

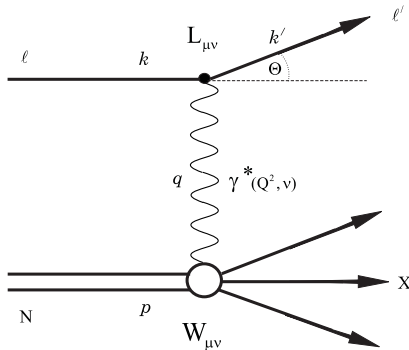
**Precision results
on the spin structure function g_1
of the proton, the deuteron and the
neutron
from the HERMES experiment**

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INFN/LNF

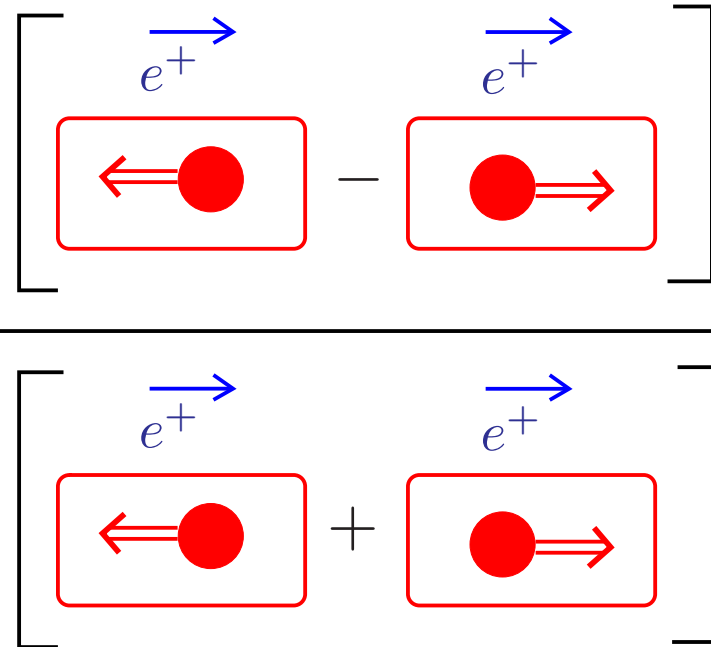
Catania, 30. September 2005





Inclusive asymmetry $A_{||}$

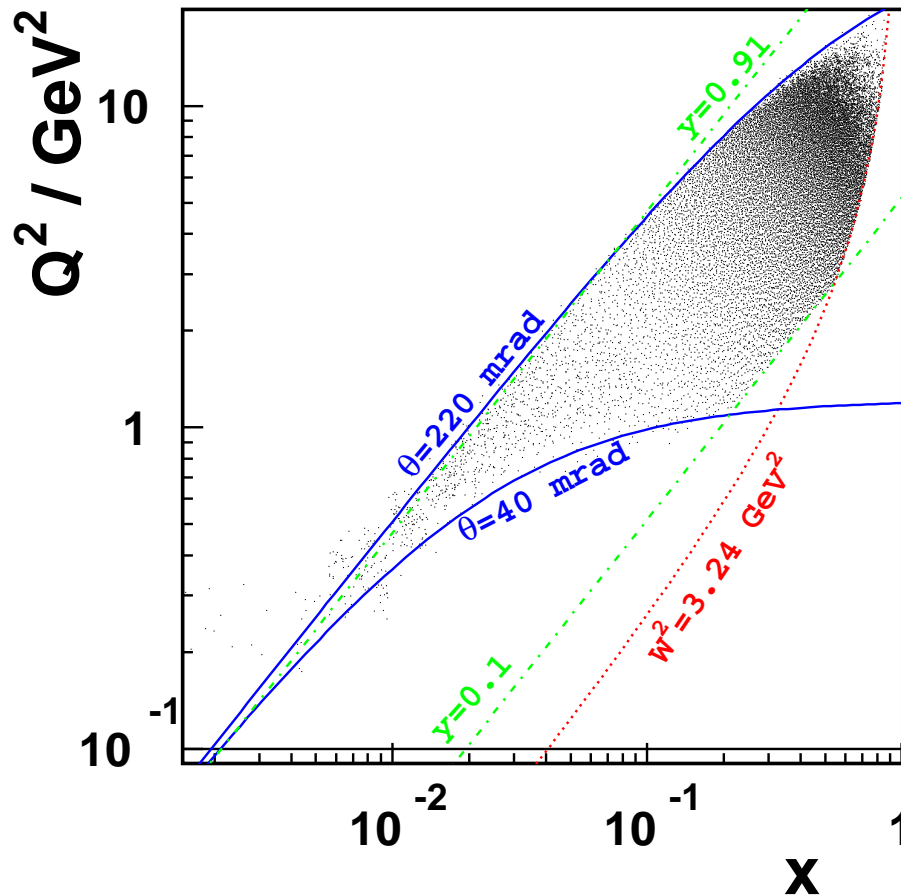
$$A_{||} = \frac{1}{\langle P_B P_z \rangle} \cdot$$



$$\sim \frac{g_1}{F_1}$$

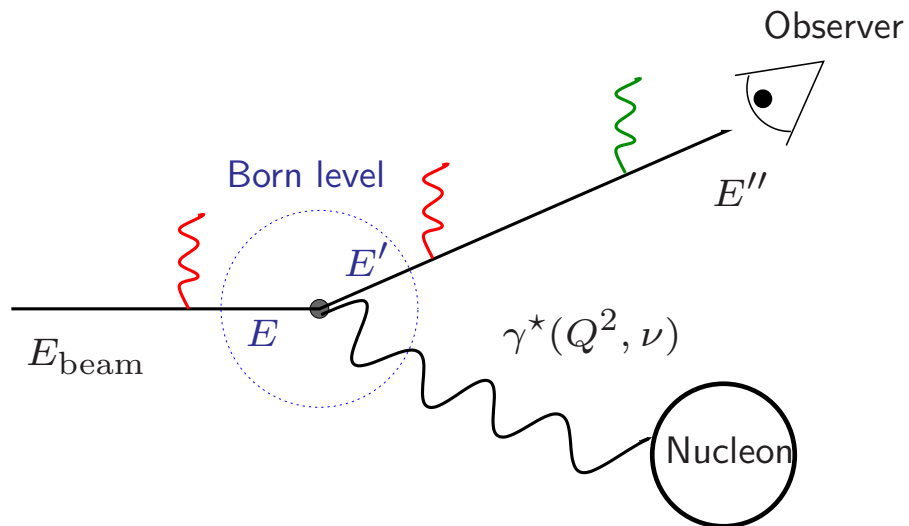
- Polarized positron beam: $\langle P_B \rangle \approx 0.53$, $E_B = 27.6$ GeV
- Polarized fixed **target**: gaseous hydrogen or deuterium, $\langle P_z \rangle \approx 0.85$

Inclusive data sample

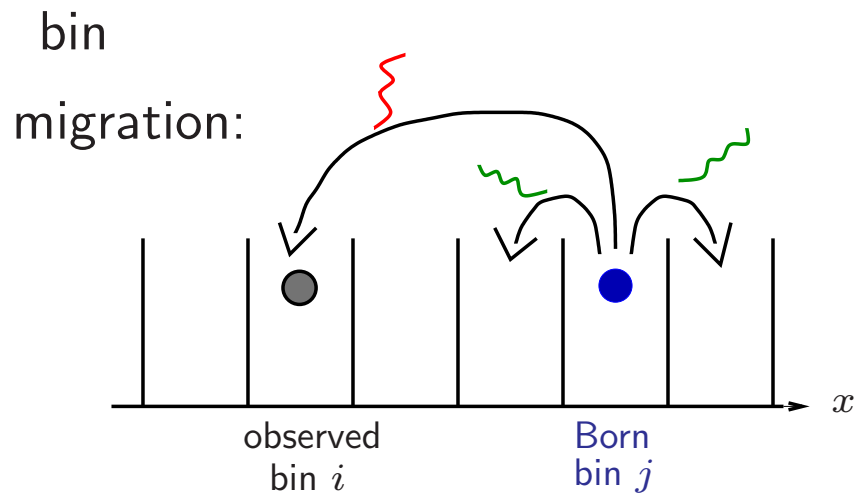


- Leading lepton
- Fiducial volume defines active area of spectrometer
- Kinem. cuts: $0.0021 < x < 0.85$, $Q^2 > 0.1 \text{ GeV}^2$
- 20 bins in x -Björken

From the observed to the true bin



QED radiative effects and detector smearing



Unfolding of kinematic migrations

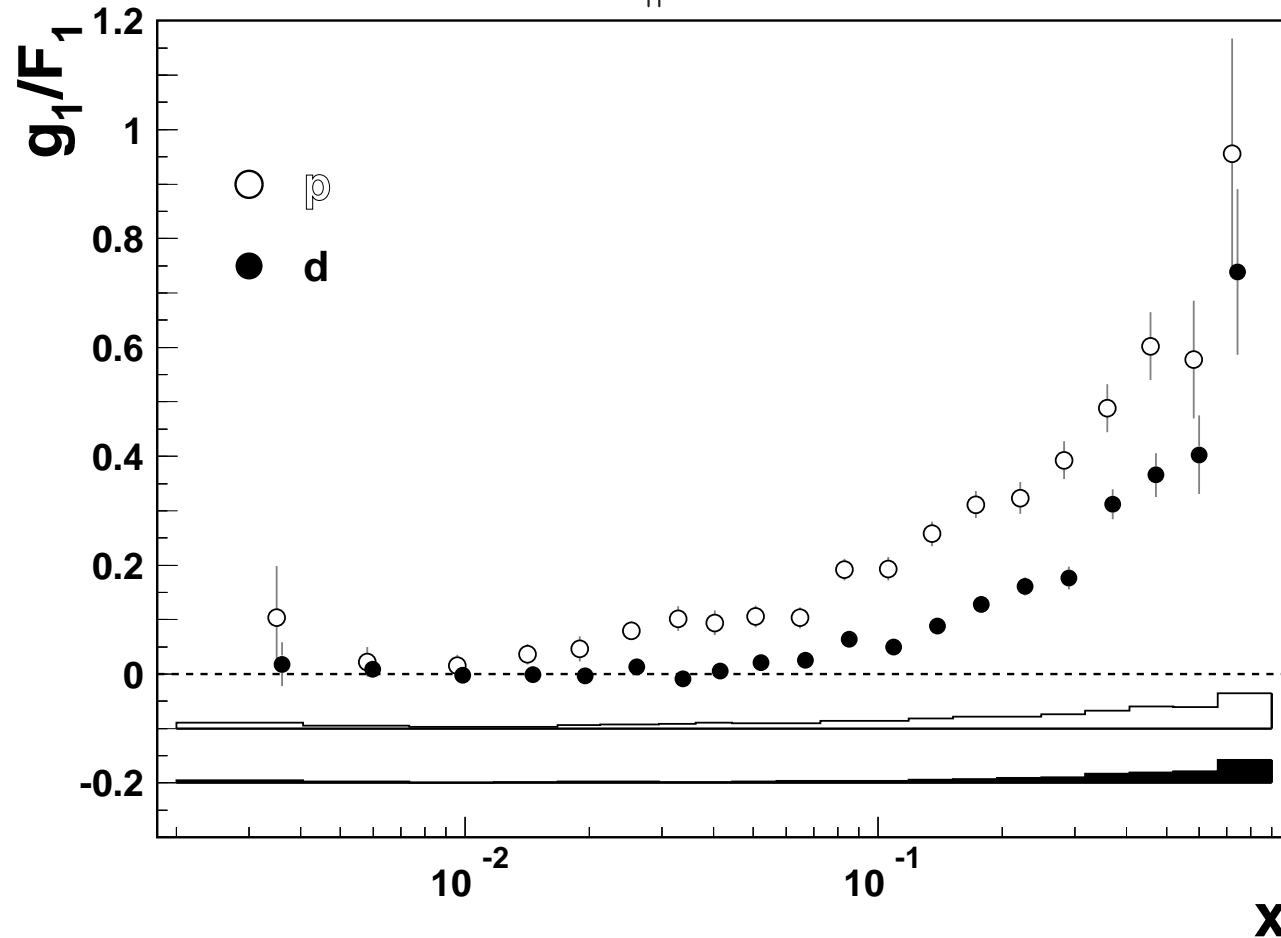
- Measured asymmetry $A_{\parallel}^{\text{meas}}$ diluted by background $\Delta\sigma$
 \Rightarrow to be removed to get asymmetry on Born level $A_{\parallel}^{\text{Born}}$:

$$A_{\parallel}^{\text{Born}}(j) = -1 + \frac{2}{\sigma_{\text{Born}}^{\text{U}}(j)} \cdot \sum_{i=1}^n [S']^{-1}(j, i) \times \left[A_{\parallel}^{\text{meas}}(i) \sigma_{\text{X}}^{\text{U}}(i) - \Delta \hat{\sigma}^{\text{P}}(i) + \sum_{k=1}^n S^{-}(i, k) \sigma_{\text{Born}}^{\text{U}}(k) \right]$$

