The HERMES *measurement of transverse single-spin asymmetries*

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on behalf of the

hermes

collaboration

Special thanks to our analysis crew:

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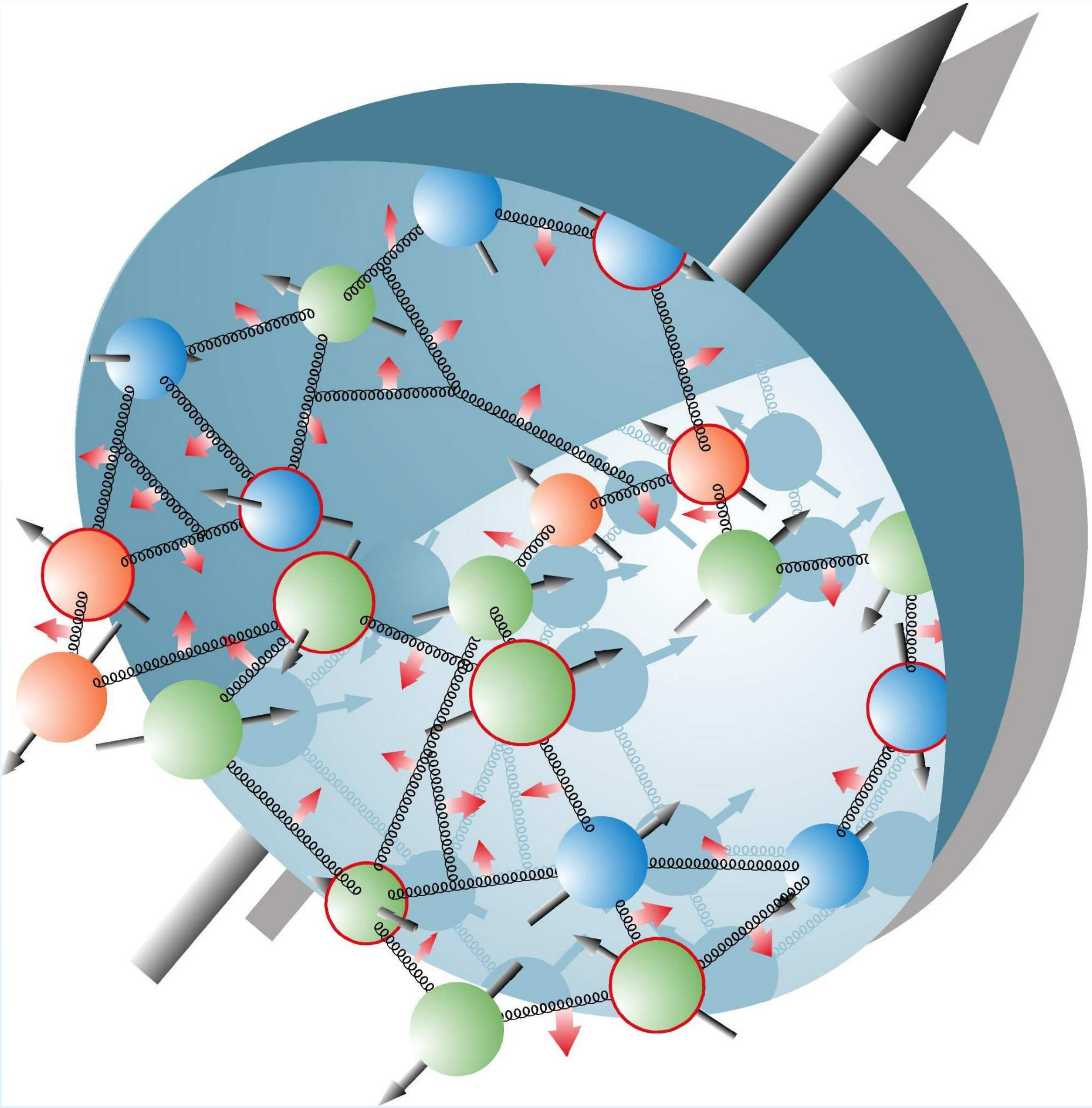
The spin structure of the nucleon:

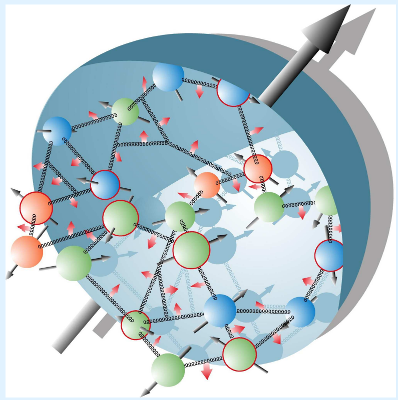


The spin structure of the nucleon:

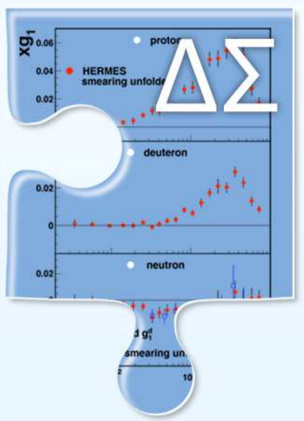
Angular momentum sum rule:

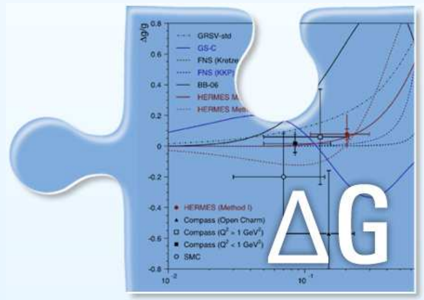
$$\frac{s_z^N}{\hbar} = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

Spin of the nucleon



HERMES contributions to the spin puzzle:



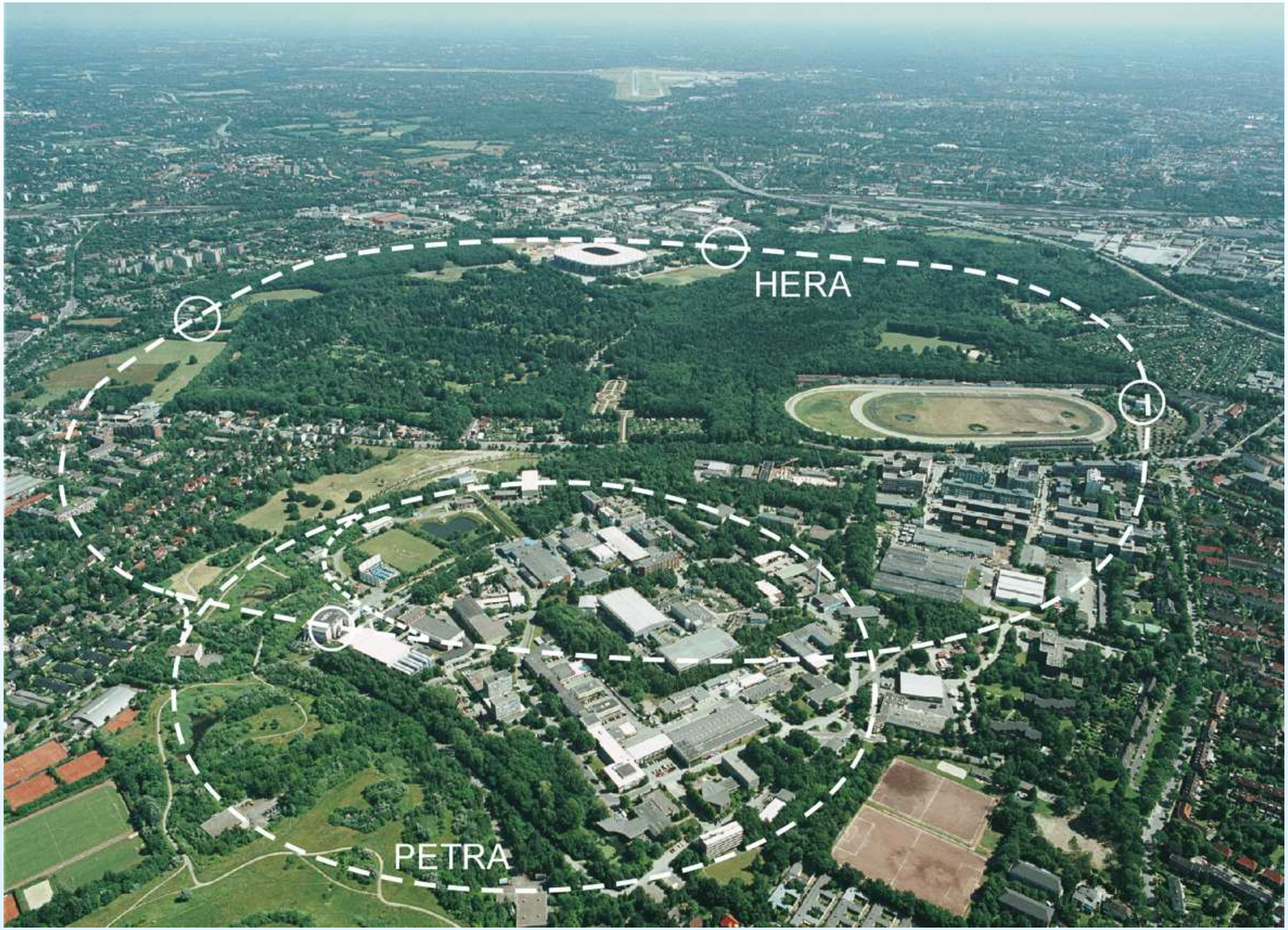


Measurement of transverse spin phenomena:

