Measurement of the Generalized GDH Integral at HERMES

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For the hermes Collaboration

The Beauty of Simplicity

consider the complex series $z_{n+1} = z_n^2 + c$
where c is some complex constant and map out the complex plane for which this series converges:

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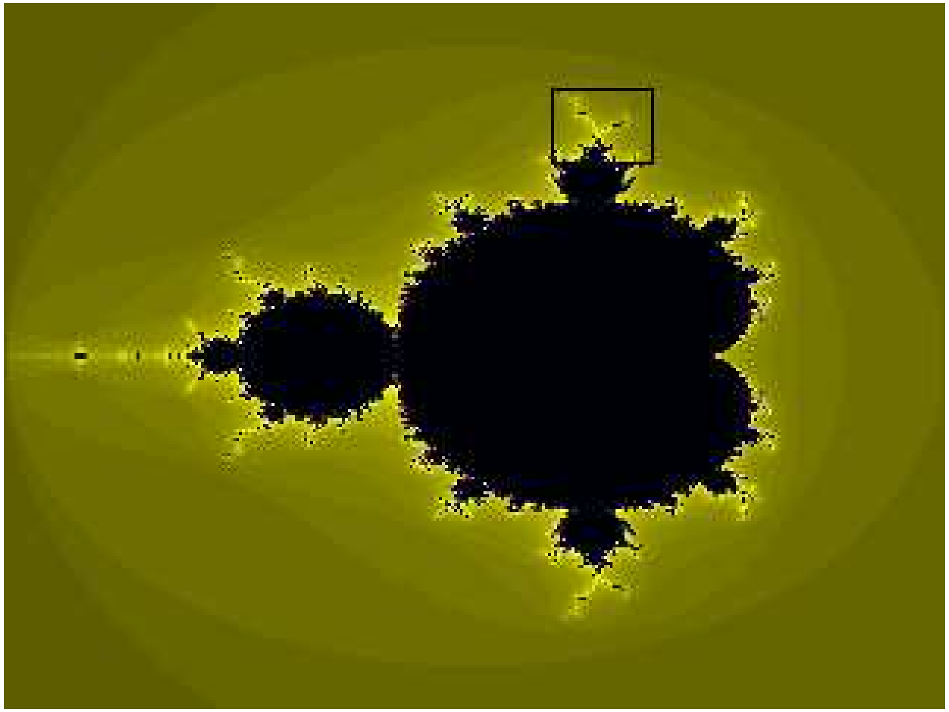

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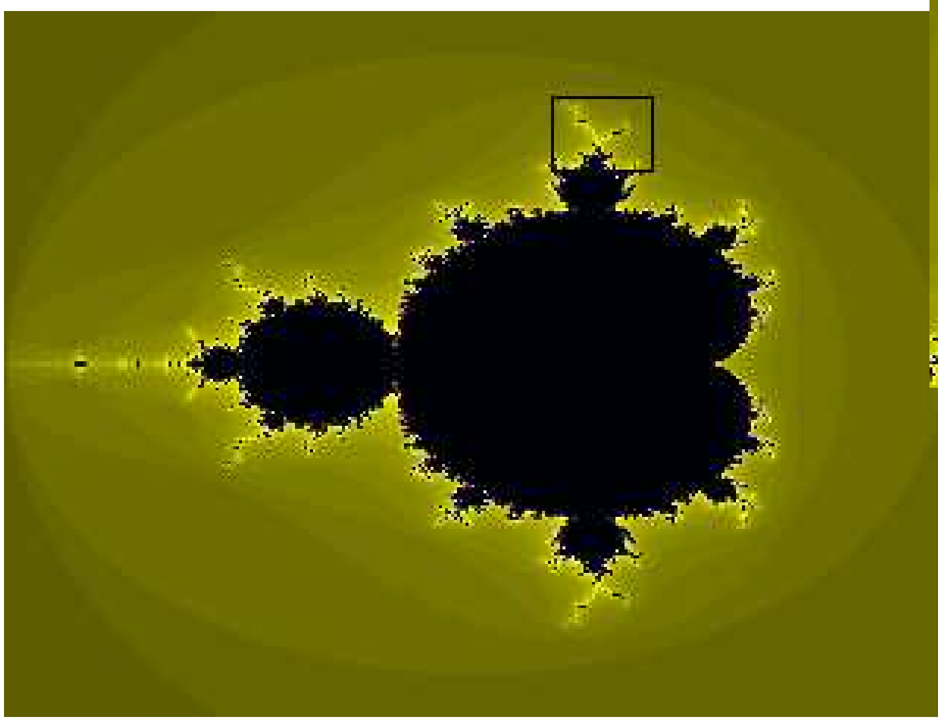

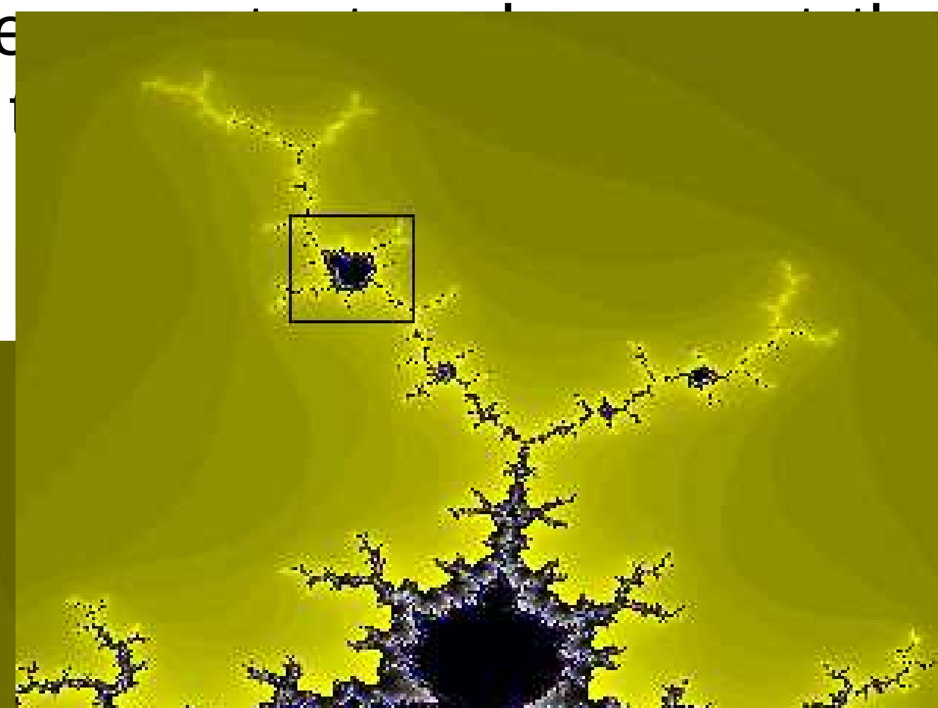

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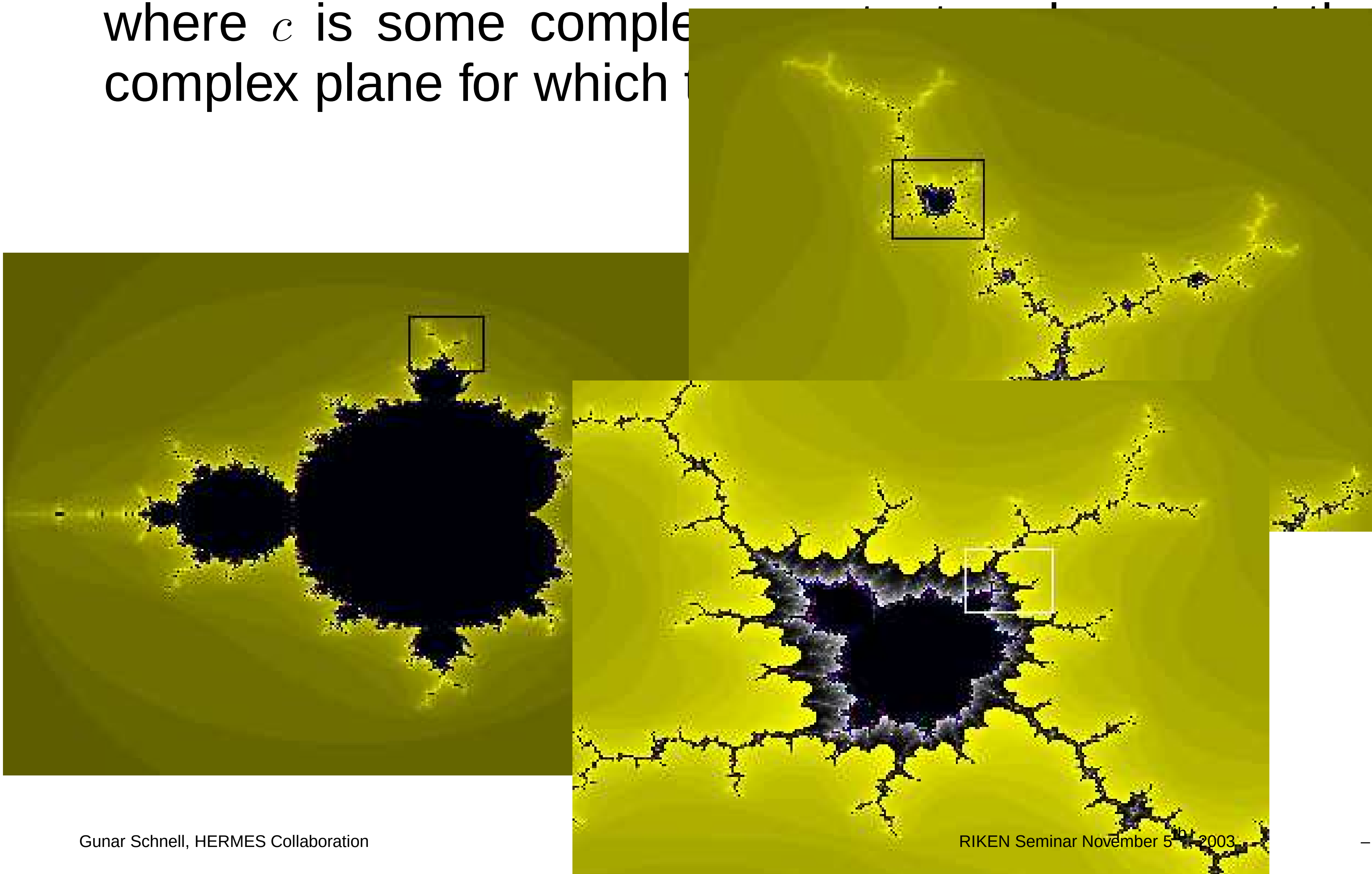