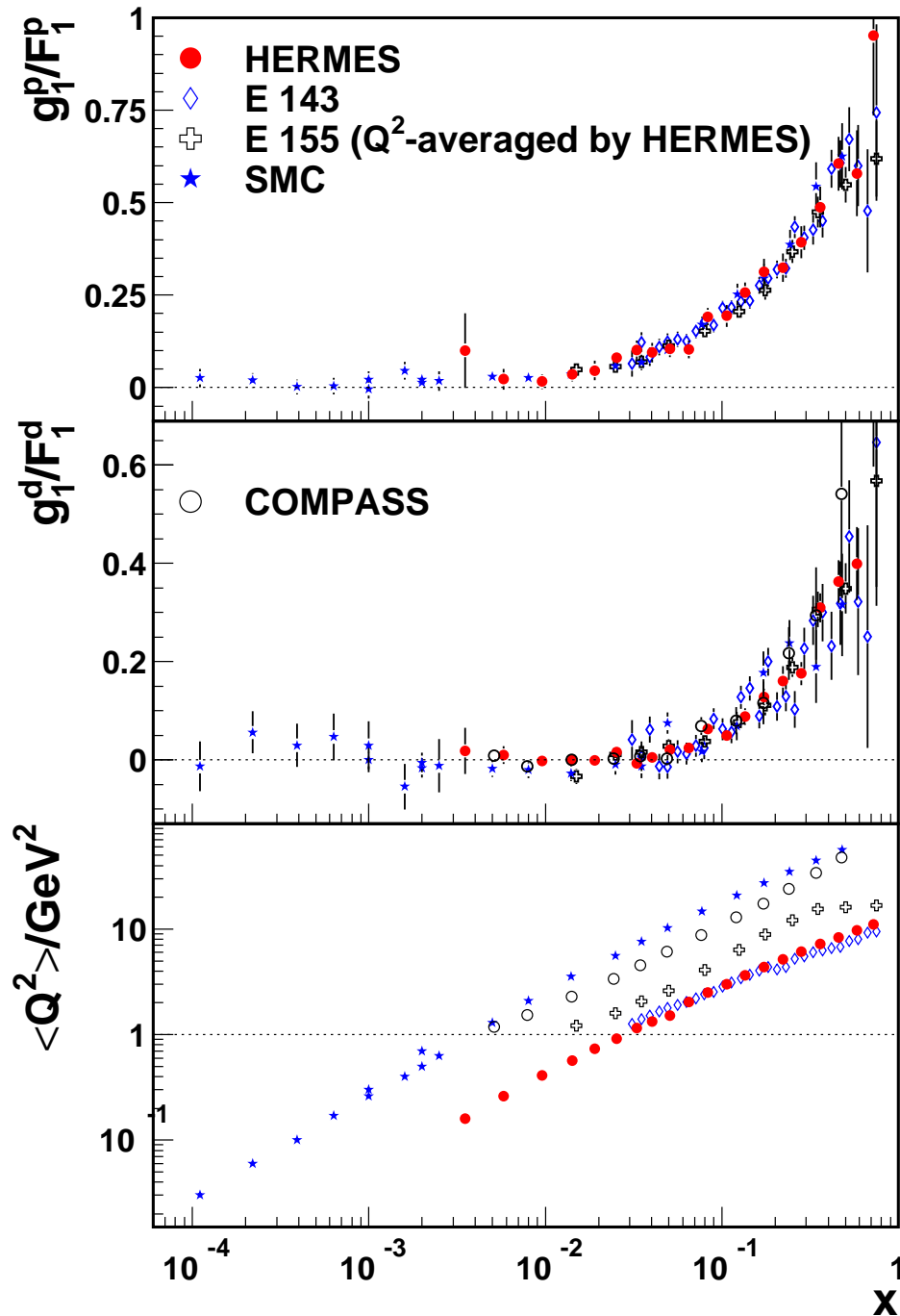
$$A_{\parallel}^{\text{Born}} \equiv A_{\parallel}^{\text{meas}} \cdot \left(1 + \frac{\Delta\sigma^{\text{U}}}{\sigma_{\text{Born}}^{\text{U}}} \right) - \frac{\Delta\sigma^{\text{P}}}{\sigma_{\text{Born}}^{\text{U}}}$$

- Monte Carlo productions:
 - ✓ Radiative corrections (RADGEN)
 - ✓ Detector model (GEANT)
 - ✓ Matrix $S(i, j)$ with event migrations from bin j to i
- Unfolding of migrations :
 - ✓ Removes *systematic* correlations, introduces *statistical* corr.
 - ✓ No iteration needed
 - ✓ Algorithm (almost) model-independent

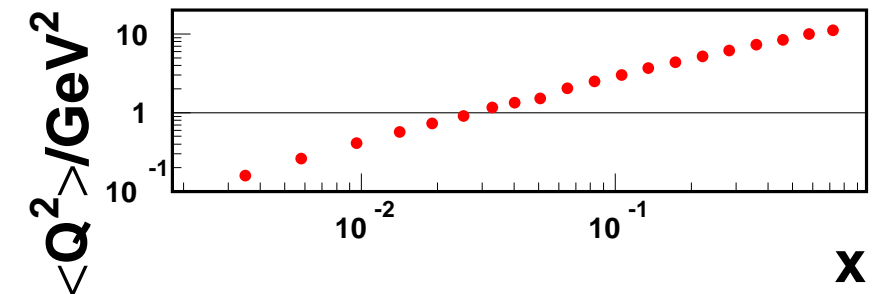
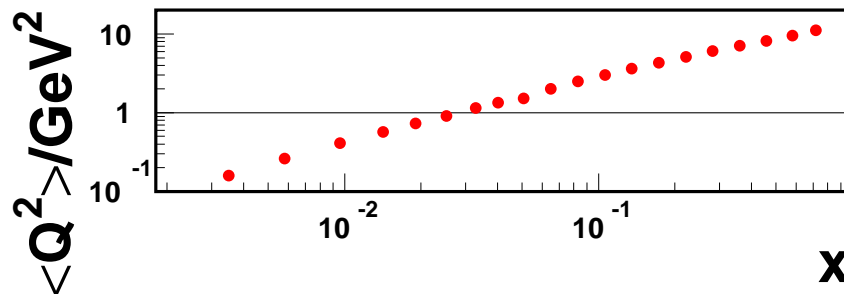
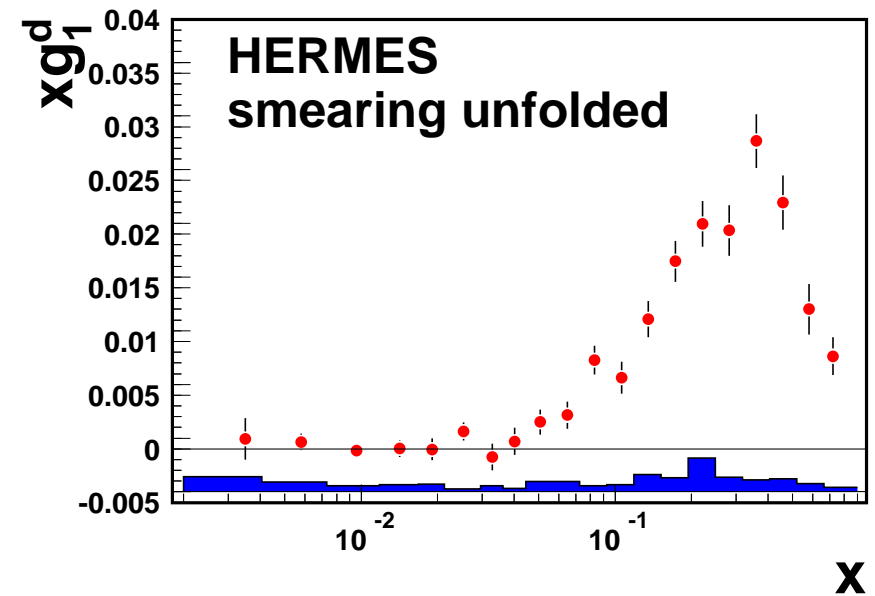
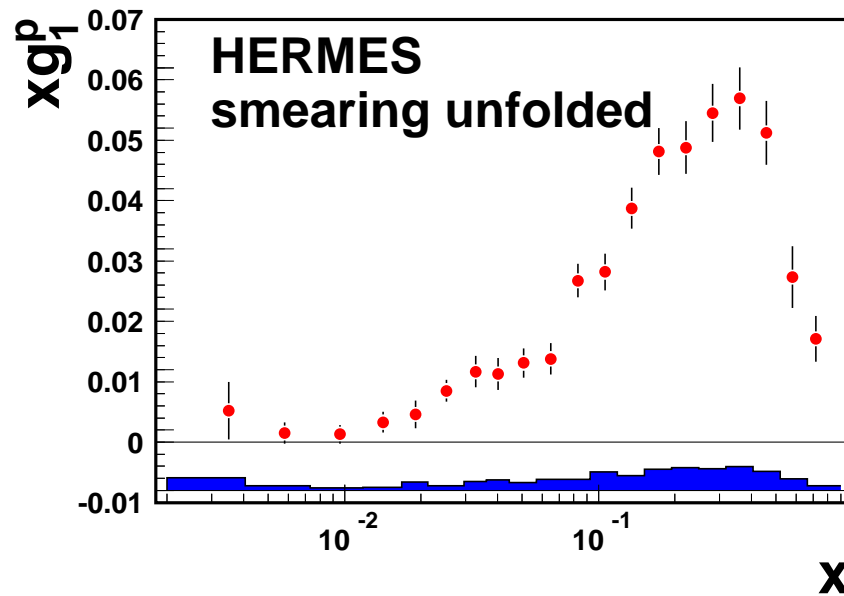
HERMES: $g_1/F_1 \sim A_{\parallel}$ (proton, deuteron)



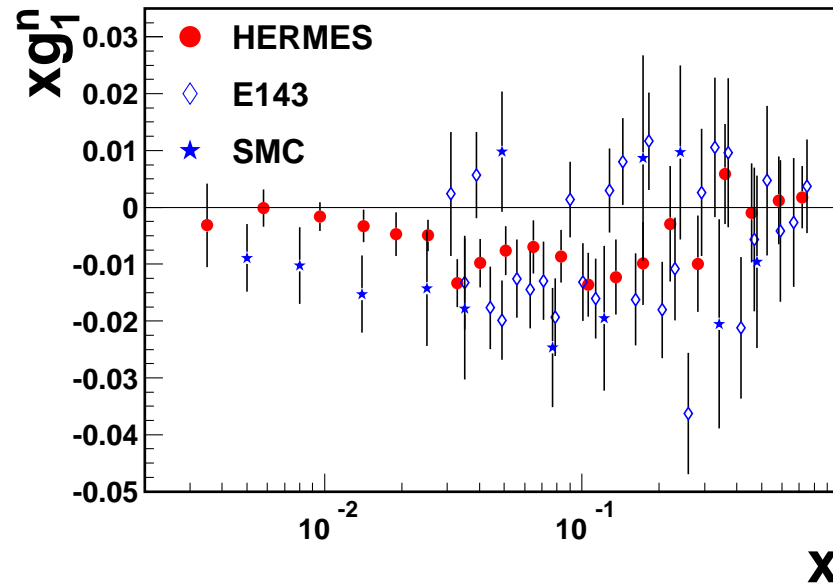
World data on $g_1/F_1 \sim A_{||}$



Spin structure function xg_1 of p and d



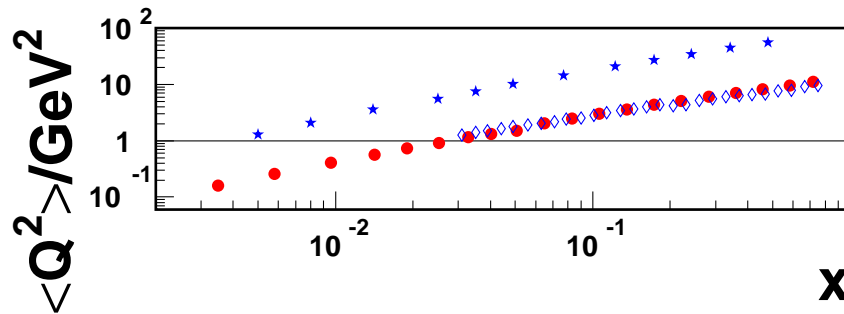
Neutron xg_1 from proton and deuteron data



Extracted from

$$g_1^d = \frac{1}{2} \left(1 - \frac{3}{2} \omega_D \right) (g_1^p + g_1^n),$$

with $\omega_D = 0.05$



Moments of g_1

- Evolution to common $Q_0^2 = 5 \text{ GeV}^2$:

$$M = \int_{x_s}^{x_f} g_1(x, Q_0^2) dx = \sum_i \left[\frac{g_1(\langle x_i \rangle, \langle Q_i^2 \rangle)}{F_1} \int_{x_i}^{x_{i+1}} F_1(x, Q_0^2) dx \right]$$

$$\delta M = \sum_i \sum_j \text{cov}(i, j) \int_{x_i}^{x_{i+1}} F_1(x, Q_0^2) dx \int_{x_j}^{x_{j+1}} F_1(x, Q_0^2) dx$$

- HERMES ($x_s = 0.021$, $x_f = 0.9$, $Q_0^2 = 5 \text{ GeV}^2$, only data $Q^2 > 1 \text{ GeV}^2$):

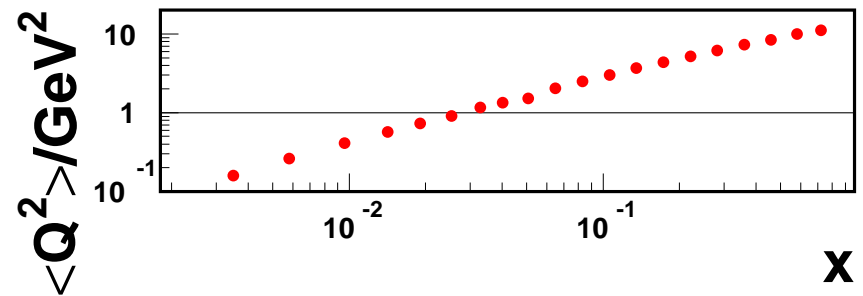
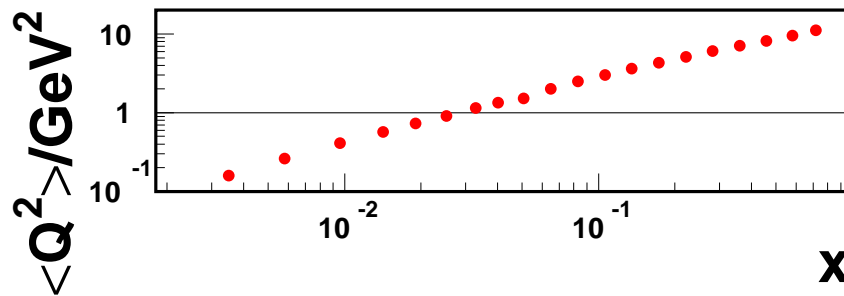
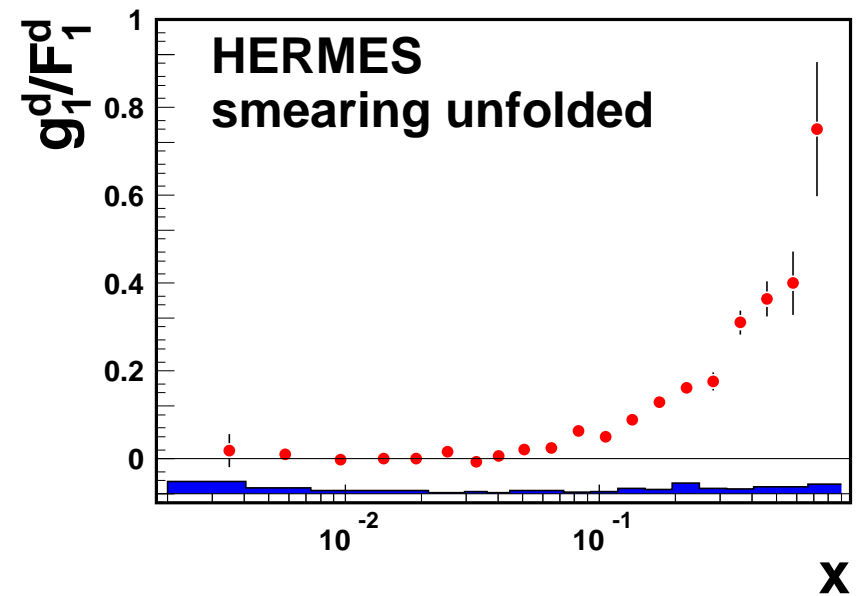
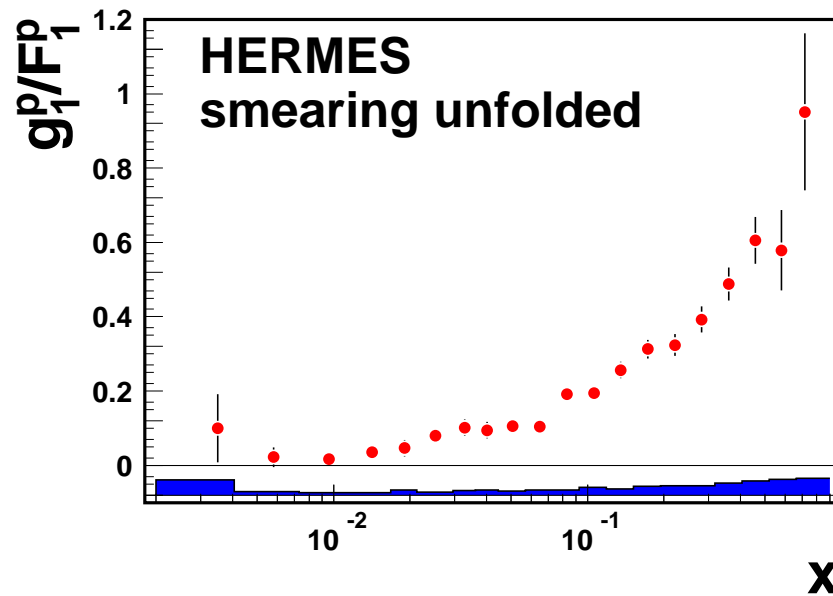
Target	M
proton	$0.1245 \pm 0.0033(\text{stat}) \pm 0.0080(\text{sys})$
deuteron	$0.0454 \pm 0.0016(\text{stat}) \pm 0.0023(\text{sys})$
"n=2d-p"	$-0.0263 \pm 0.0047(\text{stat}) \pm 0.0100(\text{sys})$

