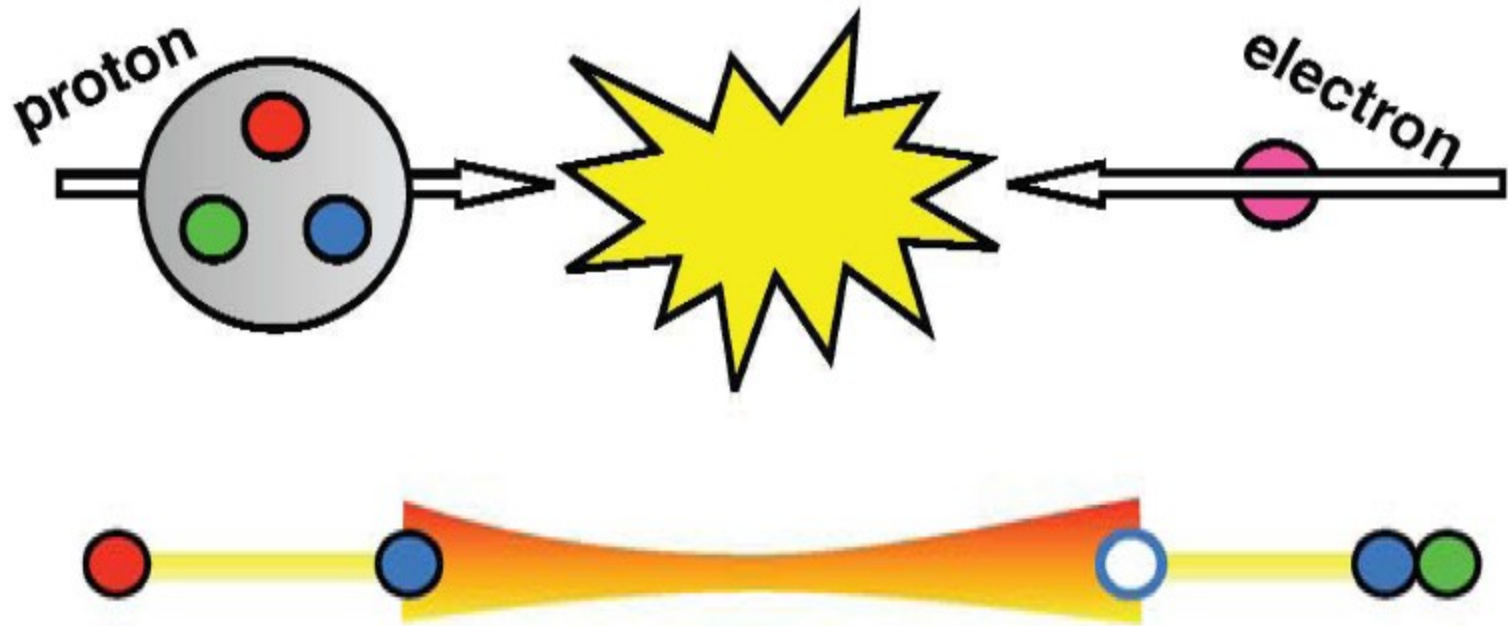
Fragmentation



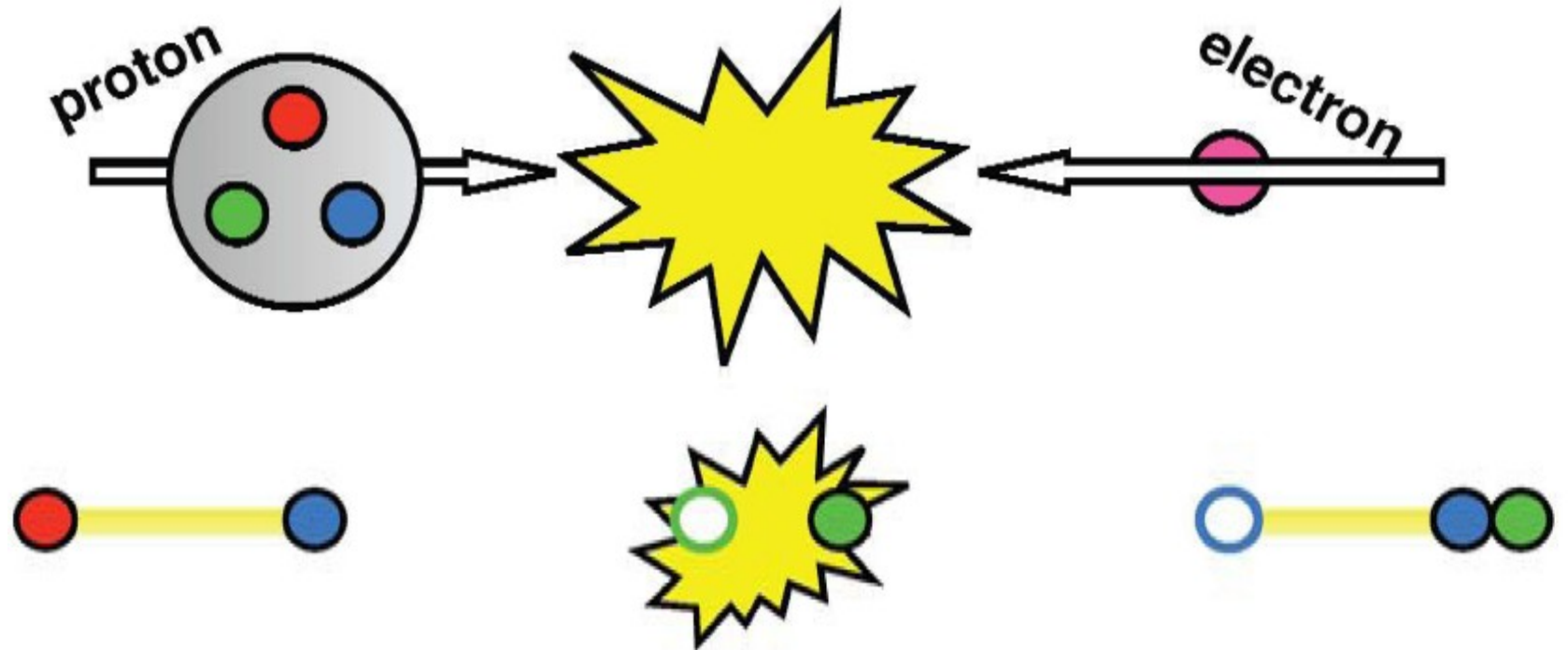
Fragmentation



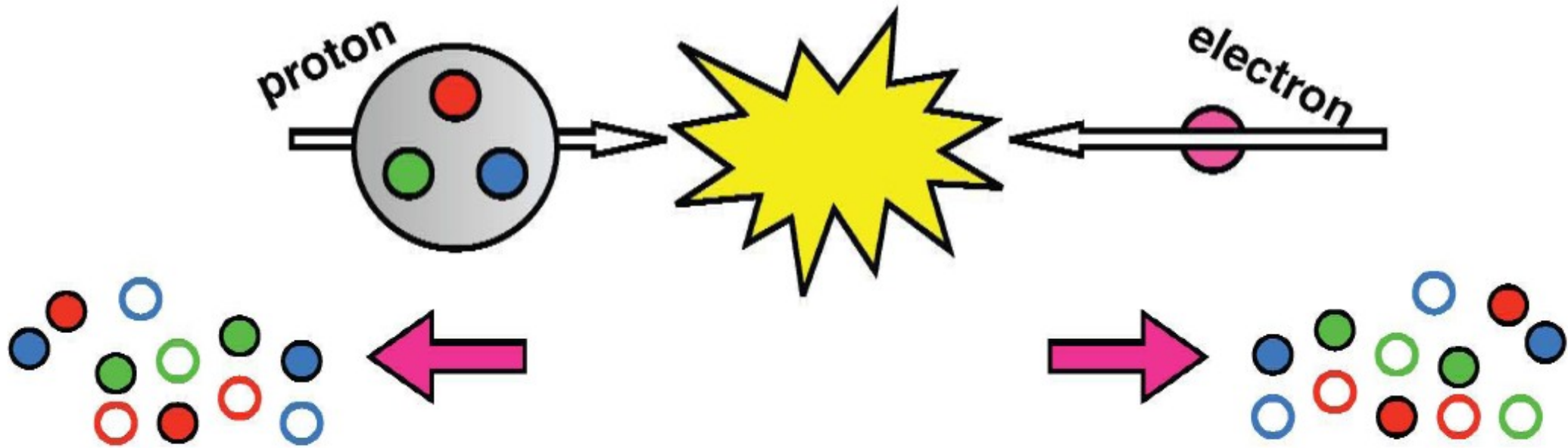
Fragmentation



Fragmentation



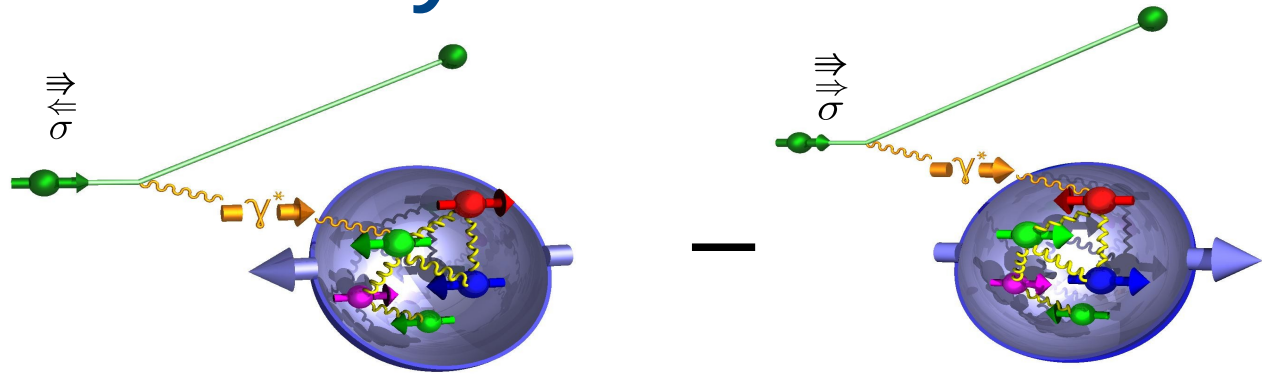
Fragmentation



Quark helicity distribution

$$A_1^h \approx \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}}$$

$$\propto \sum_q e_q^2 \Delta q(x_B, Q^2) D_q^h(z, Q^2)$$

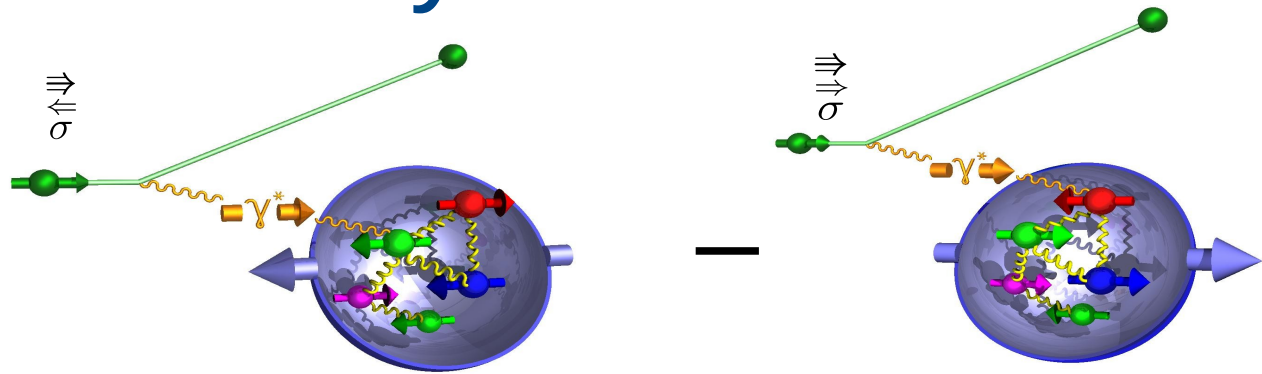


Quark helicity distribution

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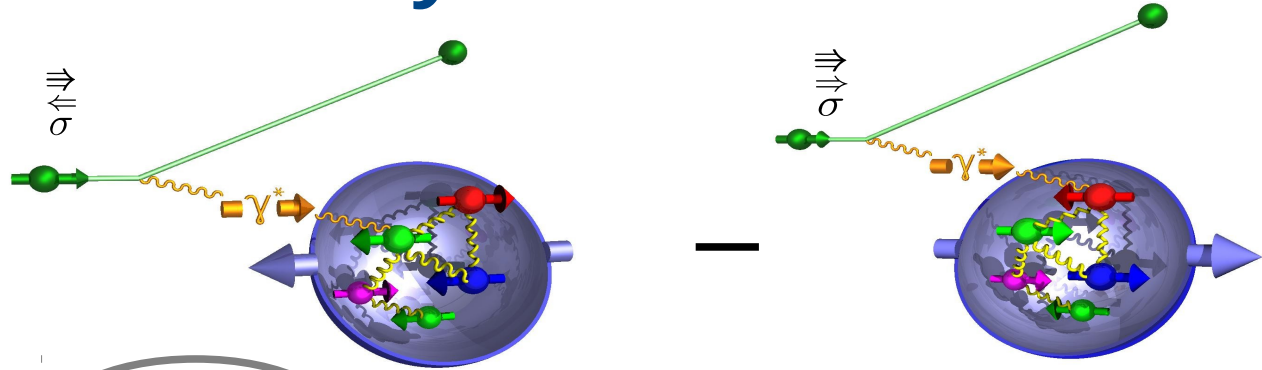
$$\propto \sum_q e_q^2 \Delta q(x_B, Q^2) D_q^h(z, Q^2)$$

proportional to square
of quark electric charge



Quark helicity distribution

$$A_1^h \approx \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}} \propto \sum_q e_q^2 \Delta q(x_B, Q^2) D_q^h(z, Q^2)$$



proportional to square of quark electric charge

favored fragmentation

$$u \rightarrow \pi^+ = |u\bar{d}\rangle$$

$$d \rightarrow \pi^- = |\bar{u}d\rangle$$

unfavored fragmentation

$$d \rightarrow \pi^+ = |u\bar{d}\rangle$$

$$u \rightarrow \pi^- = |\bar{u}d\rangle$$

Quark helicity distribution

A. Airapetian et al., Phys. Rev. Lett. **92** (2004) 012005

A. Airapetian et al., Phys. Rev. D **71** (2005) 012003

$$A_1^h \approx \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}} \propto \sum_q e_q^2 \Delta q(x_B, Q^2) D_q^h(z, Q^2)$$

proportional to square of quark electric charge

favored fragmentation

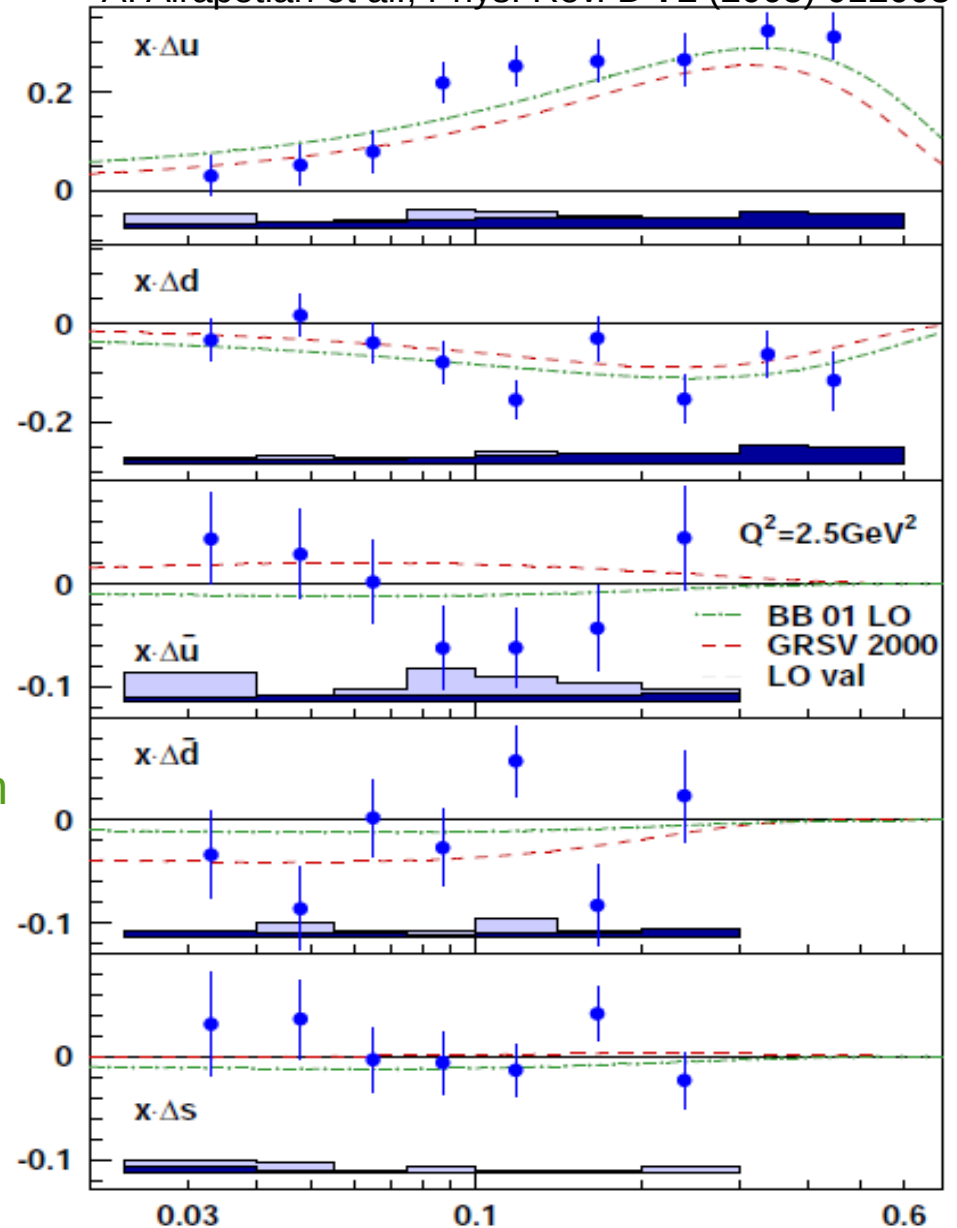
$$u \rightarrow \pi^+ = |u\bar{d}\rangle$$