➥ L_q

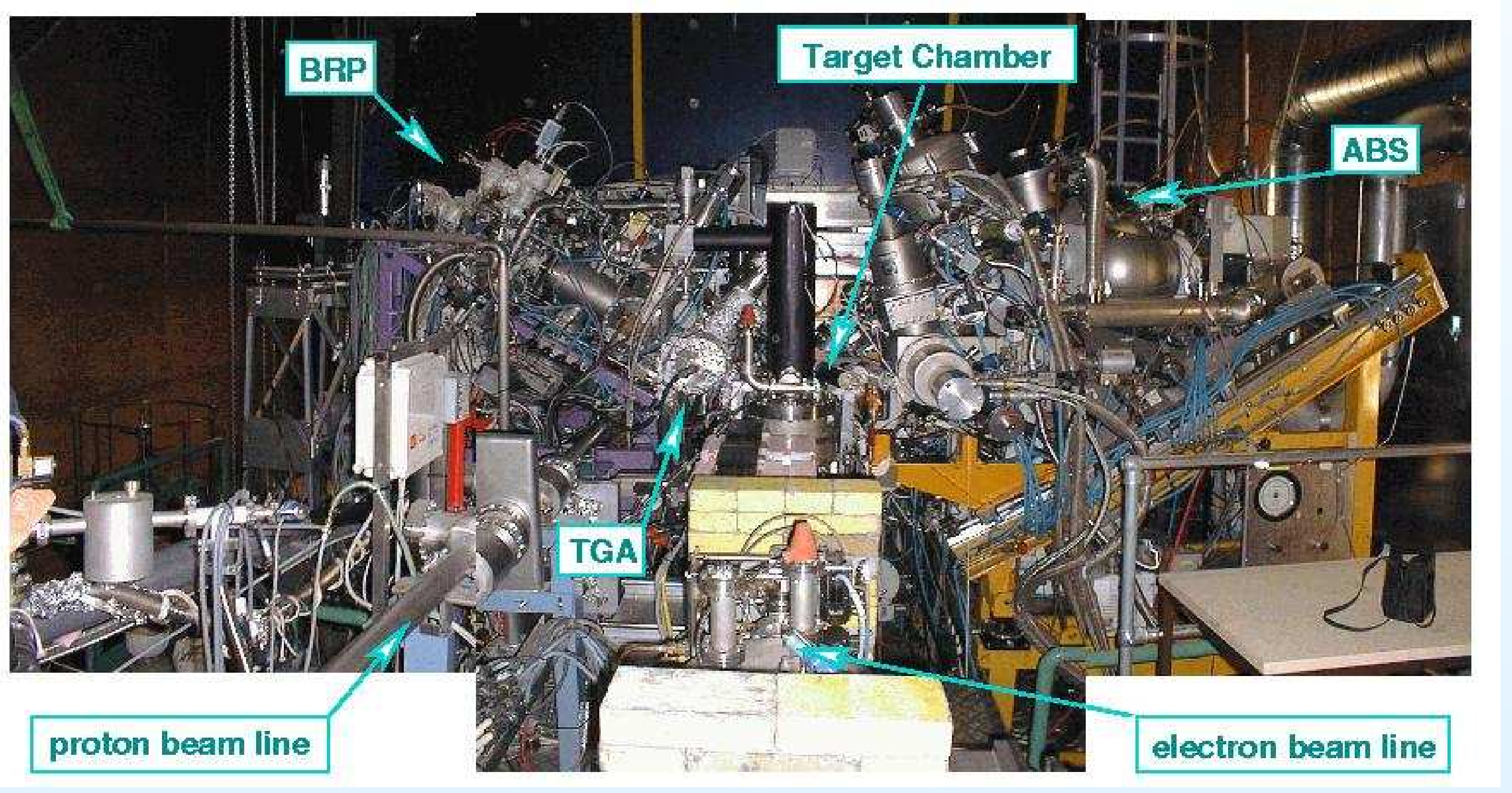
➥ **transversity measurements**

The HERMES (polarised scattering) experiment:



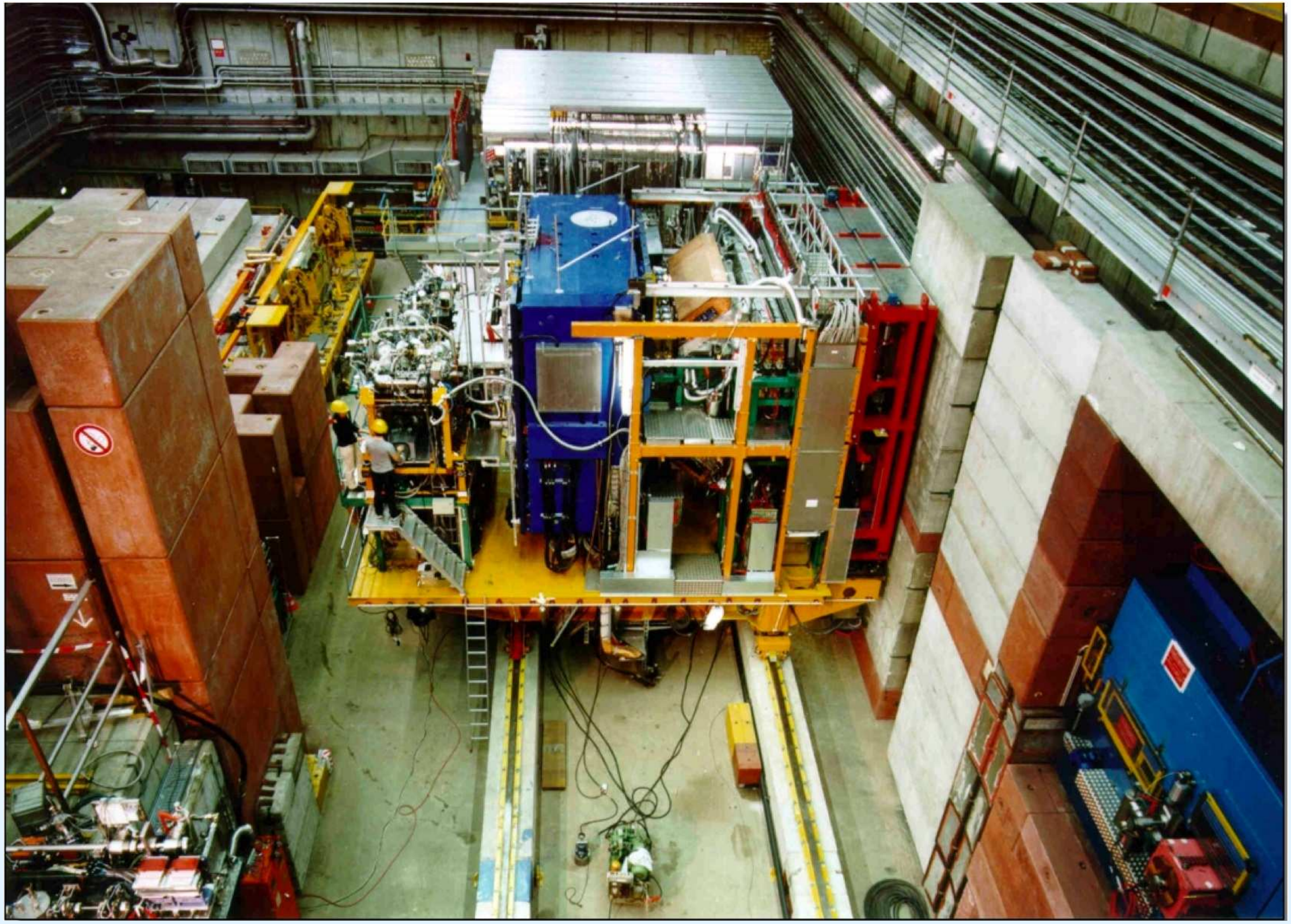
The transversely polarised target:

- polarised **gas target** internal to the HERA storage ring
- background-free measurements from highly polarised nucleons
- **2002–2005: transversely polarised hydrogen** target

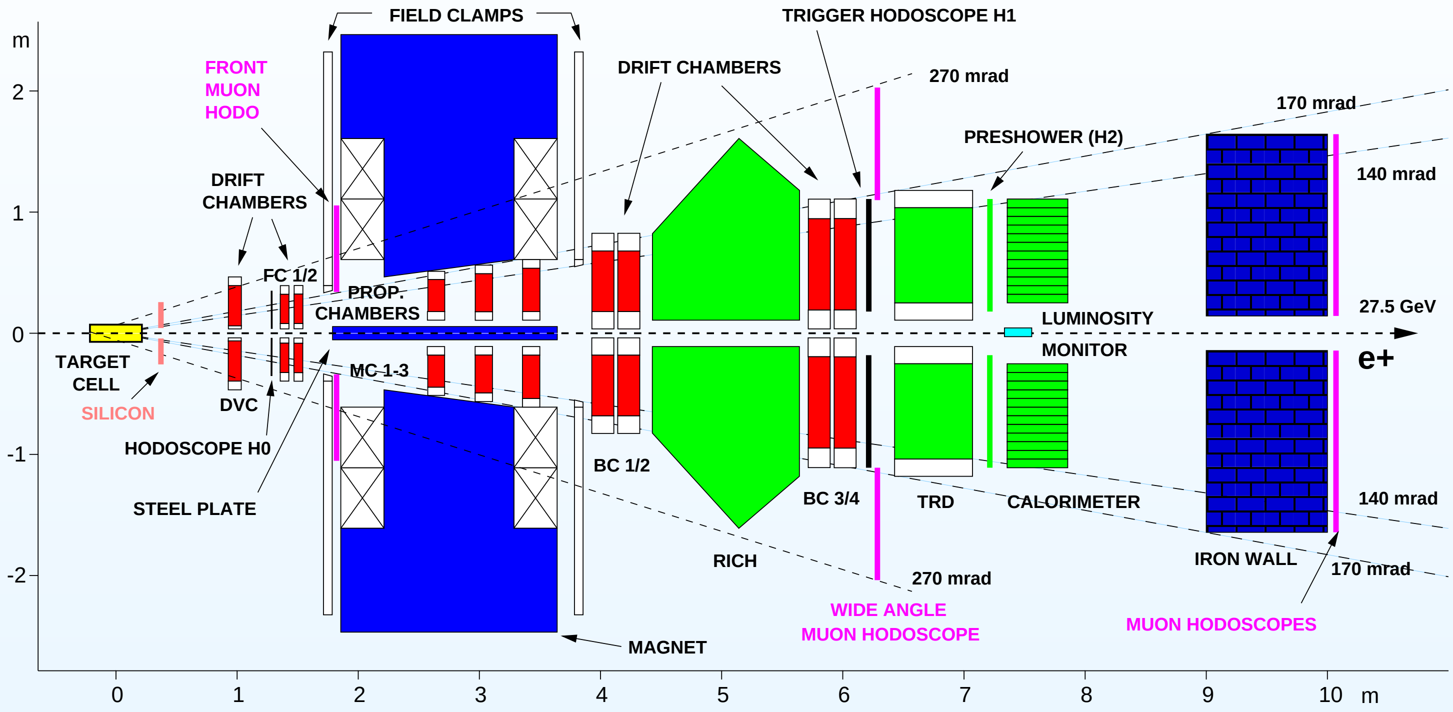


(front view of the HERMES interaction region)

The HERMES spectrometer:



The HERMES spectrometer:



- large momentum and angle acceptance: $\theta_{\text{hor.}} \leqslant 170\,\mathtt{mrad}$, $40\,\mathtt{mrad} \leqslant \theta_{\text{vert.}} \leqslant 140\,\mathtt{mrad}$
- good momentum resolution: $\Delta p/p \leqslant 0.026$
- and good angle resolution: $\Delta\theta \leqslant 0.6\,\mathtt{mrad}$
- very clean lepton-hadron separation and hadron identification

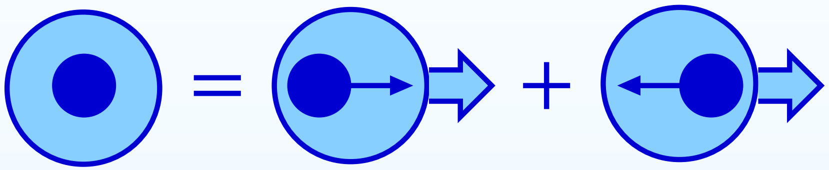
Transverse spin phenomena:

(courtesy of Alessandro Bacchetta (JLAB))

Leading twist description of quark momentum and spin:

momentum distribution $q(x)$:

measures spin average

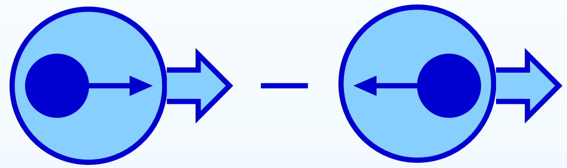


(in helicity basis)

$$F_1(x) = \frac{1}{2} \sum e_q^2 \left(q(x) - \bar{q}(x) \right)$$

helicity distribution $\Delta q(x)$:

measures helicity difference



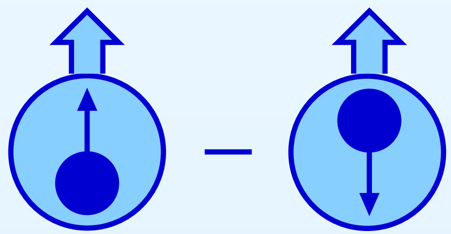
(in helicity basis)

$$g_1(x) = \frac{1}{2} \sum e_q^2 \left(\Delta q(x) + \Delta \bar{q}(x) \right)$$

transversity distribution $\delta q(x) / h_1^q(x)$:

- helicity flip amplitude
- non-relativistic quarks: $\delta q(x) = \Delta q(x)$
- no gluon transversity at nucleon target

probabilistic interpretation:



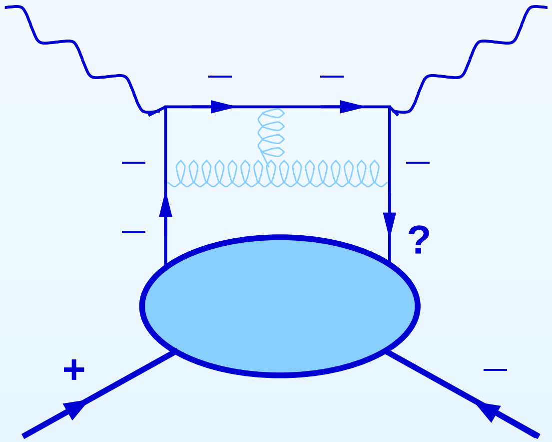
(in basis of transverse spin eigenstates)

Measurement of the transversity distribution:

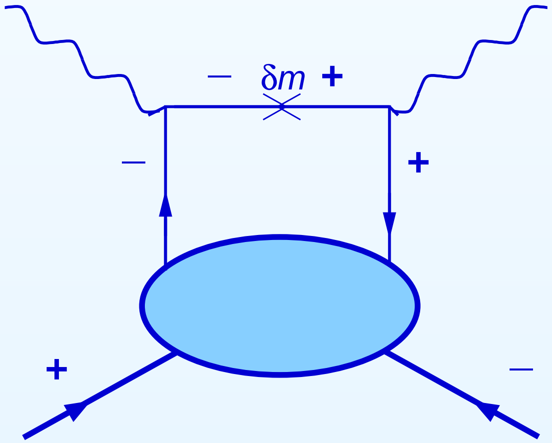
- **Chirality of the transversity distribution**
  - transversity distribution measures helicity flip

$$N^{\Uparrow} q^{\downarrow} \longrightarrow N^{\Downarrow} q^{\uparrow}$$

  - **chiral-odd** quark distribution:



cannot be determined in inclusive DIS



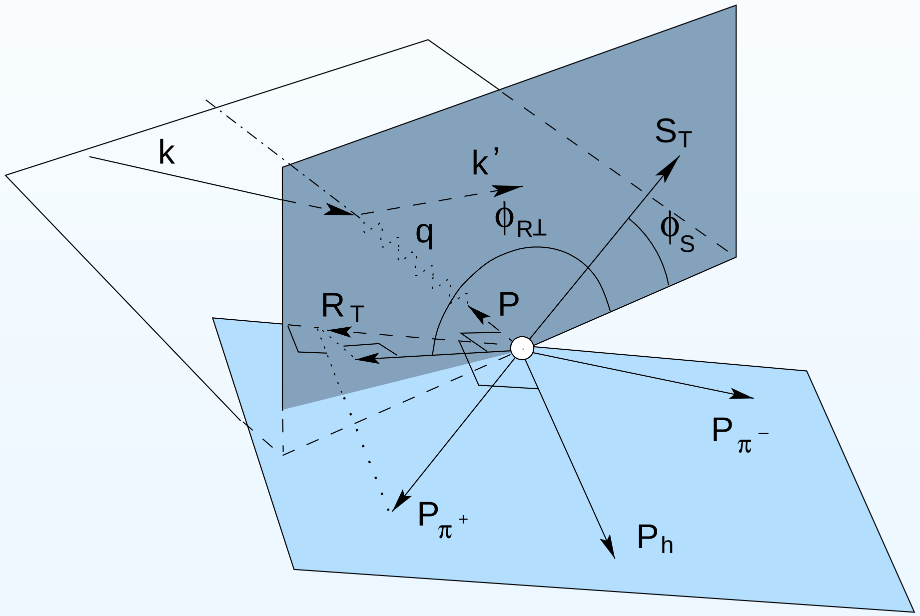
another chiral-odd function is needed

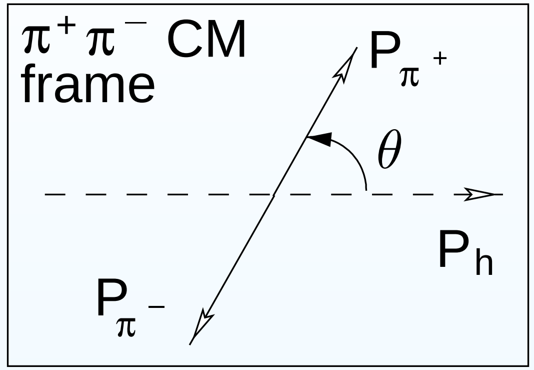
- **HERMES measurements:** transverse single-spin asymmetries semi-inclusive DIS on a transversely polarised hydrogen target

Transverse single-spin asymmetries:

- **naive time reversal odd** (naive-T-odd) **functions**
- involve interference of amplitudes with different helicities
  ➥ suppressed in perturbative QCD
  ➥ assigned to distribution and fragmentation functions
- **associated with spin/orbit effects ($\boldsymbol{S} \cdot (\boldsymbol{P_1} \times \boldsymbol{P_2})$)**
- observed in semi-inclusive DIS on a transversely polarised target:
  - **single-hadron production** ($ep^{\Uparrow} \rightarrow e'hX$):
    - $\boldsymbol{S_q} \cdot (\boldsymbol{p_q} \times \boldsymbol{P_h})$
      ➥ *Collins mechanism*, sensitive to transversity
    - $\boldsymbol{S_N} \cdot (\boldsymbol{P} \times \boldsymbol{p_q})$
      ➥ *Sivers mechanism*, sensitive to L_q
  - **dihadron production** ($ep^{\Uparrow} \rightarrow e'h_1h_2X$):
    - $\boldsymbol{S_q} \cdot (\boldsymbol{p_q} \times \boldsymbol{R})$
    - transfer of transverse quark spin to relative orbital angular momentum of hadron pair ($2\boldsymbol{R} = \boldsymbol{P_{h_1}} - \boldsymbol{P_{h_1}}$)
    - sensitive to transversity

The semi-inclusive production of $\pi^+\pi^-$ pairs:





$$\boldsymbol{P_h} \equiv \boldsymbol{P_{\pi^+}} + \boldsymbol{P_{\pi^-}}$$

$$\boldsymbol{R} \equiv \frac{\boldsymbol{P_{\pi^+}} - \boldsymbol{P_{\pi^-}}}{2}$$

$$\boldsymbol{R_T} \equiv \boldsymbol{R} - (\boldsymbol{R} \cdot \hat{\boldsymbol{P}}_{\boldsymbol{h}})\hat{\boldsymbol{P}}_{\boldsymbol{h}}$$

azimuthal angles ϕ_S and ϕ_{R_T}:

$$\phi_S \equiv \frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{S_T}}{|(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{S_T}|} \arccos\left(\frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot(\boldsymbol{q}\times\boldsymbol{S_T})}{|(\boldsymbol{q}\times\boldsymbol{k})|\,|\boldsymbol{q}\times\boldsymbol{S_T}|}\right)$$

$$\phi_{\boldsymbol{R}_\perp} \equiv \frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{R_T}}{|(\boldsymbol{q}\times\boldsymbol{k})\cdot\boldsymbol{R_T}|} \arccos\left(\frac{(\boldsymbol{q}\times\boldsymbol{k})\cdot(\boldsymbol{q}\times\boldsymbol{R_T})}{|(\boldsymbol{q}\times\boldsymbol{k})|\,|\boldsymbol{q}\times\boldsymbol{R_T}|}\right)$$

SSA in semi-inclusive $\pi^+\pi^-$ production:

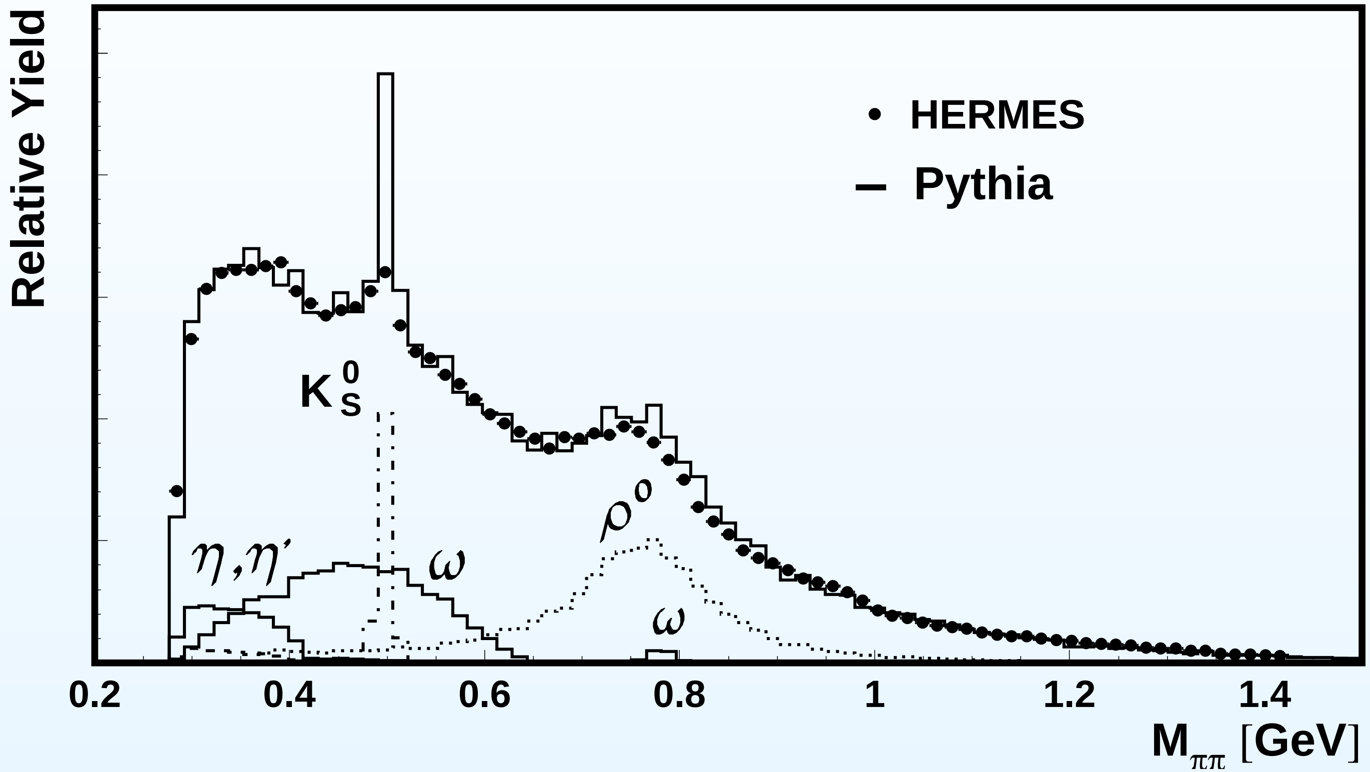
- Fourier/Legendre amplitude $\boldsymbol{A_{UT}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta}}$ of