- HERMES moments compatible with E143, E155 and SMC

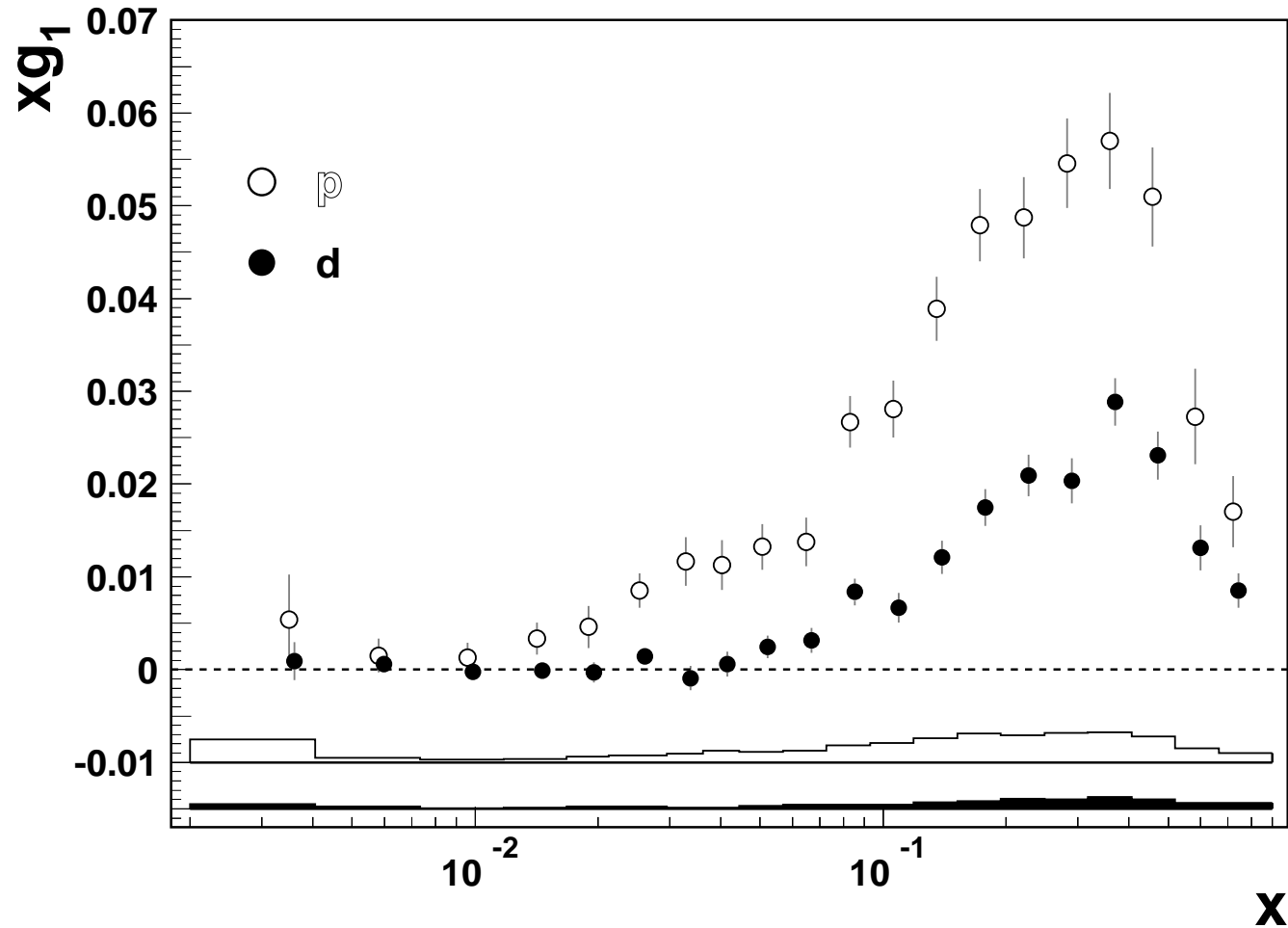
Summary

- **Final results** on spin structure function g_1 from HERMES data (proton, deuteron):
 - Corrected for QED radiative effects and detector smearing
 - Correction algorithm introduces statistical correlations between bins and removes systematic correlations
- Comparison to world data:
 - **Proton**: HERMES data comparable to most precise results (SLAC, CERN); good agreement
 - **Deuteron**: so far most precise determination; results agree with COMPASS, but show systematic disagreement with SMC data for $x \lesssim 0.02$
 - **Neutron** (from p and d): Best statistical accuracy; HERMES g_1 shows no drop-off at small- x as suggested by SMC
- Calculated **g_1 moments** in the measured range in agreement with CERN and SLAC results
- **Long HERMES publication on g_1** in preparation (soon to come)

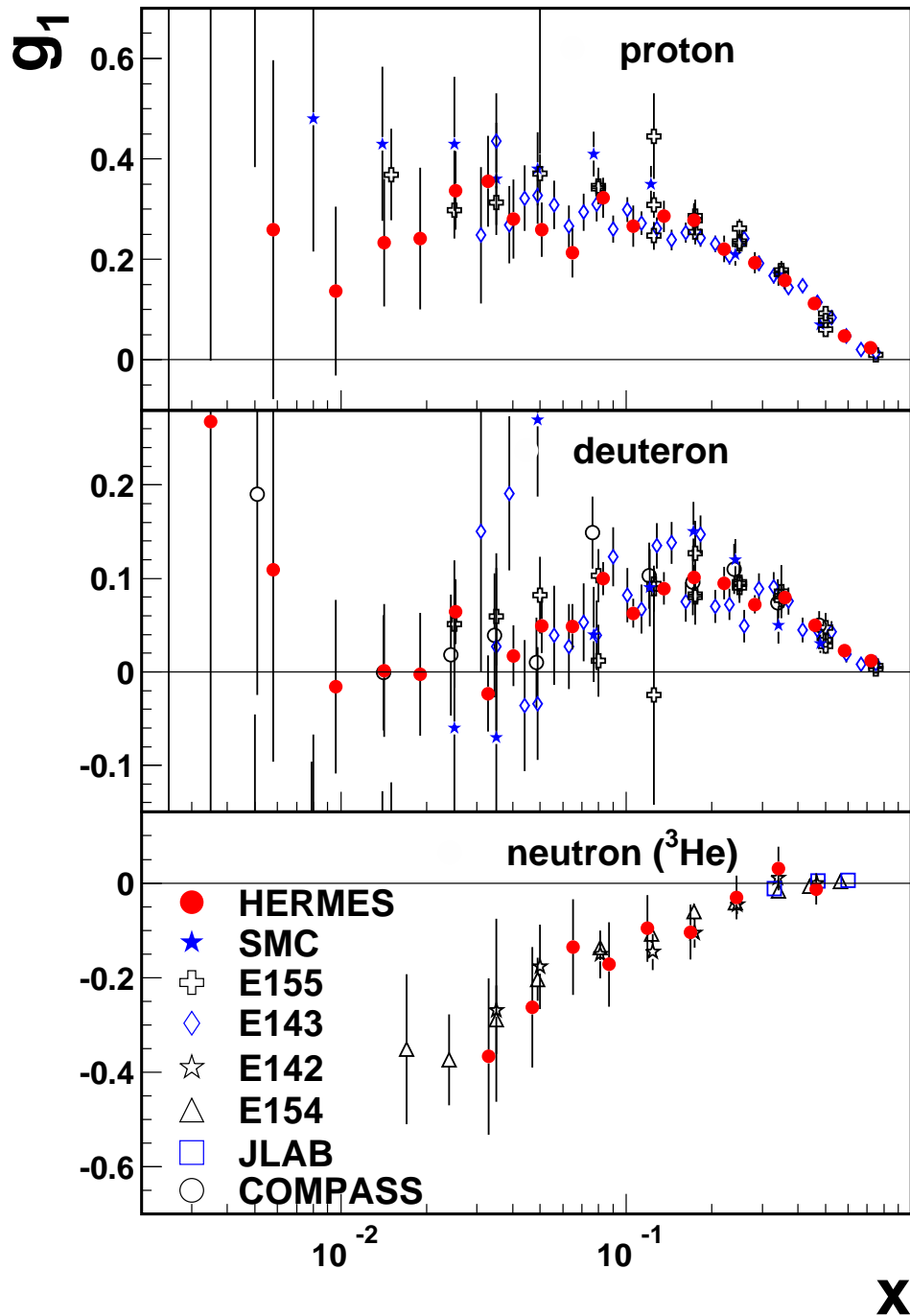
g_1/F_1 of the proton and of the deuteron



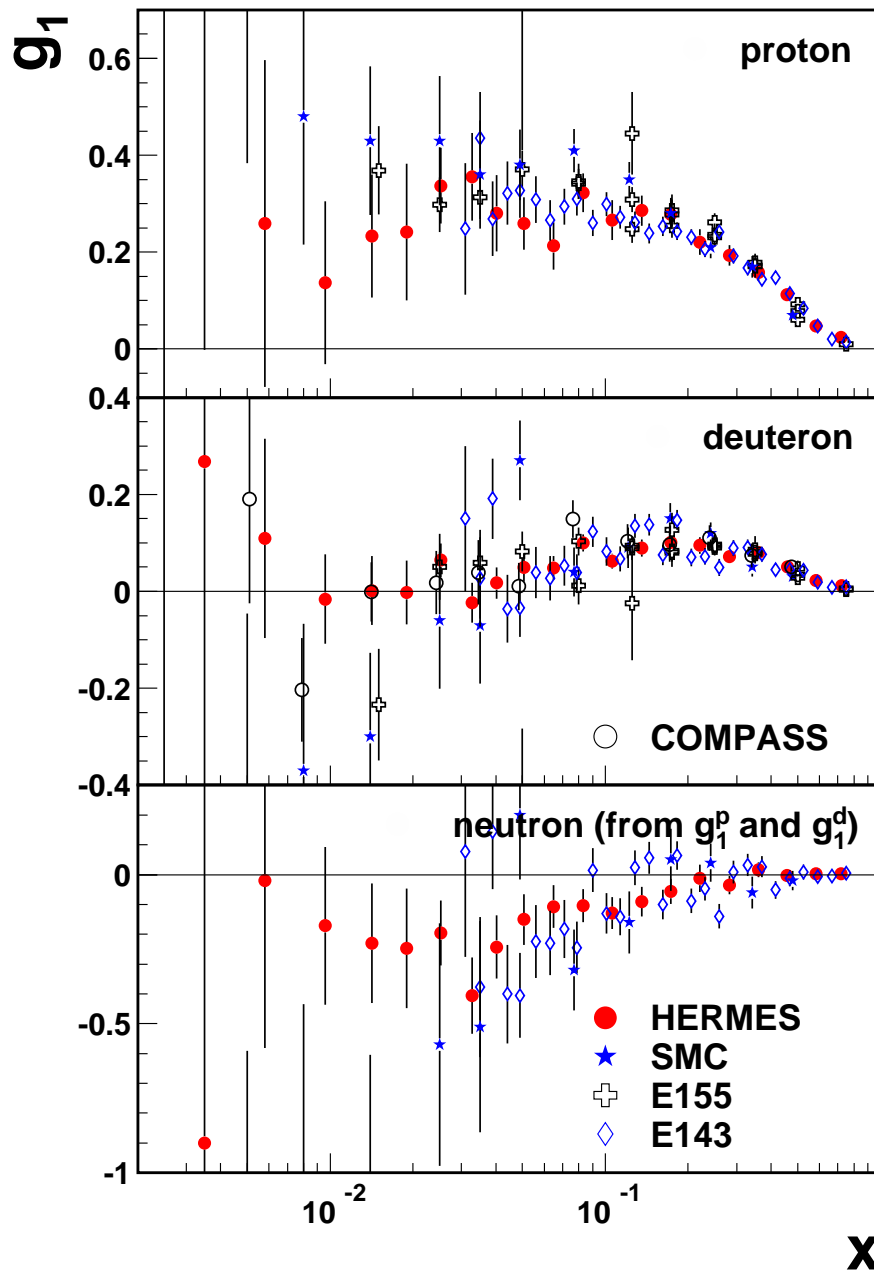
HERMES: xg_1 (proton, deuteron)