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unfavored fragmentation

$$d \rightarrow \pi^+ = |u\bar{d}\rangle$$

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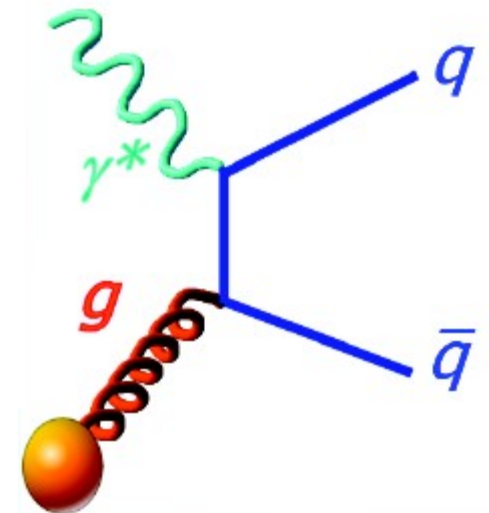


Gluon helicity distribution

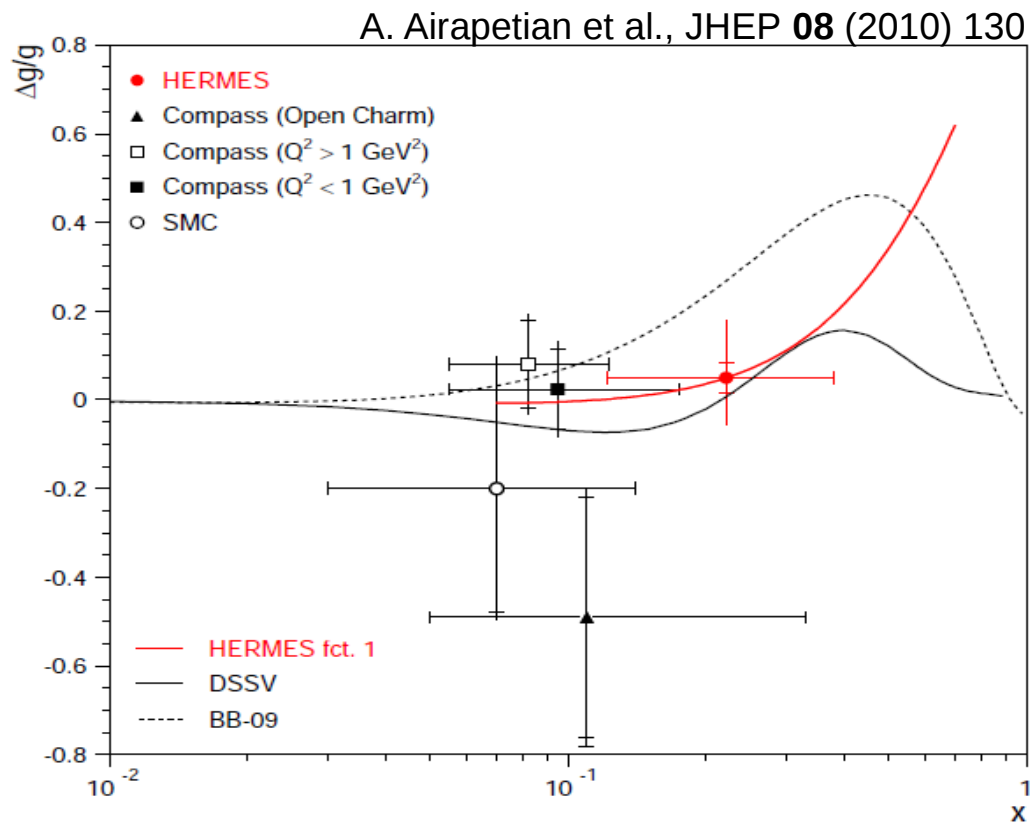
$$A_1^h \sim \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}}$$

$$\propto \frac{\Delta g(x_B, Q^2)}{g(x_B, Q^2)}$$

select high- $P_{h\perp}$ hadrons



photon-gluon fusion

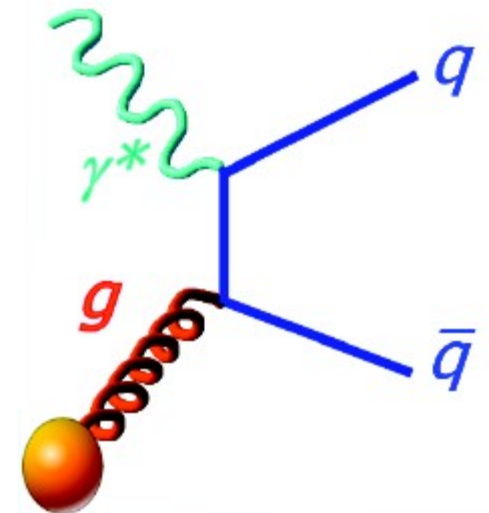


Gluon helicity distribution

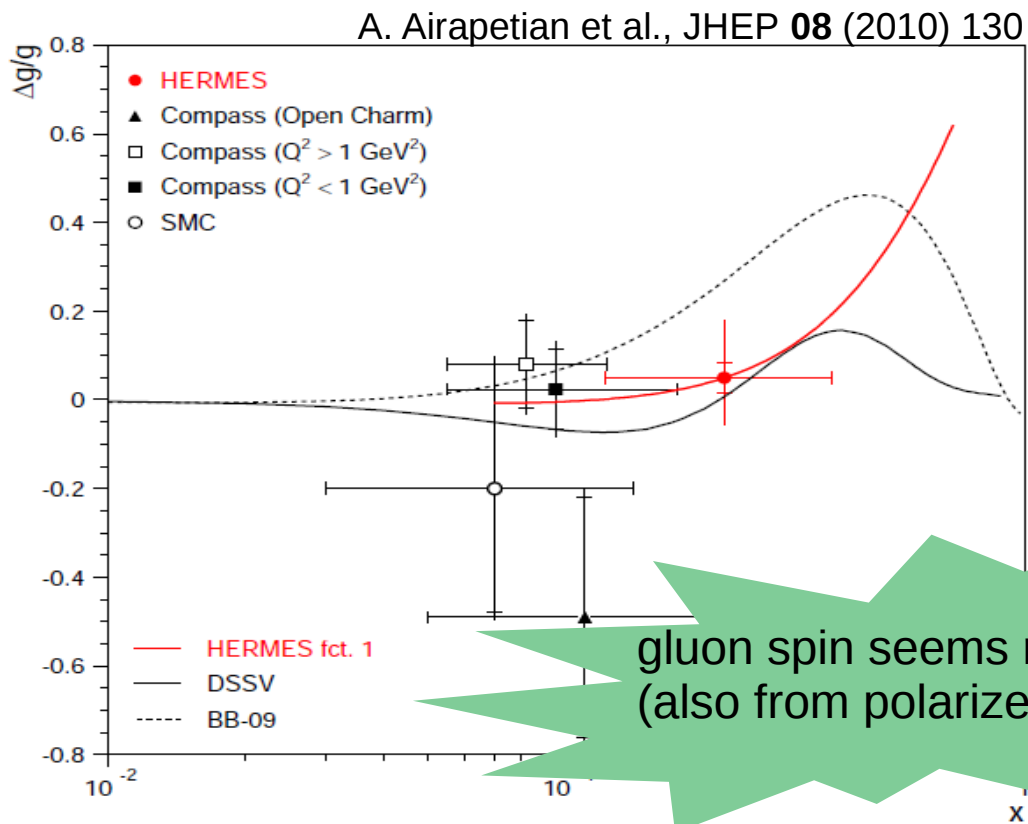
$$A_1^h \sim \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}}$$

$$\propto \frac{\Delta g(x_B, Q^2)}{g(x_B, Q^2)}$$

select high- $P_{h\perp}$ hadrons



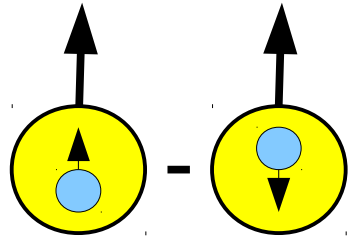
photon-gluon fusion



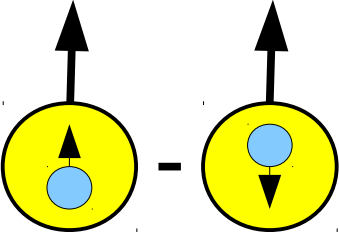
gluon spin seems rather small
(also from polarized pp data)

Transversity

$$h_{1T}^q(x, p_T^2) =$$



Transversity

$$h_{1T}^q(x, p_T^2) = \text{Diagram 1} - \text{Diagram 2}$$


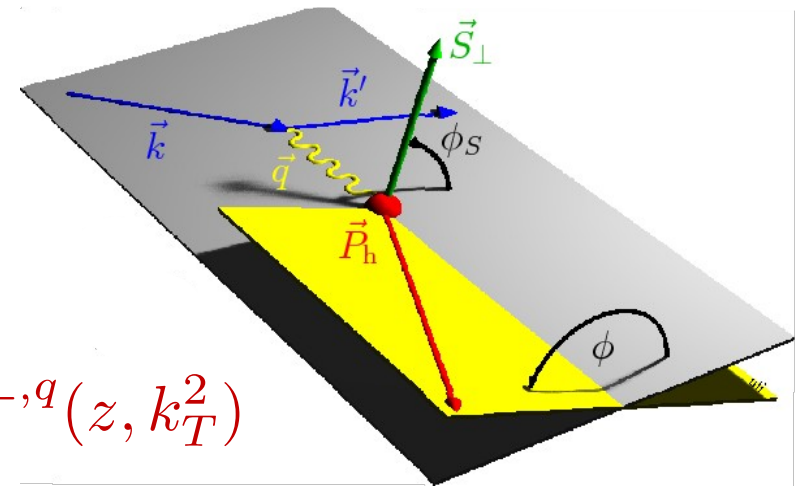
access through single-spin asymmetry on transversely polarized target

$$A_{UT} = \frac{1}{\langle |S_T| \rangle} \frac{N^\uparrow(\phi, \phi_S) - N^\downarrow(\phi, \phi_S)}{N^\uparrow(\phi, \phi_S) + N^\downarrow(\phi, \phi_S)}$$

$$\sim \sin(\phi + \phi_S) \sum_q e_q^2 \frac{\vec{k}_T \cdot \hat{P}_{h\perp}}{M_h} h_{1T}^q(x, p_T^2) \otimes H_1^{\perp, q}(z, k_T^2)$$

chiral-odd

chiral-odd



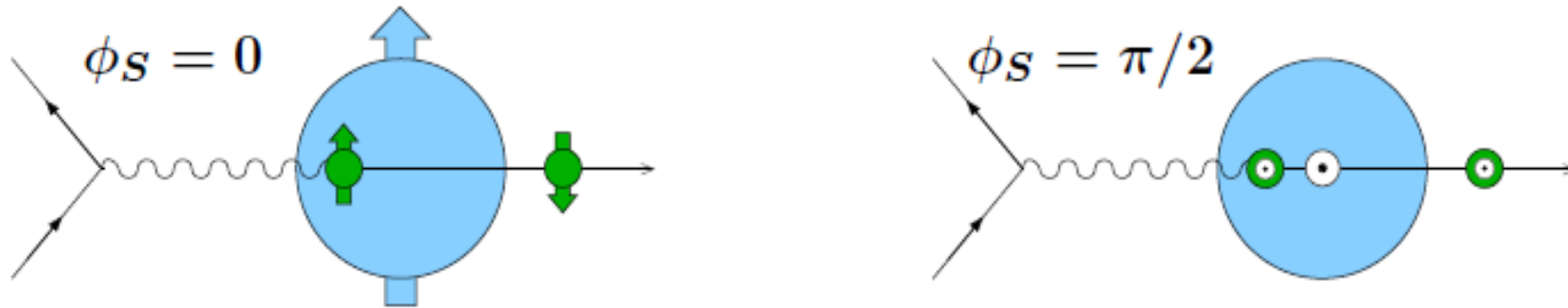
$H_1^{\perp, q}(z, k_T^2)$: Collins fragmentation function

fragmentation of a transversely polarized quark into an unpolarized hadron

Collins fragmentation functions

Artru model

polarisation component in lepton scattering plane reversed by photoabsorption:



string break, quark-antiquark pair with vacuum numbers:

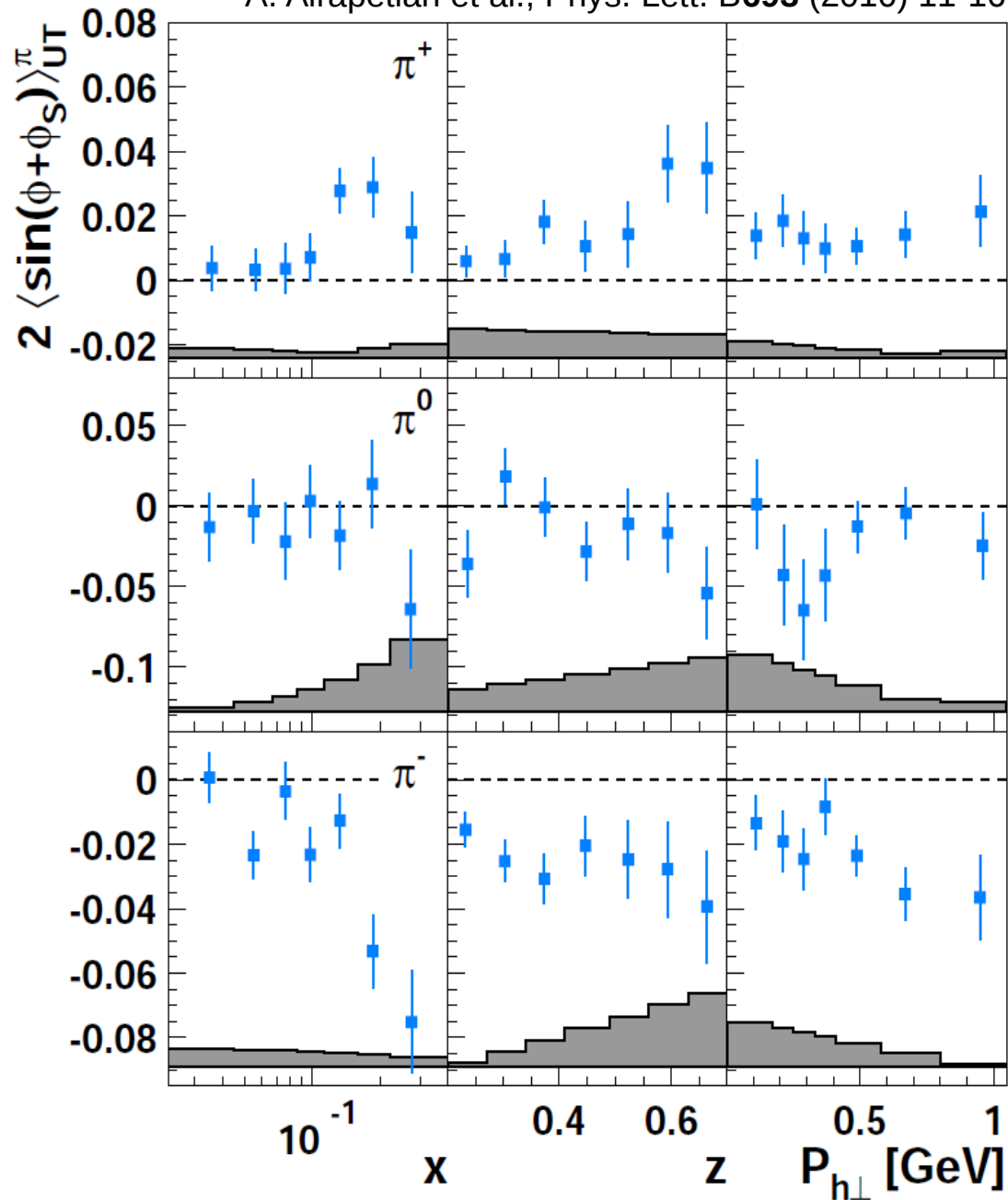


orbital angular momentum creates transverse momentum:



Collins amplitudes for pions

A. Airapetian et al., Phys. Lett. B**693** (2010) 11-16



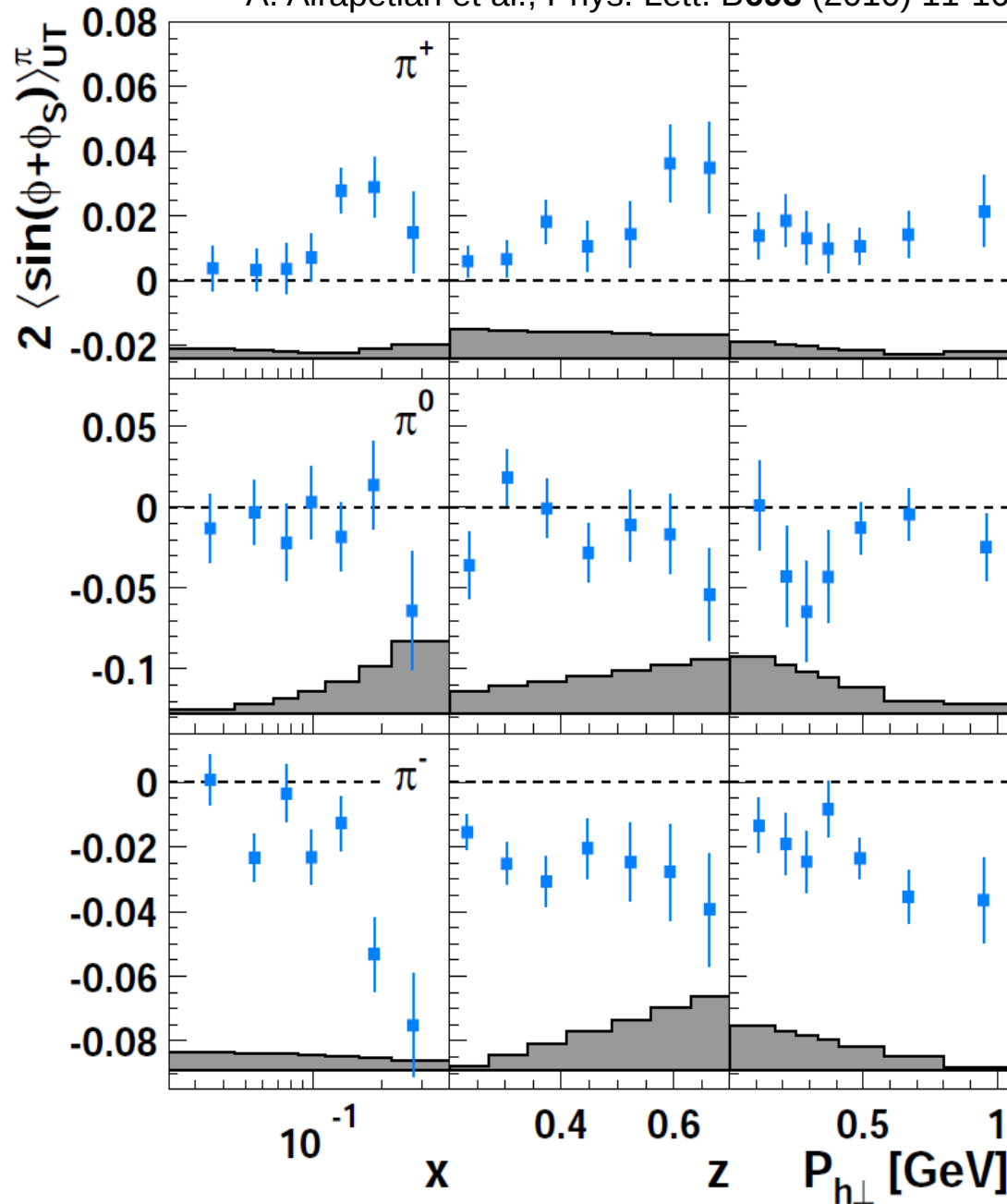
- π^\pm increasing with z
- positive for π^+
- large & negative for π^-

$$H_1^{\perp, fav} \approx -H_1^{\perp, unfav}$$

- isospin symmetry fulfilled

Collins amplitudes for pions

A. Airapetian et al., Phys. Lett. B**693** (2010) 11-16

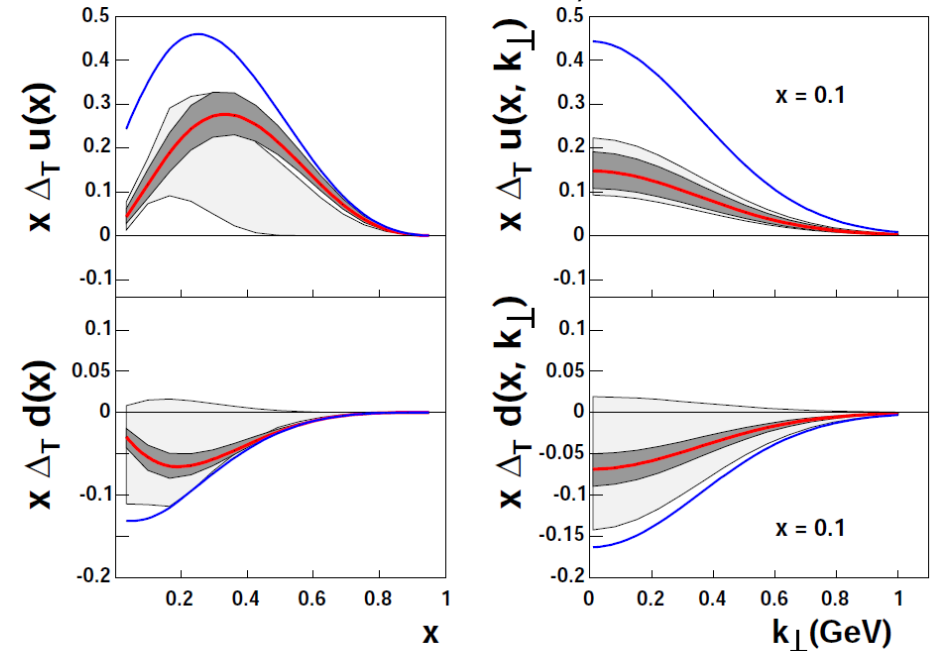


- π^\pm increasing with z
- positive for π^+
- large & negative for π^-

$$H_1^{\perp, fav} \approx -H_1^{\perp, unfav}$$

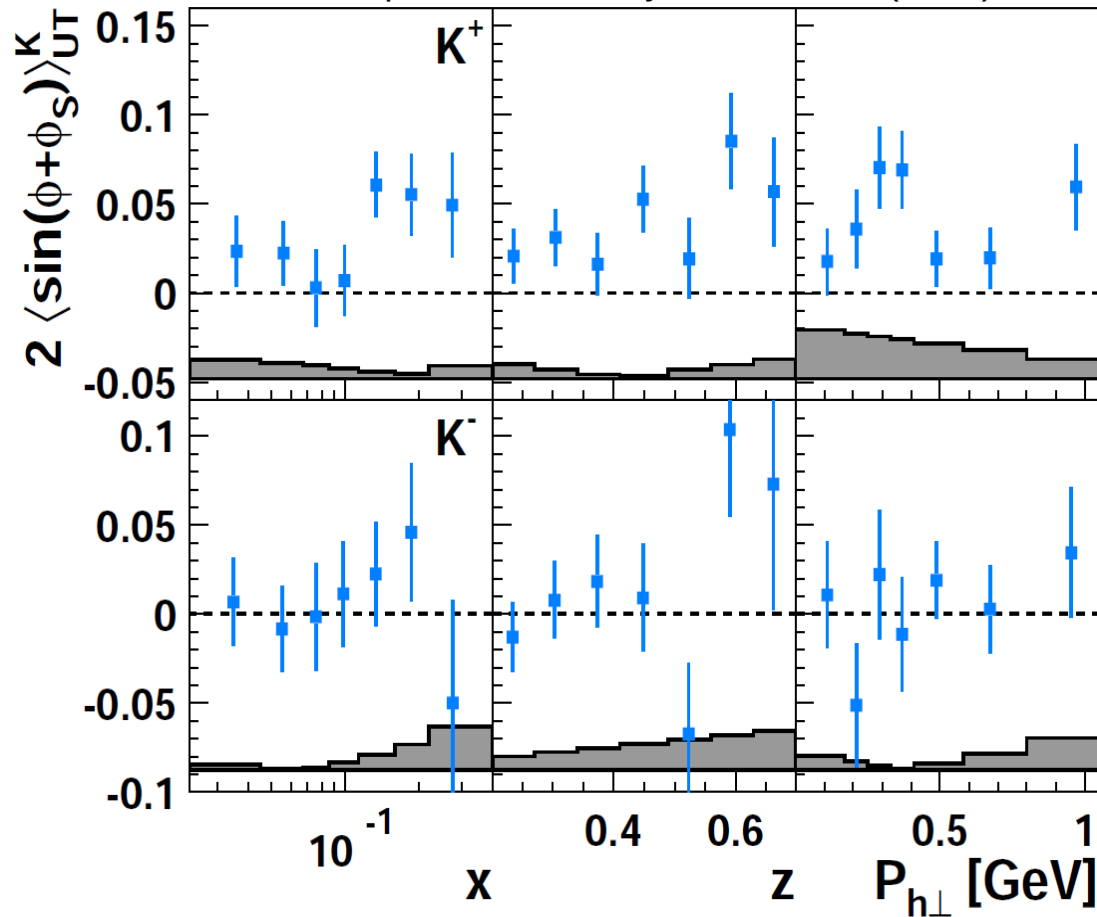
- isospin symmetry fulfilled
- data from BELLE, COMPASS & HERMES \longrightarrow extraction of h_{1T}

Anselmino et al., arXiv: 0807.0173



Collins amplitudes for kaons

A. Airapetian et al., Phys. Lett. B693 (2010) 11-16



- K^+ : increasing with z
- positive for K^+ & larger than for π^+
 - role of s-quark
 - u-dominance $\rightarrow ?$

$$H_1^{\perp, u \rightarrow K^+} > H_1^{\perp, u \rightarrow \pi^+}$$

- $K^- \approx 0$, \neq from π^-
- K^- is pure sea object:
- sea-quark transversity expected to be small

Transverse-momentum-dependent distributions

Distribution functions

leading twist

$$f_1 = \text{yellow circle with blue center}$$

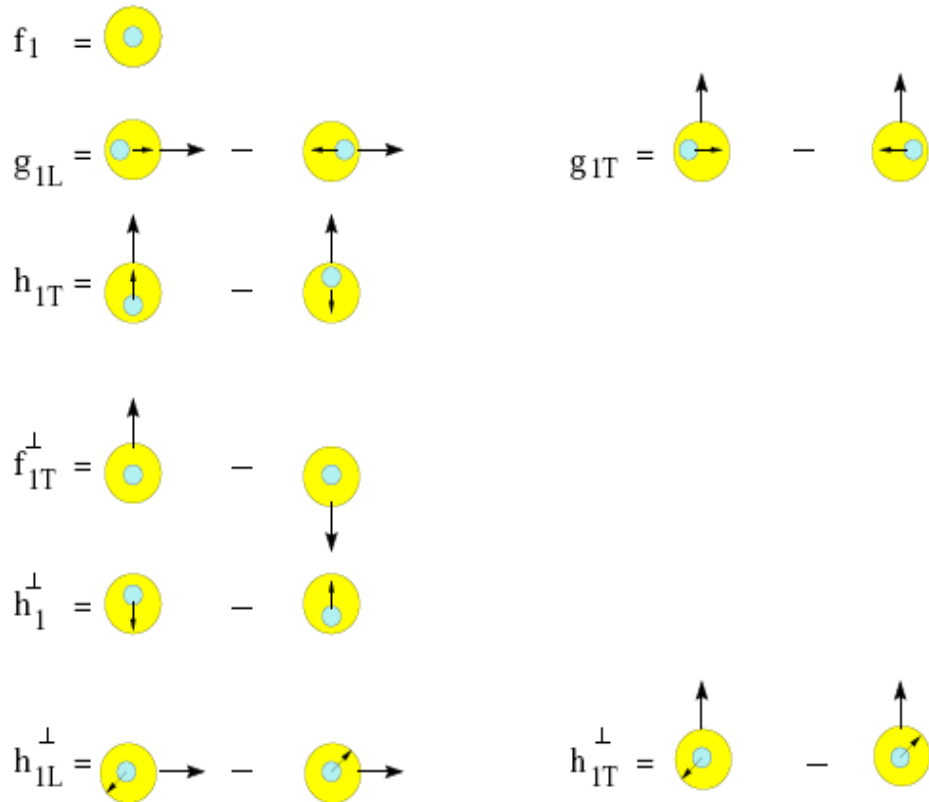
$$g_{1L} = \text{yellow circle with blue center and right arrow} - \text{yellow circle with blue center and left arrow}$$

$$h_{1T} = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

Transverse-momentum-dependent distributions

Distribution functions

leading twist

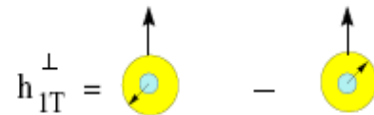
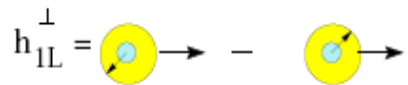
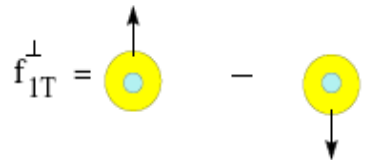
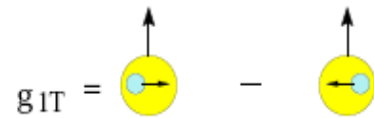
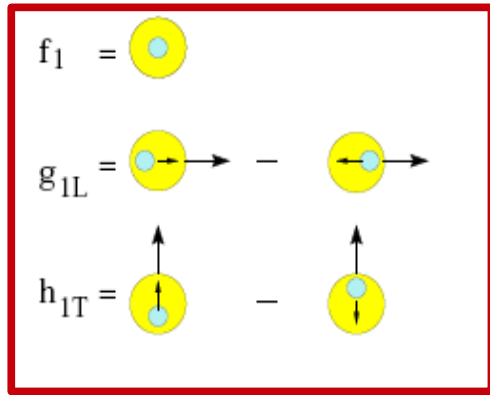


the eight leading-twist transverse-momentum-dependent parton distribution functions describing the DIS cross section for hadron production

Transverse-momentum-dependent distributions

Distribution functions

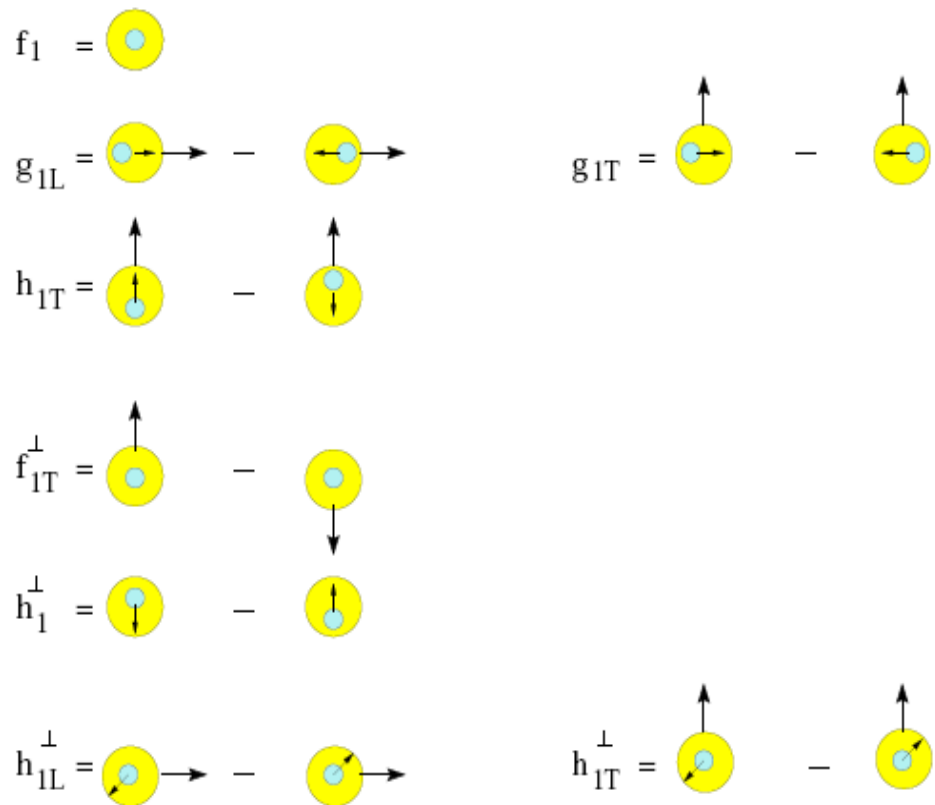
leading twist



only distribution functions that survive integration over transverse momentum

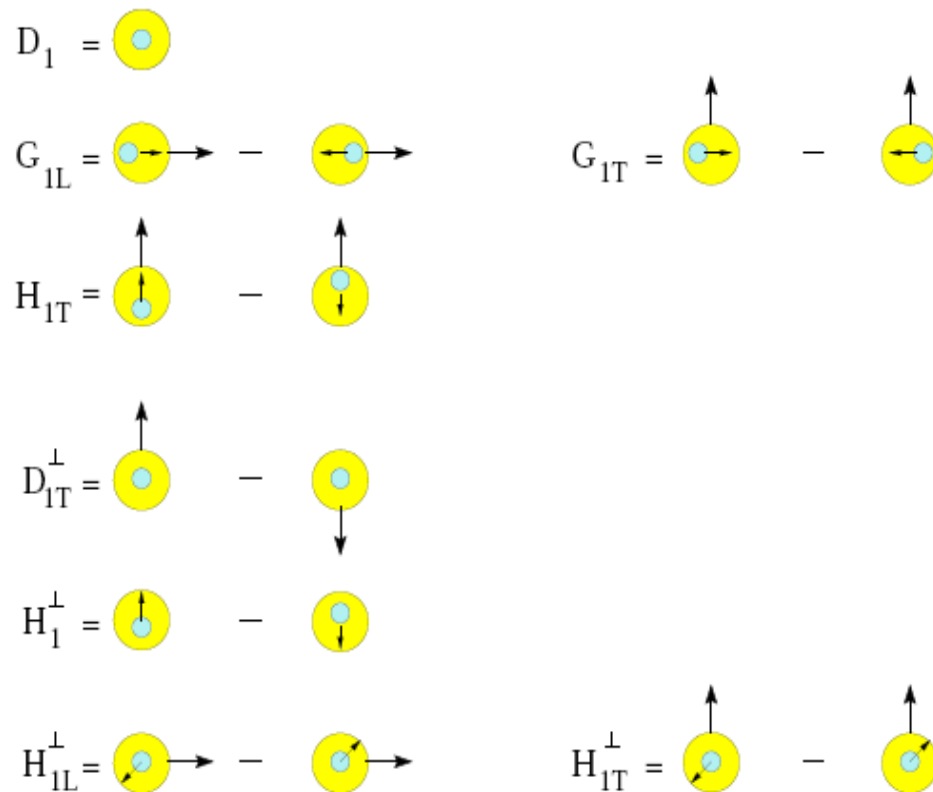
Transverse-momentum-dependent distributions

Distribution functions



leading twist

Fragmentation functions



Transverse-momentum-dependent distributions

Distribution functions

$$f_1 = \text{yellow circle with blue center}$$

$$g_{1L} = \text{yellow circle with blue center and right arrow} - \text{yellow circle with blue center and left arrow}$$

$$h_{1T} = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$f_{1T}^\perp = \text{yellow circle with blue center and up arrow} - \text{yellow circle with blue center and down arrow}$$

$$h_1^\perp = \text{yellow circle with blue center and right arrow} - \text{yellow circle with blue center and left arrow}$$

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leading twist

Fragmentation functions

$$D_1 = \text{yellow circle with blue center}$$

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chiral odd: involve helicity flip of quark → appear in pairs in cross section

Transverse-momentum-dependent distributions

Distribution functions

$$f_1 = \text{yellow circle with blue dot}$$

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$$h_1^\perp = \text{yellow circle with blue dot and down arrow} - \text{yellow circle with blue dot and up arrow}$$

$$h_{1L}^\perp = \text{yellow circle with blue dot, right arrow, and up arrow} - \text{yellow circle with blue dot, right arrow, and down arrow}$$

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chiral odd: involve helicity flip of quark → appear in pairs in cross section

T-odd: appear in pairs in spin-independent x-section & double-spin asymmetries
single in single-spin asymmetries

Transverse-momentum-dependent distributions

Distribution functions

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leading twist

Fragmentation functions

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Sivers

Boer-Mulders

chiral odd: involve helicity flip of quark → appear in pairs in cross section

T-odd: appear in pairs in spin-independent x-section & double-spin asymmetries
single in single-spin asymmetries