Simplicity in Nuclear Physics I: Duality

Lepton-Nucleon Scattering exhibits different “faces”:

- Resonance region:
  - Excitations of nucleon
  - Mesonic degrees of freedom
- Deep Inelastic Scattering (DIS) region:
  - Substructure - described in terms of (asymptotically free) partons (quarks, gluons)
  - application of perturbative QCD

DUALITY = CONNECTION between RESONANCE and DIS REGION

- GDH (GDHHY)[1] Sum Rule:

$$\int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left[\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)\right] = -\frac{2\pi^2\alpha}{M^2}\kappa^2$$

$\Rightarrow$ A low-energy limit is expressed in terms of an integral that runs over all energies

[1] Hosoda and Yamamoto are often ignored when talking about the derivation of the GDH sum rule. They actually were the first using current-algebra techniques!

Outline

- (Opening Remarks)
- Introduction
  - DIS & Parton Model
  - Concepts of Duality
  - The GDH Sum Rule
  - Generalized GDH Integral
- The HERMES experiment
- g_1 and the helicity distribution functions
- Duality in polarized case
- Measurements of the (generalized) GDH integrals
- Summary

Lepton-Nucleon Scattering

Scattering off Nucleons

- scattering off asymptotically free quarks
- pQCD applicable
- data well reproduced in models
- “well-understood”

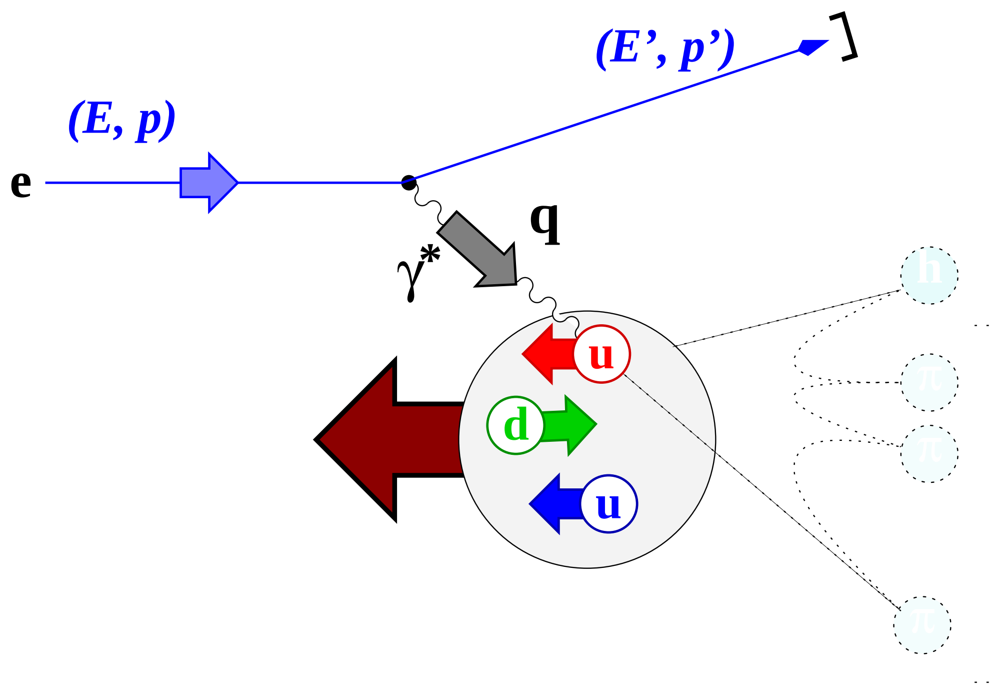
- complicated structure
- strong coupling constant $\Rightarrow$ non-perturbative regime
- hard to reproduce in models

Can we use knowledge from DIS in Resonance Region?

- DUALITY: Compare DIS and Resonance regions
- GDH: Combine DIS and Resonance regions

Lepton Deep Inelastic Scattering

use well-known probe to study hadronic structure



$$Q^2 \overset{\text{lab}}{=} 4EE' \sin^2(\frac{\Theta}{2})$$

$$\nu \overset{\text{lab}}{=} E - E'$$

$$W^2 \overset{\text{lab}}{=} M^2 + 2M\nu - Q^2$$

$$y \overset{\text{lab}}{=} \frac{\nu}{E}$$

$$x \overset{\text{lab}}{=} \frac{Q^2}{2M\nu}$$

Lepton Deep Inelastic Scattering

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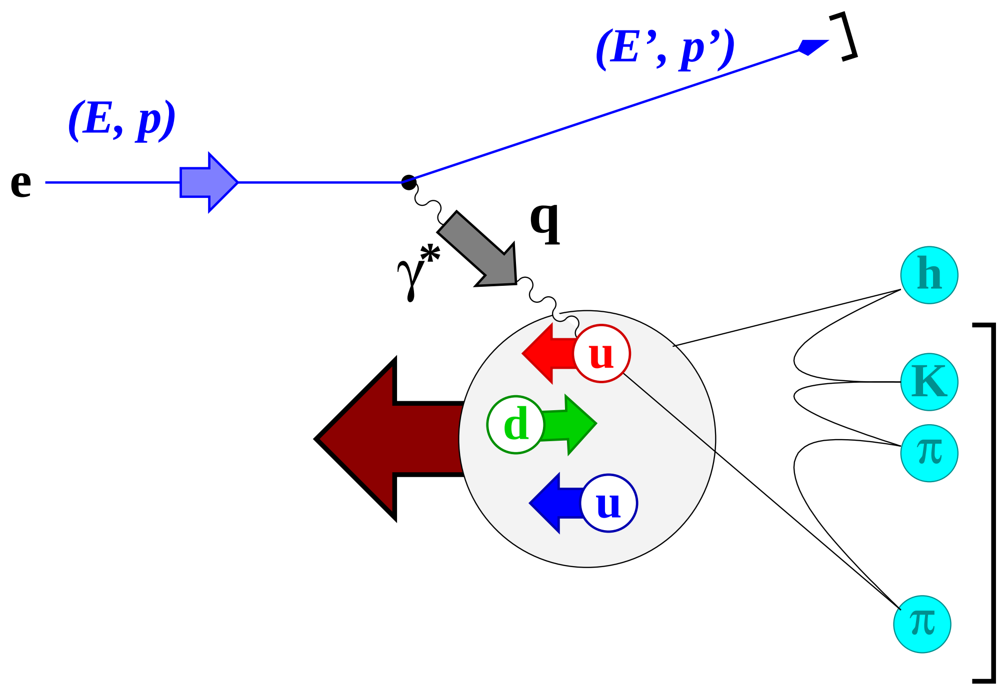

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$$z \stackrel{\text{lab}}{=} \frac{E_h}{\nu}$$

Lepton Deep Inelastic Scattering

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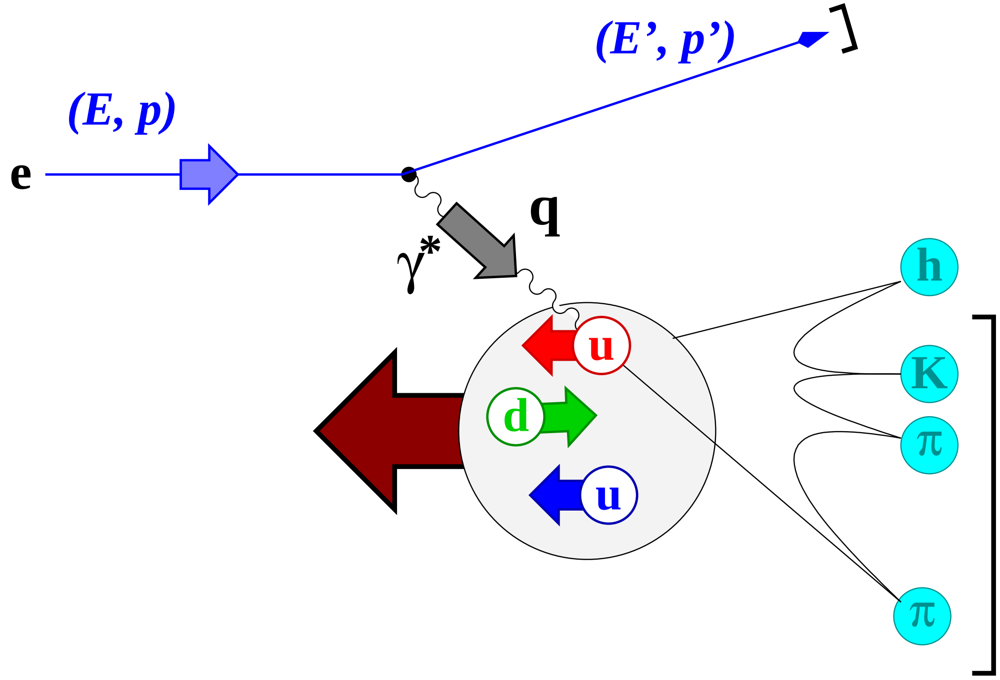

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Factorization $\Rightarrow \sigma^{ep\to ehX} = \sum_q f^{p\to q} \otimes \sigma^{eq\to eq} \otimes D^{q\to h}$