World data on g_1

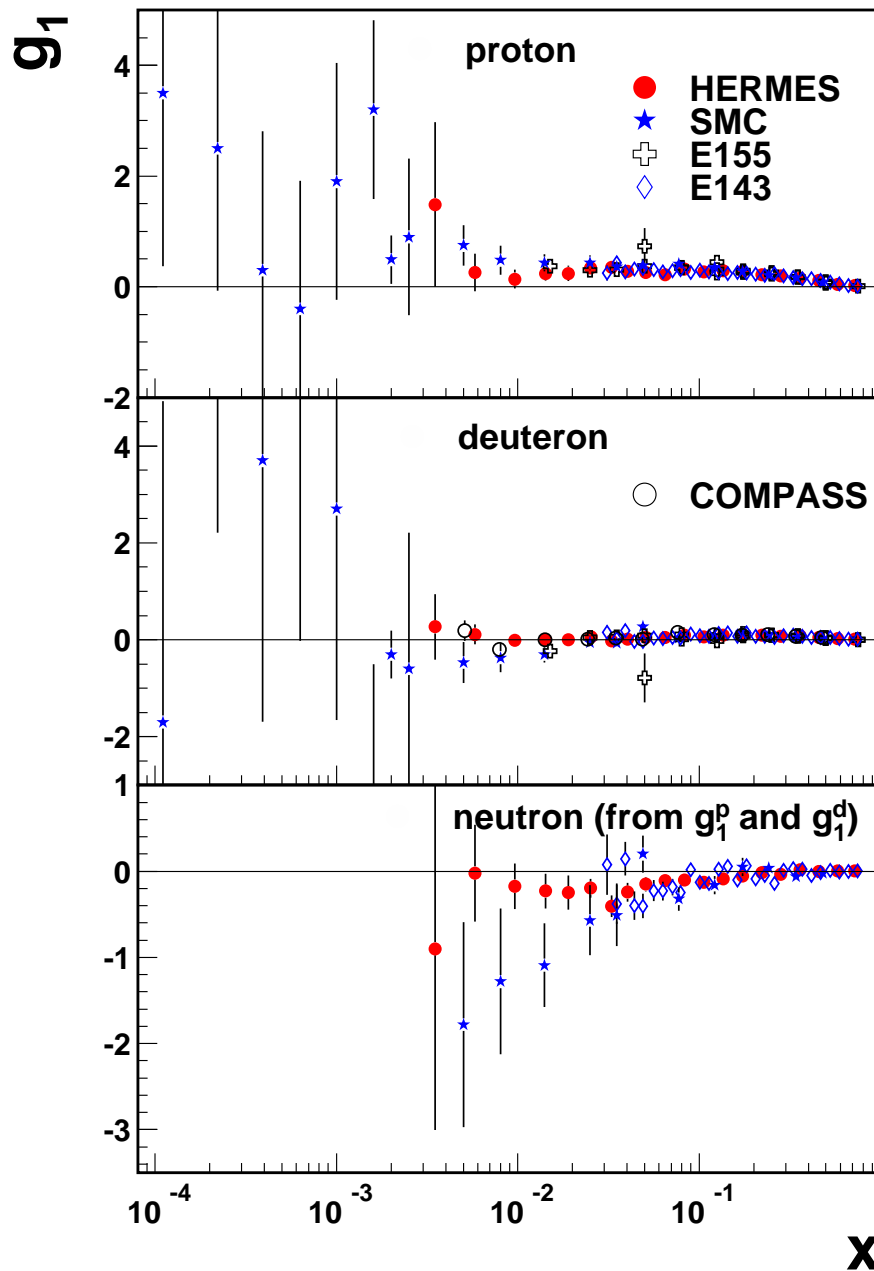


World data on g_1 (neutron)



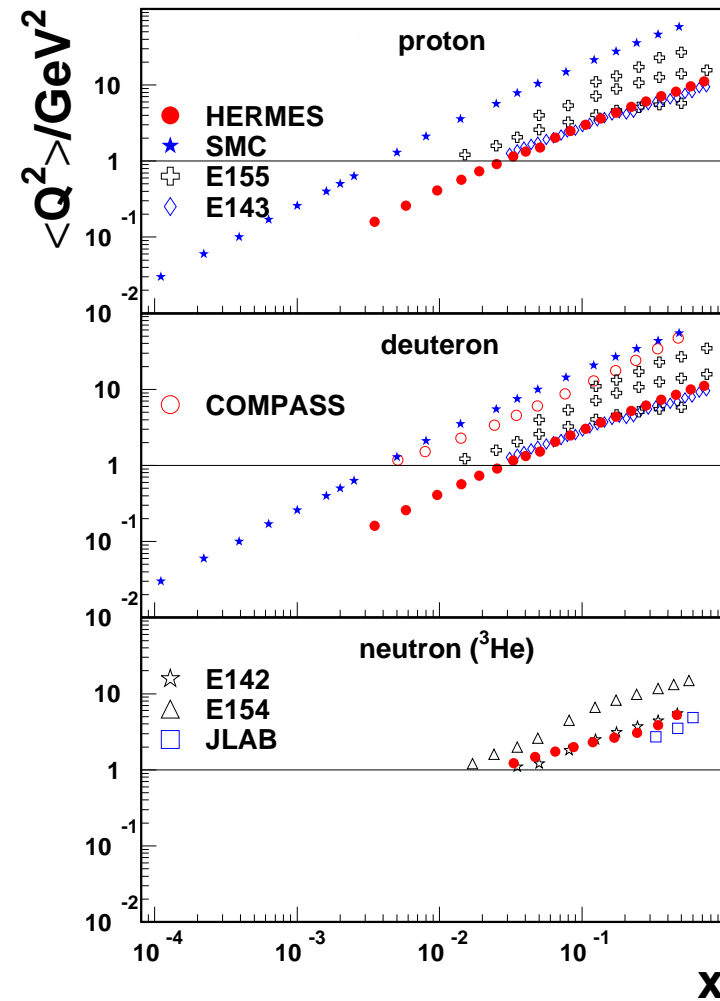
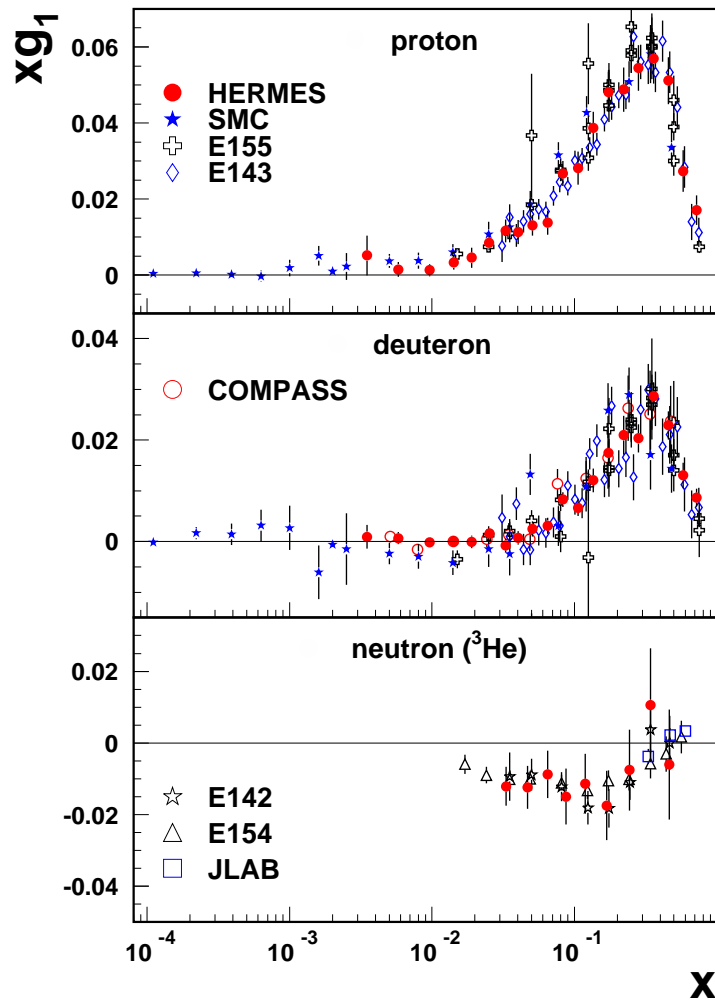
$$g_1^d = \frac{1}{2} \left(1 - \frac{3}{2}\omega_D \right) (g_1^p + g_1^n), \text{ with } \omega_D = 0.058$$

World data on g_1 (neutron)

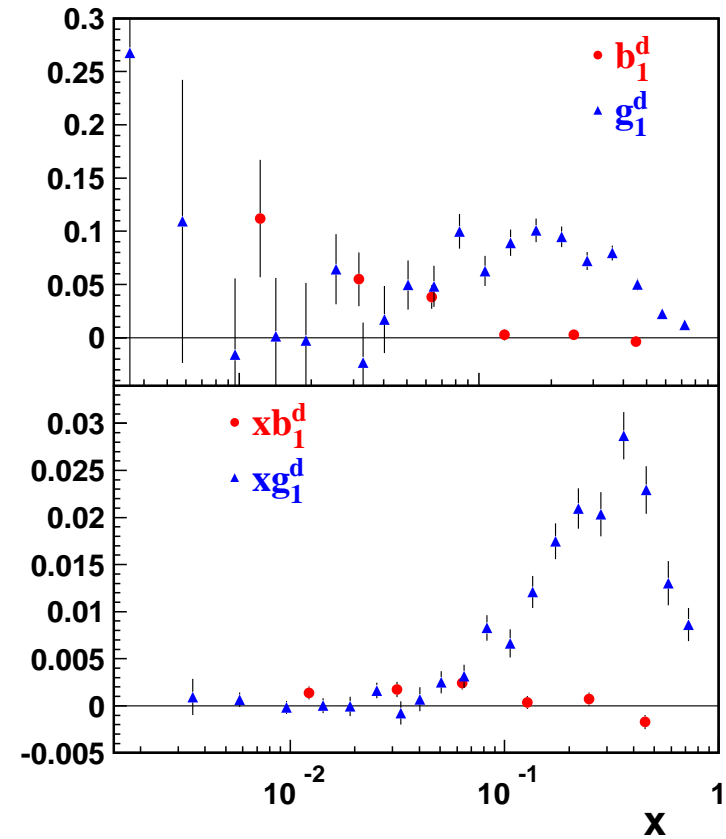
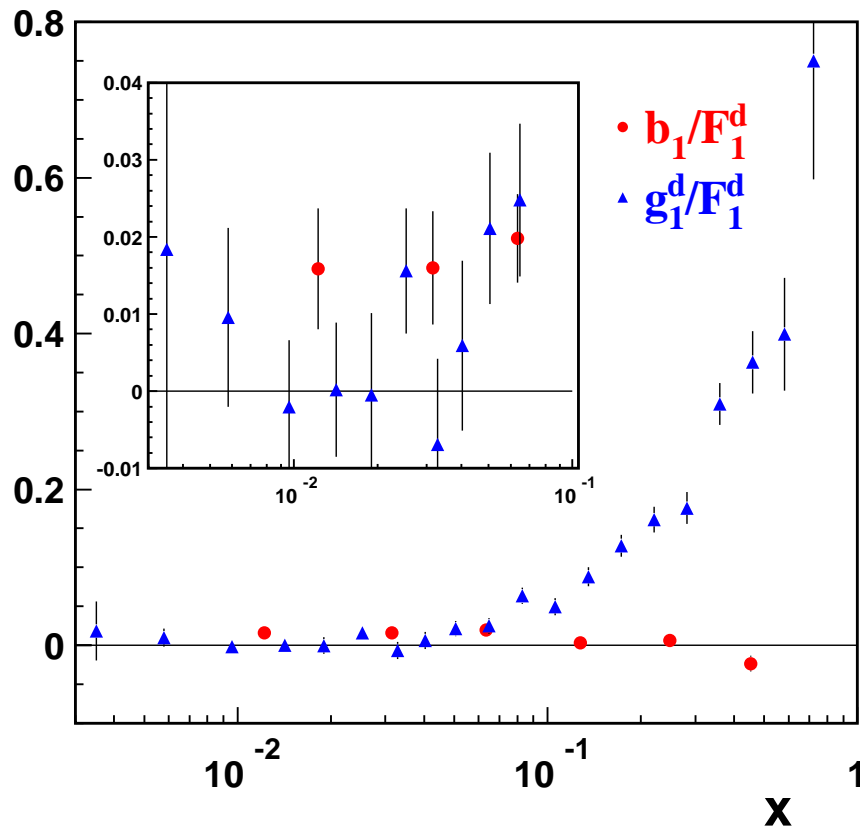


$$g_1^d = \frac{1}{2} \left(1 - \frac{3}{2}\omega_D \right) (g_1^p + g_1^n), \text{ with } \omega_D = 0.058$$