Sivers distribution function

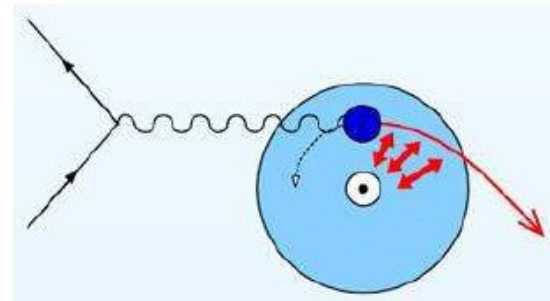
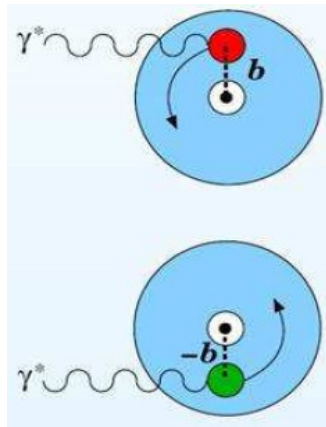
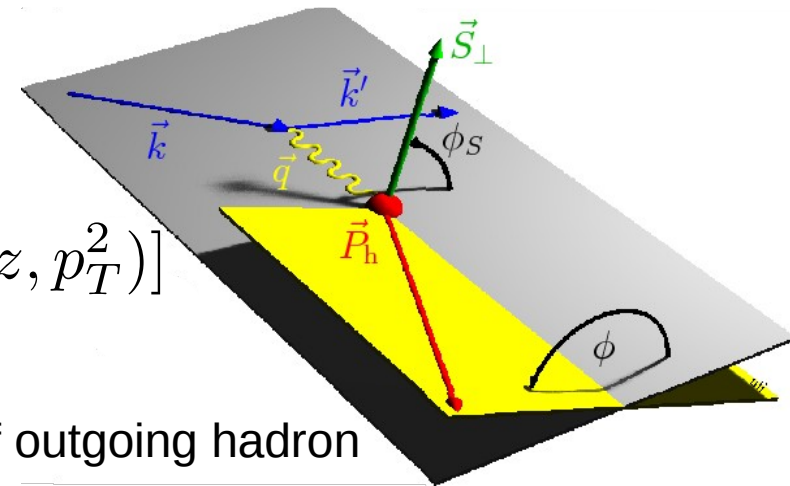
$$f_{1T}^{\perp q}(x, p_T^2) = \text{Diagram 1} - \text{Diagram 2}$$

access through single-spin asymmetry on transversely polarized target

$$A_{UT} = \frac{1}{\langle |S_T| \rangle} \frac{N^\uparrow(\phi, \phi_S) - N^\downarrow(\phi, \phi_S)}{N^\uparrow(\phi, \phi_S) + N^\downarrow(\phi, \phi_S)}$$

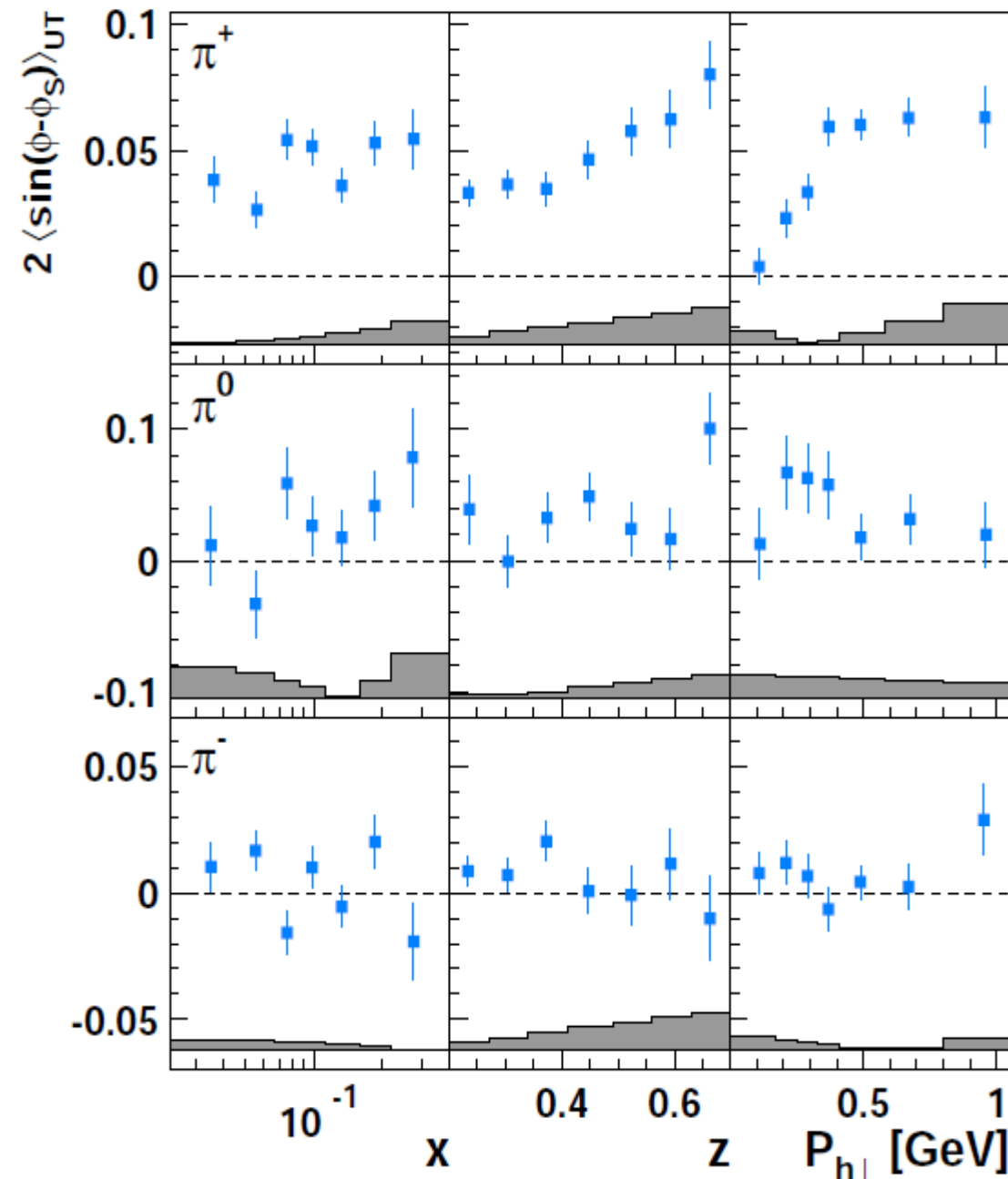
$$\sim \sin(\phi - \phi_S) \sum_q e_q \mathcal{I} \left[\frac{\vec{k}_T \cdot \hat{P}_h \perp}{M_h} f_{1T}^{\perp, q}(x, k_T^2) D_1^q(z, p_T^2) \right]$$

- requires non-zero quark orbital angular momentum
- FSI \longrightarrow left-right (azimuthal) asymmetry in direction of outgoing hadron



Sivers amplitude for pions

A. Airapetian et al., Phys. Rev. Lett. **103** (2009) 152002

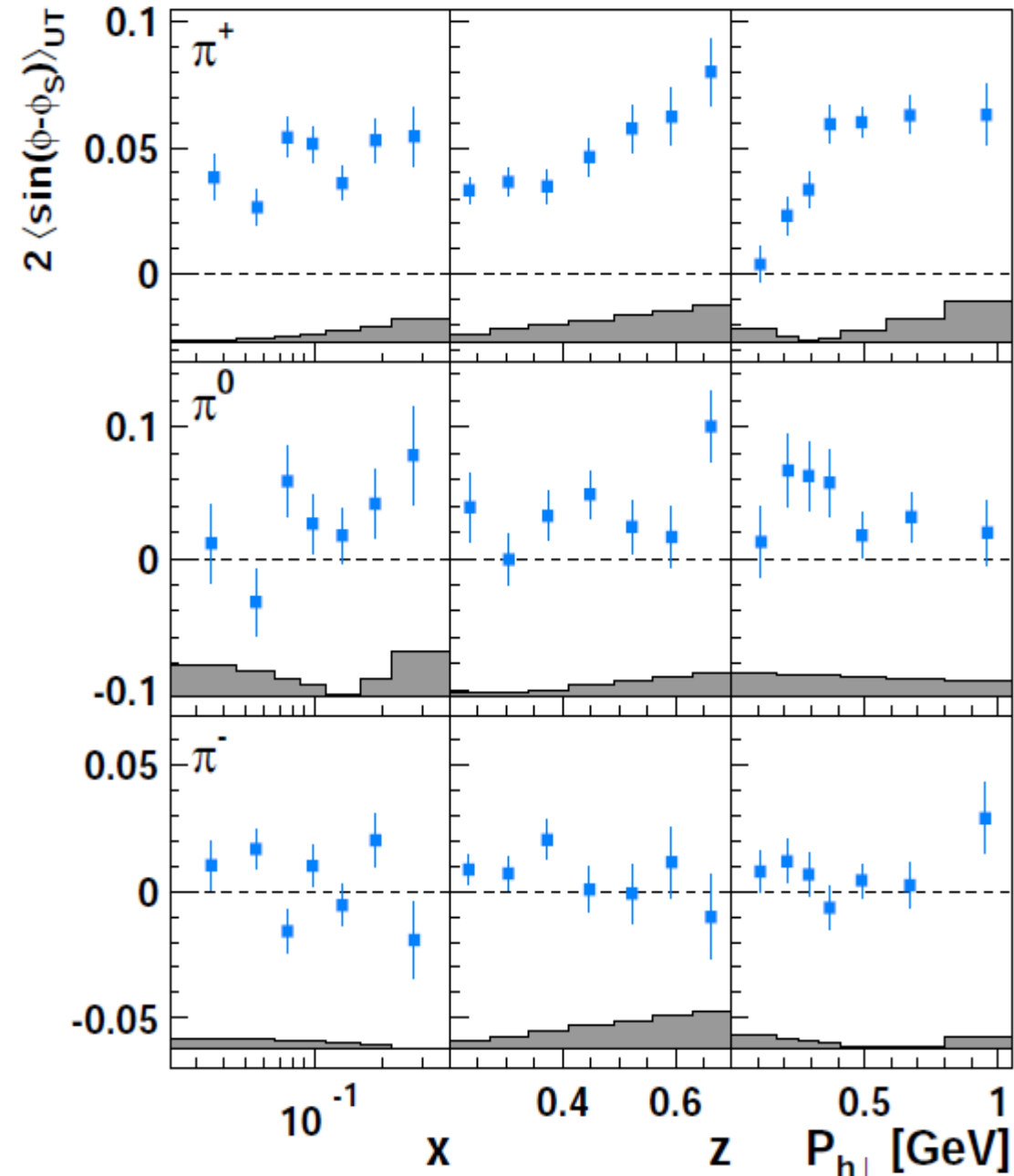


• π^+

- significantly positive
non-zero orbital
angular momentum!

Sivers amplitude for pions

A. Airapetian et al., Phys. Rev. Lett. **103** (2009) 152002



- π^+

- significantly positive non-zero orbital angular momentum!
- clear rise with z
- rise at low $P_{h\perp}$, plateau at high $P_{h\perp}$
- amplitude dominated by u-quark scattering:

$$\approx \frac{f_{1T}^{\perp,u}(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u \rightarrow \pi^+}(z, k_T^2)}$$

➡ $f_{1T}^{\perp,u}(x, p_T^2) < 0$

- π^-

- consistent with zero
- u- and d- quark cancellation
- $f_{1T}^{\perp,d}(x, p_T^2) > 0$

- π^0

- slightly positive
- isospin symmetry fulfilled

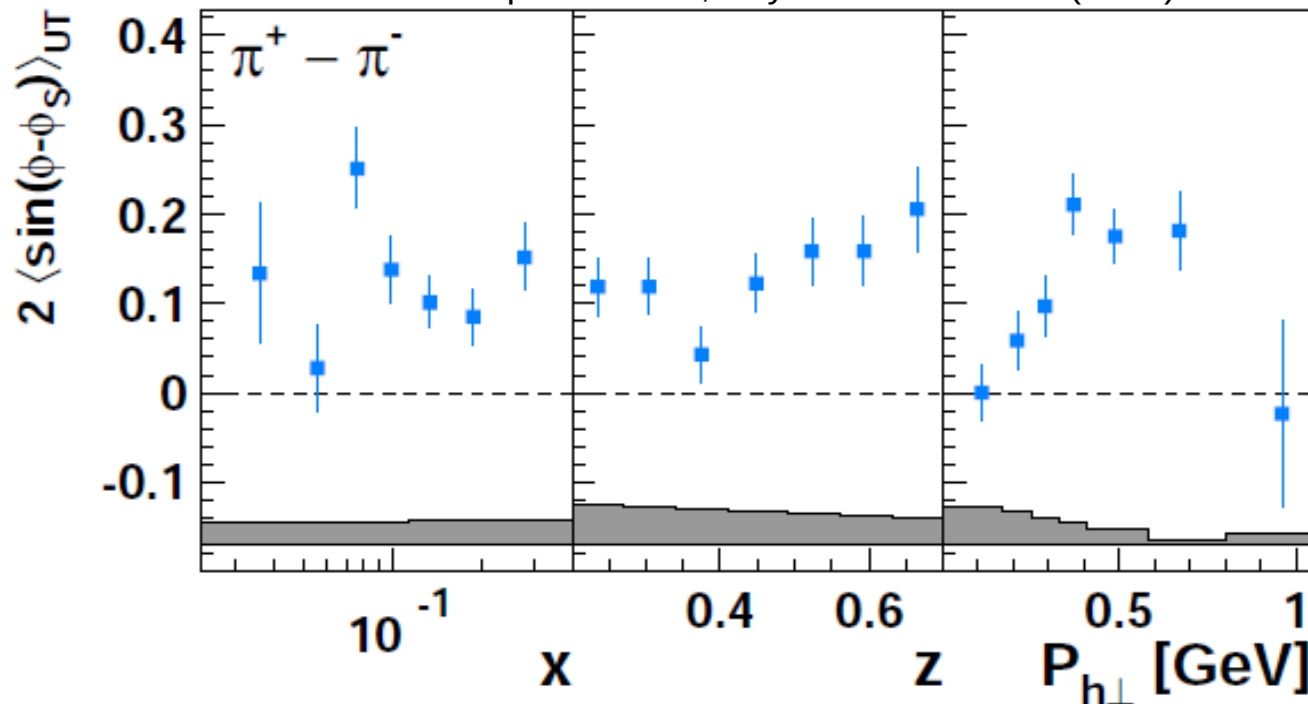
Sivers distribution for valence quarks

$$A_{UT}^{\pi^+ - \pi^-} = \frac{1}{\langle |S_T| \rangle} \frac{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-}) - (\sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-})}{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-}) + (\sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-})}$$

→ suppressed exclusive VM (ρ^0) contribution

$$\langle \sin(\phi - \phi_S) \rangle_{UT}^{\pi^+ - \pi^-} \approx - \frac{4f_{1T}^{\perp, u_v} - f_{1T}^{\perp, d_v}}{4f_1^{u_v} - f_1^{d_v}}$$

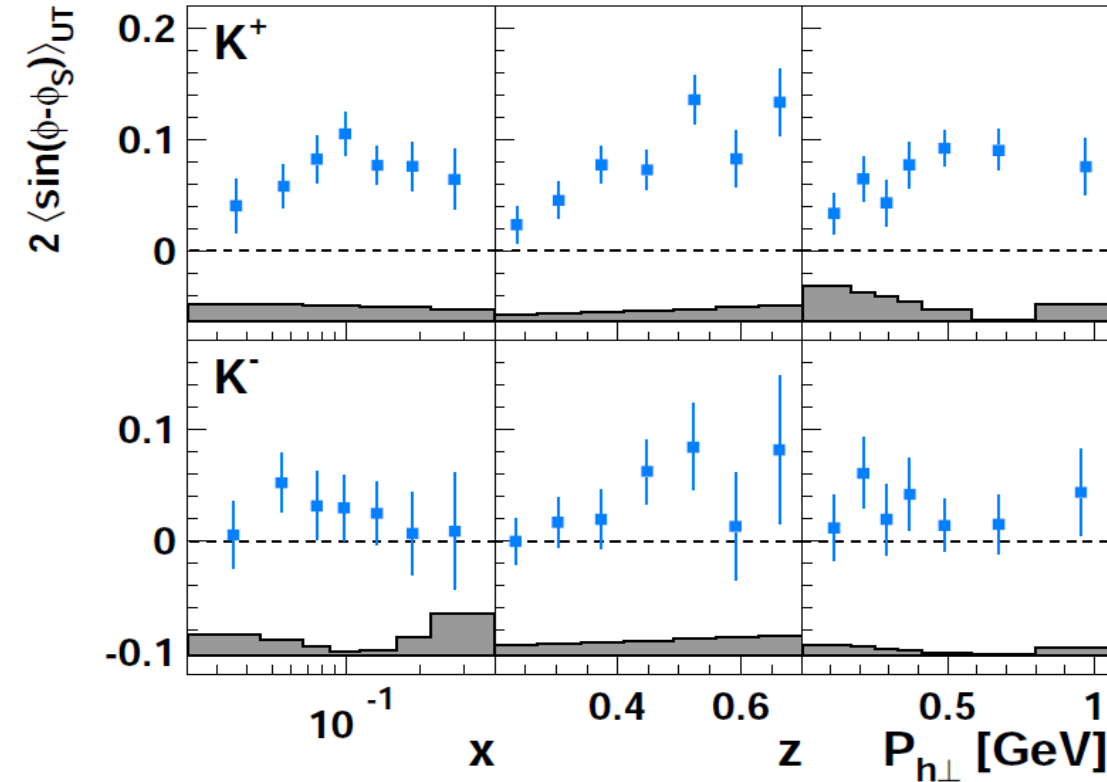
A. Airapetian et al., Phys. Rev. Lett. **103** (2009) 152002



- Sivers distribution for d-valence \gg u-valence or
- Sivers distribution for u-valence is large & < 0 (more likely)

Sivers amplitude for kaons

A. Airapetian et al., Phys. Rev. Lett. **103** (2009) 152002



- K^+

- significantly positive
- clear rise with z
- rise at low $P_{h\perp}$, plateau at high $P_{h\perp}$
- larger than π^+



non-trivial role of sea quarks?