Inclusive (polarized) DIS

$$\frac{d^2\sigma}{d\Omega dE^2} = \frac{\alpha^2 E'}{Q^2 E} L_{\mu\nu}(k,q,s) W^{\mu\nu}(P,q,S)$$

$L_{\mu\nu}$: Lepton Tensor (exactly calculable in QED)

$W^{\mu\nu}$: Hadron Tensor (parametrized in terms of structure functions)

$$= -g^{\mu\nu} F_1(x,Q^2) + \frac{p^\mu p^\nu}{\nu} F_2(x,Q^2)$$

$$+ i\epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha}{\nu} \left(S_\beta g_1(x,Q^2) + \frac{1}{\nu}(p \cdot q S_\beta - S \cdot q p_\beta) g_2(x,Q^2) \right)$$

F_1, F_2 : unpolarized structure functions

g_1, g_2 : polarized structure functions

Quark Parton Model

structure functions can be written as probability densities (on the light cone)

$$F_1(x) = \frac{1}{2}\sum_q e_q^2 q(x)$$ **momentum distribution**

$$F_2(x) = 2xF_1(x)$$ **Callan-Gross relation**

$$g_1(x) = \frac{1}{2}\sum_q e_q^2 \Delta q(x)$$ **helicity distribution**

$$g_2(x) = 0$$

including transverse momentum of partons:

$$g_2(x) = g_2^{WW}(x) + g_2^{twist-3}(x)$$ **(Wandzura-Wilczek '77)**

where $g_2^{WW}(x) = \int_x^1 \frac{dy}{y} g_1(y) - g_1(x)$

Origin of the Proton's Spin

Helicity distribution $\Delta q(x) = q^{\uparrow} - q^{\downarrow}$

$q^{\uparrow(\downarrow)}(x)$ – probability to find quark of flavor q

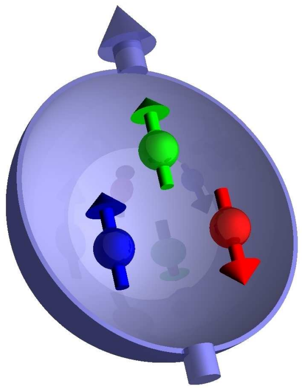
with momentum fraction x and

spin (anti)aligned to proton spin

Origin of the Proton's Spin

Helicity distribution $\Delta q(x) = q^{\uparrow} - q^{\downarrow}$

$\Downarrow$



$$\mathbf{S}_N = \frac{1}{2} \stackrel{?}{=} \frac{1}{2} \sum_q \int \Delta q(x)\, dx \stackrel{\text{def}}{=} \frac{1}{2} \Delta\Sigma$$

only valence quarks

↙ ↘

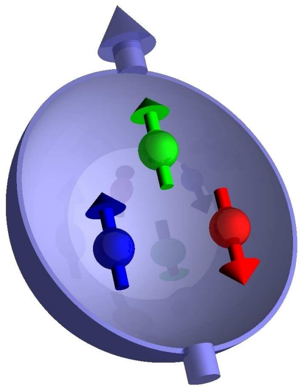
Naive Quark Model: $\Delta\Sigma = 1$

EMC('88): $\Delta\Sigma \approx 10 - 20\%$

↘ ↙

"SPIN CRISIS"

Origin of the Proton's Spin



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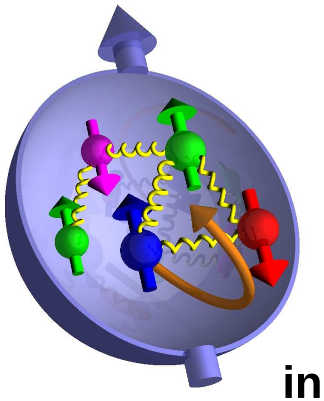


$$\boxed{\mathbf{S}_N = \frac{1}{2} \; = \; \frac{1}{2}\Delta\Sigma + \Delta G + L_z}$$

Angular Momentum Sum Rule

include also gluons, sea quarks & orbital angular momentum

Concept of Duality

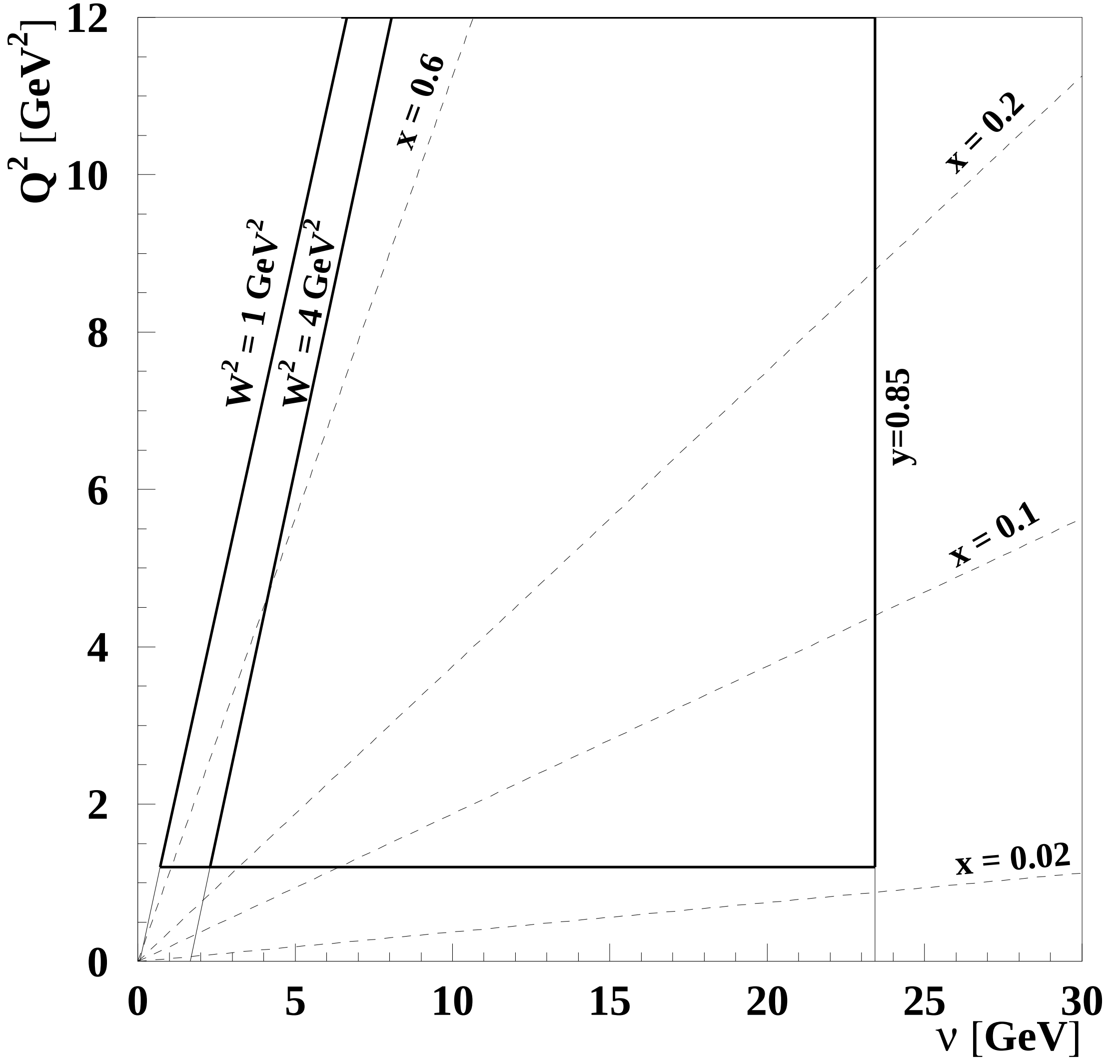
DUALITY = RELATION BETWEEN DIS AND RESONANCE REGIONS **(Bloom & Gilman ’70)**

- Curve measured in the resonance region (low W^2) is *in average* equal to the curve measured in DIS region (high W^2)
- Originally introduced for unpolarized photo-absorption cross section
- Quantified by means of ratios $R_i = \frac{I_i}{S_i}$ where