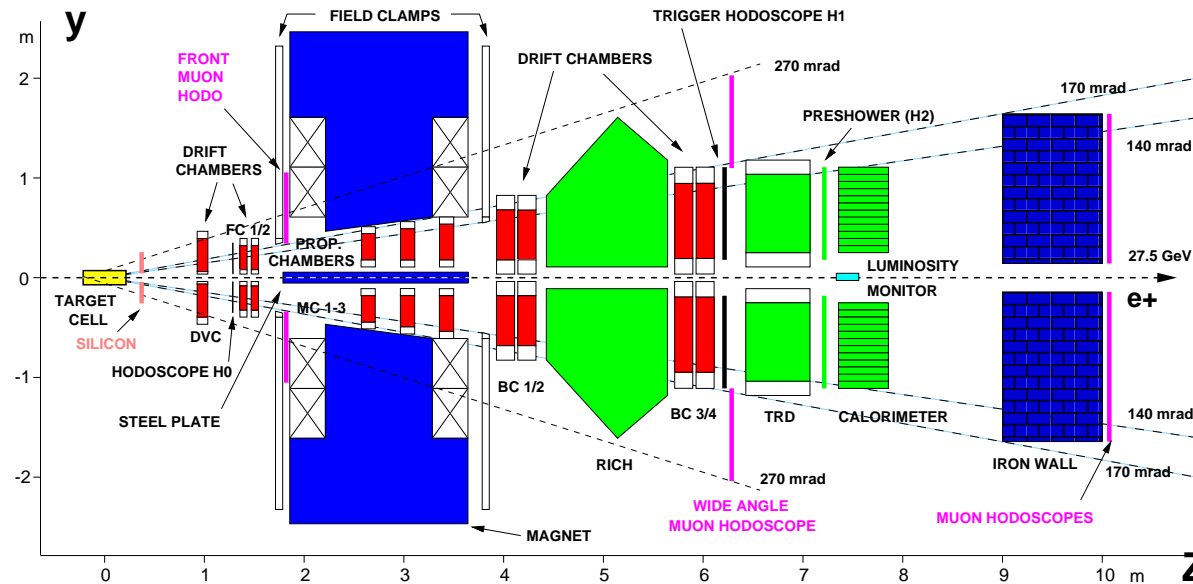
World data on xg_1



Comparison: Tensor vs. Vector asymmetry

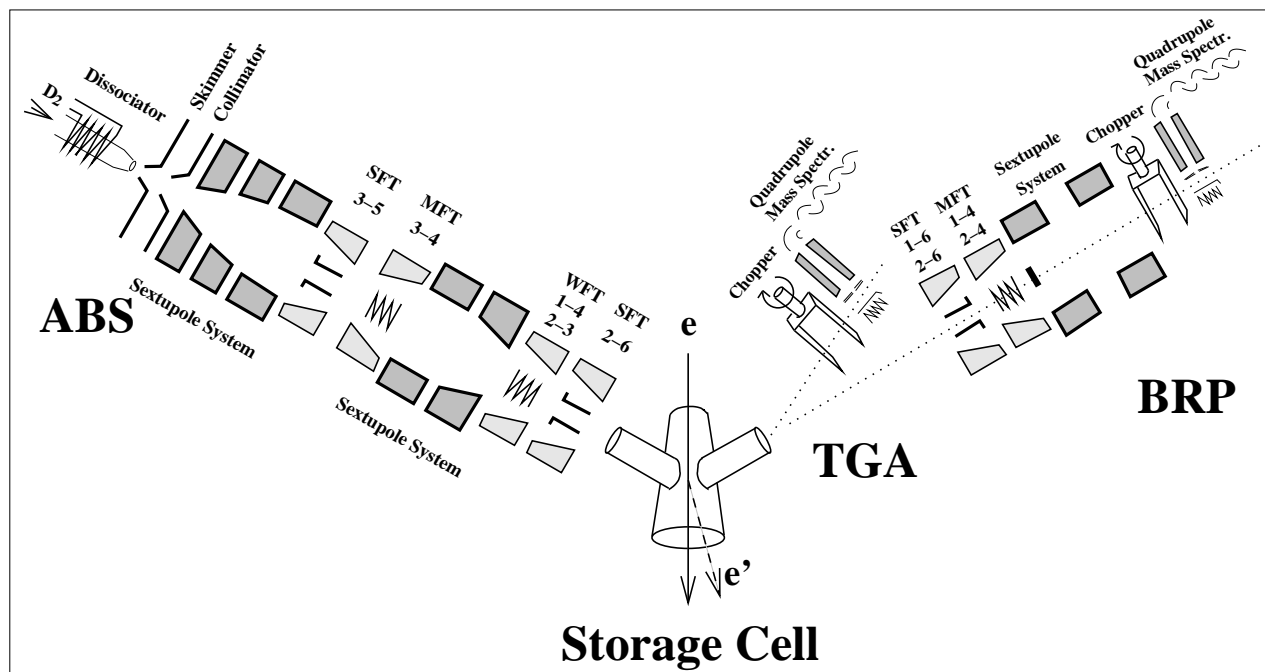


The HERMES spectrometer

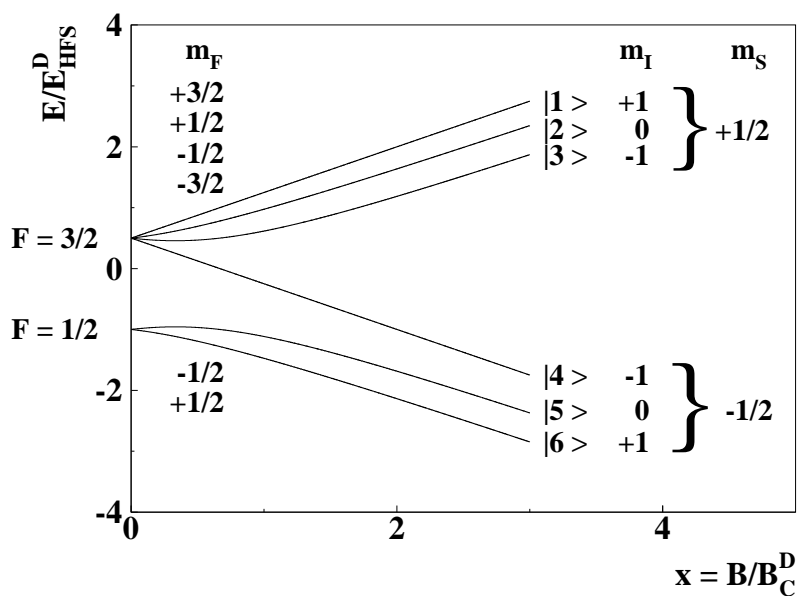


- **Acceptance:** $40 < \theta < 220$ mrad
- **Momentum resolution:** $\frac{\delta p}{p} \approx 2\%$;
Angular resolution: $0.3 - 0.6$ mrad;
- **Calorimeter:** $\frac{\delta E}{E} \approx \frac{(5.1 \pm 1.1)}{\sqrt{E[\text{GeV}]}} \%$
- **PID:** RICH, TRD, preshower, calo
- Efficiency of **electron ID:** 98-99 %
- **Hadron contamination:** $< 1\%$

The HERMES-target



Hyperfine splitting in a magnetic field for deuterium:

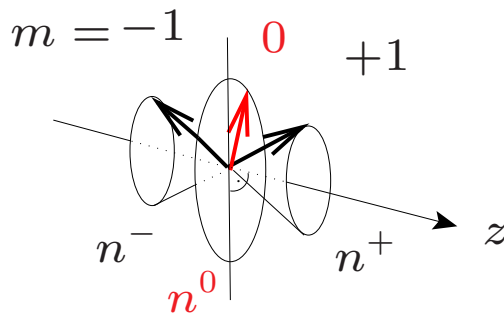


Polarized atomic gas target

Proton: $m = \pm\frac{1}{2}$

Polarizations:

Deuteron (Spin-1):



vector

$$P_z = \frac{n^+ - n^-}{n^+ + n^- + n^0}$$

tensor

$$P_{zz} = \frac{(n^+ + n^-) - 2n^0}{n^+ + n^- + n^0}$$

$$|P_z| \leq 1, \quad -2 \leq P_{zz} < 1$$

HERMES target: ABS + gas analyzing system

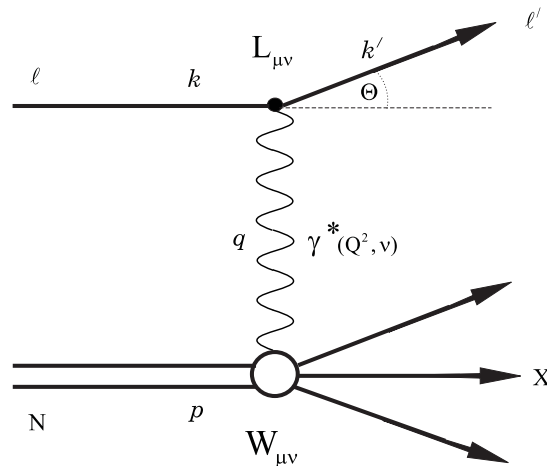
Special:

- Hyperfine states can be selected separately
- Negative P_{zz} reachable!

target state	injected	P_z	P_{zz}
vector +	n^+	+0.85	+0.80
vector -	n^-	-0.84	+0.85
tensor +	$n^+ + n^-$	-0.01	+0.89
tensor -	n^0	-0.01	-1.66

⇒ High P_{zz}
(at $P_z=0$)
reachable

Deeply inelastic lepton-nucleon scattering (inclusive measurement)



LORENTZ invariant kinematic variables

- square of four-momentum transfer

$$Q^2 = -q^2 \stackrel{\text{Lab.}}{\simeq} 2EE'(1 - \cos \Theta)$$

- Björken scale variable

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{2M\nu} \quad 0 \leq x \leq 1$$

- electron energy transferred to the nucleon

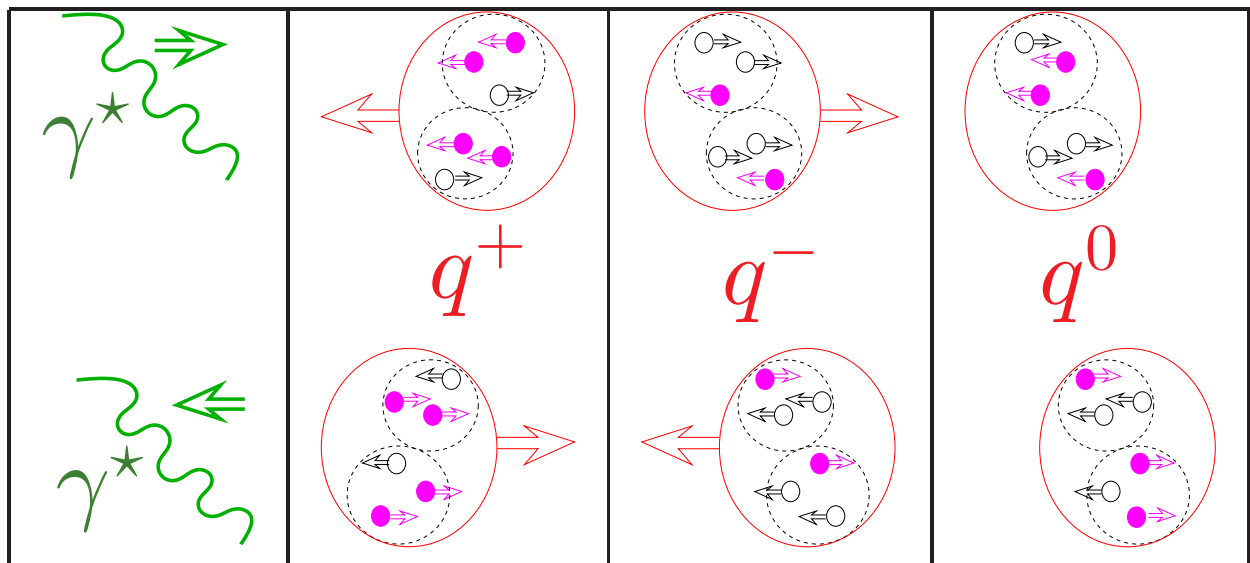
$$\nu = \frac{pq}{M} \stackrel{\text{Lab.}}{\simeq} E - E'$$

- fraction of electron energy transferred to the nucleon

$$y = \frac{pq}{pk} \stackrel{\text{Lab.}}{=} \frac{\nu}{E} \quad 0 \leq y \leq 1$$

Structure functions in the QPM

Quark densities $q(x, Q^2)$



Spin- $\frac{1}{2}$ (proton)

$$F_1 = \frac{1}{2} \sum_q e_q^2 (q^+ + q^-)$$

$$g_1 = \frac{1}{2} \sum_q e_q^2 (q^+ - q^-)$$

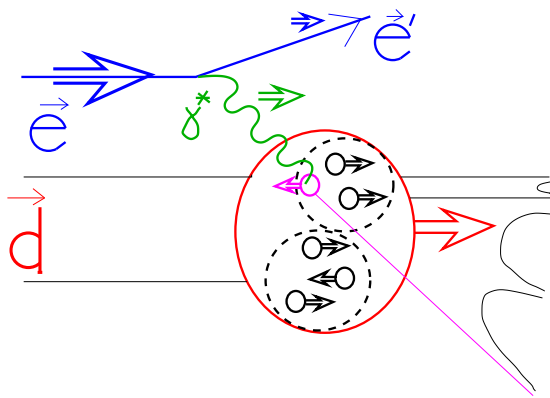
Spin-1 (deuteron)

$$F_1 = \frac{1}{3} \sum_q e_q^2 (q^+ + q^- + q^0)$$

$$g_1 = \frac{1}{2} \sum_q e_q^2 (q^+ - q^-)$$

$$b_1 = \frac{1}{2} \sum_q e_q^2 (2q^0 - (q^+ + q^-))$$

Structure functions and interaction



x-section for DIS:

$$\frac{d^2\sigma}{dE' d\Omega} \Big|_{\text{Born}} = \frac{\alpha^2}{2MQ^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$$

Leptonic and hadronic tensor each separable in

{symmetric}, spin independent and

[anti-symmetric], spin dependent part \Rightarrow

$$L_{\mu\nu} W^{\mu\nu} = \underbrace{L_{\{\mu\nu\}} W^{\{\mu\nu\}}(F_1, F_2, b_1, b_2, b_3, b_4)}_{\text{unpolarized}} + \underbrace{i L_{[\mu\nu]} W^{[\mu\nu]}(g_1, g_2)}_{\text{polarized inclusive x-section}}$$

($\Rightarrow b_1$ not sensitive to beam polarization, but implicitly dependent on target spin)

Leptonic and hadronic tensor

$$L^{\mu\nu} = L^{\{\mu\nu\}} + iL^{[\mu\nu]}(s)$$

spin *independent* and
{symmetric}

spin *dependent* and
[anti symmetric]

$$W^{\mu\nu} = W^{\{\mu\nu\}}(F_1, F_2) + iW^{[\mu\nu]}(g_1, g_2) +$$

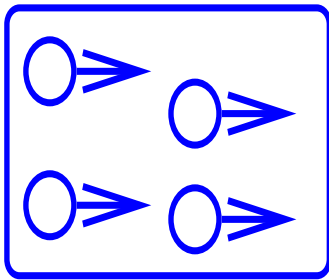
$$+ W^{\{\mu\nu\}}(b_1, b_2, b_3, b_4)$$

implicitly *dependent on target spin*
(additionally and *only for Spin-1*)

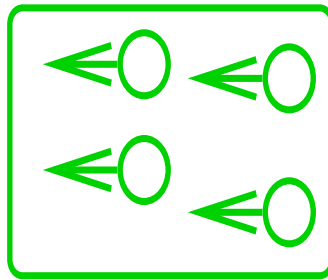
Inclusive spin physics at HERMES

Measure (E, θ) of pol. DIS electron off pol. target:

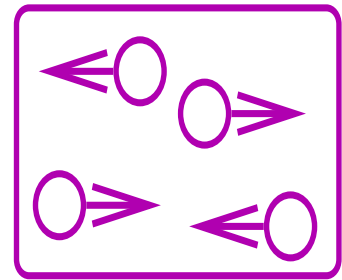
$$P_z = +1$$



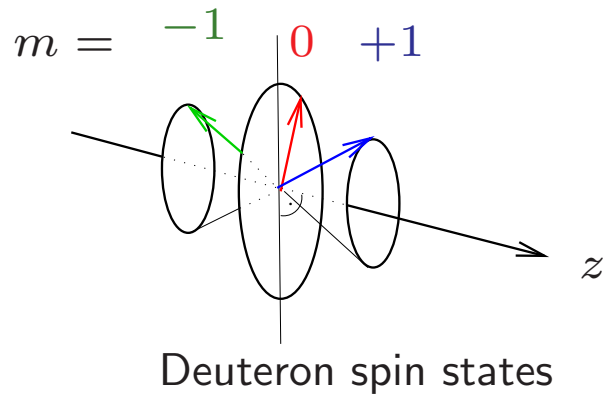
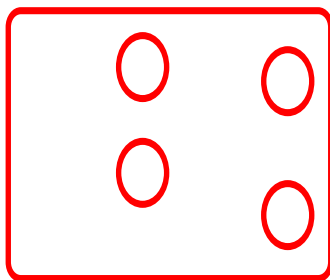
$$P_z = -1$$



$$P_z = 0$$



$$P_z = 0 \quad P_{zz} = -2$$



$$A_{\parallel} = \frac{1}{\langle P_B P_z \rangle} \cdot \frac{\left(\frac{N^{\leftarrow}}{L^{\leftarrow}} \right) - \left(\frac{N^{\rightarrow}}{L^{\rightarrow}} \right)}{\left(\frac{N^{\leftarrow}}{L^{\leftarrow}} \right) + \left(\frac{N^{\rightarrow}}{L^{\rightarrow}} \right)} \sim D \frac{g_1}{F_1}$$

$$A_{zz} = \frac{1}{\langle P_{zz} \rangle} \cdot \frac{\frac{2}{3} \left(\frac{N^{\leftarrow}}{L^{\leftarrow}} + \frac{N^{\rightarrow}}{L^{\rightarrow}} + \frac{N^{\leftrightarrow}}{L^{\leftrightarrow}} \right) - 2 \left(\frac{N^0}{L^0} \right)}{\frac{2}{3} \left(\frac{N^{\leftarrow}}{L^{\leftarrow}} + \frac{N^{\rightarrow}}{L^{\rightarrow}} + \frac{N^{\leftrightarrow}}{L^{\leftrightarrow}} \right) + \left(\frac{N^0}{L^0} \right)} = -\frac{2 b_1}{3 F_1}$$

Inclusive asymmetries

- Measured cross section:

$$\sigma = \sigma^U \left[1 - P_B P_z A_{\parallel} + \frac{1}{2} P_{zz} A_{zz} \right]$$

- Inclusive **vector asymmetry** :

$$A_{\parallel} = \frac{1}{\langle P_B P_z \rangle} \cdot \frac{\left(\frac{N_{\leftarrow}^{\rightarrow}}{L_{\leftarrow}^{\rightarrow}} \right) - \left(\frac{N_{\rightarrow}^{\rightarrow}}{L_{\rightarrow}^{\rightarrow}} \right)}{\left(\frac{N_{\leftarrow}^{\rightarrow}}{L_{\leftarrow}^{\rightarrow}} \right) + \left(\frac{N_{\rightarrow}^{\rightarrow}}{L_{\rightarrow}^{\rightarrow}} \right)}$$

$$\frac{g_1}{F_1} = \frac{1}{1 + \gamma^2} \cdot \left(\frac{A_{\parallel}}{D} + (\gamma - \eta) A_2 \right)$$

Kinematic variables:

$$\gamma = \frac{\sqrt{Q^2}}{\nu}, \quad \eta = \eta(x, Q^2), \quad D = \frac{P_{\gamma^*}}{P_B}$$

- Inclusive **tensor asymmetry** :

$$A_{zz} = \frac{1}{\langle P_{zz} \rangle} \cdot \frac{\frac{2}{3} \left(\frac{N_{\leftarrow}^{\rightarrow}}{L_{\leftarrow}^{\rightarrow}} + \frac{N_{\rightarrow}^{\rightarrow}}{L_{\rightarrow}^{\rightarrow}} + \frac{N_{\leftarrow}^{\leftrightarrow}}{L_{\leftarrow}^{\leftrightarrow}} \right) - 2 \left(\frac{N^0}{L^0} \right)}{\frac{2}{3} \left(\frac{N_{\leftarrow}^{\rightarrow}}{L_{\leftarrow}^{\rightarrow}} + \frac{N_{\rightarrow}^{\rightarrow}}{L_{\rightarrow}^{\rightarrow}} + \frac{N_{\leftarrow}^{\leftrightarrow}}{L_{\leftarrow}^{\leftrightarrow}} \right) + \left(\frac{N^0}{L^0} \right)} = -\frac{2 b_1}{3 F_1}$$

Vector asymmetry

Virtual photon asymmetries:

$$A_1(x) = \frac{\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}}{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}} = \frac{g_1(x) - \gamma^2 g_2(x)}{F_1(x)}$$

$$A_2(x) = \frac{\sigma_{\text{TL}}}{\sigma_{\text{T}}} = \frac{\gamma(g_1(x) + g_2(x))}{F_1(x)}$$

\Rightarrow write A_{\parallel} as:

$$A_{\parallel} = D(A_1 + \eta A_2)$$

Used parameterizations

$$A_2 \sim M_p \frac{x}{\sqrt{Q^2}}$$

$$F_2^p = F_2^p(\text{ALLM}), \quad F_2^d = \frac{1}{2} F_2^p \underbrace{\left(\frac{F_2^n}{F_2^p} \right)}_{\text{NMC}}$$

$$R = R(1990)$$

with

$$R = \sigma_L / \sigma_T$$

and using

$$F_1 = F_2 \frac{1 + \gamma}{2x(1 + R)}$$

Kinematic variables:

$$\gamma = \frac{\sqrt{Q^2}}{\nu}$$

$$\eta = \eta(x, Q^2)$$

$$D = \frac{P_{\gamma^*}}{P_{\text{beam}}}$$

Monte Carlo productions

$$A_{\text{Born}} = A_X \cdot \left(1 + \frac{\Delta\sigma^{\text{U}}}{\sigma_{\text{Born}}^{\text{U}}} \right) - \frac{\Delta\sigma^{\text{P}}}{\sigma_{\text{Born}}^{\text{U}}}$$

- Two MC samples with same input parameterizations $F_2(\text{ALLM})$, $R \equiv \sigma_L/\sigma_T(1990)$, A_{Born}
 1. eXperimental MC: $\Delta\sigma^{\text{P(U)}}$
 - \rightsquigarrow Observed cross section
 - ✓ Radiative corrections (RADGEN)
 - ✓ Tracking through detector (GEANT)
 - \Rightarrow contains info about bin migrations:
 - ✓ Migration matrix $M(i, j)$
 - ✓ Background from outside acceptance $\Delta\hat{\sigma}$
 2. Born MC: $\sigma_{\text{Born}}^{\text{U}}$
 - \rightsquigarrow Born cross section within HERMES acceptance
- MC data 9 times higher statistical accuracy as real data
- MC data subject to same cuts as real data

Unfolding of kinematic migrations

- Unfolding: Removes **systematic** correlations at the cost of introducing **statistical** correlations
- Unfolding algorithm:
 - ✓ No iteration needed
 - ✓ Use **eXp** and **Born** MC to correct asymmetry
- Smearing matrix $S(i, j)$:

$$S(i, j) \equiv \frac{M(i, j)}{\sigma_{\text{Born}}(j)}$$

\leadsto invert \rightarrow “un-smearing” matrix!

- A_X corrected in every bin for unpolarized and polarized background (not re-sorting event by event)

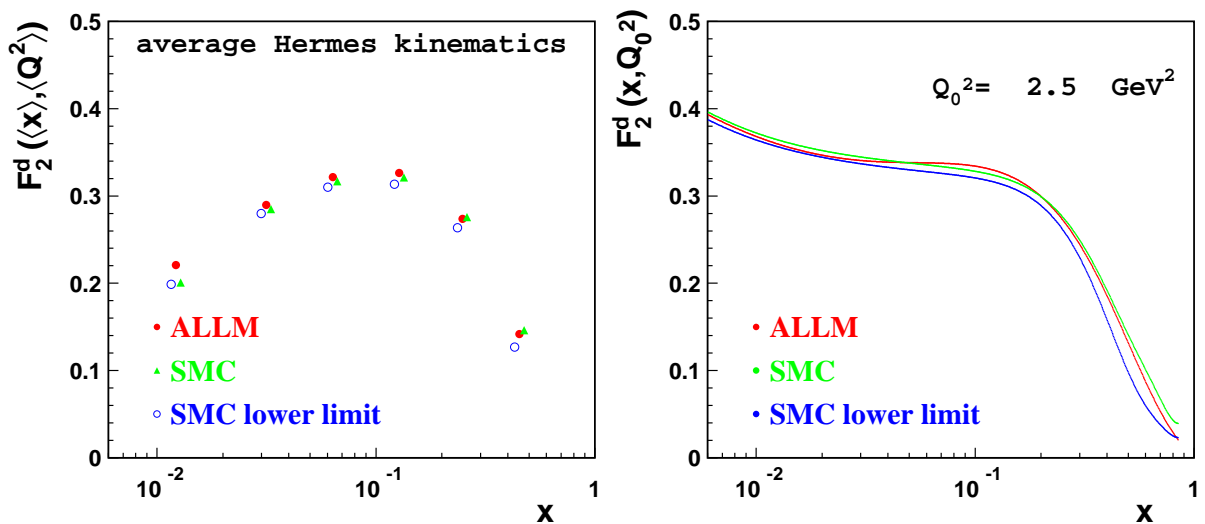
$$A_{\text{Born}}(j) = -2 + \frac{6}{\sigma_{\text{Born}}^{\text{U}}(j)} \cdot \sum_{i=1}^n [S']^{-1}(j, i) \times$$
$$\left[\frac{1}{2} A_X(i) \sigma_X^{\text{U}}(i) - \frac{1}{2} \Delta \hat{\sigma}^{\text{P}}(i) + \sum_{k=1}^n S^-(i, k) \sigma_{\text{Born}}^{\text{U}}(k) \right]$$

Model independence of unfolding

$$S(i, j) \equiv \frac{M(i, j)}{\sigma_{\text{Born}}(j)}$$

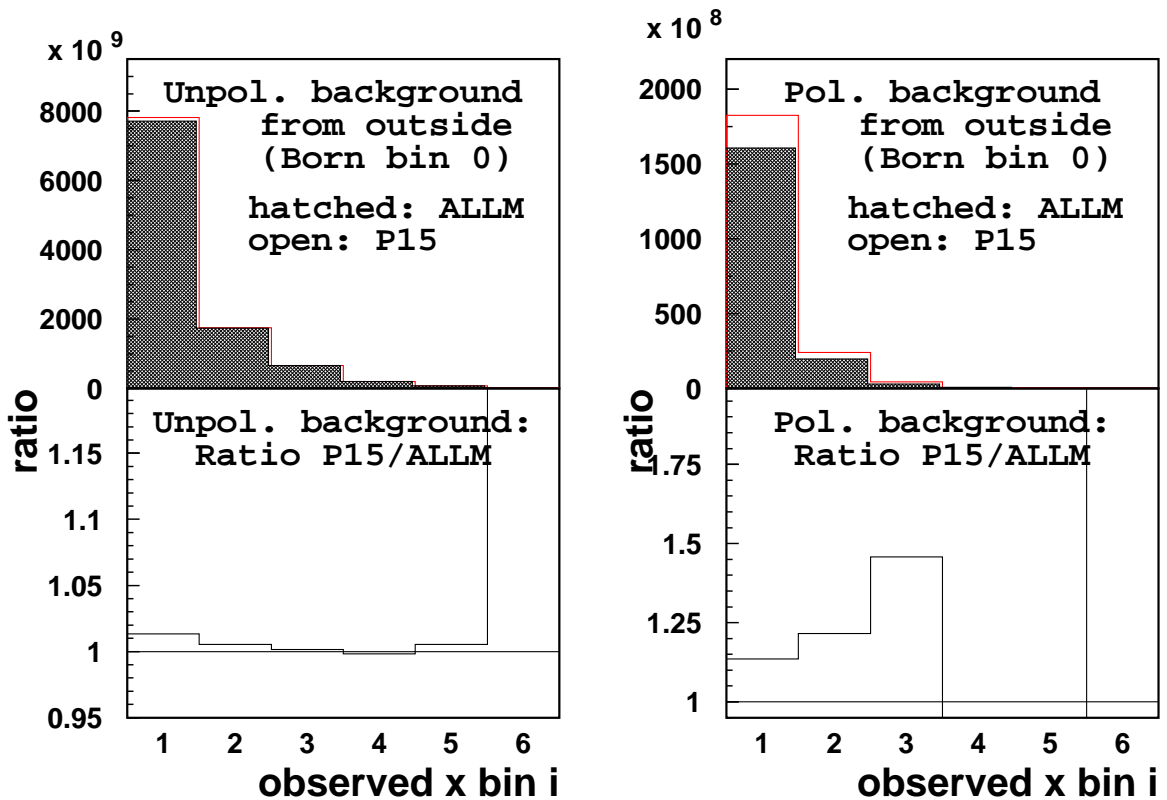
- Both ● and ● scale with # of generated events in Born bin j
- # of generated events controlled by input parameterizations

⇒ algorithm is independent of MC input parameterization
 except for **polarized background entering acceptance** $\Delta \hat{\sigma}^P$
 ⇒ close-to model independent