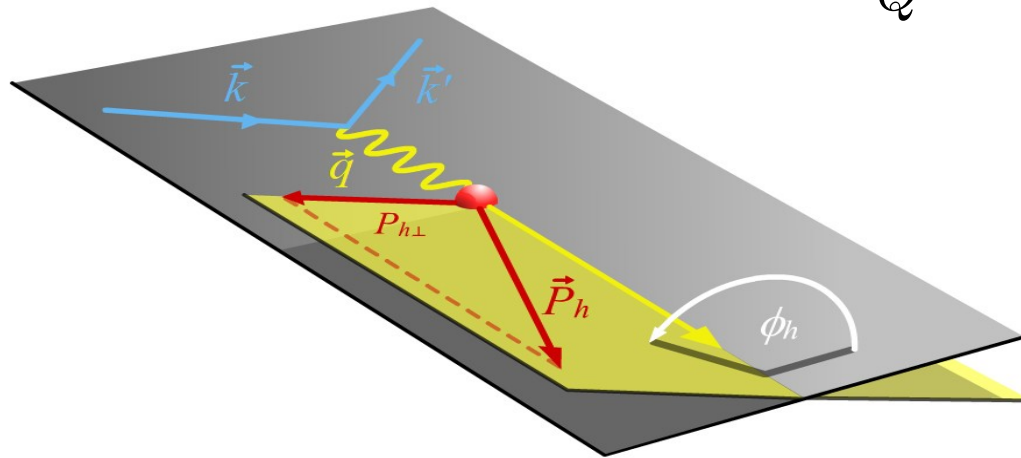
- K^-

- slightly positive

Spin-independent semi-inclusive non-collinear DIS cross section

$$\frac{d\sigma}{dx dy dz dP_{h\perp}^2 d\phi_h} = \frac{\alpha^2}{xyQ^2} \left(1 + \frac{\gamma^2}{2x}\right) \left\{ A(y) F_{UU,T} + B(y) F_{UU,L} \right. \\ \left. + C(y) \cos \phi_h F_{UU}^{\cos \phi_h} + B(y) \cos 2\phi_h F_{UU}^{\cos 2\phi_h} \right\}$$

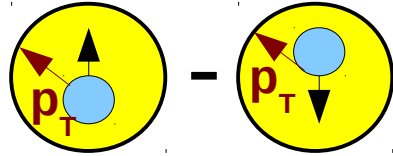
$$\gamma = \frac{2Mx}{Q}, \quad F = F(x, Q, z, \vec{P}_{h\perp})$$



Spin-independent semi-inclusive non-collinear DIS cross section

leading twist

$$F_{UU}^{\cos 2\phi_h} = \mathcal{I} \left[- \frac{2(\hat{P}_{h\perp} \cdot \vec{p}_T)(\hat{P}_{h\perp} \cdot \vec{k}_T) - \vec{p}_T \cdot \vec{k}_T}{M_h M} h_1^\perp H_1^\perp \right]$$



Boer-Mulders DF

- chiral odd
- naïve-T-odd

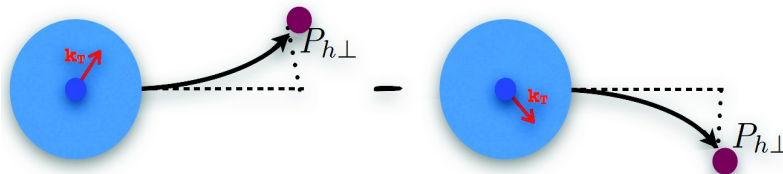
Collins FF

- chiral odd
- naïve-T-odd

sub-leading twist

$$F_{UU}^{\cos \phi_h} = \frac{2M}{Q} \mathcal{I} \left[- \frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M} f_1 D_1 - \frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M_h} \frac{p_T^2}{M^2} h_1^\perp H_1^\perp + \dots \right]$$

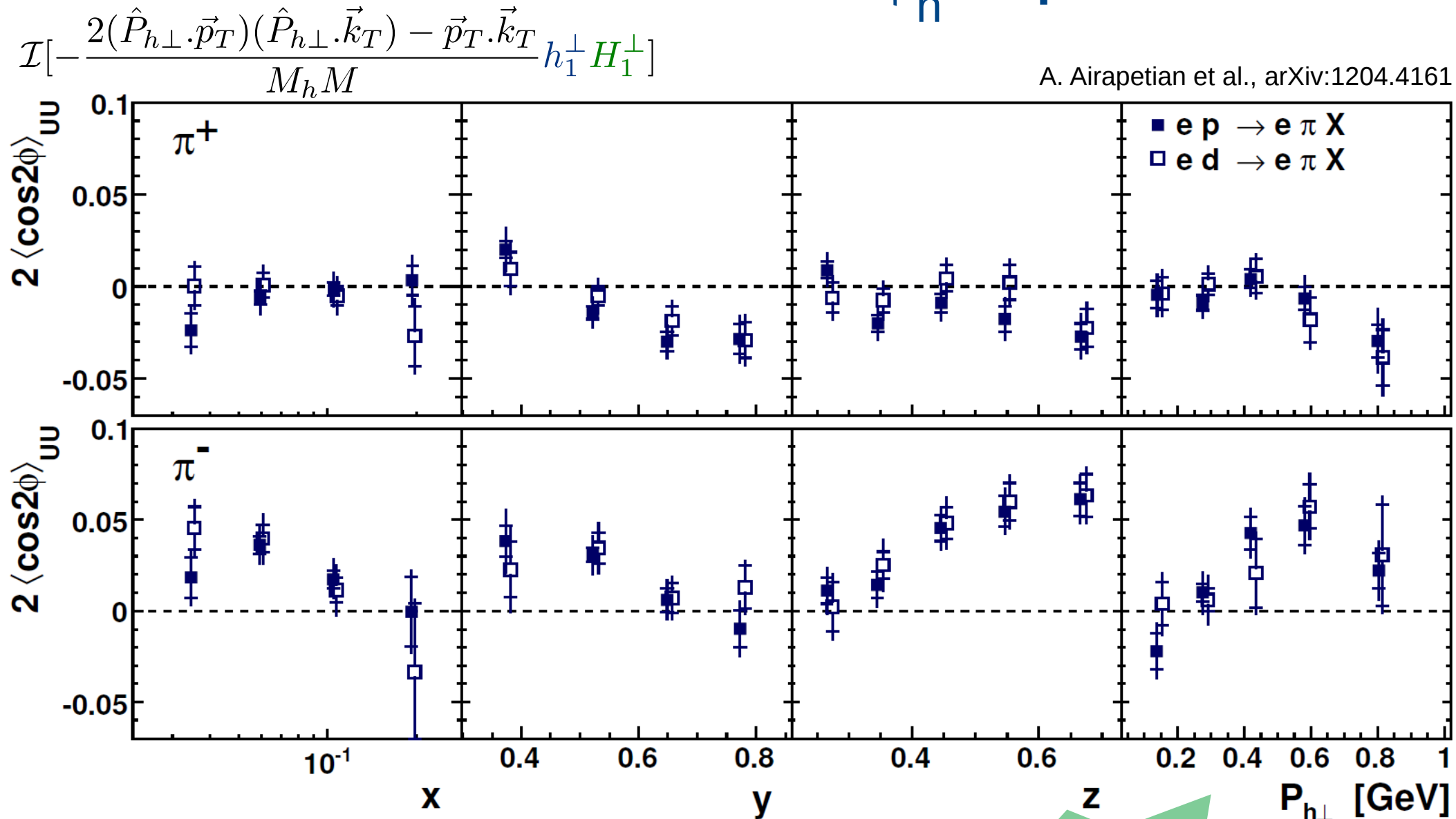
Cahn effect



quark-gluon-quark correlations

Results for $\langle \cos 2\phi_h \rangle$: pions

A. Airapetian et al., arXiv:1204.4161



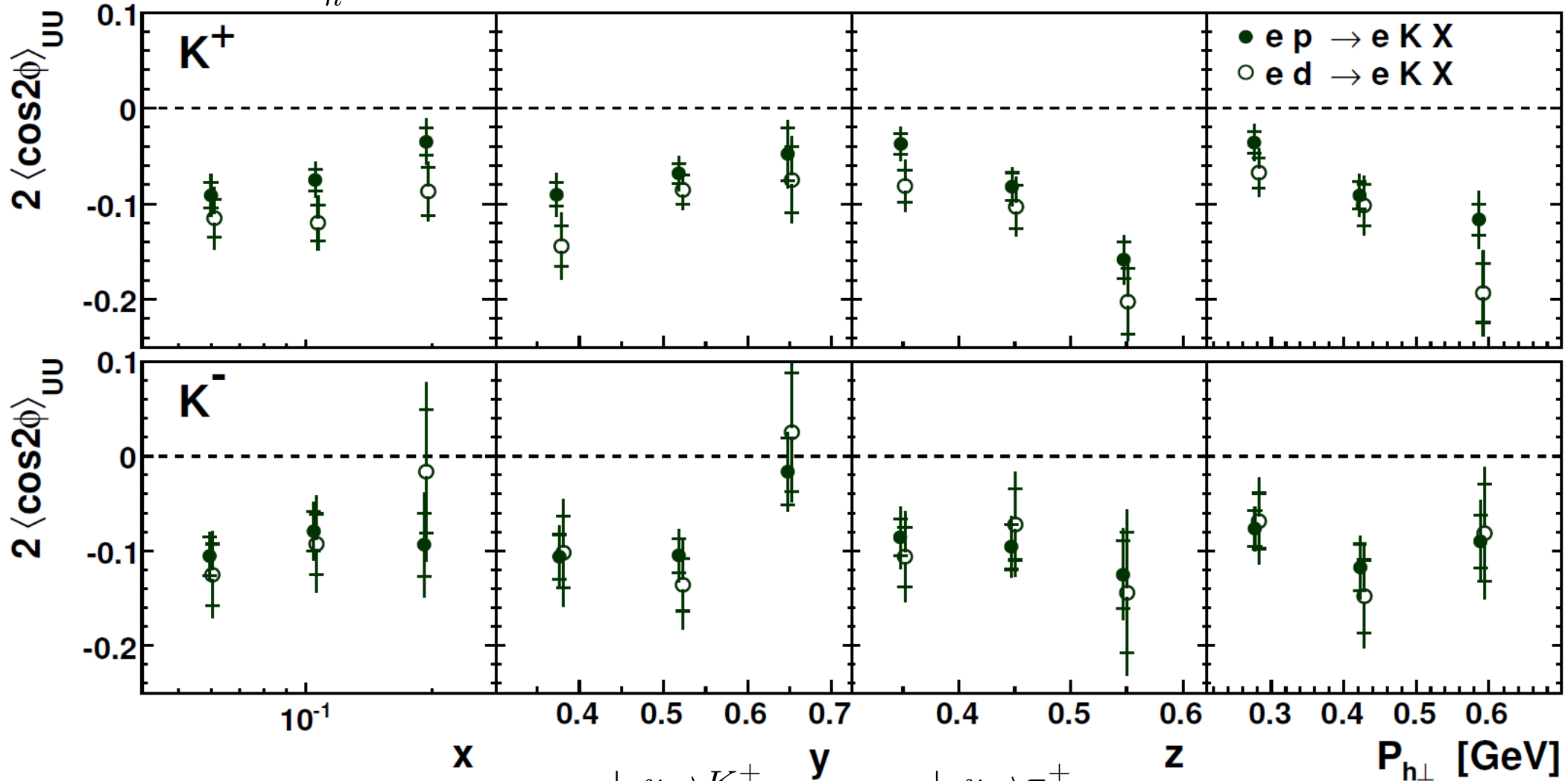
- H-D comparison: $h_1^{\perp,u} \approx h_1^{\perp,d}$
- $\pi^- > 0 \longleftrightarrow \pi^+ \leq 0$: $H_1^{\perp, fav} \approx -H_1^{\perp, unfav}$

non-zero orbital
angular momentum!

Results for $\langle \cos 2\phi_h \rangle$: kaons

$$\mathcal{I}\left[-\frac{2(\hat{P}_{h\perp}\cdot\vec{p}_T)(\hat{P}_{h\perp}\cdot\vec{k}_T) - \vec{p}_T\cdot\vec{k}_T}{M_h M} h_1^\perp H_1^\perp\right]$$

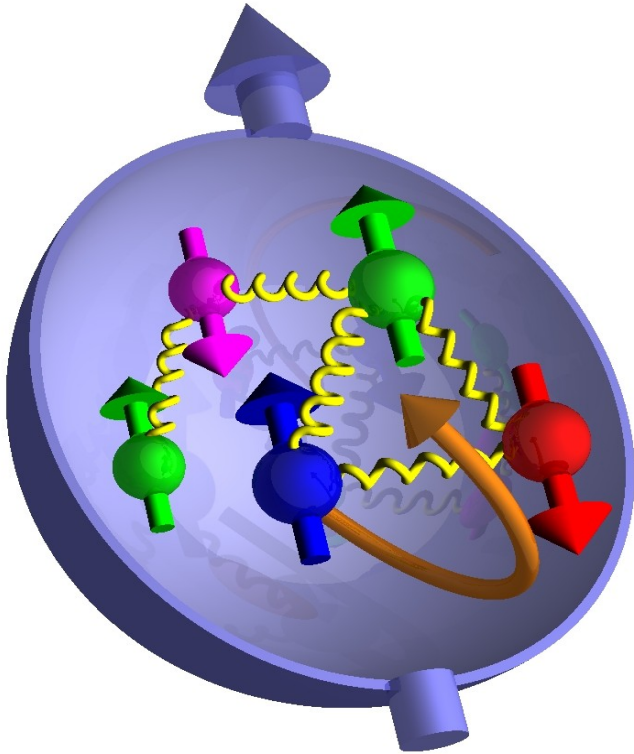
A. Airapetian et al., arXiv:1204.4161



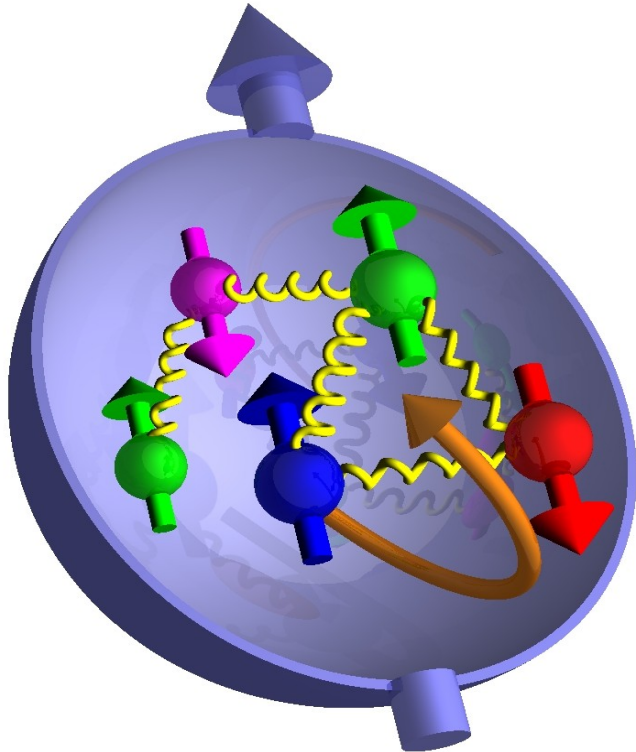
- $K^+ < 0$: - Artru model: $\text{sign } H_1^\perp, u \rightarrow K^+ = \text{sign } H_1^\perp, u \rightarrow \pi^+$
- $K^- \approx K^+$: - u-dominance $\xrightarrow{?} H_1^\perp, u \rightarrow K^+ \approx H_1^\perp, u \rightarrow K^-$
- role of sea-quarks

Spin budget of proton

$$\frac{1}{2} = \underbrace{\frac{1}{2} \Delta \Sigma + L^Q}_{J^Q} + J^g$$



Spin budget of proton



$$\frac{1}{2} = \underbrace{\frac{1}{2} \Delta \Sigma + L^Q}_{J^Q} + J^g$$

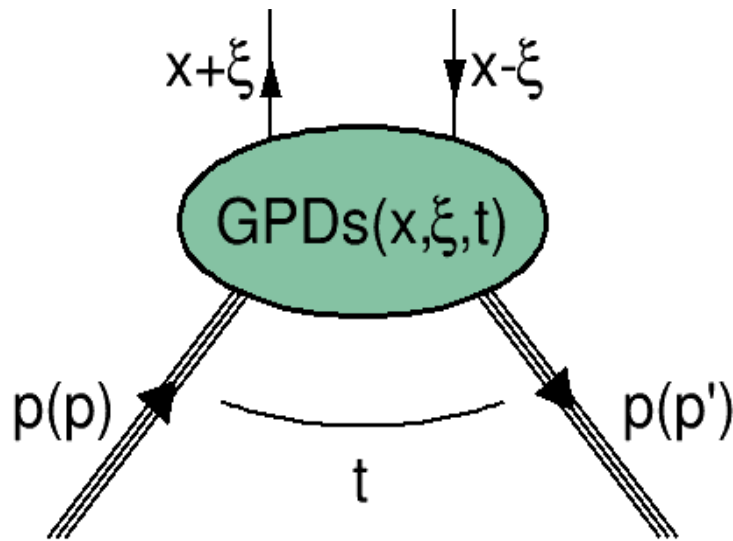
$$J^q = \lim_{t \rightarrow 0} \frac{1}{2} \int_{-1}^1 dx x [H^q + E^q]$$

GPDs
 X. Ji, Phys. Rev. Lett. **78** (1997) 610



$$L^Q = \sum_q L^q = \sum_q (J^q - \Delta q)$$

Generalized Parton Distributions (GPDs)



- x =average longitudinal momentum fraction
- 2ξ =average longitudinal momentum transfer
- t = squared momentum transfer to nucleon

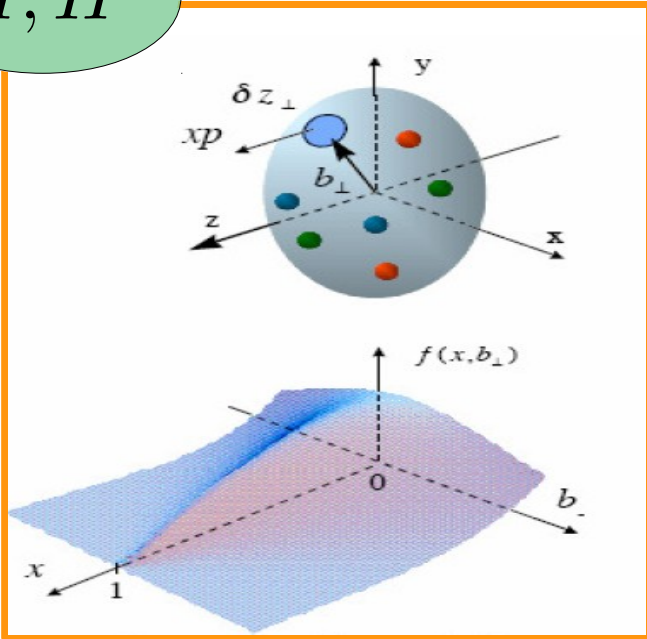
Four quark helicity-conserving GPDs at twist-2