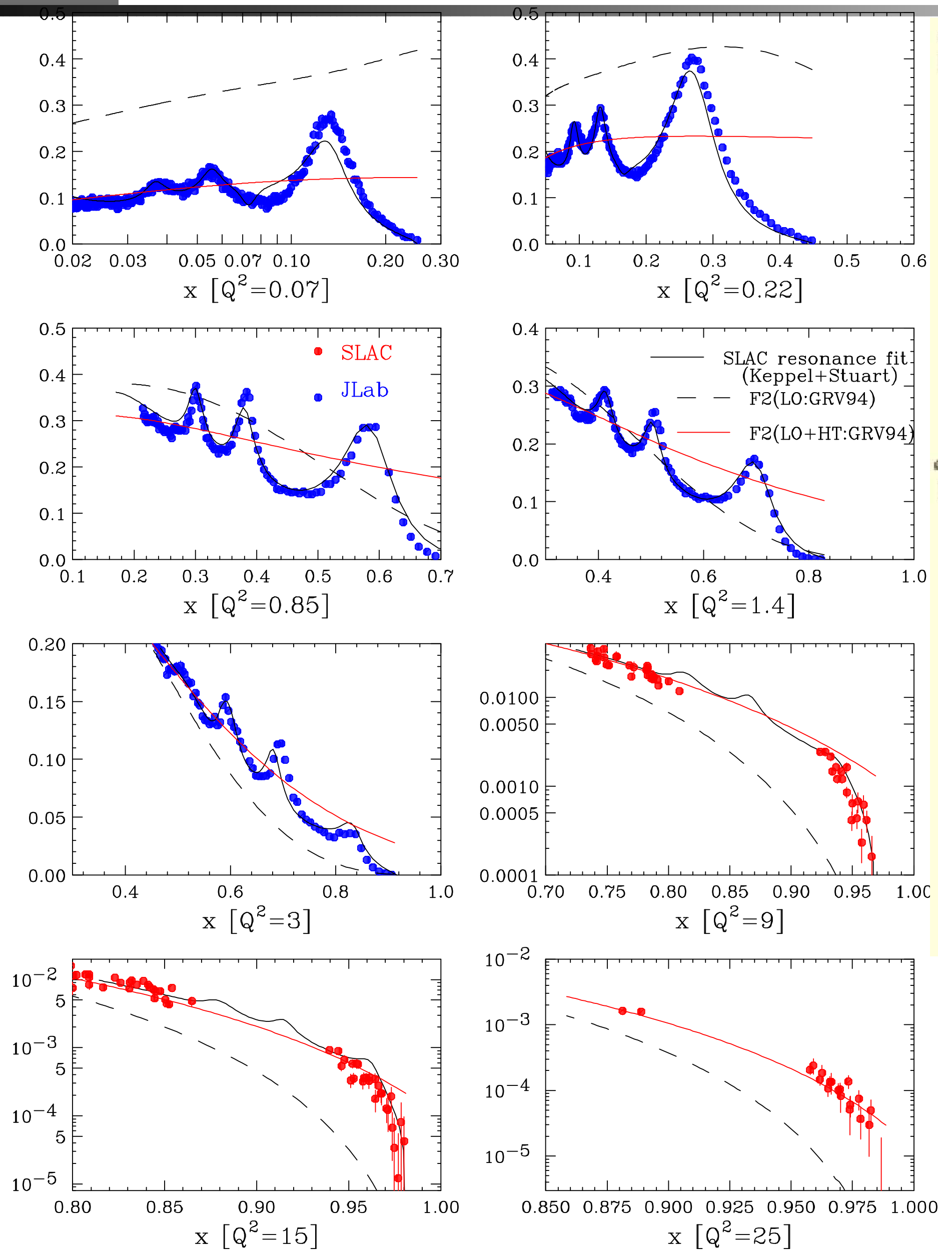
$$I_i(Q^2) = \int_{x_{min_i}}^{x_{max_i}} F_2^{Res}(x)dx$$

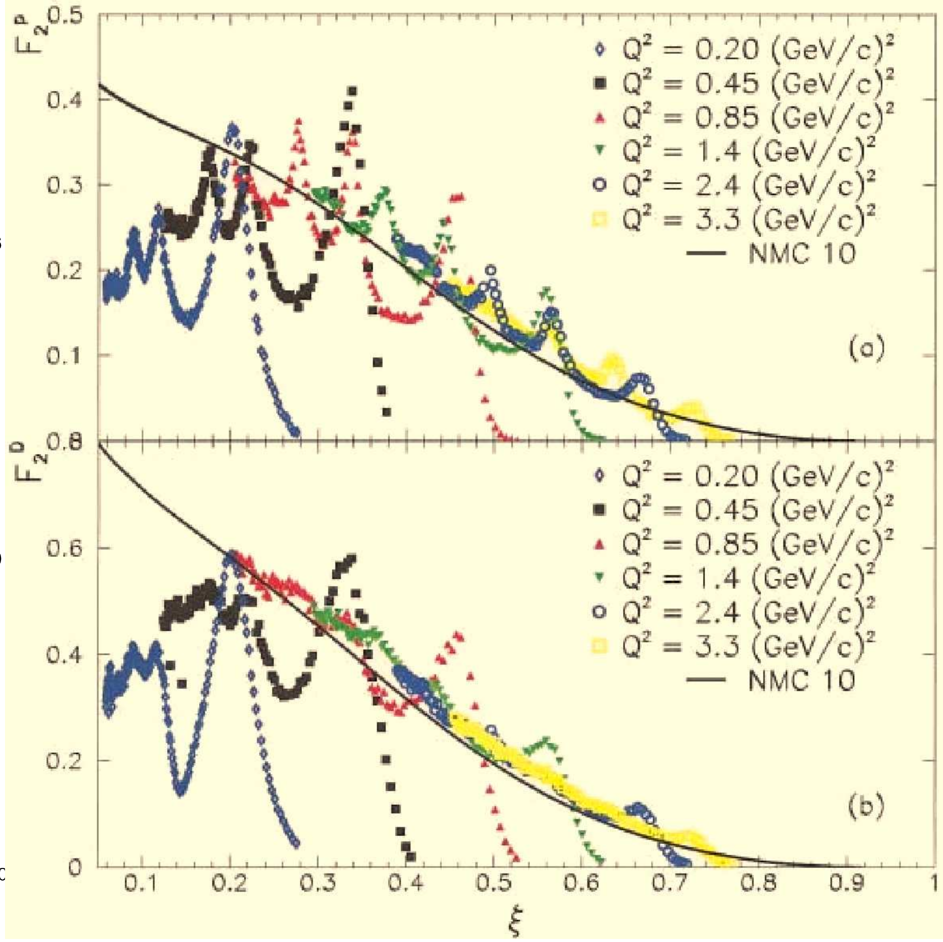
$$S_i(Q^2) = \int_{x_{min_i}}^{x_{max_i}} F_2^{DIS}(x)dx$$

Kinematic Plane



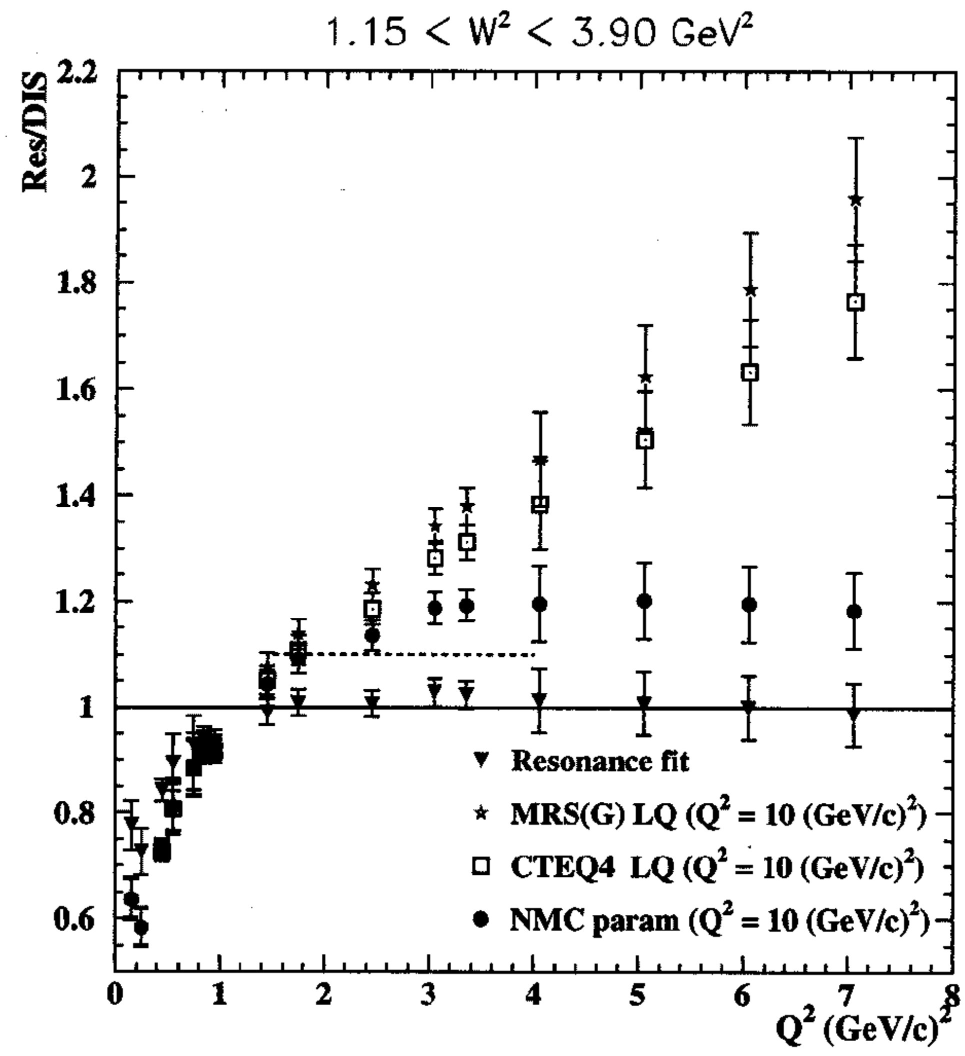
Duality in Unpolarized Case





Resonance spectra at diff. Q^2 appear at diff. x on DIS curve.

Duality in Unpolarized Case II



- Duality observed for $Q^2 \geq 1.5 (GeV/c)^2$
- Holds for individual resonance contributions as well as for whole range $1.15 \leq W^2 \leq 3.9 (GeV/c)^2$
- Holds for phenomenological DIS fits but not for leading order fits

Duality in Polarized Case

- Duality extensively studied for unpolarized case
- Duality hardly explored for spin-dependent photoabsorption cross section
- There is no *a-priori* reason to have the same situation as in unpolarized case
- Duality expected to fail at low Q^2 since for the proton the Ellis-Jaffe sum rule (integral over DIS region) and the GDH sum rule (real photon limit of integral over DIS+Resonance region) have opposite signs

The GDH Sum Rule

Relates anomalous contribution κ to the magnetic moment of the nucleon with total absorption cross section for circularly polarized real photons on polarized nucleons

$\Rightarrow$ A low-energy limit is expressed in terms of an integral that runs over all energies

$$I_{GDH} = \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left[\sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu)\right] = -\frac{2\pi^2\alpha}{M^2}\kappa^2$$

ν_0: **pion production threshold**

$\sigma_{\frac{1}{2}(\frac{3}{2})}$: **polarized photoabsorption cross section with total helicity in initial state equal** $\frac{1}{2}(\frac{3}{2})$

GDH Sum Rule II

- κ for proton (neutron, deuteron): $1.79\ (-1.91,\ -0.143)$

$$\begin{aligned} I^p_{GDH} &= -204\ \mu\text{b} \\ I^n_{GDH} &= -233\ \mu\text{b} \\ I^d_{GDH} &= -0.65\ \mu\text{b} \end{aligned}$$

- hard to verify experimentally because of up-to-now limited range in photon energies
- results for low-energy part of integral in conjunction with Regge extrapolations $\Rightarrow$ sizeable contributions from higher energies and multi-pion photoproduction needed
- expected NOT to fail EXCEPT in case of existence of $J = 1$ fixed poles

The Generalized GDH Integral

use virtual photons instead of real ($Q^2 = 0$) photons

$$
\begin{aligned}
I(Q^2) &\equiv \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left[\sigma_{\frac{1}{2}}(\nu, Q^2) - \sigma_{\frac{3}{2}}(\nu, Q^2)\right] \\
&= \frac{16\pi^2\alpha}{Q^2} \int_0^{x_0} dx \frac{g_1(x, Q^2) - \gamma^2 g_2(x, Q^2)}{\sqrt{1+\gamma^2}} \\
&= \frac{8\pi^2\alpha}{M} \int_0^{x_0} \frac{dx}{x} \frac{A_1(x, Q^2) F_1(x, Q^2)}{K}
\end{aligned}
$$

K**: virtual photon flux factor (in Gilman convention)**

$\gamma = Q^2/\nu^2$

A_1**: longitudinal cross-section asymmetry for virtual-photon absorption**

Connections with other Sum Rules

- in leading-twist approximation, $\gamma \rightarrow 0$, Burkhardt-Cottingham sum rule holds, i.e.