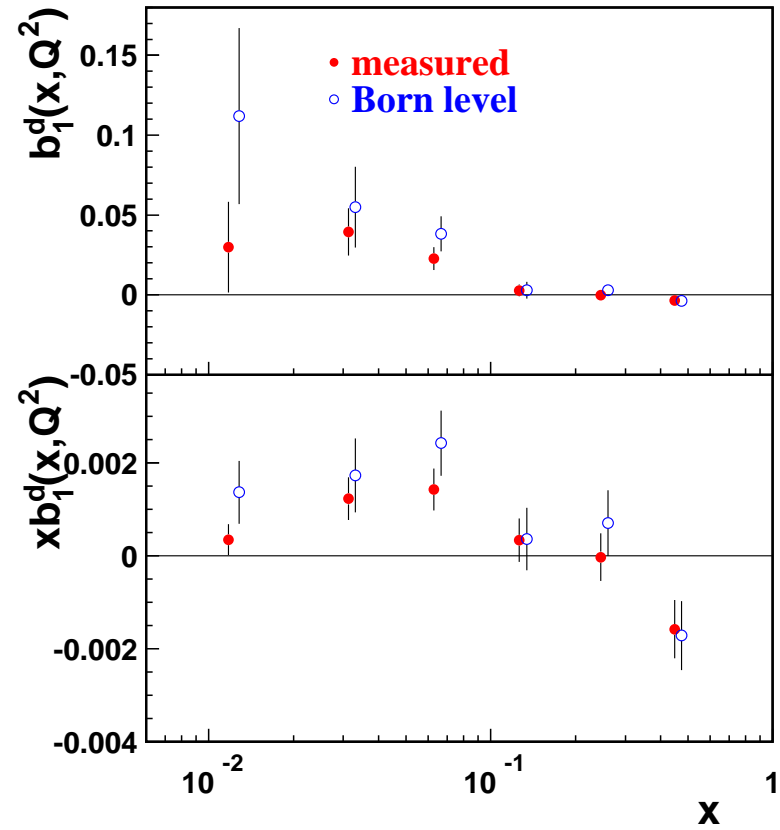
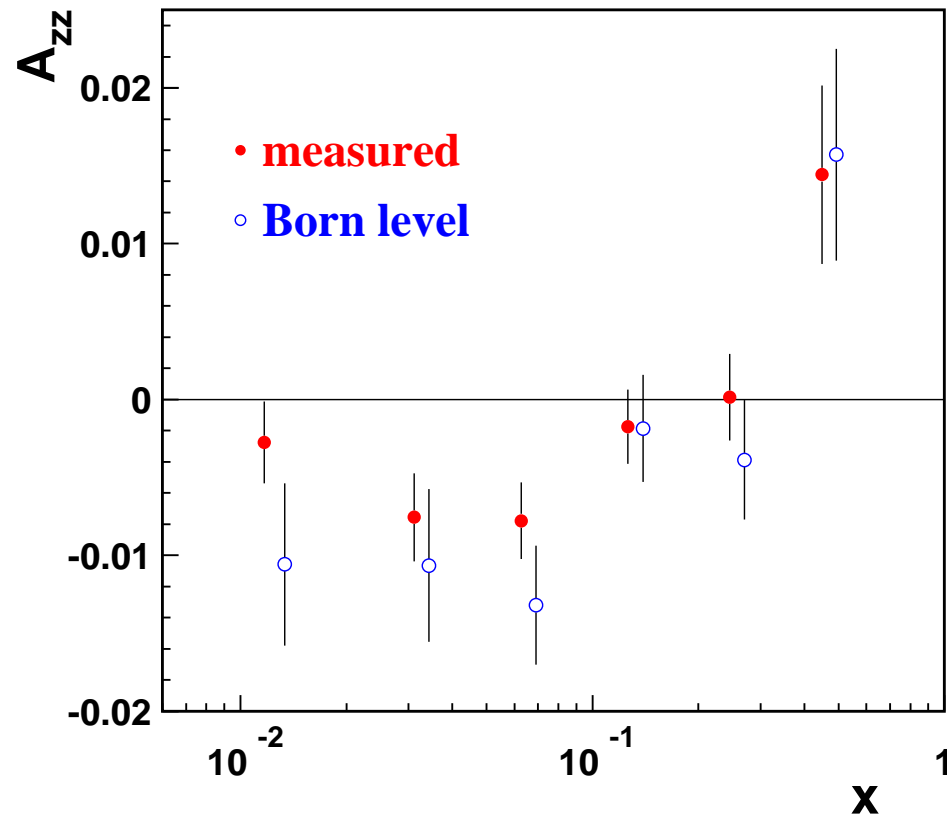
Impact of MC parameterizations

- Compare $\Delta \hat{\sigma}$ from $F_2(\text{ALLM})$ and $F_2(\text{SMC})$



- Unfolding results compatible within statistics
- No systematic error assigned

Comparison: Measured and Born level



Covariance and correlation matrix

Covariance matrix for unfolded asymmetry:

$$\text{cov}(j, i) = \sum_{k=1}^n \frac{\partial A_{\text{Born}}(j)}{\partial A_X(k)} \frac{\partial A_{\text{Born}}(i)}{\partial A_X(k)} \delta A_X^2(k)$$

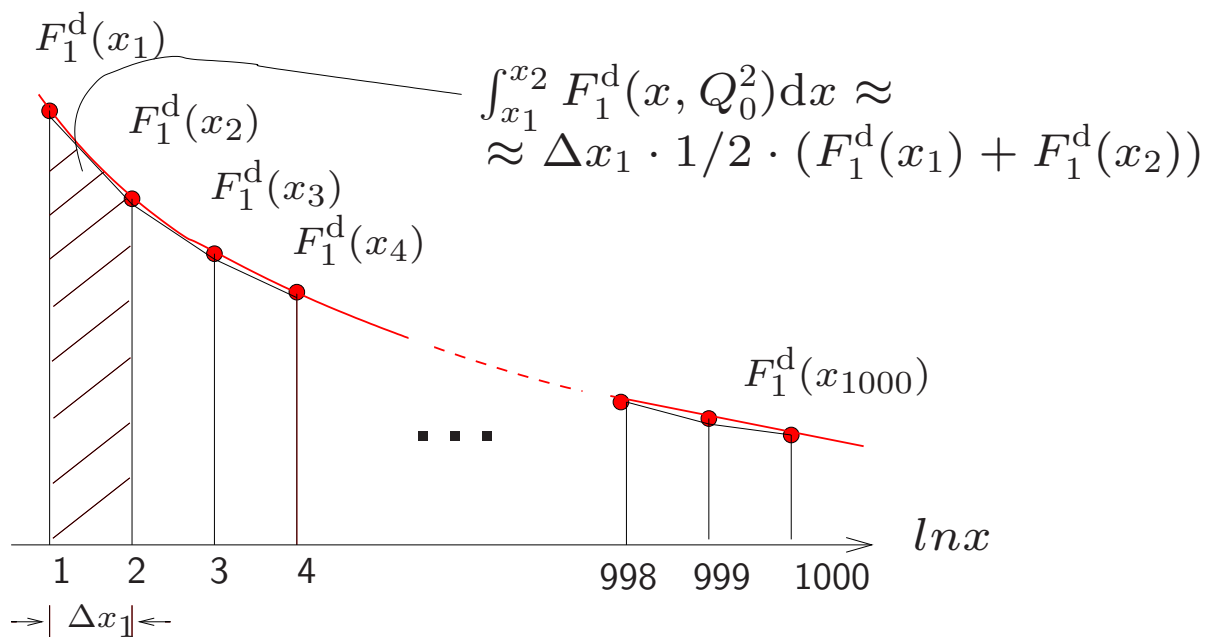
⇒ Correlation matrix:

$$\text{corr}(j, i) = \frac{\text{cov}(j, i)}{\delta A_{\text{Born}}(j) \delta A_{\text{Born}}(i)}$$

with

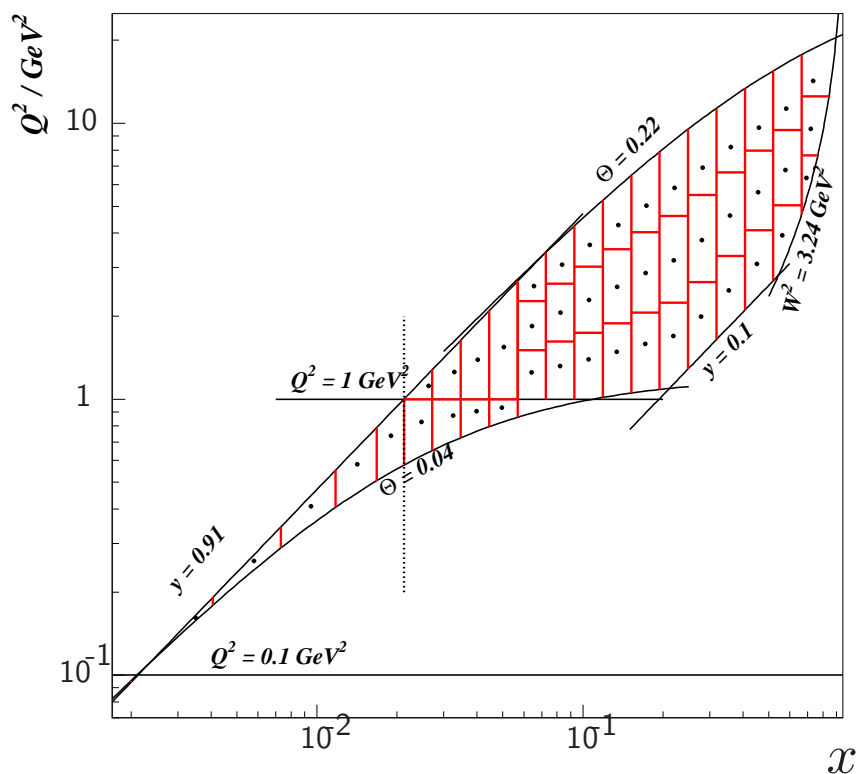
$$\text{cov}(j, j) = \delta A_{\text{Born}}^2(j)$$

$$\text{corr}(j, j) = 1$$



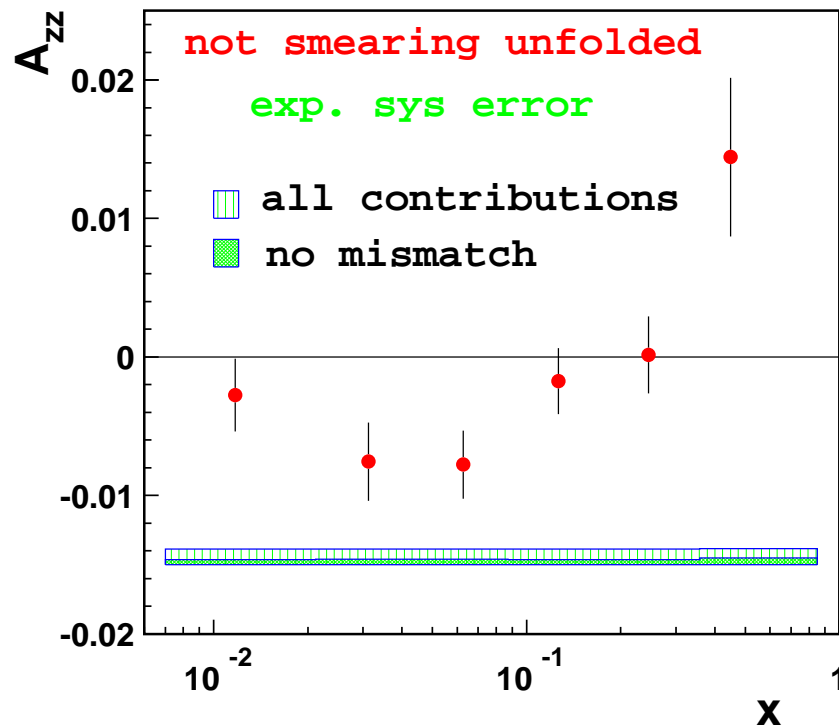
2-dimensional binning in x and Q^2

- 46 bins in x and Q^2 (20 in x , at most 3 in Q^2):



- Benefit of binning in Q^2 :
 - Higher average Q^2
 - Improvement of statistical power after unfolding (diagonal element of correlation matrix)
- $A_{||}$ unfolded in x and Q^2 , then Q^2 -average of g_1/F_1

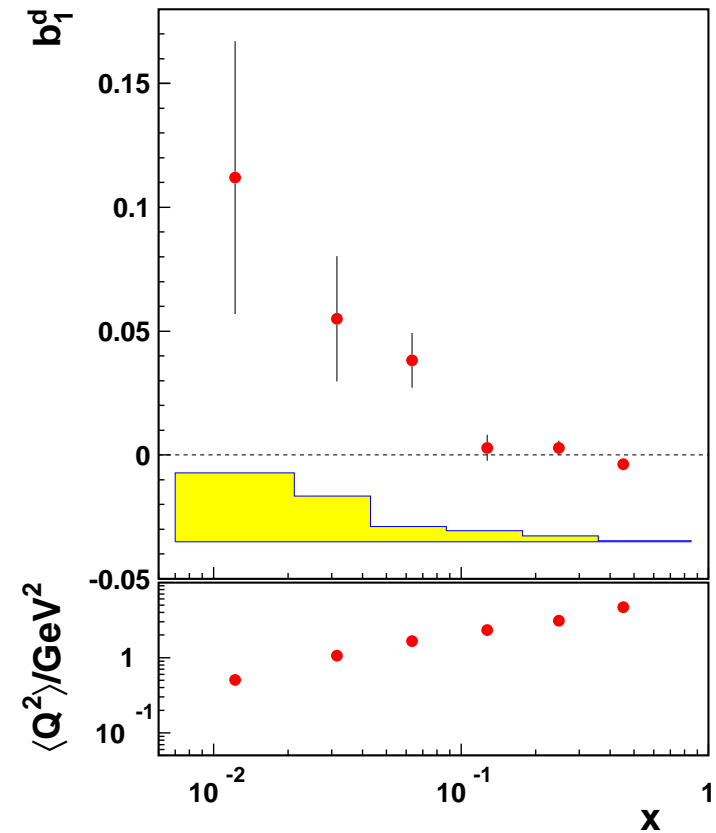
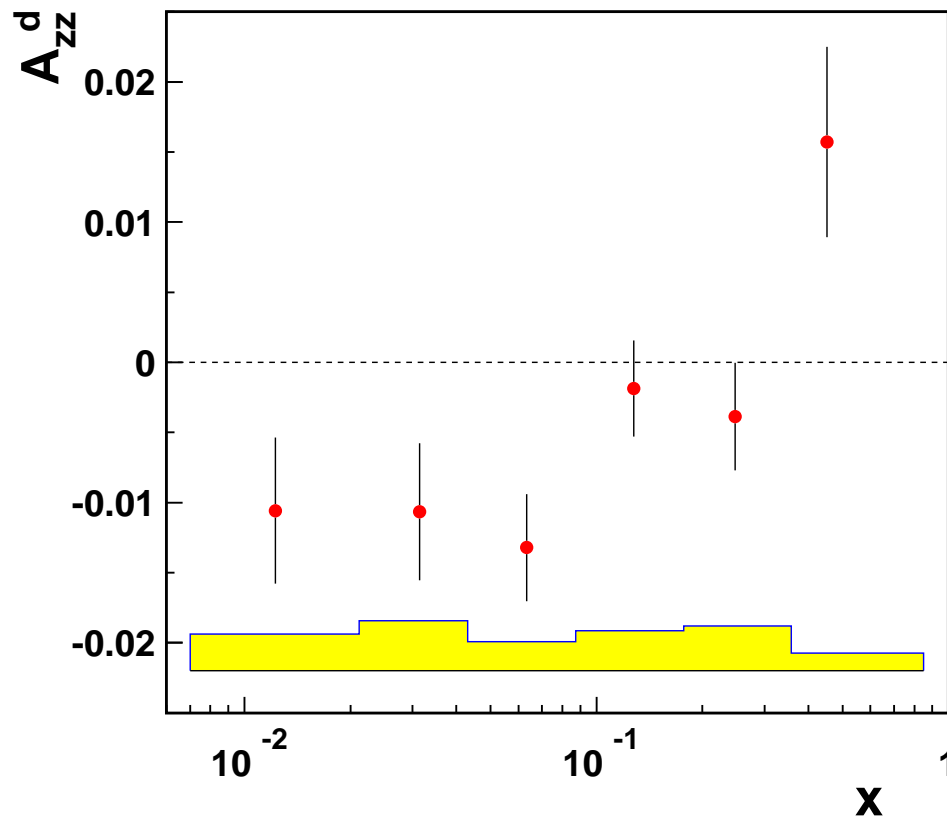
Measured tensor asymmetry