$H^q(x, \xi, t)$	$E^q(x, \xi, t)$	spin independent
$\tilde{H}^q(x, \xi, t)$	$\tilde{E}^q(x, \xi, t)$	spin dependent
proton helicity non-flip	proton helicity flip	

Generalized Parton Distributions (GPDs)

H, \tilde{H}

unpolarized nucleon



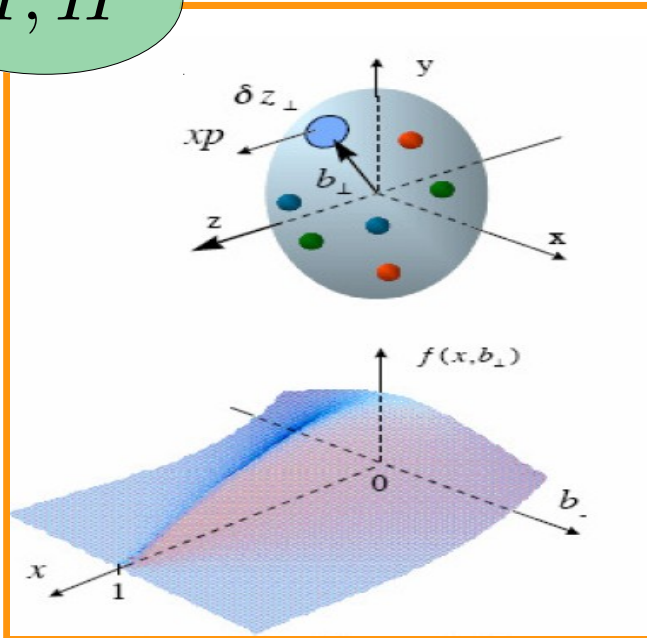
helicity-(in)dependent probability distribution of quarks as a function of their longitudinal fractional momentum and transverse position

M. Burkardt, Phys. Rev. D **62** (2000) 071503

Generalized Parton Distributions (GPDs)

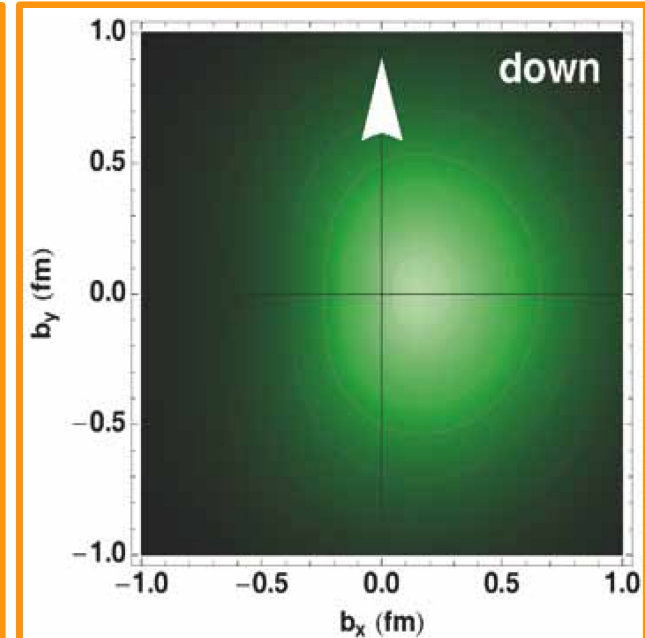
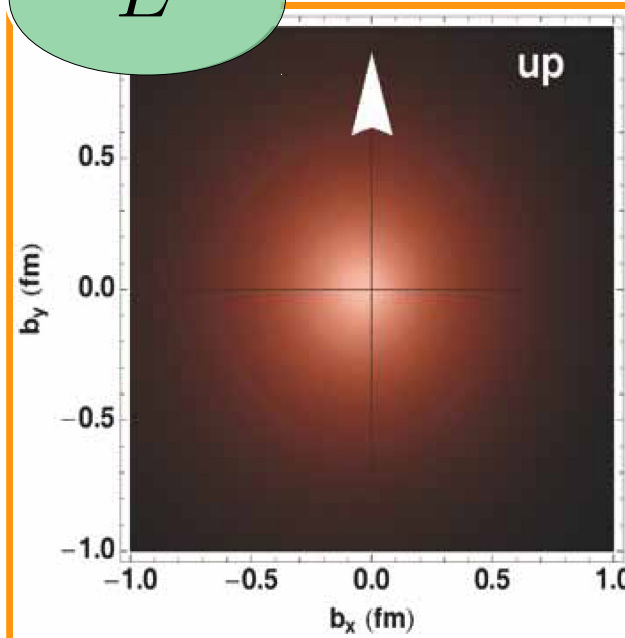
H, \tilde{H}

unpolarized nucleon



E

transversely polarized nucleon



pictures taken from A. Bacchetta and M. Contalbrigo, *Il Nuovo Saggiatore* **28** (2012) 1-2

helicity-(in)dependent probability distribution of quarks as a function of their longitudinal fractional momentum and transverse position

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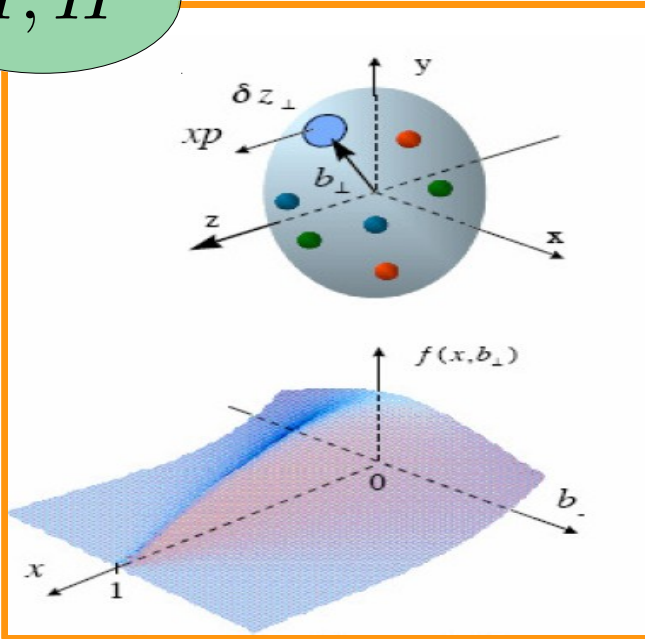
distortion of quark probability distribution compared to unpolarized nucleon

M. Burkardt, *Phys. Rev. D* **66** (2002) 114005

Generalized Parton Distributions (GPDs)

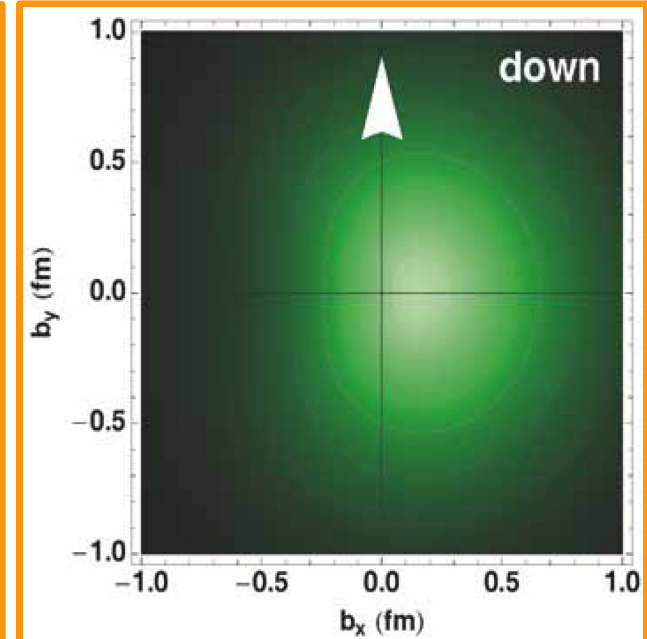
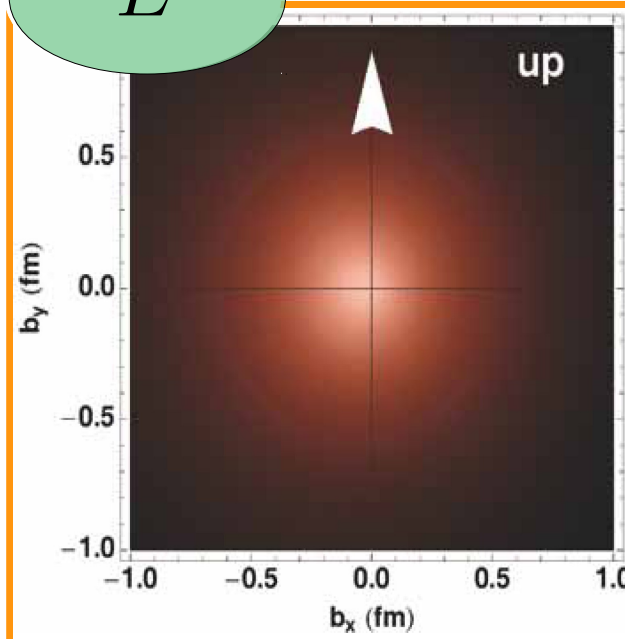
H, \tilde{H}

unpolarized nucleon



E

transversely polarized nucleon



pictures taken from A. Bacchetta and M. Contalbrigo, *Il Nuovo Saggiatore* **28** (2012) 1-2

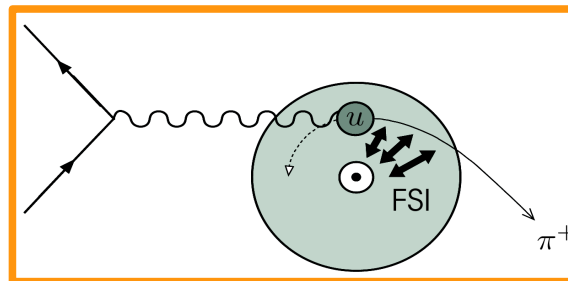
helicity-(in)dependent probability distribution of quarks as a function of their longitudinal fractional momentum and transverse position

M. Burkardt, *Phys. Rev. D* **62** (2000) 071503

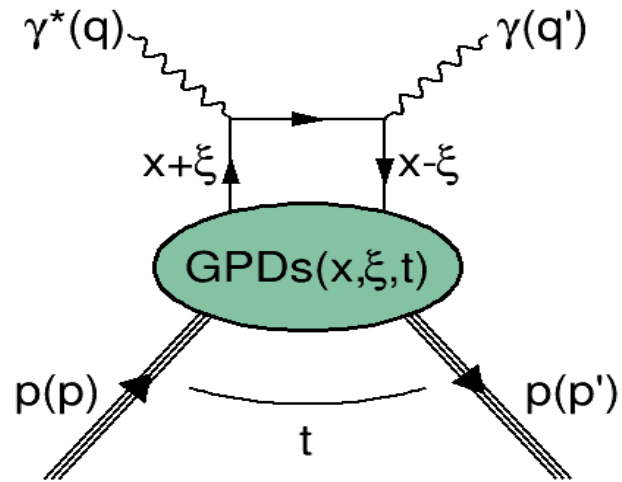
distortion of quark probability distribution compared to unpolarized nucleon

M. Burkardt, *Phys. Rev. D* **66** (2002) 114005

qualitative link to Sivers distribution



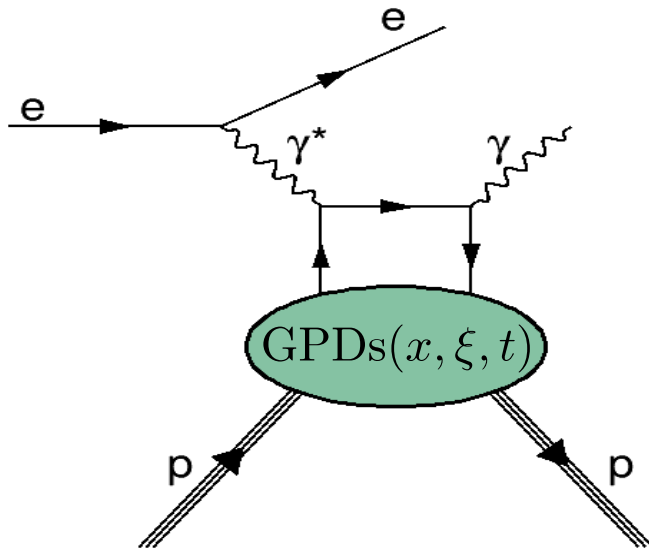
Deeply virtual Compton scattering



$$\xi \approx \frac{x_B}{2 - x_B}$$

Exclusive lepto-production of real photons

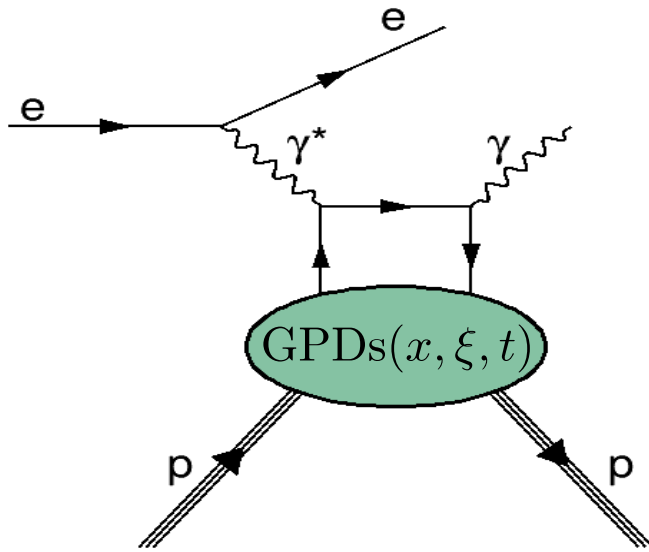
exclusive deep-inelastic scattering



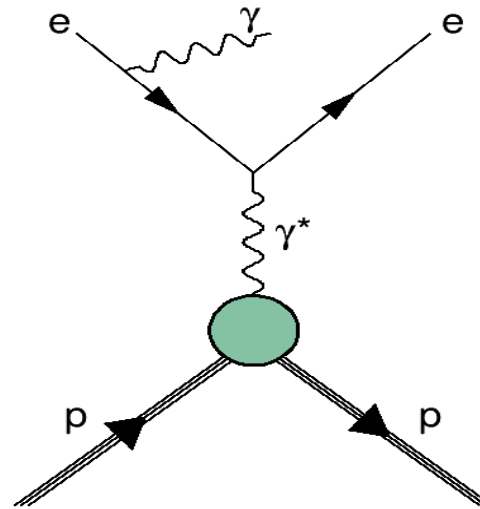
DVCS

Exclusive lepto-production of real photons

exclusive deep-inelastic scattering



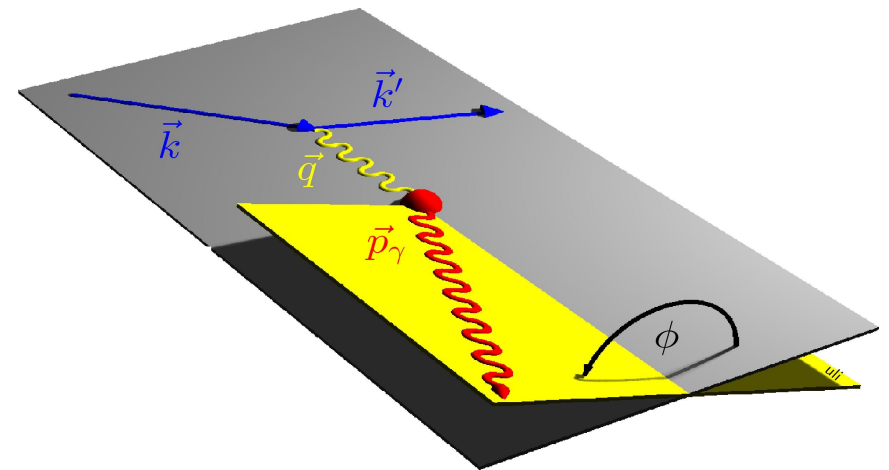
DVCS



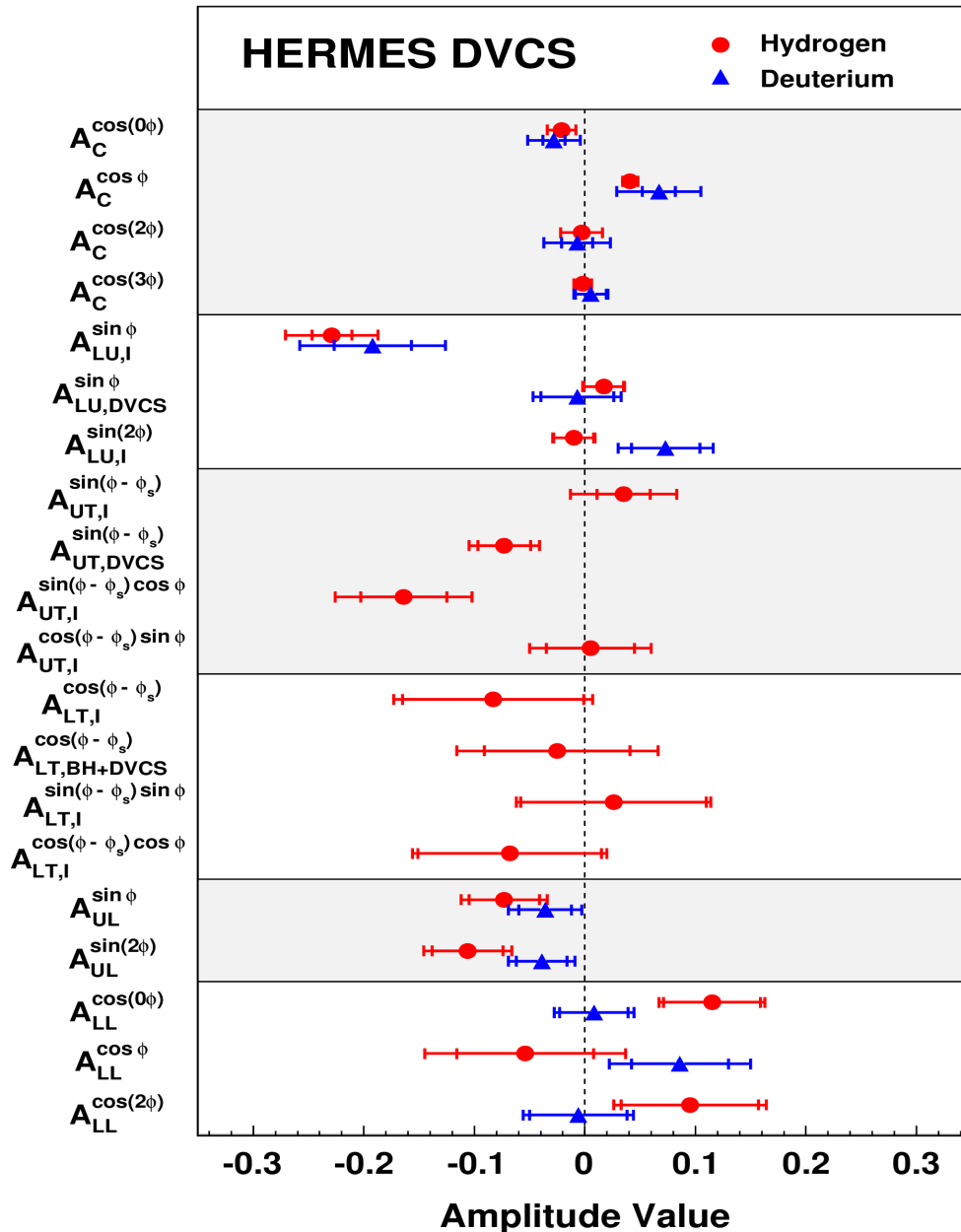
Bethe-Heitler

$$d\sigma \propto |\tau_{BH}|^2 + |\tau_{DVCS}|^2 + \tau_{BH} \tau_{DVCS}^* + \tau_{DVCS} \tau_{BH}^*$$