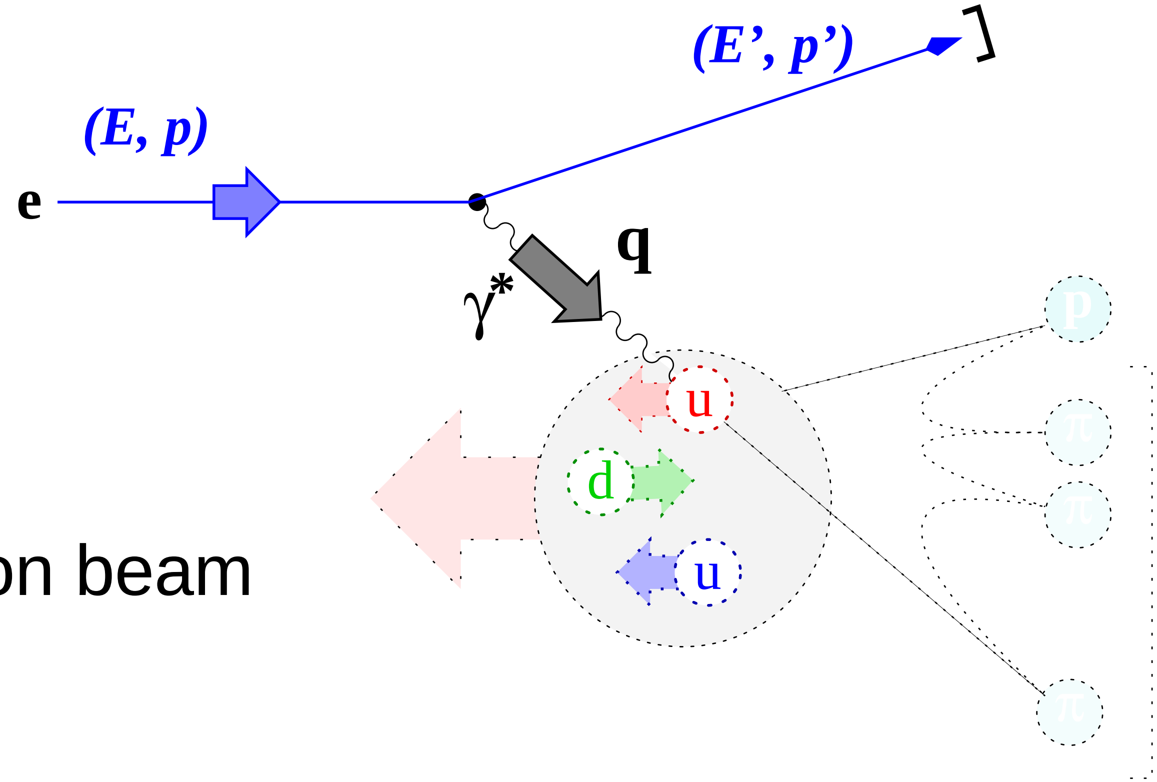
$$\int_0^1 g_2(x,Q^2)dx = 0$$

$$I_{GDH}(Q^2)_{\gamma^2\rightarrow 0} = \frac{16\pi^2\alpha}{Q^2}\int_0^{x_0} dx \frac{g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)}{\sqrt{1+\gamma^2}} = \frac{16\pi^2\alpha}{Q^2}\Gamma_1(Q^2)$$

- $\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2)dx$ Ellis-Jaffe Integral
- difference between proton and neutron $\Rightarrow$ Bjorken SR:

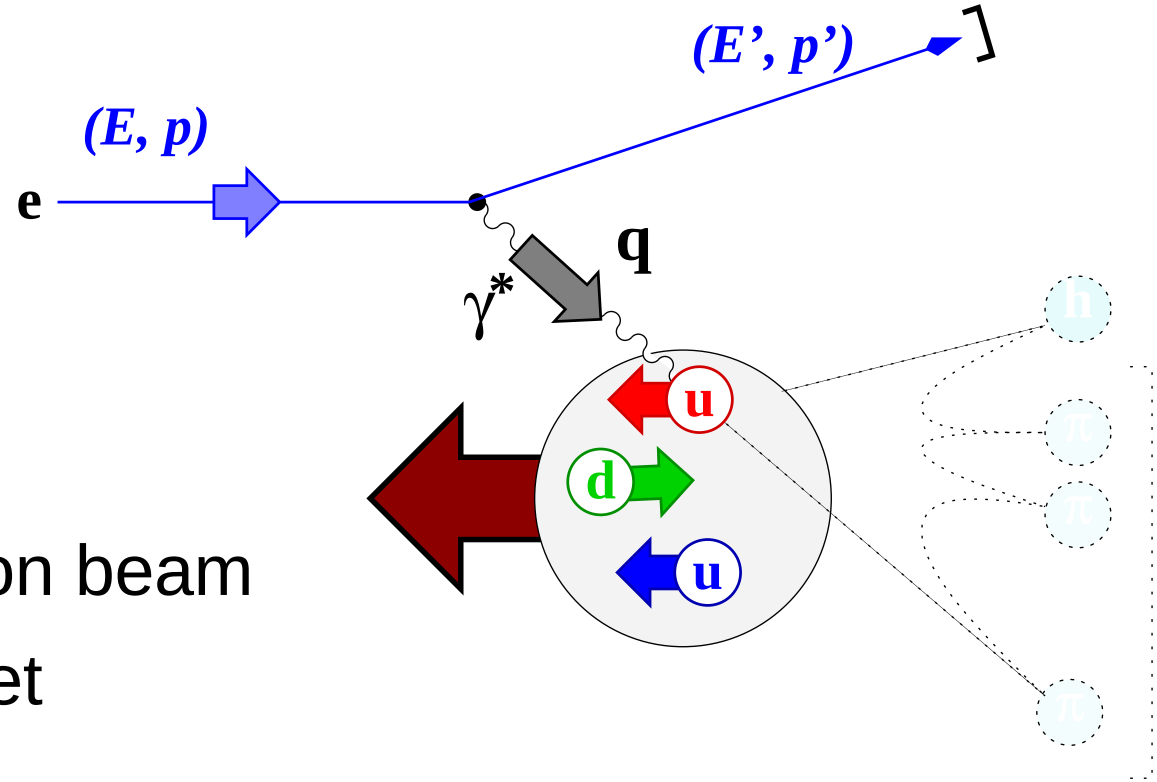
$$\frac{Q^2}{16\pi^2\alpha}\{I^p_{GDH}(Q^2) - I^n_{GDH}(Q^2)\}_{\gamma^2\rightarrow 0} = \Gamma_1^P(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6}g_a$$