Experimental systematic uncertainties:

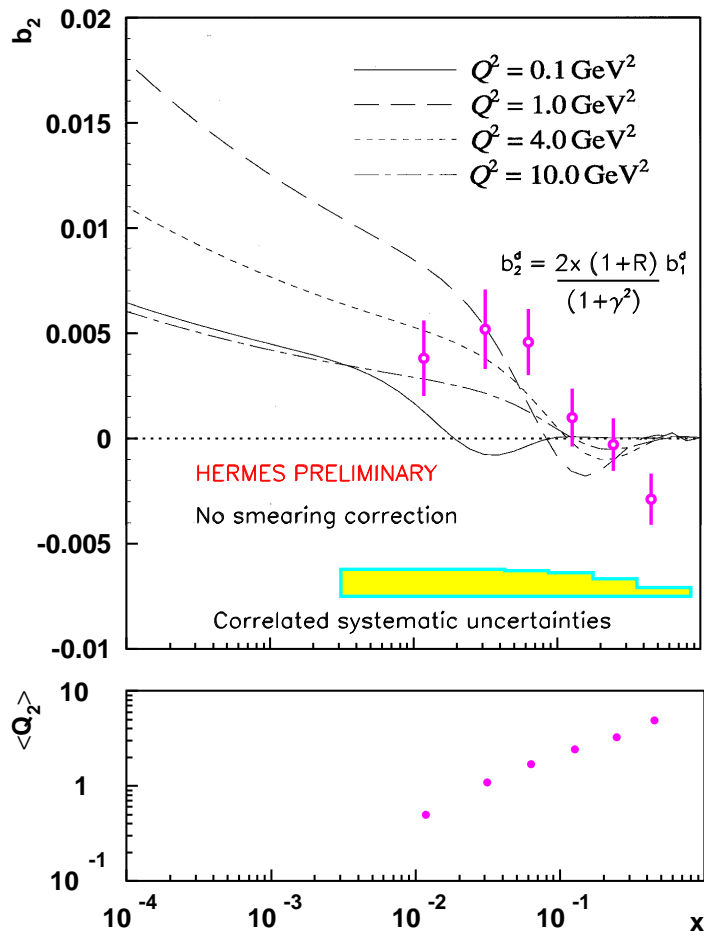
- Target nuclear polarization, density, residual shell electron polarization
- Hadron contamination
- “Mismatch” between two methods to obtain vector averaged data sample

Final results: Tensor analysis



Systematic uncertainties: experimental + electromagnetic background + detector misalignment (MC studies)

b_1^d, b_2^d and model calculations



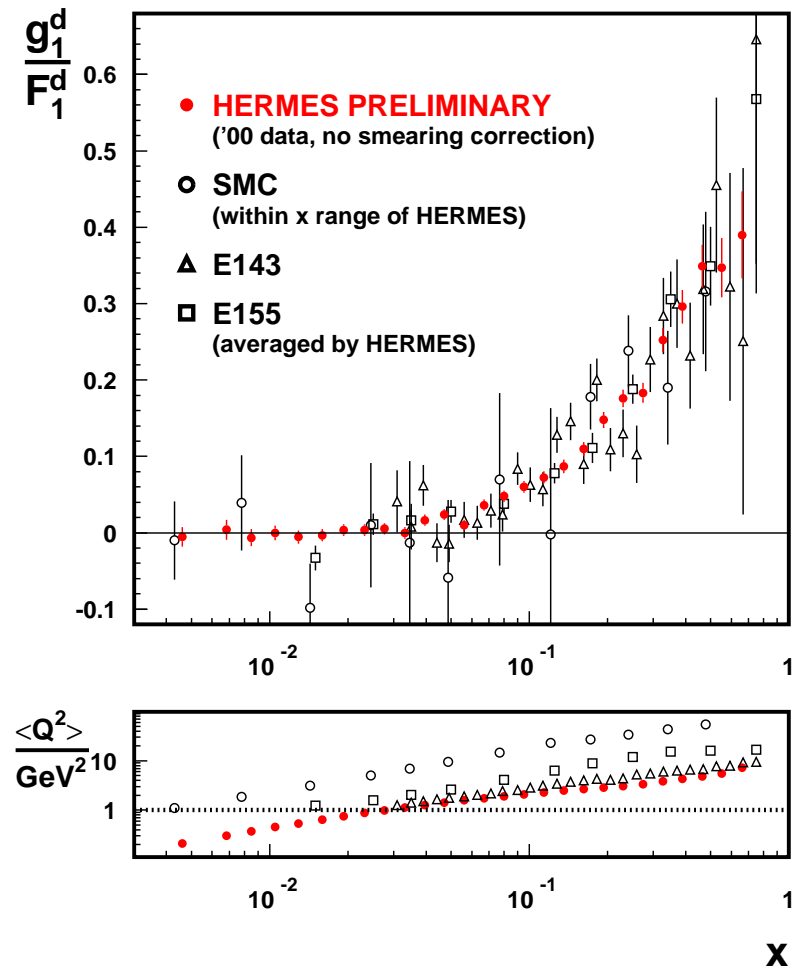
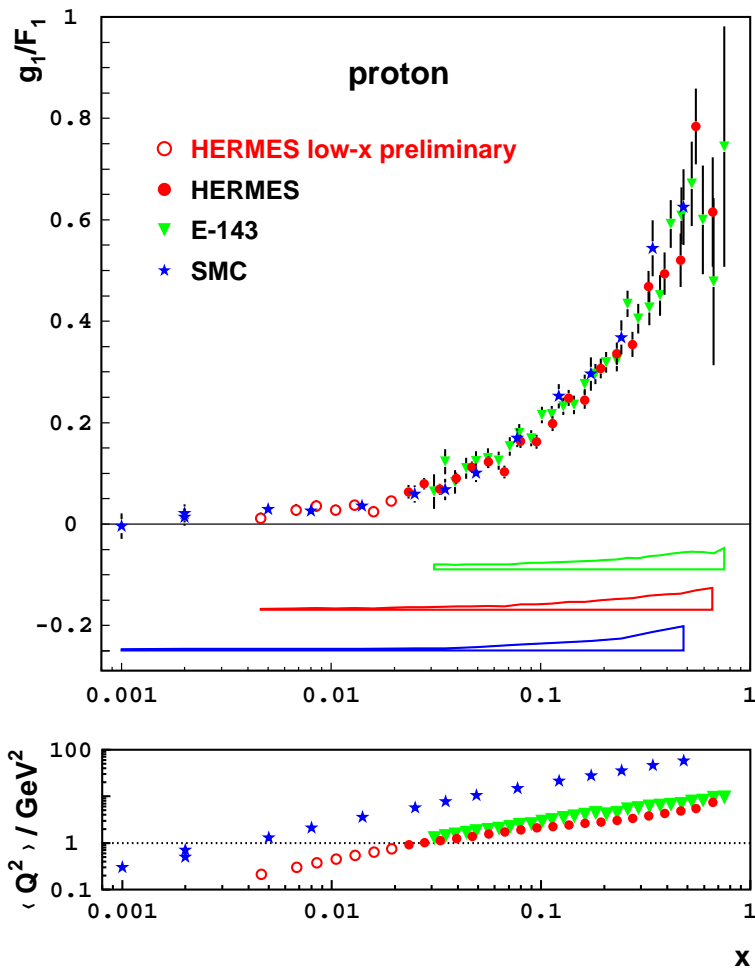
$\mathcal{O}(b_1^d) \xleftrightarrow{\checkmark}$ latest model calculations

- deuteron: D-state admixture
 \Rightarrow el. quadrupole moment $\neq 0$
- \hookrightarrow double scattering mechanisms with a significant contribution to b_1 at small x
 (e.g. Nikolaev *et al.*, *Phys. Lett. B* **398** (1997) 245)
- Callan-Gross relation \Rightarrow

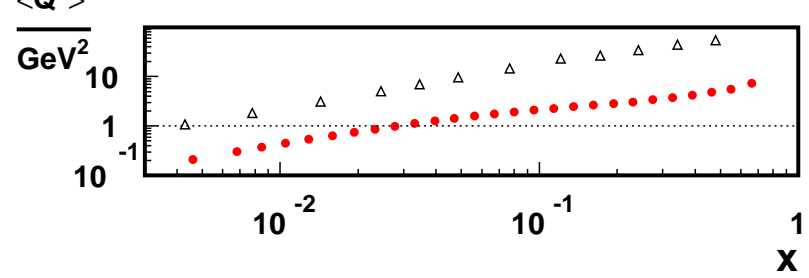
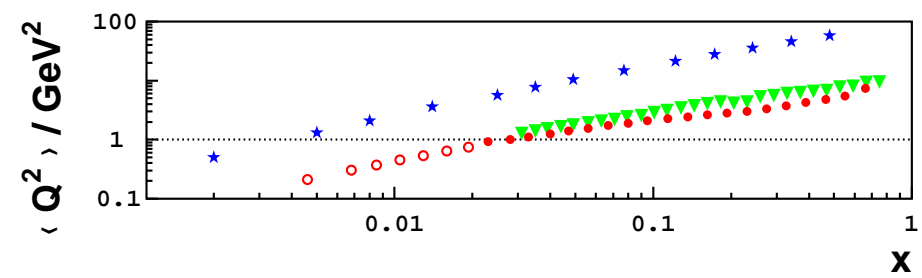
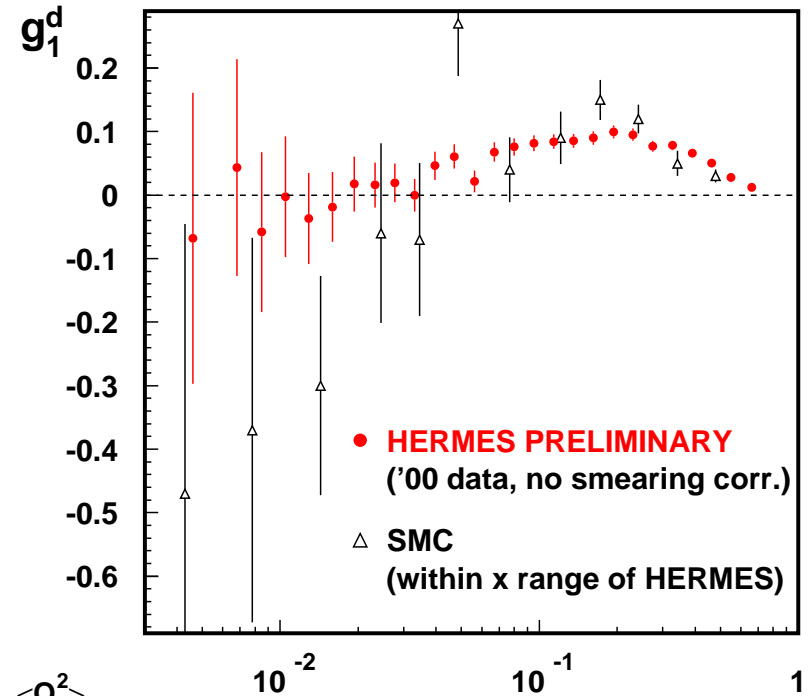
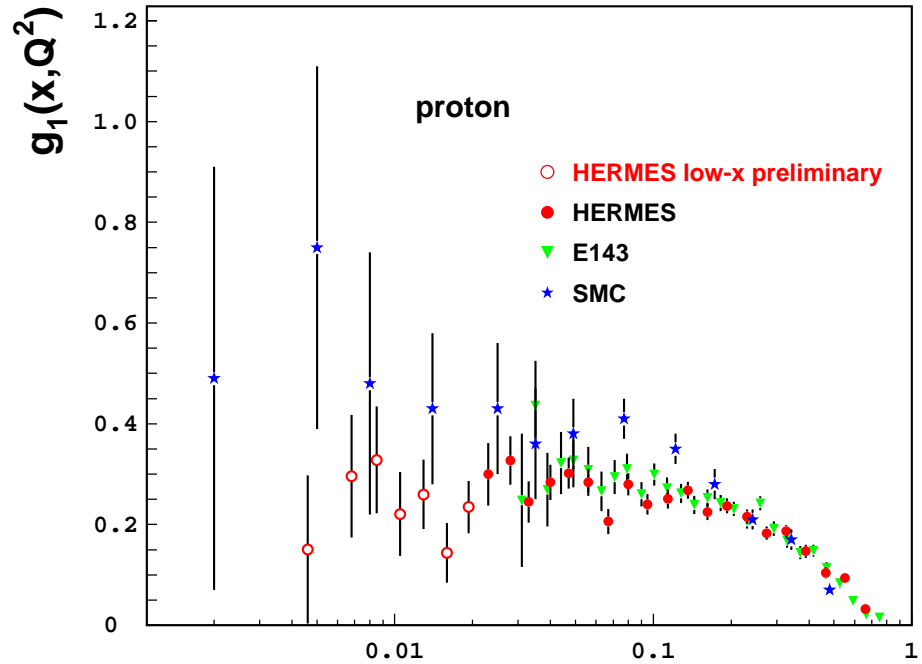
$$b_2^d = \frac{2x(1+R)}{1+\gamma^2} b_1^d$$

Theory curves: Bora *et al.*, *Phys. Rev. D* **57** (1998) 6906

Old HERMES release: g_1/F_1 of p and d



Old HERMES release: g_1 of p and d



Old HERMES release: g_1 world data