- $|\tau_{BH}|$: calculable (form factors)
- $|\tau_{BH}| \gg |\tau_{DVCS}|$ at HERMES
- interference term:
through azimuthal asymmetries



DVCS at HERMES



beam-charge asymmetry
[JHEP 07 \(2012\) 32](#)
[Nucl. Phys. B 829 \(2010\) 1](#)

beam-helicity asymmetry
[JHEP 07 \(2012\) 32](#)
[Nucl. Phys. B 829 \(2010\) 1](#)

transverse target-spin asymmetry
[JHEP 06 \(2008\) 066](#)

double spin (LT) asymmetry
[Phys. Lett. B 704 \(2011\) 15](#)

longitudinal target-spin asymmetry
[JHEP 06 \(2010\) 019](#)
[Nucl. Phys. B 842 \(2011\) 265](#)

double spin (LL) asymmetry
[JHEP 06 \(2010\) 019](#)
[Nucl. Phys. B 842 \(2011\) 265](#)

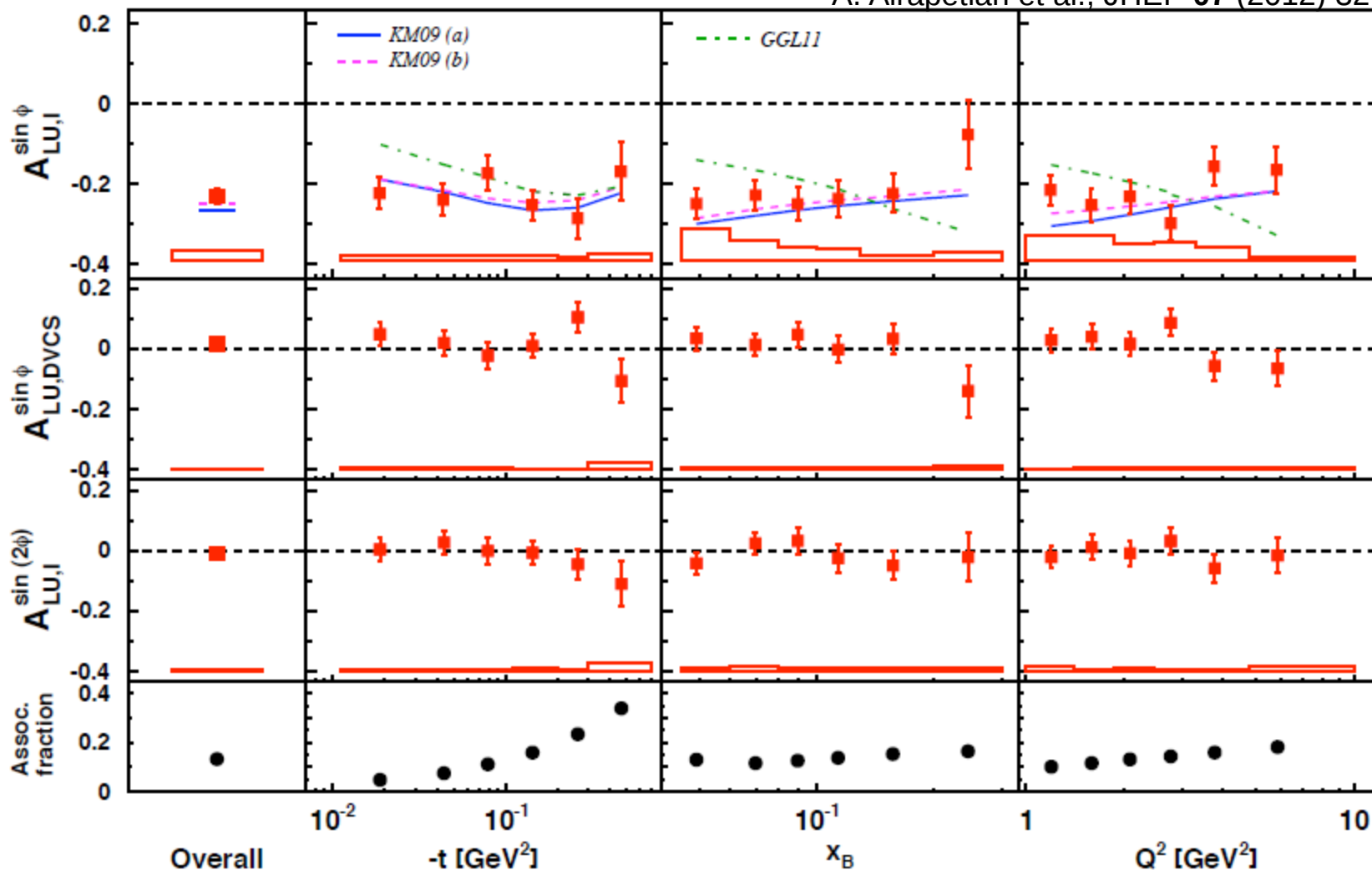
GPD H

GPD E

GPD \tilde{H}

Charged-separated beam-helicity asymmetry

A. Airapetian et al., JHEP 07 (2012) 32

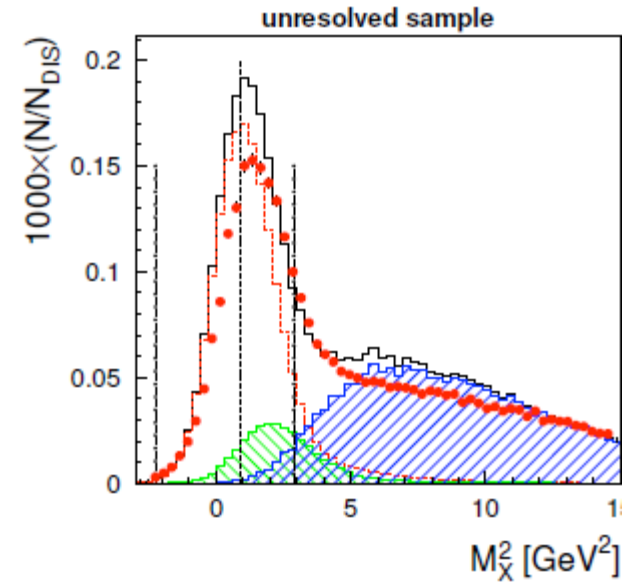
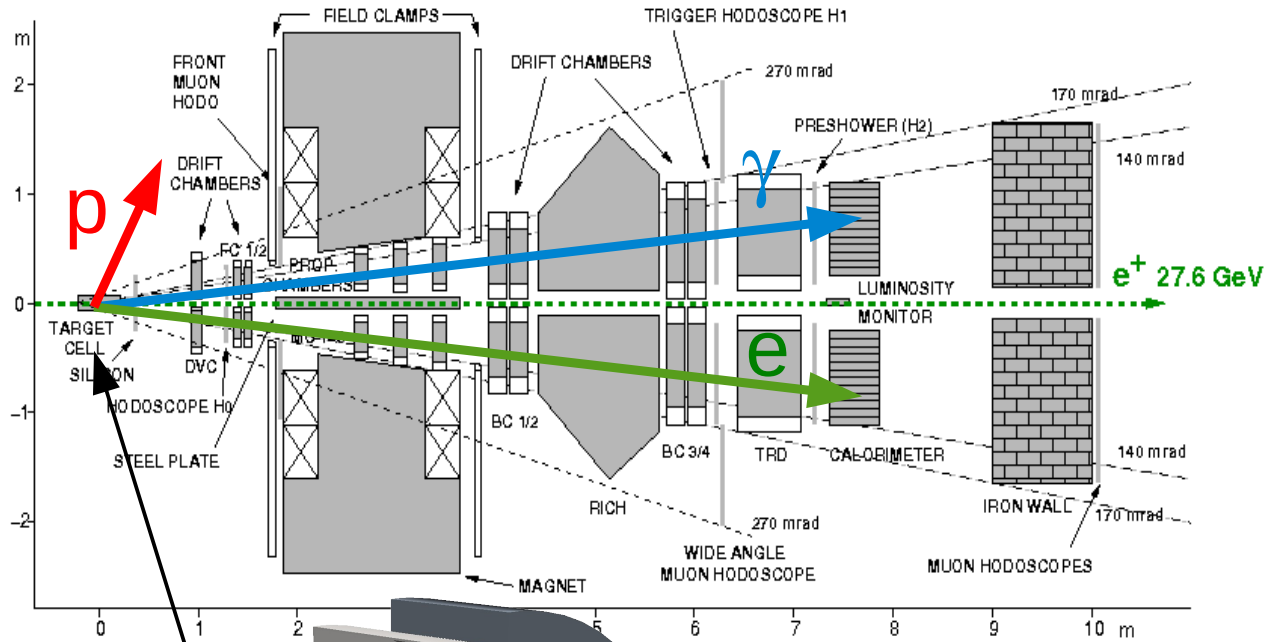


KM09: Nucl. Phys. B
841 (2010) 1:
 fit to HERMES, ZEUS,
 H1 data
 Fit to HERMES, ZEUS,
 H1, Jefferson Lab
 data

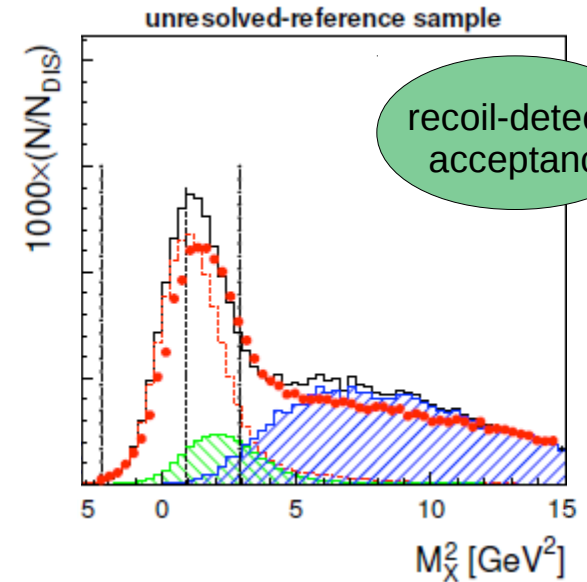
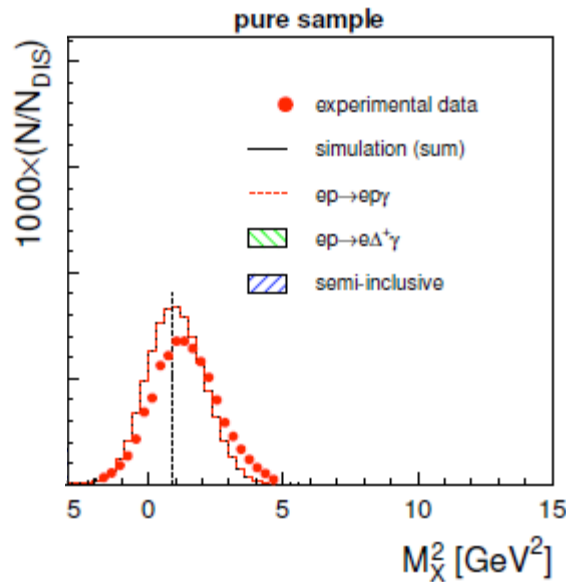
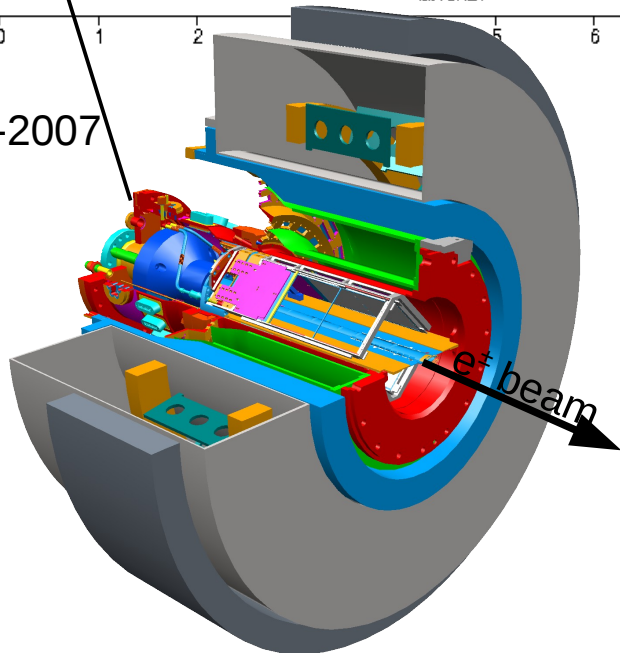
GGL11: Phys. Rev. D
84 (2011) 034007

- data collected from 1996-2007 (74% of data from 2006-2007)
- additional 3.2% scale uncertainty from beam polarization