Experimental Prerequisits



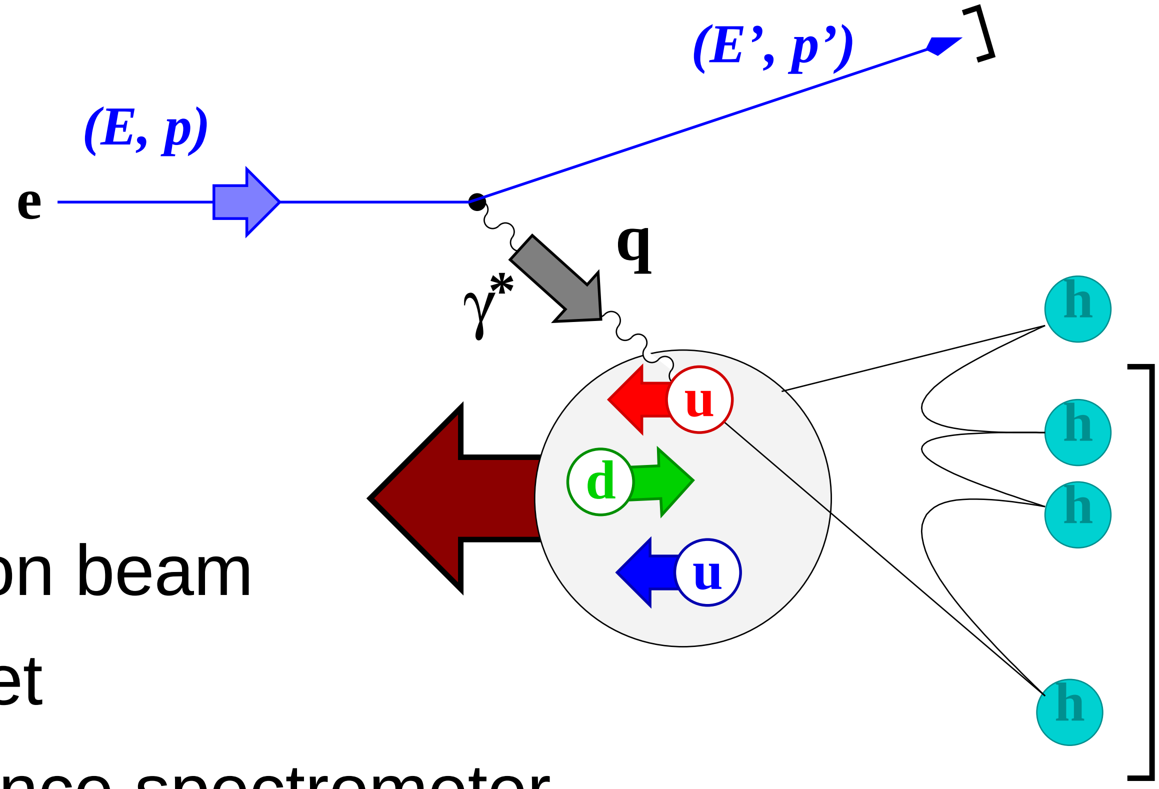
- Polarized lepton beam

Experimental Prerequisits



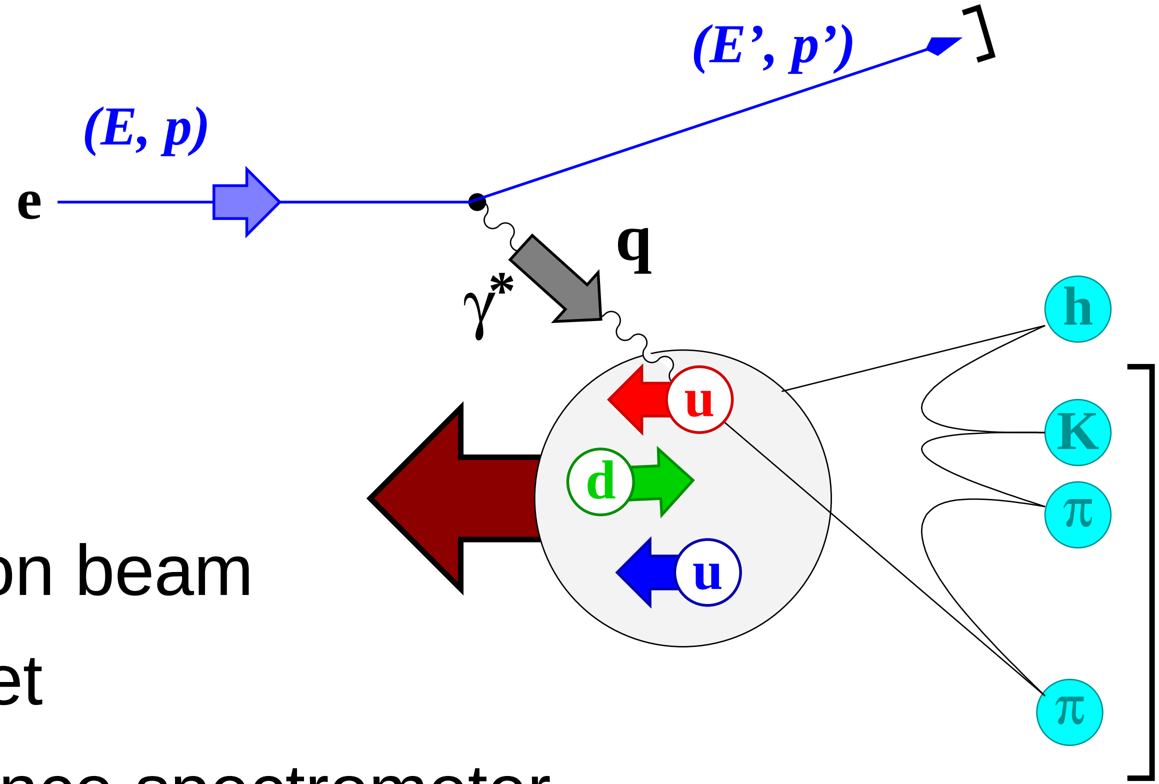
- Polarized lepton beam
- Polarized target

Experimental Prerequisits



- Polarized lepton beam
- Polarized target
- Large acceptance spectrometer

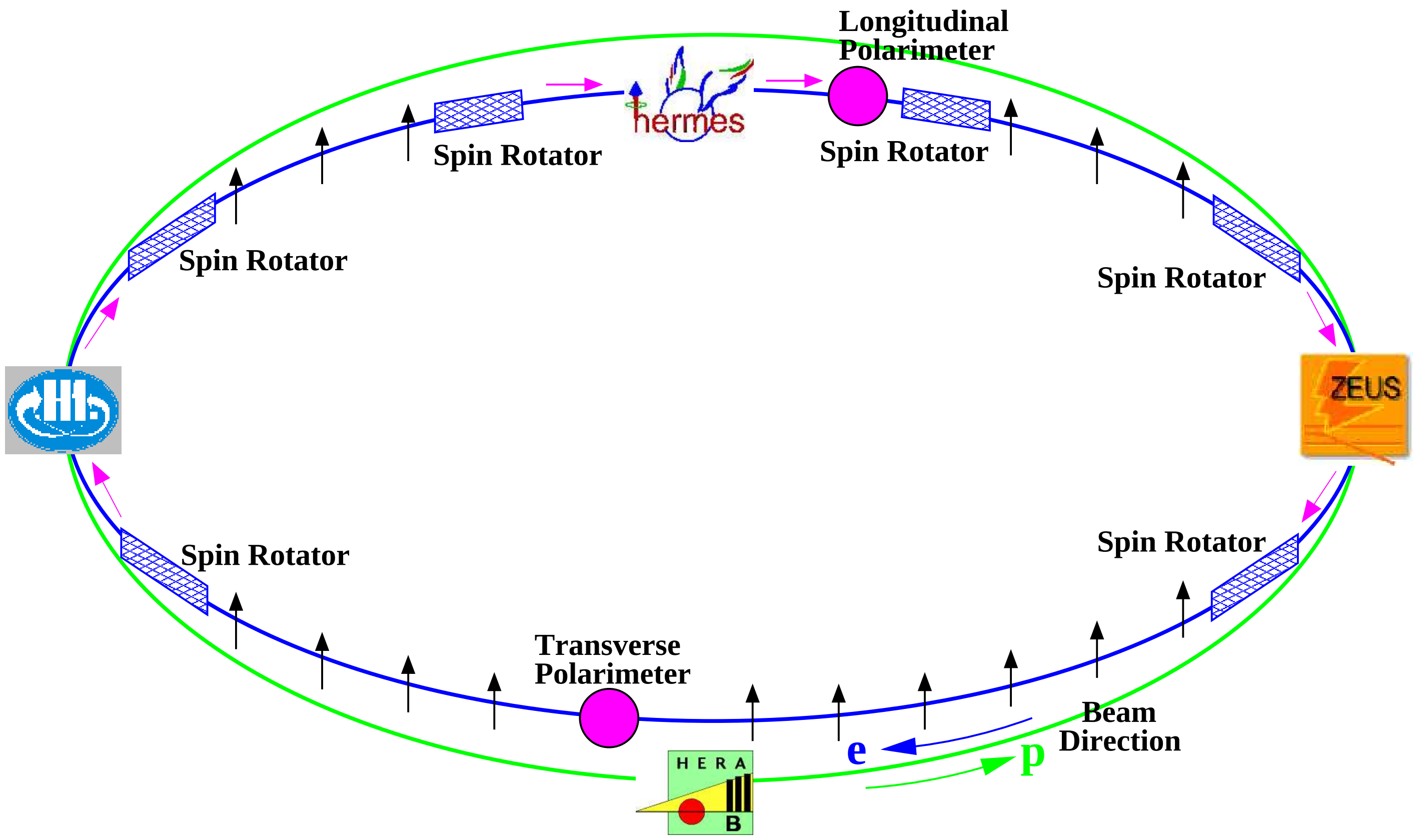
Experimental Prerequisits



- Polarized lepton beam
- Polarized target
- Large acceptance spectrometer
- Good **P**article **ID**entification (PID)