$$A_{UT}\left(x,y,z,\phi_S,\phi_{R\perp},\cos\theta,M_{\pi\pi}\right) = \frac{1}{|\boldsymbol{S}_T|}\frac{\mathrm{d}^7\sigma_{\mathrm{U}\Uparrow} - \mathrm{d}^7\sigma_{\mathrm{U}\Downarrow}}{\mathrm{d}^7\sigma_{\mathrm{U}\Uparrow} + \mathrm{d}^7\sigma_{\mathrm{U}\Downarrow}}$$

- provides signal for
  - transversity distribution $h_1^q(x)$
  - dihadron fragmentation function $\boldsymbol{H_{1,q}^{\sphericalangle}(z,M_{\pi\pi},\cos\theta)}$:
    - leading-twist
    - chiral-odd
    - naive-T-odd
- at leading twist, in leading order in α_s, integrated over $P_{h\perp}$:

$$A_{UT}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta} = -\frac{(1-y)}{\left(1-y+\frac{y^2}{2}\right)}\,\frac{1}{2}\sqrt{1-4\frac{M_\pi^2}{M_{\pi\pi}^2}}\;\frac{\sum_q e_q^2\,h_1^q(x)\,H_{1,q}^{\sphericalangle,sp}(z,M_{\pi\pi})}{\sum_q e_q^2\,f_1^q(x)\,D_{1,q}(z,M_{\pi\pi})}$$

The $M_{\pi\pi}$ spectrum:



- sizable contribution from spin-1 resonances
- dominant **contributions to** $H_{1,q}^{\sphericalangle}(z, M_{\pi\pi}, \cos\theta)$:
  - s**-waves** components, e.g. $\pi^+\pi^-$pair in non-resonant state
  - p**-waves** components, e.g. ρ^0 decay ($\rho^0 \rightarrow \pi^+\pi^-$)

Evaluation of the asymmetry:

- all $\pi^+\pi^-$ pairs have been selected from $ep^{\Uparrow} \rightarrow e'h_1h_2X$
- kinematic requirements:

$$\begin{array}{rcl} 1\,\mathtt{GeV}^2 < & Q^2 & \\ (0.1 \leqslant) & y & < 0.85 \\ 10\,\mathtt{GeV}^2 < & W^2 & \\ 2\,\mathtt{GeV} < & M_X & \\ 1\,\mathtt{GeV} < & \boldsymbol{P}_h & < 15\,\mathtt{GeV} \end{array}$$

- for every bin in
  - x, z $(M_{\pi\pi} \in [0.5{,}1.0])$
  - $M_{\pi\pi}$ $(M_{\pi\pi} \in [0.5{,}1.0])$
- evaluation in $(\phi_{R\perp} + \phi_S) \times \theta$ binning:

$$A_{U\perp}(\phi_{R\perp},\phi_S,\theta) = \frac{1}{|\boldsymbol{S}_T|}\frac{N^{\Uparrow}(\phi_{R\perp},\phi_S,\theta) - N^{\Downarrow}(\phi_{R\perp},\phi_S,\theta)}{N^{\Uparrow}(\phi_{R\perp},\phi_S,\theta) + N^{\Downarrow}(\phi_{R\perp},\phi_S,\theta)}$$

The extraction of $A_{UT}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta}$:

$$A_{UT}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta} \sim \frac{\sum_q e_q^2\, h_1^q(x)\, H_{1,q}^{\sphericalangle,sp}(z,M_{\pi\pi})}{\sum_q e_q^2\, f_1^q(x)\, D_{1,q}(z,M_{\pi\pi})}$$

- focus on **sp- and pp-interference** ($M_{\pi\pi} < 1.5$ GeV):

$$\begin{aligned} D_{1,q} &\simeq D_{1,q} + D_{1,q}^{sp}\cos\theta + D_{1,q}^{pp}\frac{1}{4}(3\cos^2\theta - 1) \\ H_{1,q}^{\sphericalangle} &\simeq H_{1,q}^{\sphericalangle,sp} + H_{1,q}^{\sphericalangle,pp}\cos\theta, \end{aligned}$$

- symmetrisation around $\theta = \pi/2$:

$$\theta \to \theta' \equiv \left|\left|\theta - \frac{\pi}{2}\right| - \frac{\pi}{2}\right|$$

- $D_{1,q}^{sq}$ and $H_{1,q}^{\sphericalangle,pp}$ contributions drop out
- reducing the statistical uncertainty on $A_{UT}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta}$

Functional form of the χ^2 fit:

- extraction of $a \equiv A_{UT}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta}$ in a linear fit

$$A_{U\perp}(\phi_{R\perp}+\phi_S,\theta') = \sin(\phi_{R\perp}+\phi_S)\frac{a\sin\theta'}{1+b\frac{1}{4}(3\cos^2\theta'-1)}$$

- while varying b within positivity limits

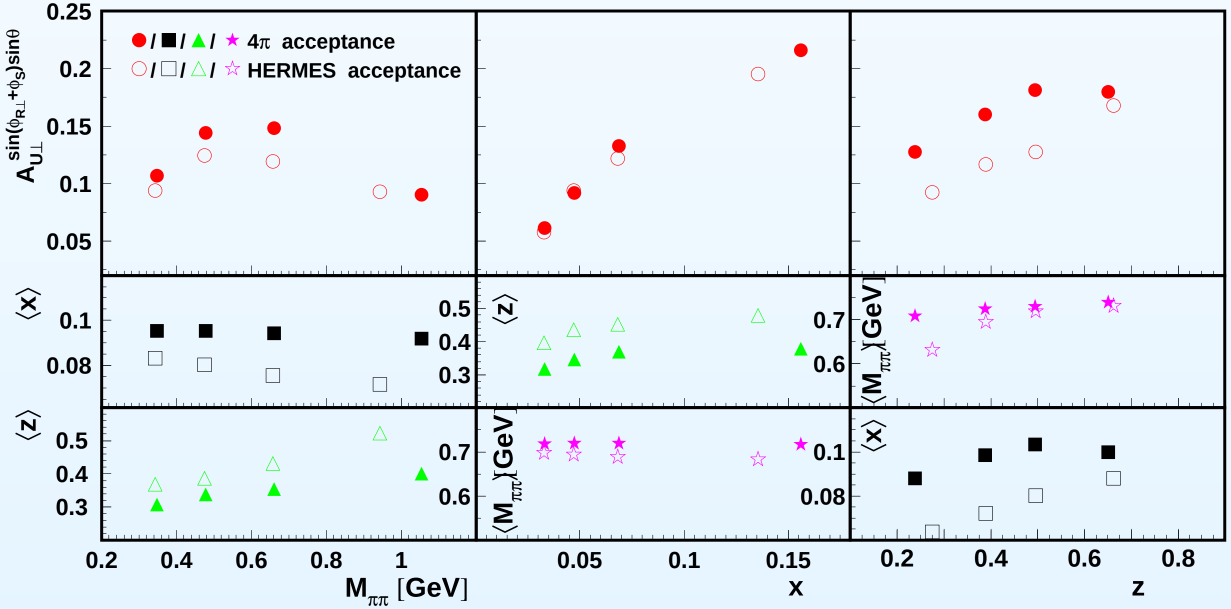
$$-\frac{3D_{1,q}^{p}(z,M_{\pi\pi})}{2D_{1,q}(z,M_{\pi\pi})} \leq b \leq \frac{3D_{1,q}^{p}(z,M_{\pi\pi})}{D_{1,q}(z,M_{\pi\pi})}$$

- limits estimated with PYTHIA6 (tuned for HERMES kinematics)
- systematic uncertainty due to “b-scan”:
  - central value in the ranges of a ➥ SSA amplitude
  - standard deviation ➥ systematic uncertainty

Influence of the experimental acceptance:

$$N^{\Uparrow(\Downarrow)}(\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}) \propto \int \mathrm{dx\,dy\,dz\,d}^2\boldsymbol{P_{h\perp}}\,\epsilon(\mathrm{x,y,z},\boldsymbol{P_{h\perp}},\phi_{\mathrm{R}\perp},\phi_{\mathrm{S}},\theta,\mathrm{M}_{\pi\pi}) \times \sigma_{U\Uparrow(\Downarrow)}(x,y,z,\boldsymbol{P_{h\perp}},\phi_{R\perp},\phi_S,\theta,M_{\pi\pi}),$$

Estimation of acceptance effects:

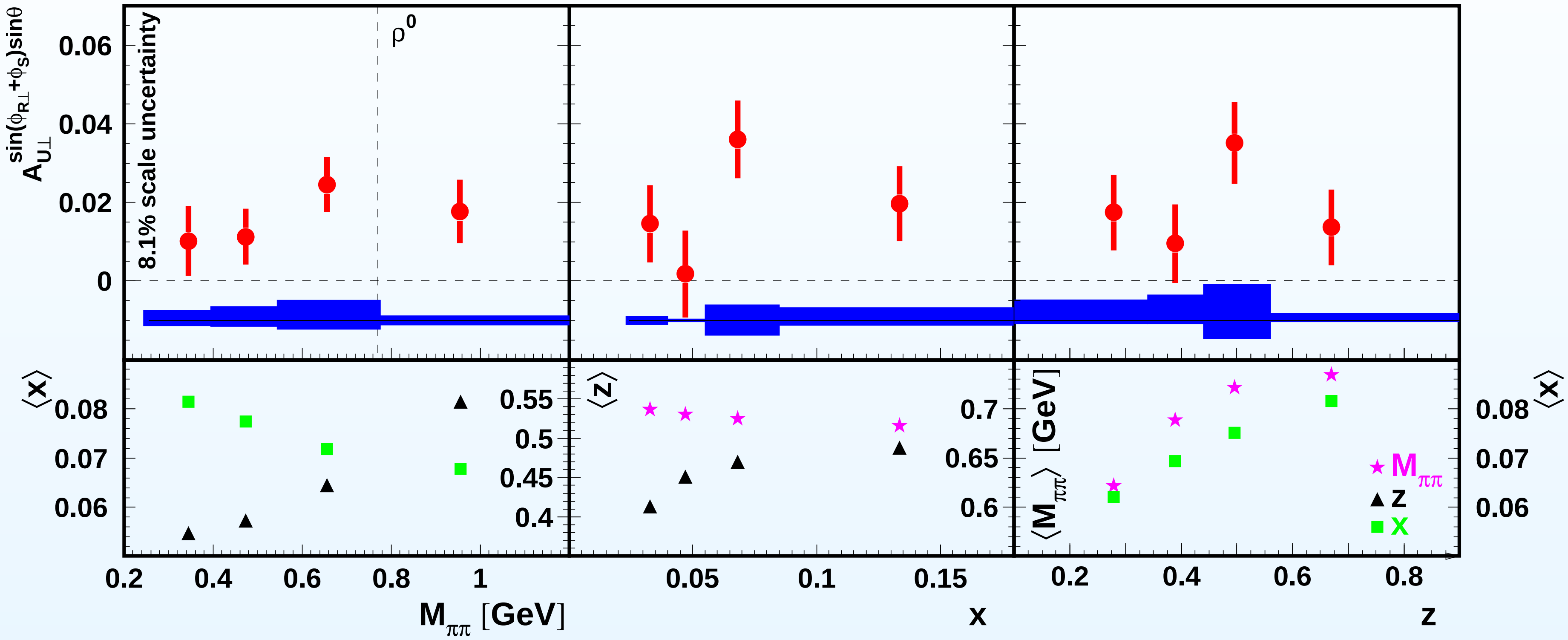


models f_1 by Gluck et al. (**Eur.Phys.J.C5:461-470,1998**),

h_1 by Schweitzer et al. (**Phys.Rev.D64:034013,2001**),

$D_1, H_1^{\sphericalangle}$ by Bacchetta and Raddici (**Phys.Rev.D74:114007,2006**)

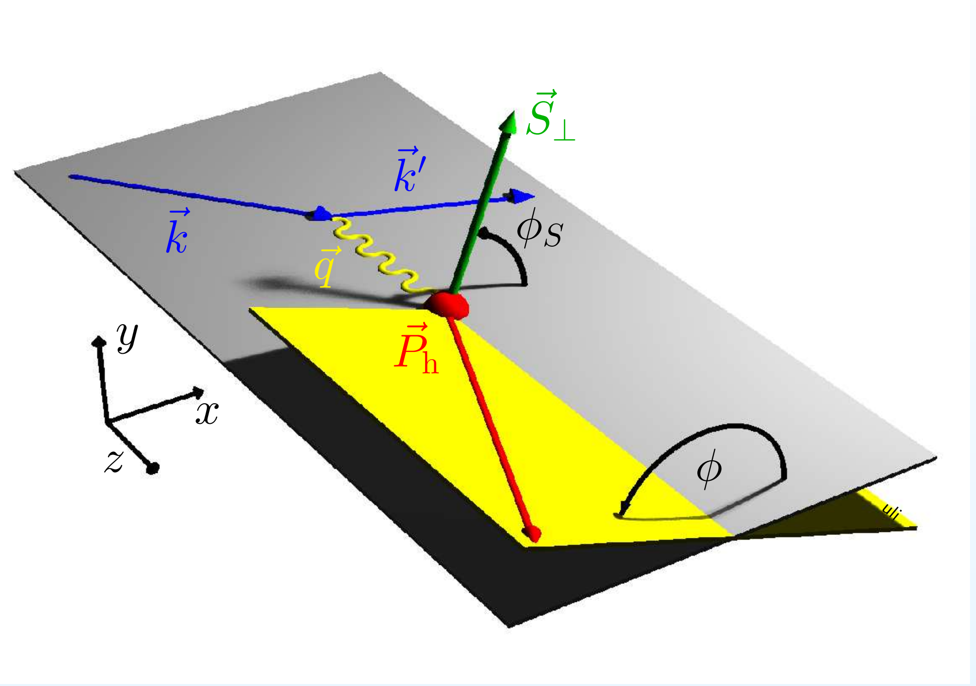
Published Results (**JHEP 0806:017,2008**):



- $A_{U\perp}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta} = 0.018 \pm 0.005_{\mathrm{stat}} \pm 0.002_{\mathrm{b-scan}} + 0.004_{\mathrm{acc}}$
- additional 8.1% scale uncertainty (target polarisation)
- first evidence for $H_{1,q}^{\sphericalangle}$
- transversity can be studied in dihadron production

SSA in single-hadron production:

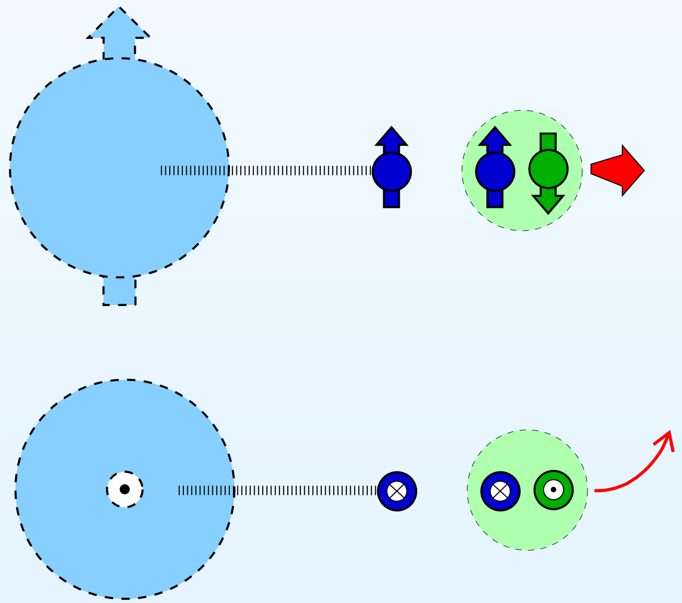
- **single-hadron production** ($ep^{\Uparrow} \rightarrow e'hX$):



- **azimuthal asymmetry** in the momentum distribution of the produced hadrons (transverse to the nucleon spin)
- non-vanishing $P_{\mathrm{h}\perp}$ is caused by
  - $S_q \cdot (p_q \times P_h)$ ➥ **Collins mechanism**
  - $S_N \cdot (P \times p_q)$ ➥ **Sivers mechanism**

The Collins mechanism:

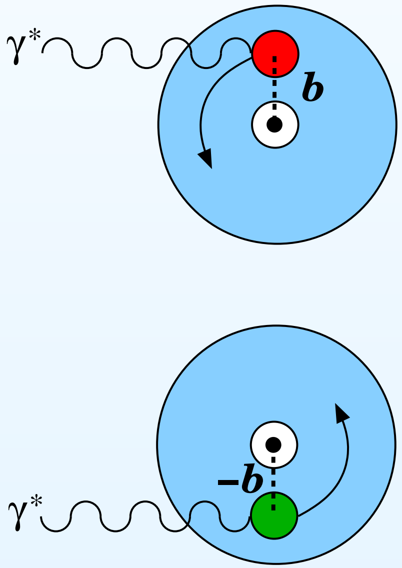
- **Collins fragmentation function $H_1^{\perp q}$**
- **chiral-odd** partner for the transversity measurement
- correlation between the transverse polarisation of the fragmenting quark and the transverse momentum $P_{h\perp}$ of the produced (unpolarised) hadron

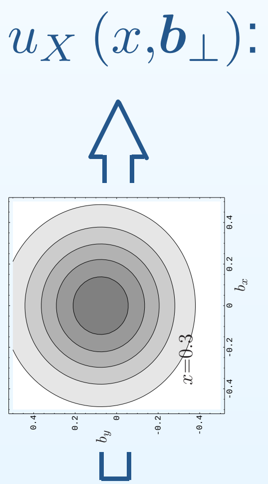


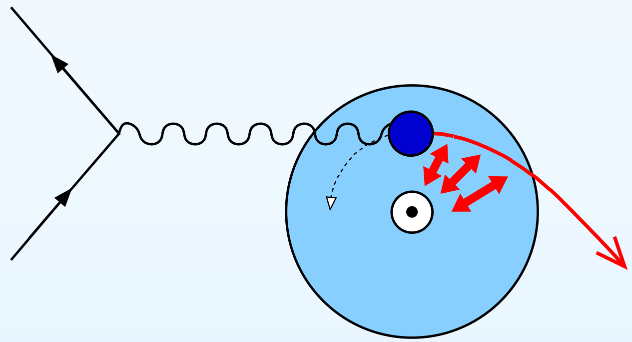
- **naive time reversal odd** $\Leftrightarrow$ final state interactions
  ➥ **transverse single-spin asymmetry**

The Sivers mechanism:

- non-zero **Sivers distribution** $f_{1T}^{\perp}$ involves non-zero Compton amplitude $N^{\Uparrow}q^{\uparrow} \to N^{\Downarrow}q^{\uparrow}$
- **orbital angular momentum of quarks:**
  (M. Burkardt, (**Phys.Rev.D66:114005,2002**))







- **final state interactions (naive-T-odd):**

  left-right asymmetry of quark distribution

  ➥ left-right asymmetry of momentum distribution of produced hadron

The Collins and Sivers amplitudes:

- $\boldsymbol{A^h_{\mathsf{UT}}}$ for hadron type h:

$$A^h_{\mathsf{UT}} = \frac{\sigma^{\Uparrow} - \sigma^{\Downarrow}}{\sigma^{\Uparrow} + \sigma^{\Downarrow}}$$

Collins amplitude

$$\propto -2\,|\boldsymbol{S}_T|\,\sin(\phi + \phi_S)\,\frac{\sum_q e_q^2\, \boldsymbol{h_1^q(x) \otimes H_1^{\perp q}(z)}}{\sum_q e_q^2\, q(x)\, D_1^q(z)}$$

↑ **distinguishable**

Sivers amplitude

Signature ↓

$$-\ 2\,|\boldsymbol{S}_T|\,\sin(\phi - \phi_S)\,\frac{\sum_q e_q^2\, \boldsymbol{f_{1T}^{\perp q}(x) \otimes D_1^q(z)}}{\sum_q e_q^2 q(x)\, D_1^q(z)}$$

- convolution over intrinsic transverse momenta

The convolution over intrinsic transverse momenta:

- transverse target cross section contains a convolution integral $\mathcal{I}$ over intrinsic transverse momenta $\boldsymbol{p}_\perp$ and $\boldsymbol{k}_\perp$:

$$\mathcal{I}(\ldots) \equiv \int \mathrm{d}^2\boldsymbol{p}_\perp \, \mathrm{d}^2\boldsymbol{k}_\perp \, \delta^{(2)}\left(\boldsymbol{p}_\perp - \frac{\boldsymbol{P}_{h\perp}}{z} - \boldsymbol{k}_\perp\right)(\ldots)$$

- e.g. Sivers SSA amplitude:

$$2\left\langle \sin(\phi - \phi_S)\right\rangle^h_{\mathrm{UT}} = -2\frac{\sum_q e_q^2 \, \mathcal{I}\left[\frac{\boldsymbol{p}_\perp \widehat{\boldsymbol{P}}_{h\perp}}{M_{\mathrm{N}}} f_{1T}^{\perp,q}(x, p_\perp^2) D_1^{q\to h}(z, z^2 k_\perp^2)\right]}{\sum_q e_q^2 f_1^q(x) D_1^{q\to h}(z)}$$

- **Disentangling the convolution integral**
  - using $P_{h\perp}$-weighted SSA
  - using several model assumptions

$P_{h\perp}$-weighted SSA:

- The ***weighted* SSA** are defined as count-rate asymmetries of the form:

$$\tilde{A}^h_{UT}(\phi,\phi_S) = \frac{1}{\langle S_\perp \rangle} \frac{\sum_{i=1}^{N_h^\Uparrow} \frac{P_{h\perp,i}}{z_i M_N} - \sum_{i=1}^{N_h^\Downarrow} \frac{P_{h\perp,i}}{z_i M_N}}{N_h^\Uparrow + N_h^\Downarrow}$$

$P_{h\perp}$-weighted SSA:

- The ***weighted SSA amplitudes*** do not involve convolution integrals over intrinsic transverse momenta:

$$2\left\langle \frac{P_{h\perp}}{zM_{\mathrm{N}}} \sin\left(\phi - \phi_S\right) \right\rangle_{\mathrm{UT}}^{h} = -2 \frac{\sum_q e_q^2 \; f_{1T}^{\perp(1),q}(x) \; D_1^{q\rightarrow h}(z)}{\sum_q e_q^2 \; f_1^q(x) \; D_1^{q\rightarrow h}(z)}$$

$P_{h\perp}$-weighted SSA:

- The ***weighted SSA amplitudes*** do not involve convolution integrals over intrinsic transverse momenta:

$$2\left\langle \frac{P_{h\perp}}{zM_{\mathrm{N}}} \sin(\phi - \phi_S) \right\rangle_{\mathrm{UT}}^{h} = -2 \frac{\sum_q e_q^2\, f_{1T}^{\perp(1),q}(x)\; D_1^{q\to h}(z)}{\sum_q e_q^2\, f_1^q(x)\; D_1^{q\to h}(z)}$$

- **Purity formalism:** w.l.o.g. binning in x and integrating over z:

$$\left\langle \frac{P_{h\perp}}{zM_{\mathrm{N}}} \sin(\phi - \phi_S) \right\rangle_{\mathrm{UT}}^{h}(x) = -\frac{\sum_q e_q^2\, f_{1T}^{\perp(1),q}(x)\; \int dz\; D_1^{q\to h}(z)}{\sum_q e_q^2\, f_1^q(x)\, \int dz\; D_1^{q\to h}(z)}$$

$P_{h\perp}$-weighted SSA:

- The ***weighted* SSA *amplitudes*** do not involve convolution integrals over intrinsic transverse momenta:

$$2\left\langle \frac{P_{h\perp}}{zM_{\mathrm{N}}}\sin(\phi-\phi_S)\right\rangle^h_{\mathrm{UT}} = -2\frac{\sum_q e_q^2\, f_{1T}^{\perp(1),q}(x)\; D_1^{q\to h}(z)}{\sum_q e_q^2\, f_1^q(x)\; D_1^{q\to h}(z)}$$

- **Purity formalism:** w.l.o.g. binning in x and integrating over z:

$$\left\langle \frac{P_{h\perp}}{zM_{\mathrm{N}}}\sin(\phi-\phi_S)\right\rangle^h_{\mathrm{UT}}(x) = -\sum_q \mathcal{P}_q^h(x)\frac{f_{1T}^{\perp(1),q}(x)}{f_1^q(x)}$$

where the ***purity*** $\mathcal{P}_q^h(x) = \frac{e_q^2 f_1^q(x)\int dz\; D_1^{q\to h}(z)}{\sum_{q'} e_{q'}^2 f_1^{q'}(x)\int dz\; D_1^{q'\to h}(z)}$ gives the probability that an observed event came from scattering of a certain quark flavour.

Problems using $P_{h\perp}$-weighted SSA:

- **complete integration over $P_{h\perp}$:**
  Can the integration to ∞ be approximated by an integration up to certain cut-off value $P_{h\perp}^2 \ll Q^2$?
- **Possibly large acceptance effects:**
  - **correction with multidimensional UNFOLDING:** appears to work in 5D (e.g. Boer-Mulders function), but not in 6D
  - **multi-parameter fit:**

- evaluation of the **full kinematic dependence** $(x, Q^2, z, P_{h\perp})$
- through a **multi-parameter fit** (e.g. 48) to the full set of semi-inclusive events
- folding with $\sigma_{UU}\left(x, Q^2, z, P_{h\perp}\right)$ in 4π
  ➥ acceptance-corrected results
- Monte Carlo tuned to data used for $\sigma_{UU}\left(x, Q^2, z, P_{h\perp}\right)$
- method has been studied in Monte Carlo

The Gaussian ansatz:

- ***unweighted SSA amplitudes***:

$$2\left\langle\sin\left(\phi-\phi_S\right)\right\rangle_{\text{UT}}^{h} = -2\,\frac{\sum_q e_q^2\,\mathcal{I}\left[\frac{\boldsymbol{p}_\perp\widehat{\boldsymbol{P}}_{h\perp}}{M_{\text{N}}}f_{1T}^{\perp,q}(x,p_\perp^2)D_1^{q\to h}(z,z^2k_\perp^2)\right]}{\sum_q e_q^2 f_1^q(x)D_1^{q\to h}(z)}$$

- use of model assumptions, e.g. **Gaussian ansatz**

$$\left\langle\sin\left(\phi-\phi_S\right)\right\rangle_{\text{UT}}^{h} = -\frac{\sqrt{\pi}}{2M_N}R_S\left\langle p_\perp^2\right\rangle_S\cdot\frac{\sum_q e_q^2 f_{1T}^{\perp,q}(x)D_1^q(z)}{\sum_q e_q^2 f_1^q(x)D_1^q(z)}$$

$$\frac{1}{R_S^2} \equiv \left\langle k_\perp^2\right\rangle+\left\langle p_\perp^2\right\rangle_S$$

- $x-$ or z-dependence and flavour-dependence of R_S and $\left\langle p_\perp^2\right\rangle_S$
- problems with flavour decomposition

Selection of semi-inclusive events:

- pions and charged kaons have been selected from $ep^{\Uparrow} \rightarrow e'hX$
- kinematic requirements:

$$\begin{array}{rcl} 1\,\mathtt{GeV}^2 < & Q^2 & \\ (0.1 \leqslant) & y & < 0.95 \\ 0.023 < & x & < 0.4 \\ 10\,\mathtt{GeV}^2 < & W^2 & \\ 2\,\mathtt{GeV} < & \boldsymbol{P}_h & < 15\,\mathtt{GeV} \\ 0.2 < & z & < 0.7 \\ 0.02\,\mathtt{rad} < & \theta_{\gamma^* h} & \end{array}$$

- **mean kinematics:**
  $\left\langle Q^2 \right\rangle = 2.4\,\mathtt{GeV}^2$, $\langle x \rangle = 0.094$, $\langle z \rangle = 0.36$, $\langle P_{h\perp} \rangle = 0.41\,\mathtt{GeV}$
- evaluation of **lepton-beam asymmetries**
- due to unknown $R = \sigma_L/\sigma_T$ for semi-inclusive DIS measurements

Charged hadron identification:

- hadron identification with dual-radiator RICH
- **observed** (most probable) **hadron fluxes $\boldsymbol{I}$**:

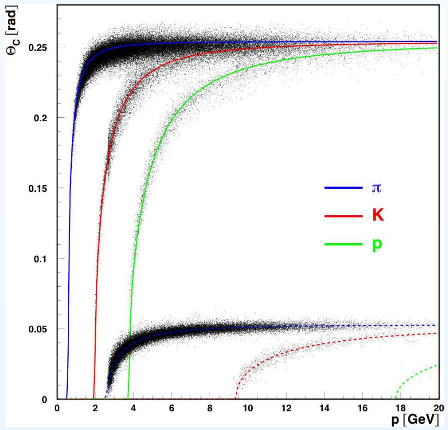
$$\begin{pmatrix} N_\pi \\ N_K \\ N_p \end{pmatrix} = \begin{pmatrix} \mathcal{P}_\pi^\pi & \mathcal{P}_\pi^K & \mathcal{P}_\pi^p & \mathcal{P}_\pi^X \\ \mathcal{P}_K^\pi & \mathcal{P}_K^K & \mathcal{P}_K^p & \mathcal{P}_K^X \\ \mathcal{P}_p^\pi & \mathcal{P}_p^K & \mathcal{P}_p^p & \mathcal{P}_p^X \end{pmatrix} \cdot \begin{pmatrix} I_\pi \\ I_K \\ I_p \\ I_X \end{pmatrix}$$

true hadron types $\boldsymbol{N}$, RICH PID event weights $\mathcal{P}$

- for each detected hadron track three event weights are assigned:
  - event weight as true pion
  - event weight as true kaon
  - event weight as true proton

Čerenkov radiation:

$$\theta = \arccos \frac{1}{\beta n}$$



SiO_2: $n = 1.03$

C_4F_{10}: $n = 1.0014$

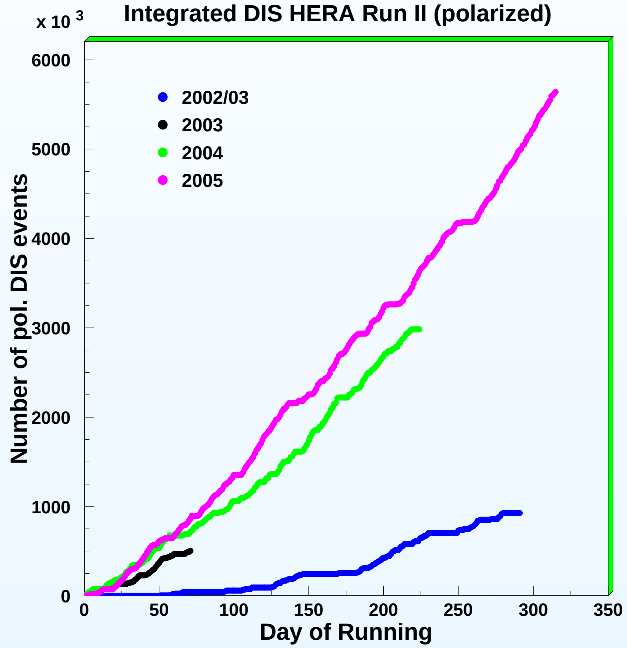
Simultaneous extraction of unweighted amplitudes:

- maximum likelihood fits are used for pions charged kaons:

$$\mathrm{F}\left(2\,\langle\sin(\phi\pm\phi_S)\rangle^h_{UT},\ldots,\phi,\phi_S\right)=$$
$$\frac{1}{2}\Big(1+P^z_\alpha\big(2\,\langle\sin(\phi+\phi_S)\rangle^h_{UT}\cdot\sin(\phi+\phi_S)+$$
$$2\,\langle\sin(\phi-\phi_S)\rangle^h_{UT}\cdot\sin(\phi-\phi_S)+$$
$$2\,\langle\sin(3\phi-\phi_S)\rangle^h_{UT}\cdot\sin(3\phi-\phi_S)+$$
$$2\,\langle\sin(2\phi-\phi_S)\rangle^h_{UT}\cdot\sin(2\phi-\phi_S)+$$
$$2\,\langle\sin\phi_S\rangle^h_{UT}\cdot\sin\phi_S\big)\Big)$$

- the logarithm of the likelihood function $\mathcal{L}=\prod(\mathrm{F_i})^{w_i}$ is maximised with respect to the SSA amplitudes (w_i RICH PID event weights)

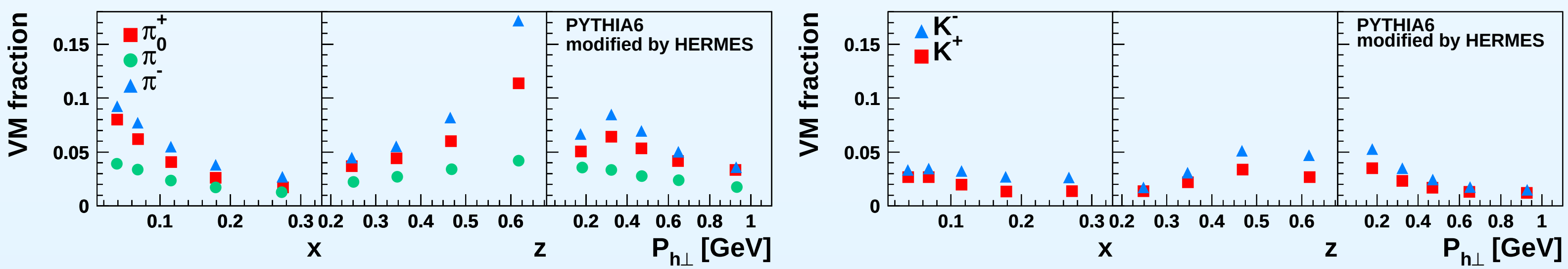
polarised H target:



$e^\pm$	$8.4\cdot10^6$
π^+	$7.4\cdot10^5$
π^0	$2.1\cdot10^5$
π^-	$5.1\cdot10^5$
K^+	$1.2\cdot10^5$
K^-	$2.2\cdot10^4$

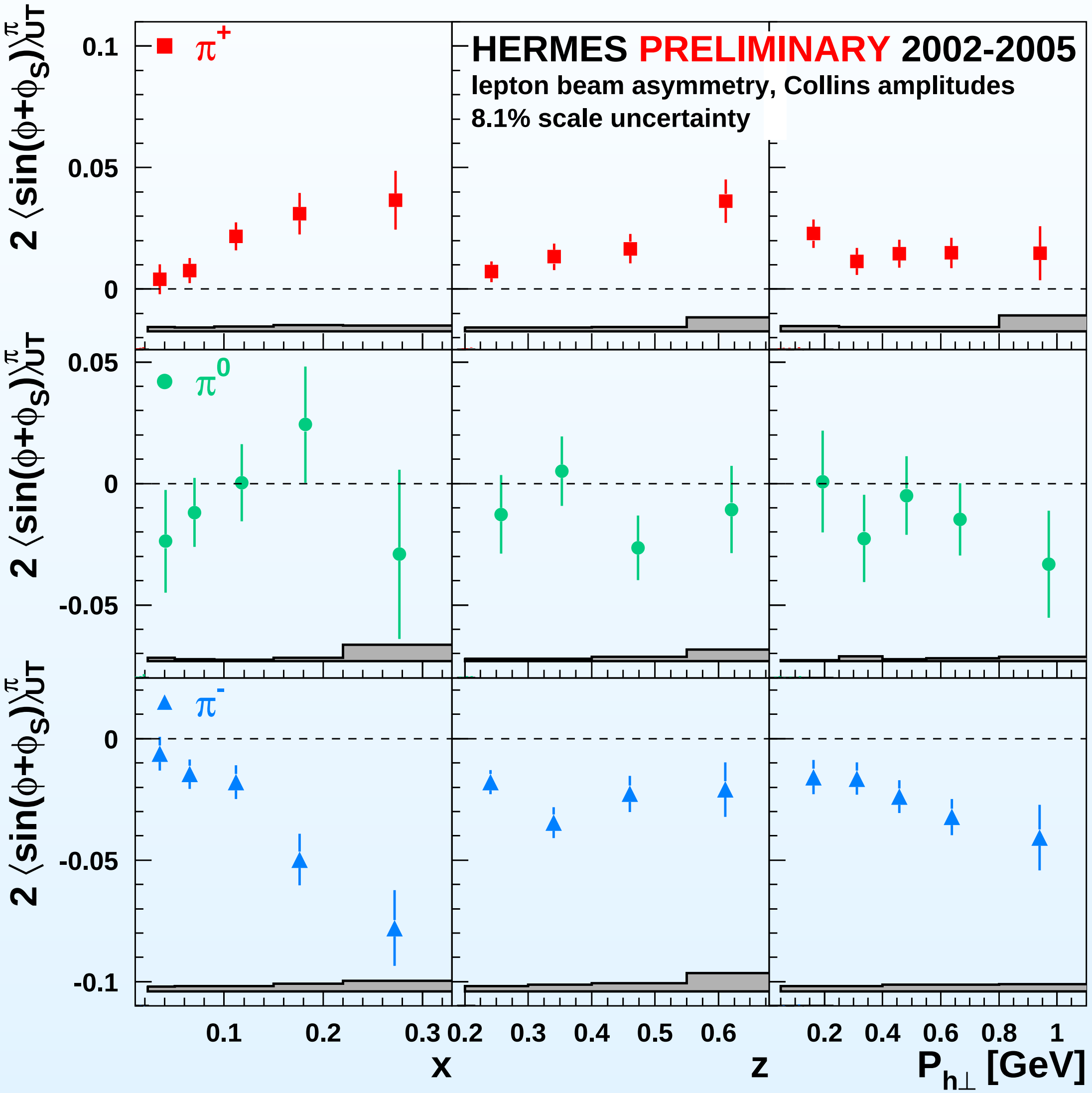
Systematic uncertainties:

- **scaling uncertainty** due to uncertainty in the target
- **Contributions to the systematic uncertainty:**
  1. acceptance effects
  2. QED radiative effects and detector smearing
  3. hadron misidentification due to the RICH PID
  4. contribution of $\cos\phi$ and $\cos(2\phi)$ amplitudes in the spin-independent cross-section
  5. contributions from subleading longitudinal asymmetries
- exclusive channels do not dominate:



➥ **no correction for exclusive contributions**

The Collins amplitudes for pions:



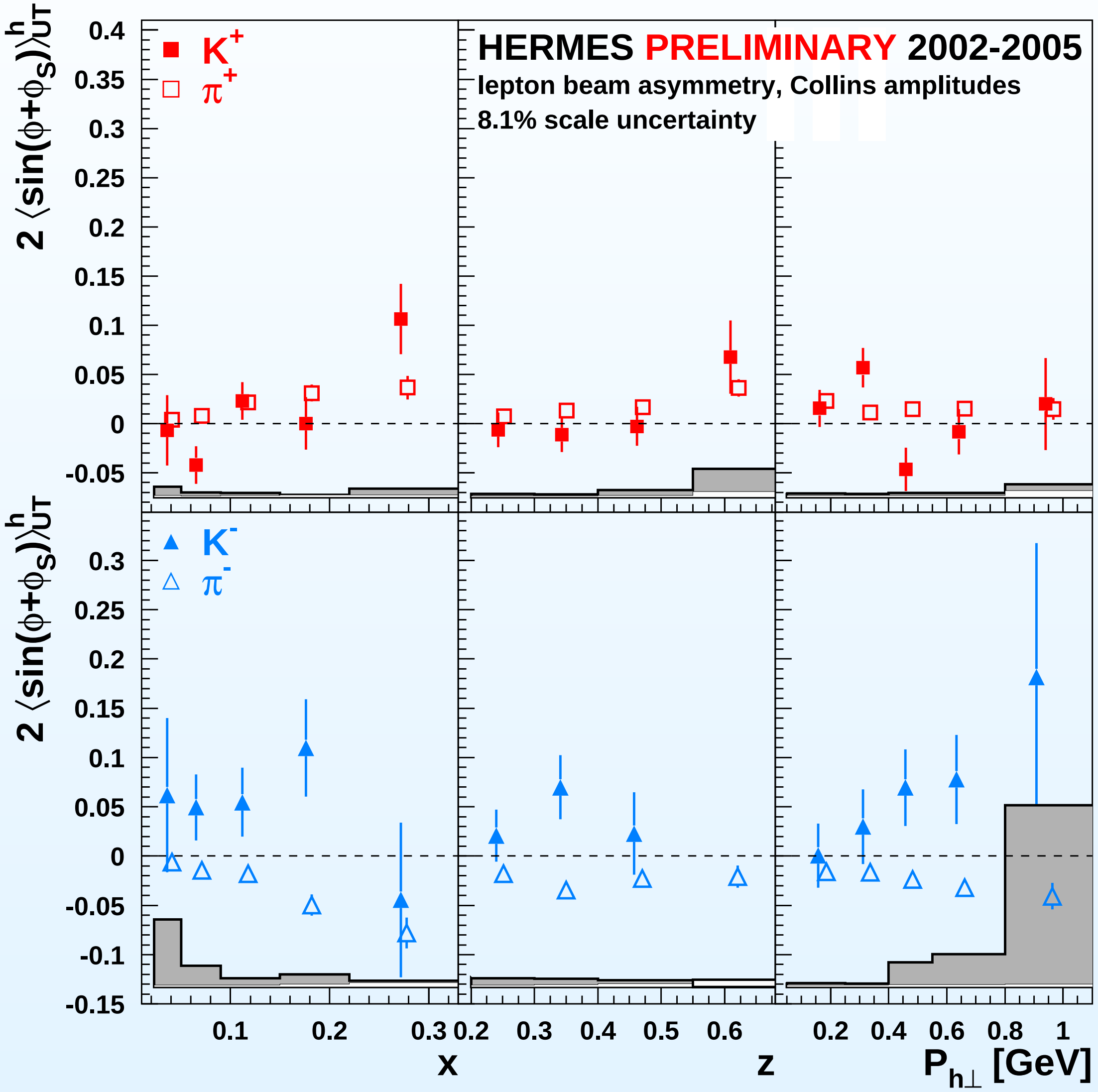
Results of the Collins amplitude:

$$h_1^q(x) \otimes H_1^{\perp q}(z)$$

from 2002–2005 data:

- positive amplitudes for π^+
- large negative π^- amplitudes is unexpected
- $H_1^{\perp,\text{unfav}}(z) \approx -H_1^{\perp,\text{fav}}(z)$
- isospin symmetry of π-mesons is fulfilled
- information from another process on the Collins fragmentation function (BELLE) permits **extraction of transversity** (e.g. Anselmino et al, **Phys.Rev.D75:054032,2007**)

The Collins amplitudes for charged kaons:



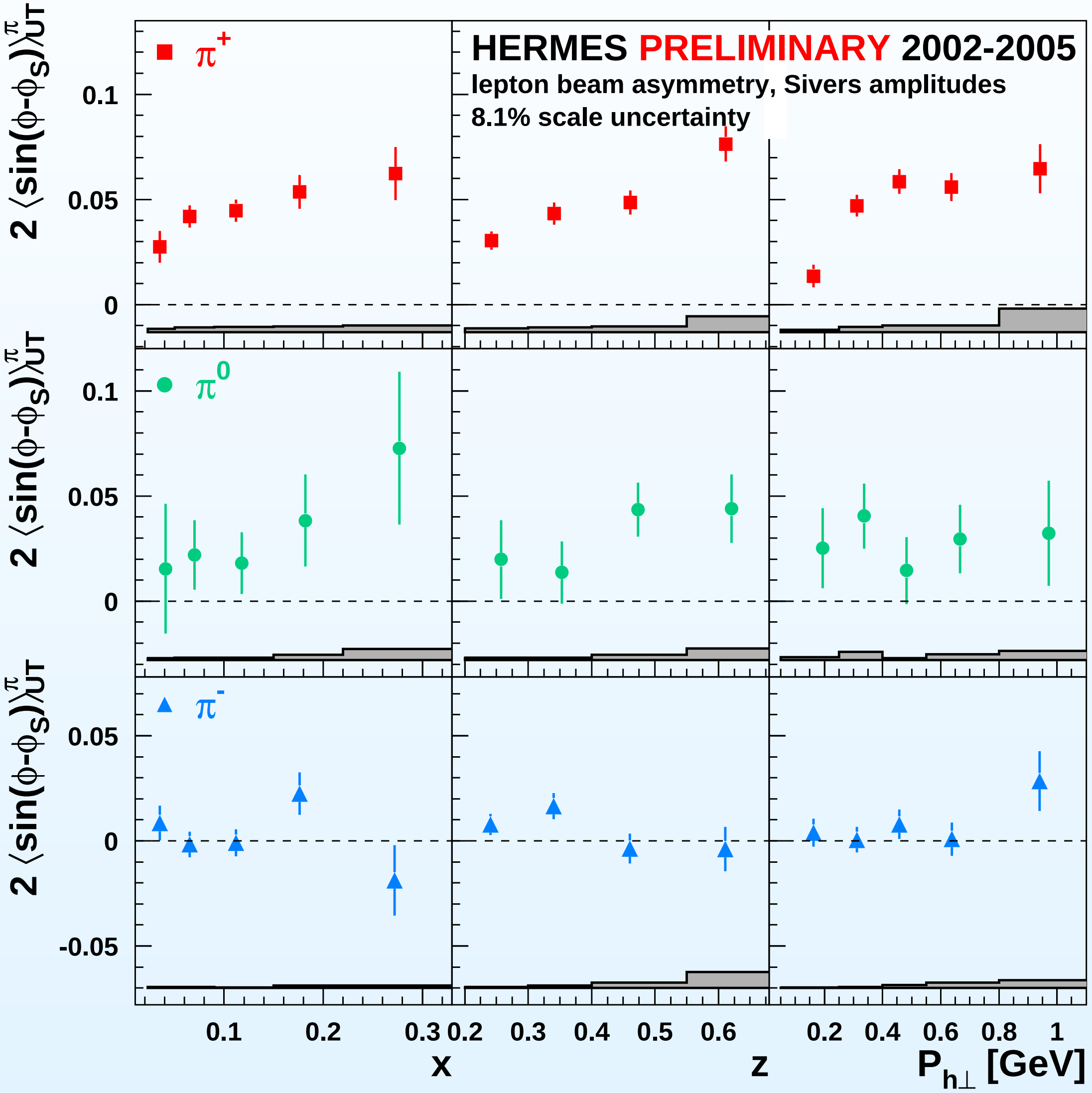
Results of the Collins amplitude:

$$h_1^q(x) \otimes H_1^{\perp q}(z)$$

from 2002–2005 data:

- no significant non-zero Collins amplitudes for both K^+ and K^-
- Collins amplitudes for K^+ are within statistical accuracy consistent with π^+

The Sivers amplitudes for pions:



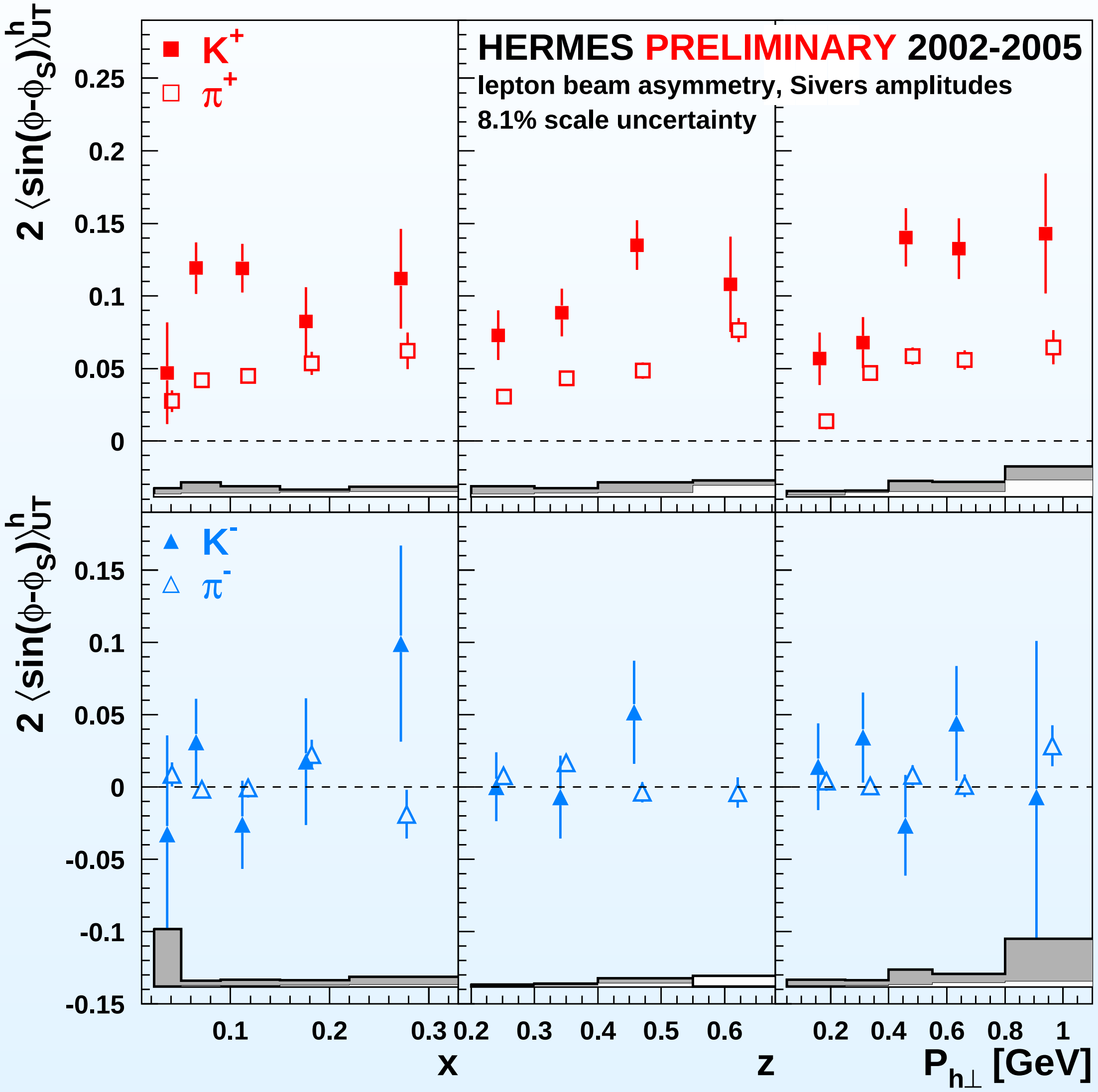
Results of the Sivers amplitude:

$$f_{1T}^{\perp q}(x) \otimes D_1^q(z).$$

from 2002–2005 data:

- significantly positive for π^+
- implies non-zero L_z^q
- π^-amplitude consistent with zero
- isospin symmetry of π-mesons is fulfilled
- **extraction of the Sivers function** is possible as spin-independent fragmentation function $D_1^q(z)$ is known

The Sivers amplitude for charged kaons:



Results of the Sivers amplitude:

$$f_{1T}^{\perp q}(x) \otimes D_1^{\perp q}(z).$$

from 2002–2005 data:

- significantly positive for K^+
- implies non-zero L_z^q
- K^-amplitude consistent with zero
- K^+ amplitude larger than π^+ amplitude
  $\Rightarrow s\bar{s}$ contribution to Sivers mechanism may be important:
  $K^+ = |u\bar{s}\rangle \quad \pi^+ = |u\bar{d}\rangle$

In a nutshell:

- (most) precise data on a transversely polarised hydrogen target
- significant Collins amplitudes for π-mesons
  ➥ enables quantitative extraction of transversity distribution
- significant Sivers amplitudes for $\pi^+$and K^+
  ➥ clear (and first) evidence of a naive-T-odd parton distribution
  ➥ enables quantitative extraction of the Sivers function
- first evidence for a naive-T-odd dihadron fragmentation function
  ➥ provides alternative probe for transversity distribution
- **under construction:**
  - multi-dimensional SSA amplitudes
  - $P_{h\perp}$-weighted SSA amplitudes
  - extraction of the Cahn effect
  - and the Boer-Mulders function
    (c.f. Gunar Schnell's talk on Wednesday)