DVCS event selection

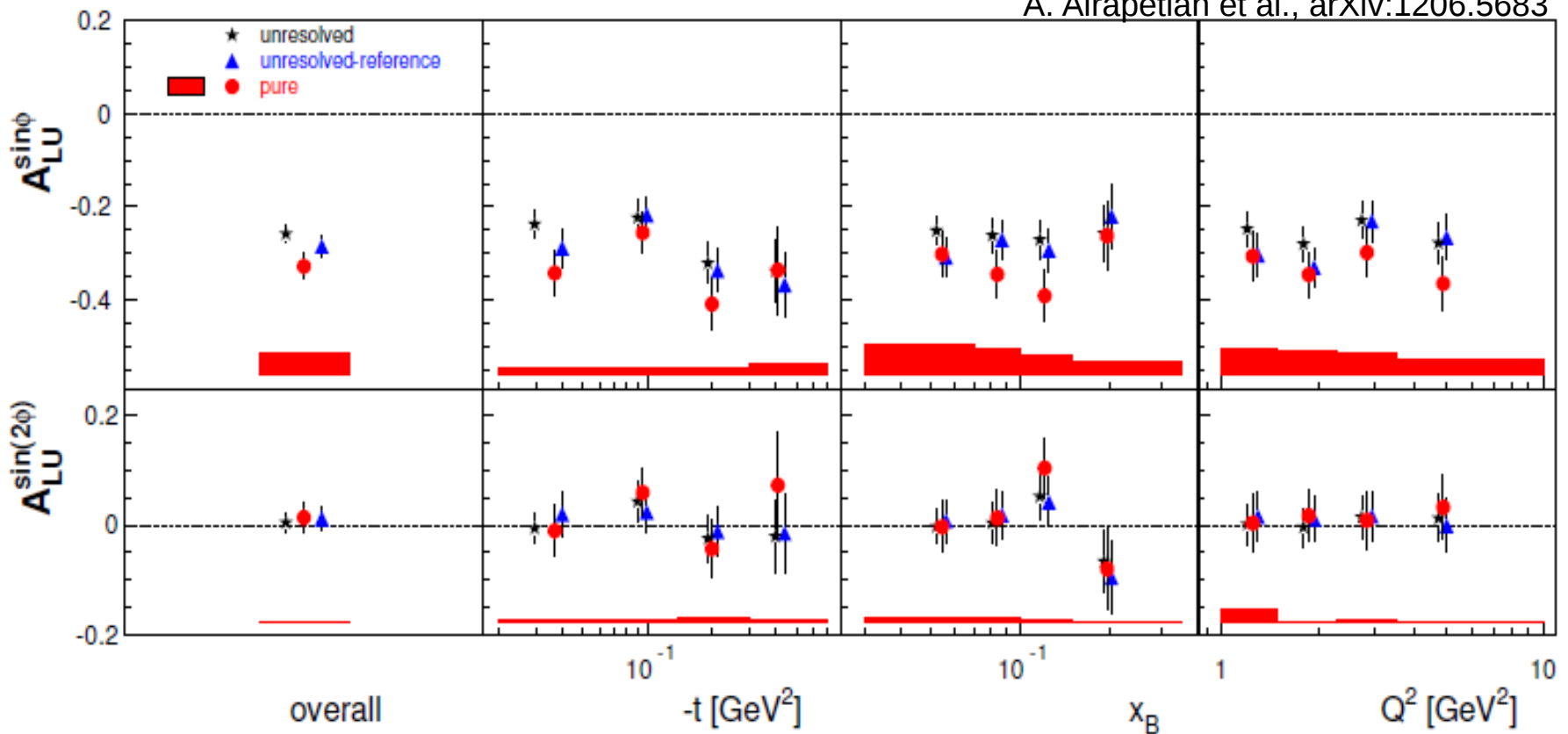


2006-2007

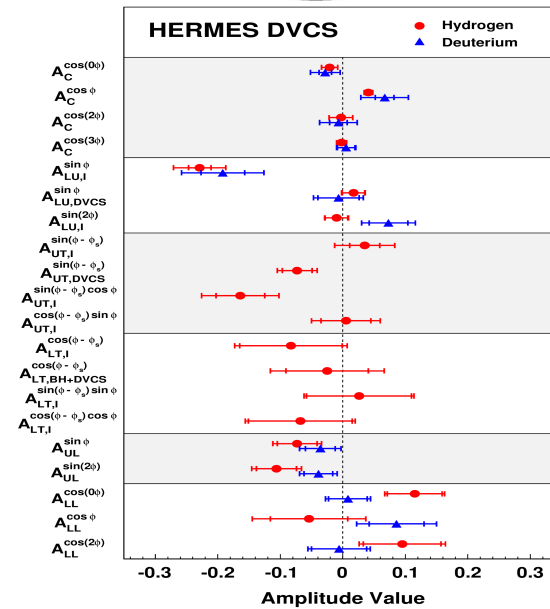
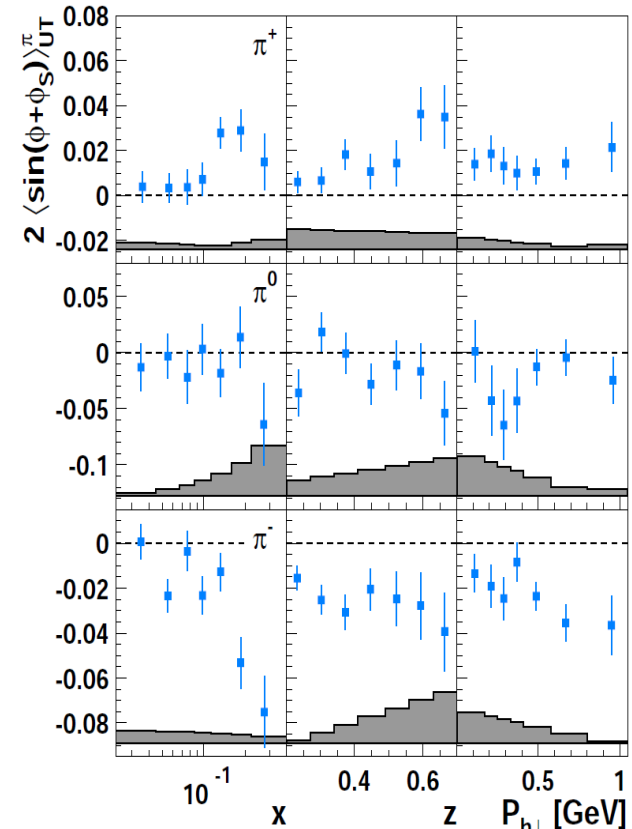
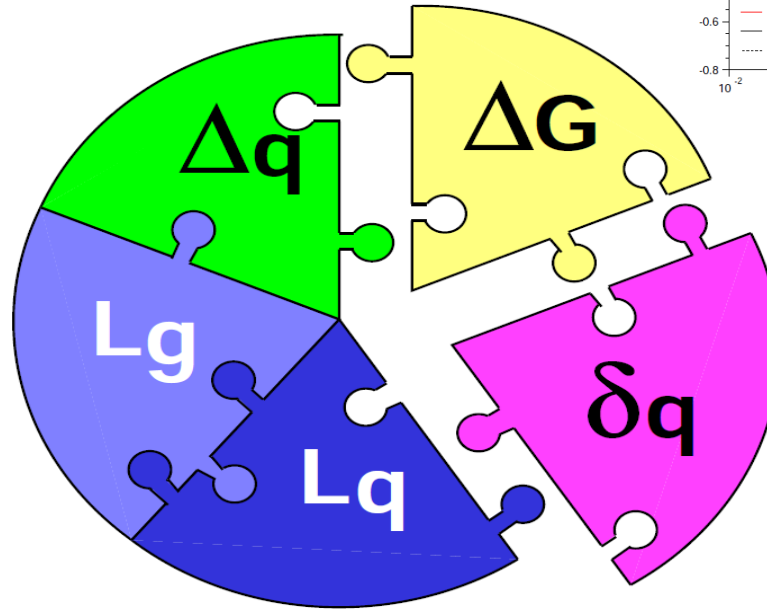
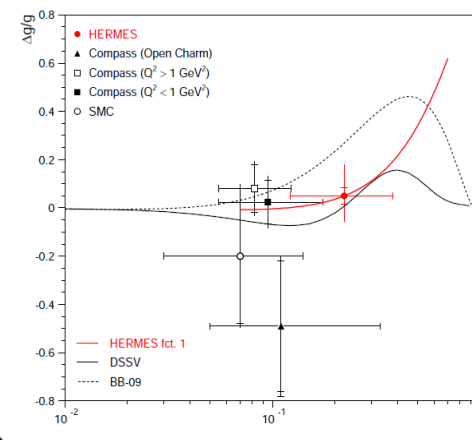
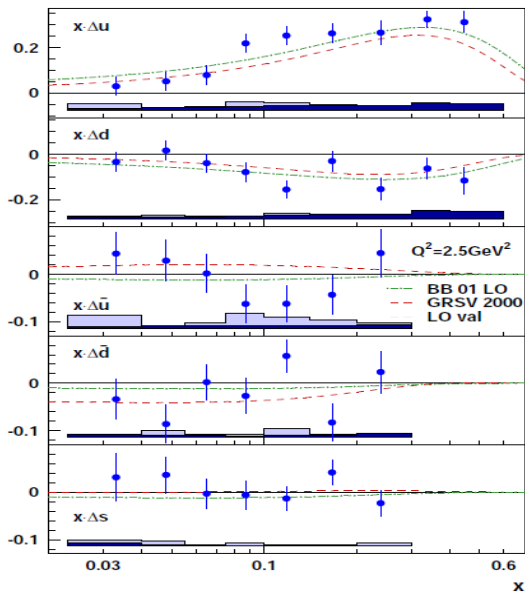


Beam-helicity asymmetry

A. Airapetian et al., arXiv:1206.5683



- additional 1.96 % scale uncertainty from beam polarization



Thank you for your attention!



Back up

Extraction of the cosine moments

$$\langle \cos(n\phi_h) \rangle \stackrel{th.}{=} \frac{\int d\phi_h \cos(n\phi_h) \sigma_{UU}(\omega, \phi_h)}{\int d\phi_h \sigma_{UU}(\omega, \phi_h)} \quad \omega = (x, y, z, P_{h\perp}^2)$$

$$\langle \cos(n\phi_h) \rangle \stackrel{exp.}{=} \frac{\int d\phi_h \cos(n\phi_h) \epsilon_{acc}(\omega, \phi_h) \epsilon_{rad}(\omega, \phi_h) \sigma_{UU}(\omega, \phi_h)}{\int d\phi_h \epsilon_{acc}(\omega, \phi_h) \epsilon_{rad}(\omega, \phi_h) \sigma_{UU}(\omega, \phi_h)}$$

extraction is challenging!

azimuthal modulations also possible due to

- detector geometrical acceptance
- higher-order QED effects

Extraction of the cosine moments

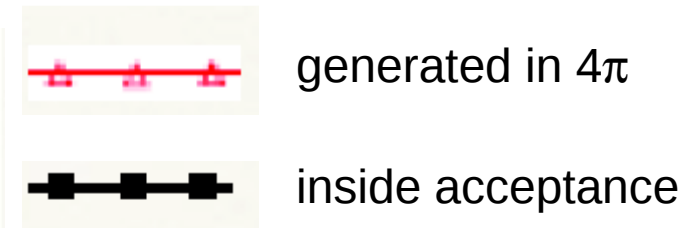
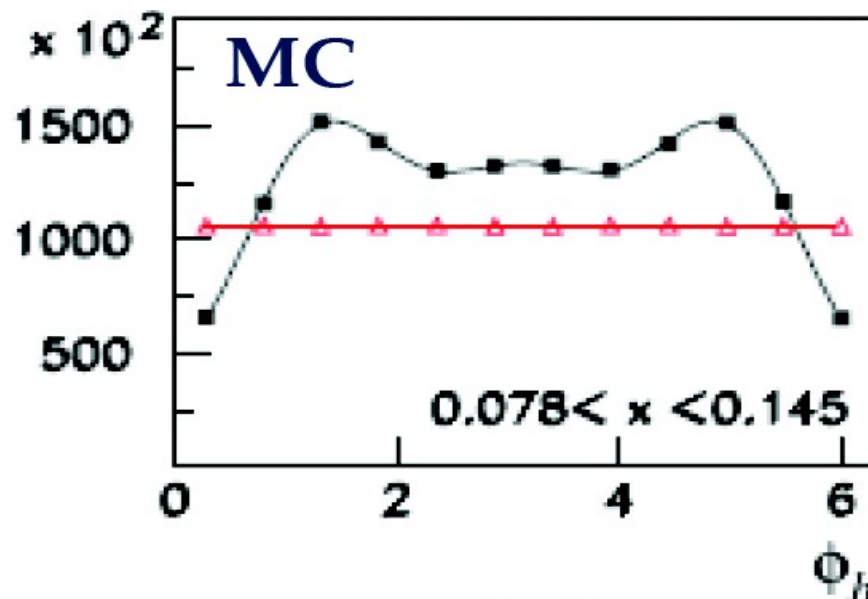
$$\langle \cos(n\phi_h) \rangle \stackrel{th.}{=} \frac{\int d\phi_h \cos(n\phi_h) \sigma_{UU}(\omega, \phi_h)}{\int d\phi_h \sigma_{UU}(\omega, \phi_h)} \quad \omega = (x, y, z, P_{h\perp}^2)$$

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Extraction of the cosine moments

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extraction is challenging!

azimuthal modulations also possible due to

- detector geometrical acceptance
- higher-order QED effects



fully differential analysis needed
unfolding procedure with 400 x 12 bins

BINNING

400 kinematic bins x 12 ϕ -bins

Variable	Bin limits						#
x	0.023	0.042	0.078	0.145	0.27	1	5
y	0.3	0.45	0.6	0.7	0.85		4
z	0.2	0.3	0.45	0.6	0.75	1	5
P_{hT}	0.05	0.2	0.35	0.5	0.75		4

Extraction of the cosine moments

$$\langle \cos(n\phi_h) \rangle \stackrel{th.}{=} \frac{\int d\phi_h \cos(n\phi_h) \sigma_{UU}(\omega, \phi_h)}{\int d\phi_h \sigma_{UU}(\omega, \phi_h)} \quad \omega = (x, y, z, P_{h\perp}^2)$$

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extraction is challenging!

azimuthal modulations also possible due to

- detector geometrical acceptance
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unfolding

fully differential analysis needed
unfolding procedure with 400 x 12 bins

$$\langle \cos(n\phi_h) \rangle \approx \left. \frac{\int d\phi_h \cos(n\phi_h) \sigma_{UU}(\omega, \phi_h)}{\int d\phi_h \sigma_{UU}(\omega, \phi_h)} \right|_{\text{bin } i}$$

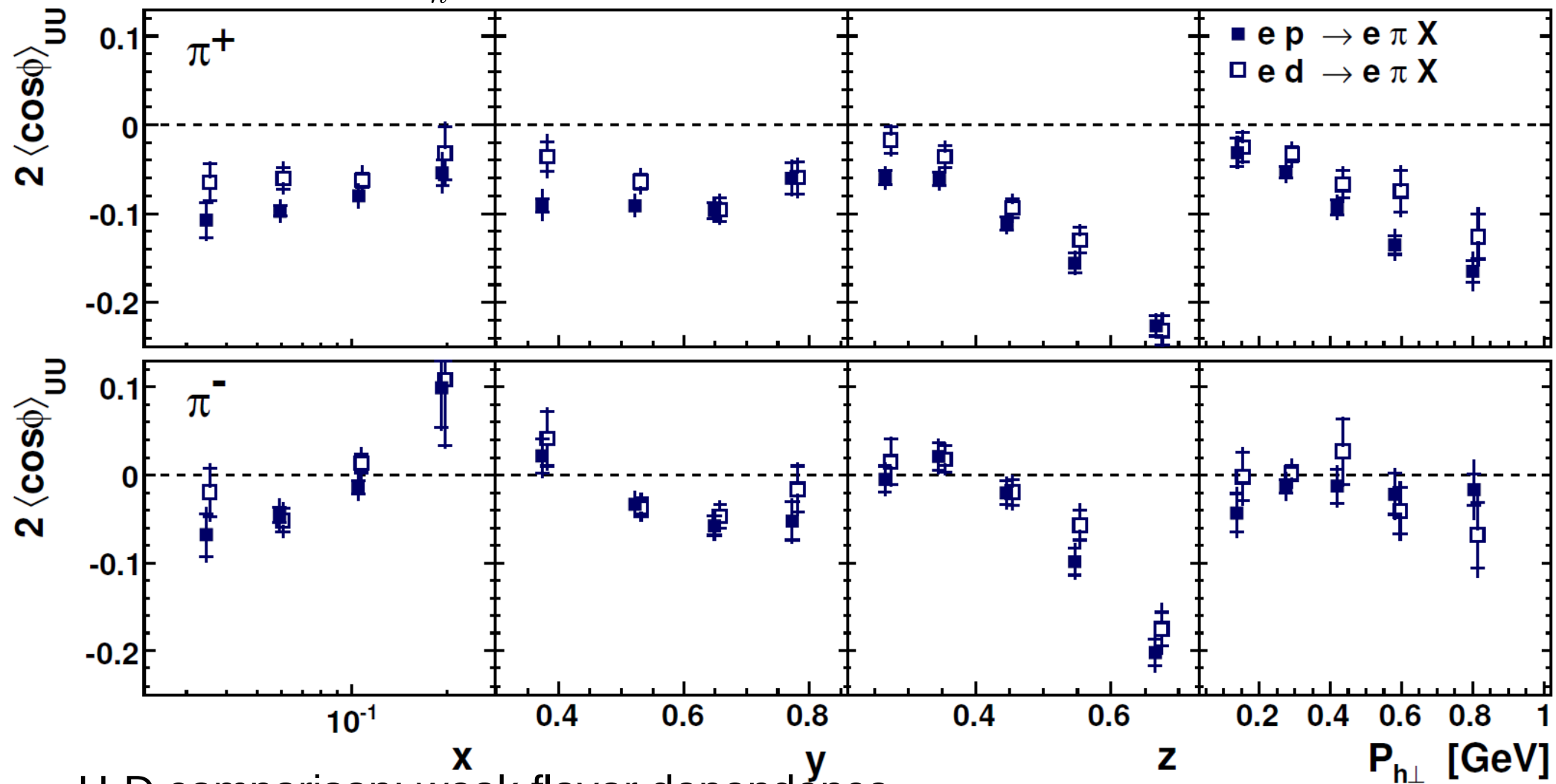
BINNING
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Results for $\langle \cos \phi_h \rangle$: pions

$$\mathcal{I}\left[-\frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M} f_1 D_1 - \frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M_h} \frac{p_T^2}{M^2} h_1^\perp H_1^\perp + \dots\right]$$

A. Airapetian et al., arXiv:1204.4161

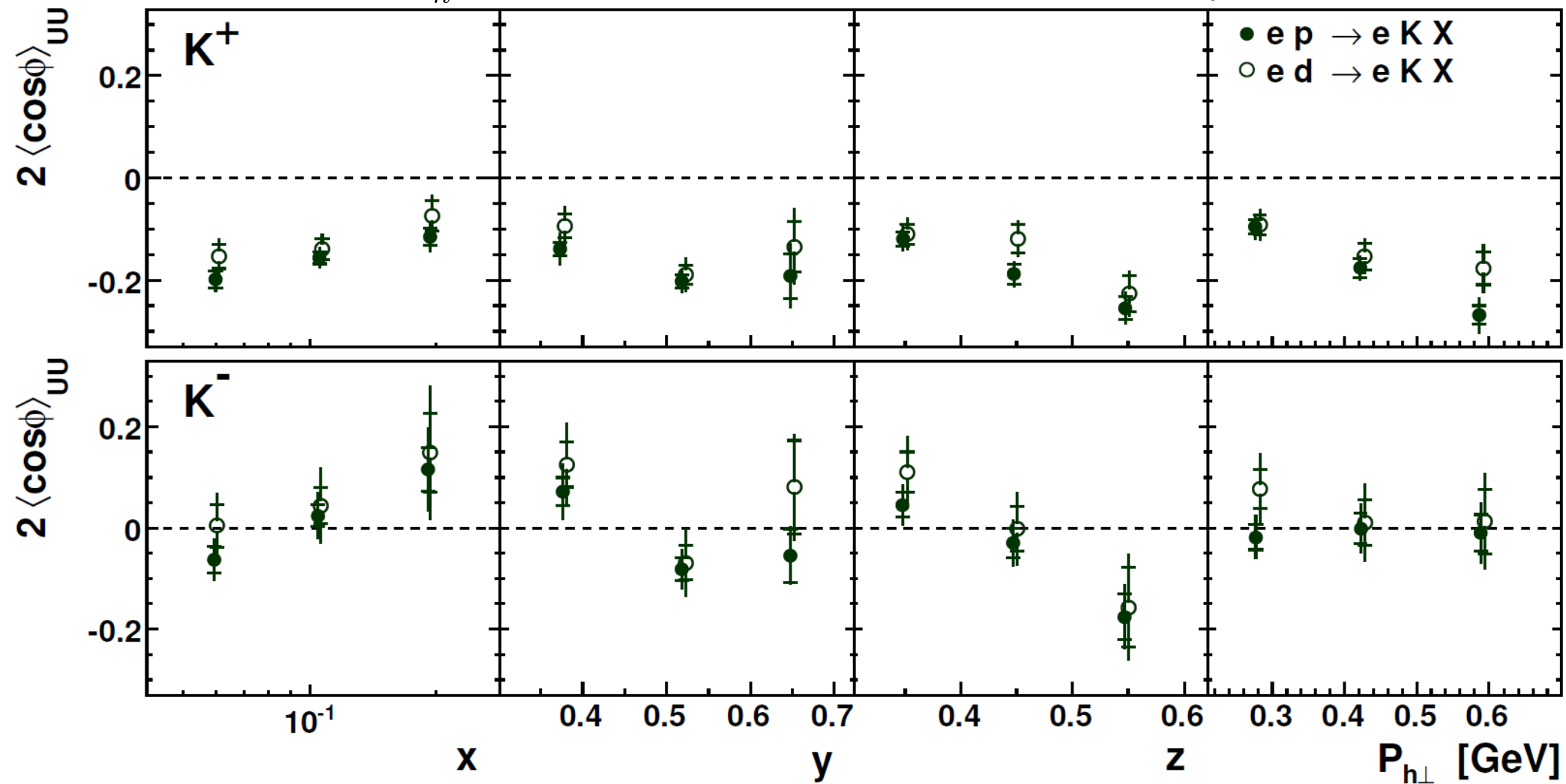


- H-D comparison: weak flavor dependence
- magnitude increases with z
- π^+ : magnitude increases with $P_{h\perp}$

Results for $\langle \cos \phi_h \rangle$: kaons

$$\mathcal{I}\left[-\frac{\hat{P}_{h\perp} \cdot \vec{p}_T}{M} f_1 D_1 - \frac{\hat{P}_{h\perp} \cdot \vec{k}_T}{M_h} \frac{p_T^2}{M^2} h_1^\perp H_1^\perp + \dots\right]$$

A. Airapetian et al., arXiv:1204.4161



- $K^+ < 0$, larger in magnitude than π^+
- $K^- \approx 0$