Polarized Beam at HERA

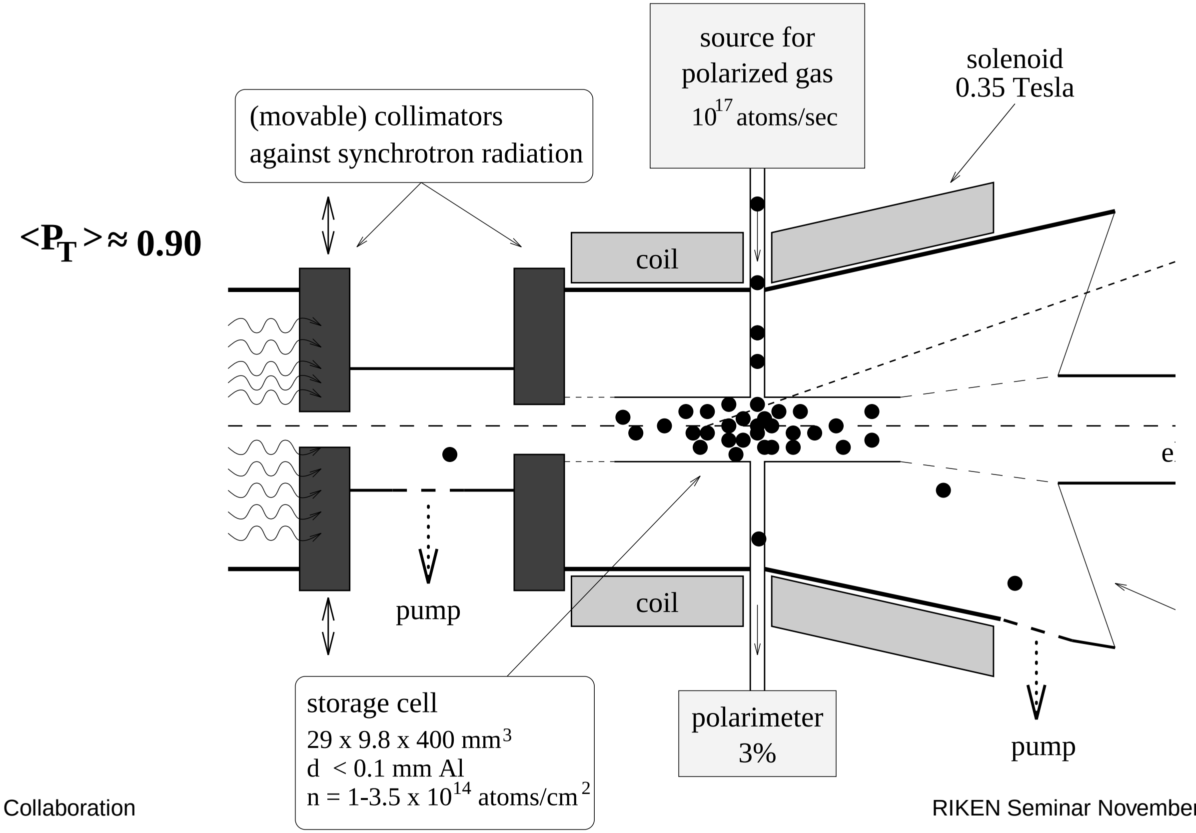
Polarized Beam at HERA



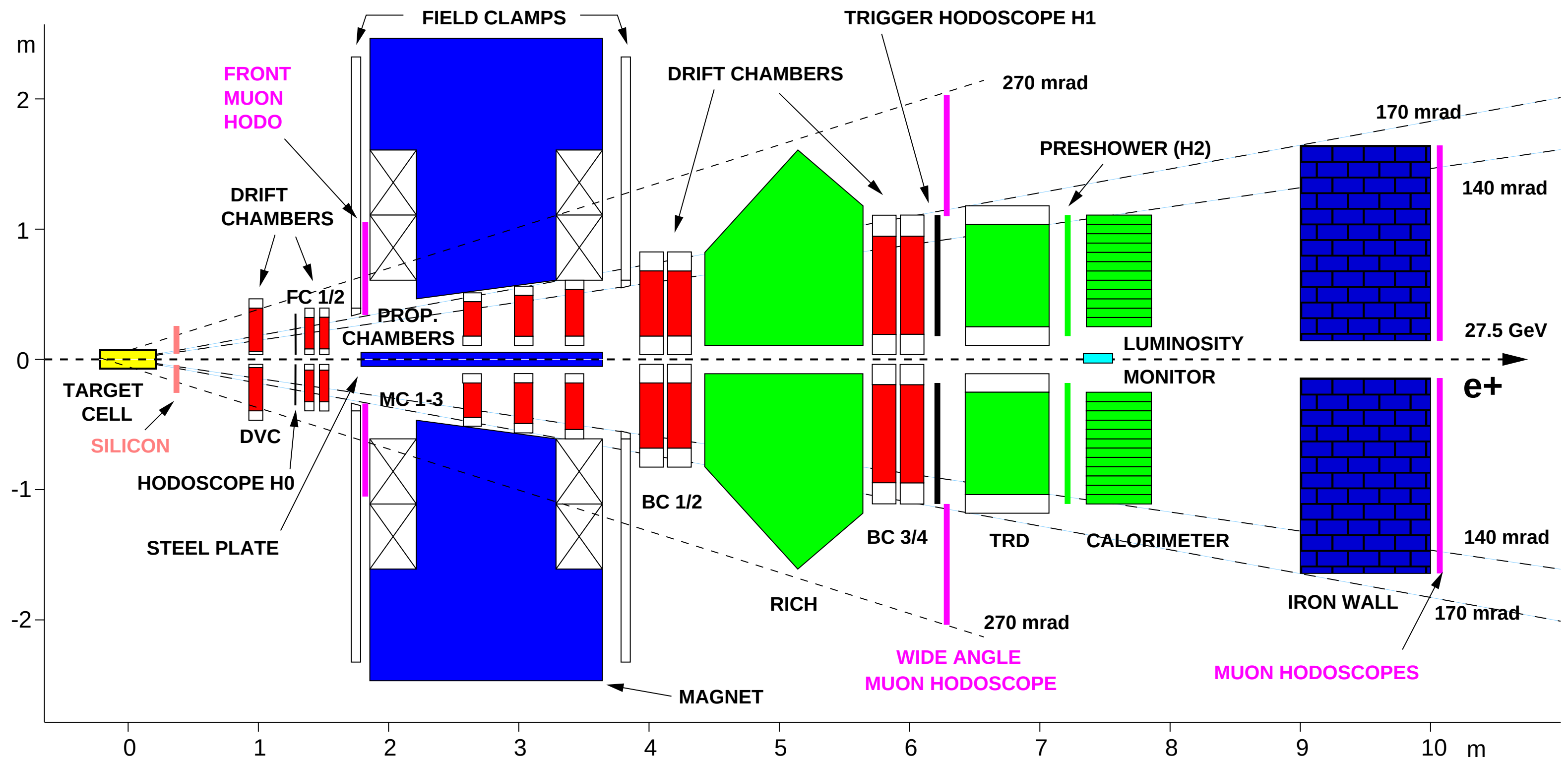
- 27.5 GeV e^+/e^- beam
- Self-polarizing through Sokolov-Ternov-Effect
- Average beam polarization of about 55%

HERMES Internal Gas Target

- Storage cell with atomic beam source
- Pure target (NO dilution)
- Polarized or unpolarized targets possible
- Different gas targets available (H, D, He, N, Kr ...)

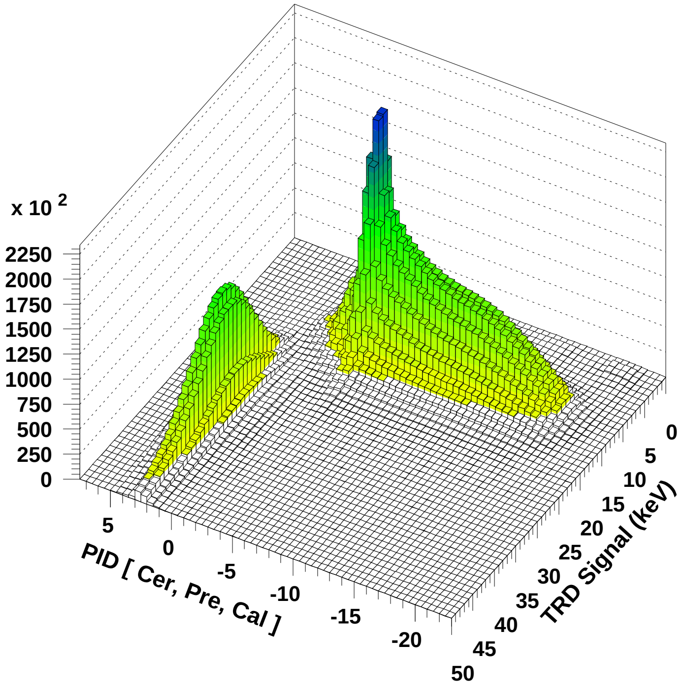


The HERMES Spectrometer



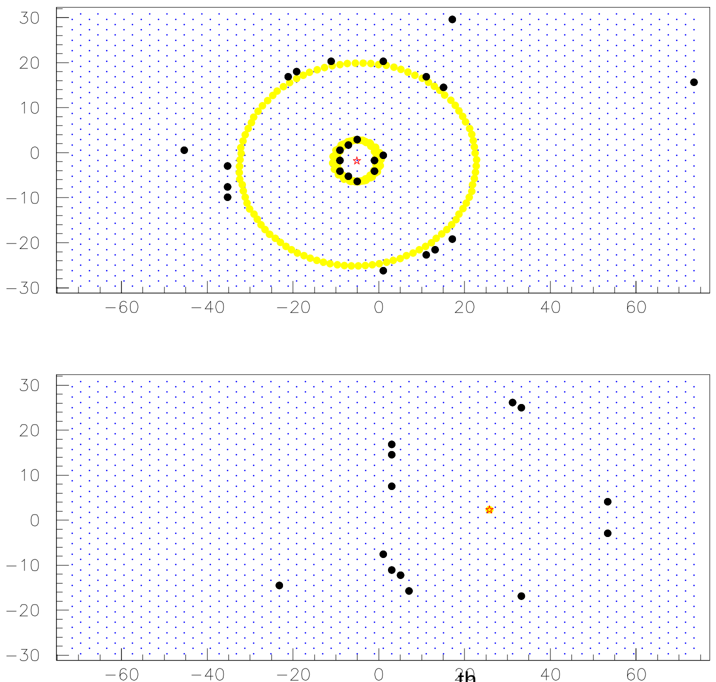
- Internal storage cell: pure gas target
- Forward acceptance spectrometer: 40 mrad $\leq \Theta \leq$ 220 mrad
- Tracking: 57 tracking planes: $\delta P/P = (0.7 - 1.3)\%$, $\delta\Theta \leq 0.6$ mrad
- PID: Cherenkov (RICH after 1997), TRD, Preshower, Calorimeter

Particle Identification



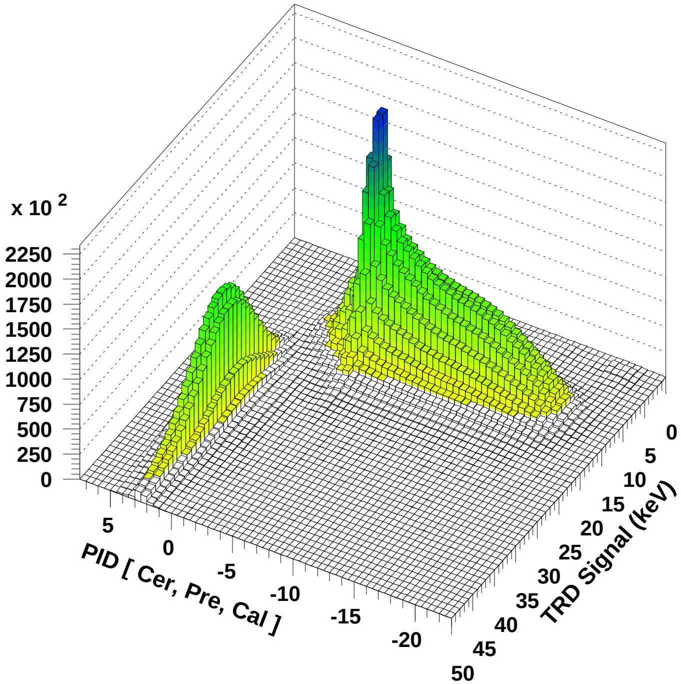
Excellent e^+/e^- identification:

- Efficiency $\geq 98\%$
- Hadron contamination $\leq 1\%$



After 1997 use dual radiator
Ring Imaging CHerenkov

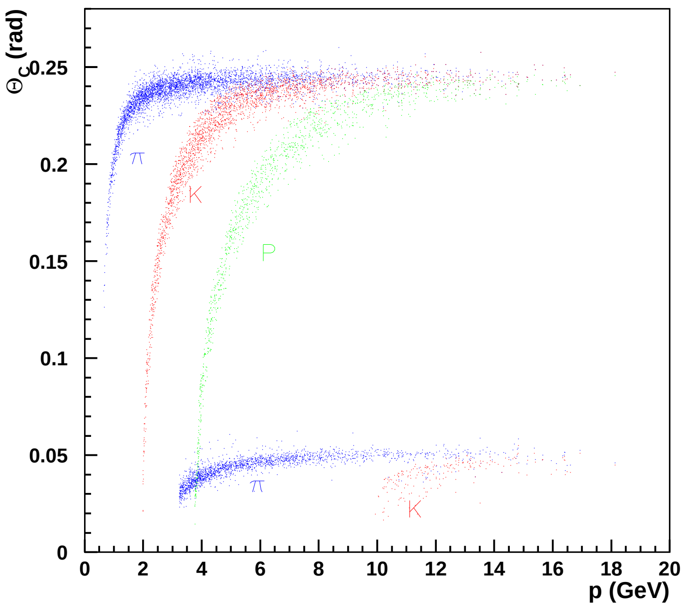
Particle Identification



Excellent e^+/e^- identification:

- Efficiency $\geq 98\%$
- Hadron contamination $\leq 1\%$

After 1997 use dual radiator
Ring Imaging CHerenkov
$\hookrightarrow$ very good hadron identification
in the range $2\ GeV \leq P_h \leq 15\ GeV$



Inclusive Asymmetries

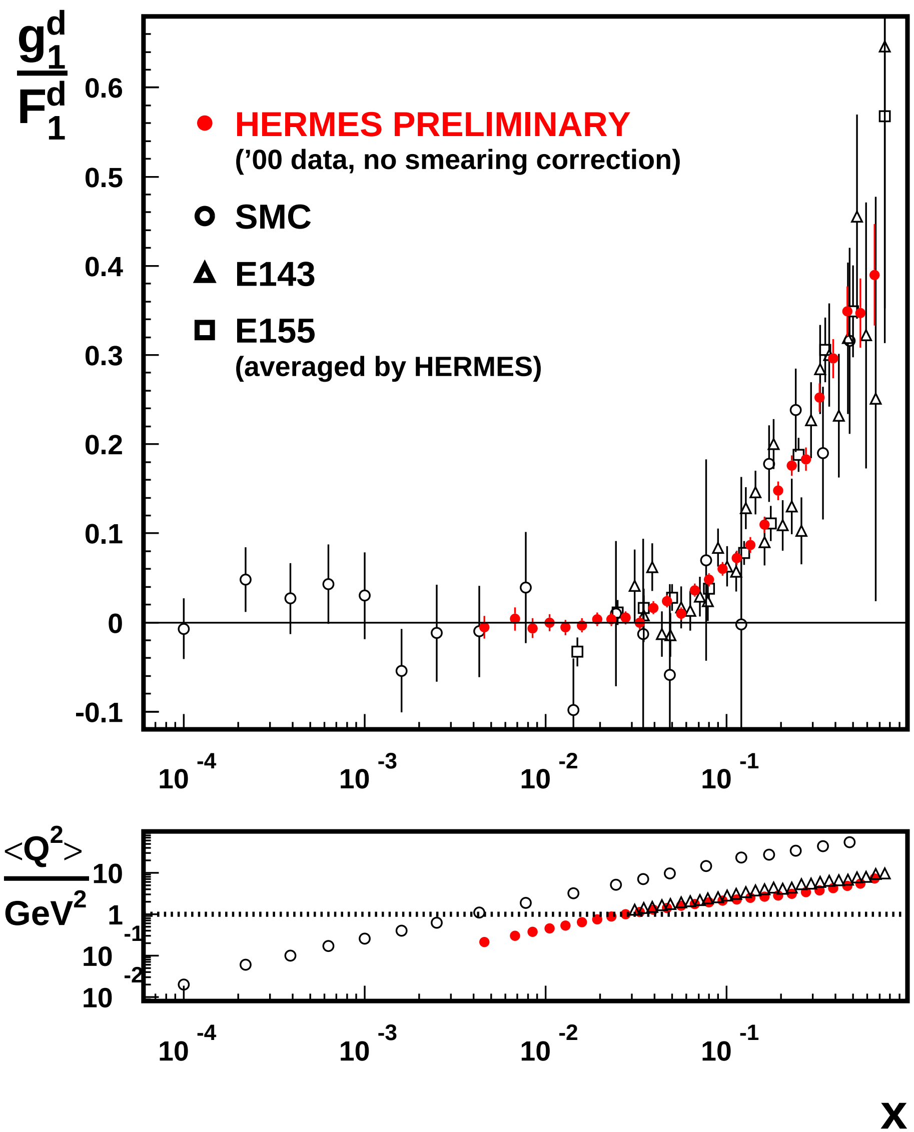
measure double spin asymmetries:

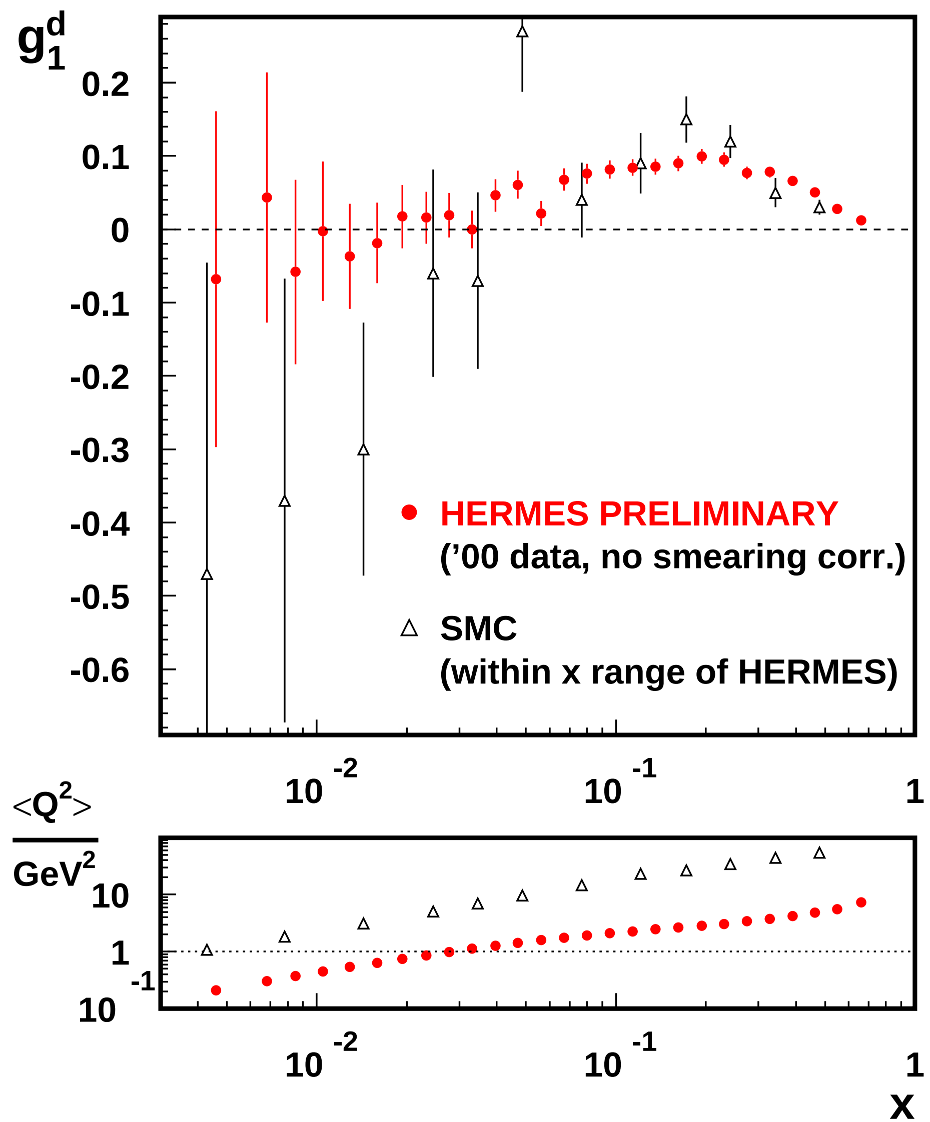
$$A_\parallel = \frac{1}{\langle P_T P_B \rangle} \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = D(A_1 - \eta A_2)$$

$$\boxed{A_1 = \frac{\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}}{\sigma_{\frac{1}{2}} + \sigma_{\frac{3}{2}}} = (1-\gamma^2)\frac{g_1}{F_1}}$$

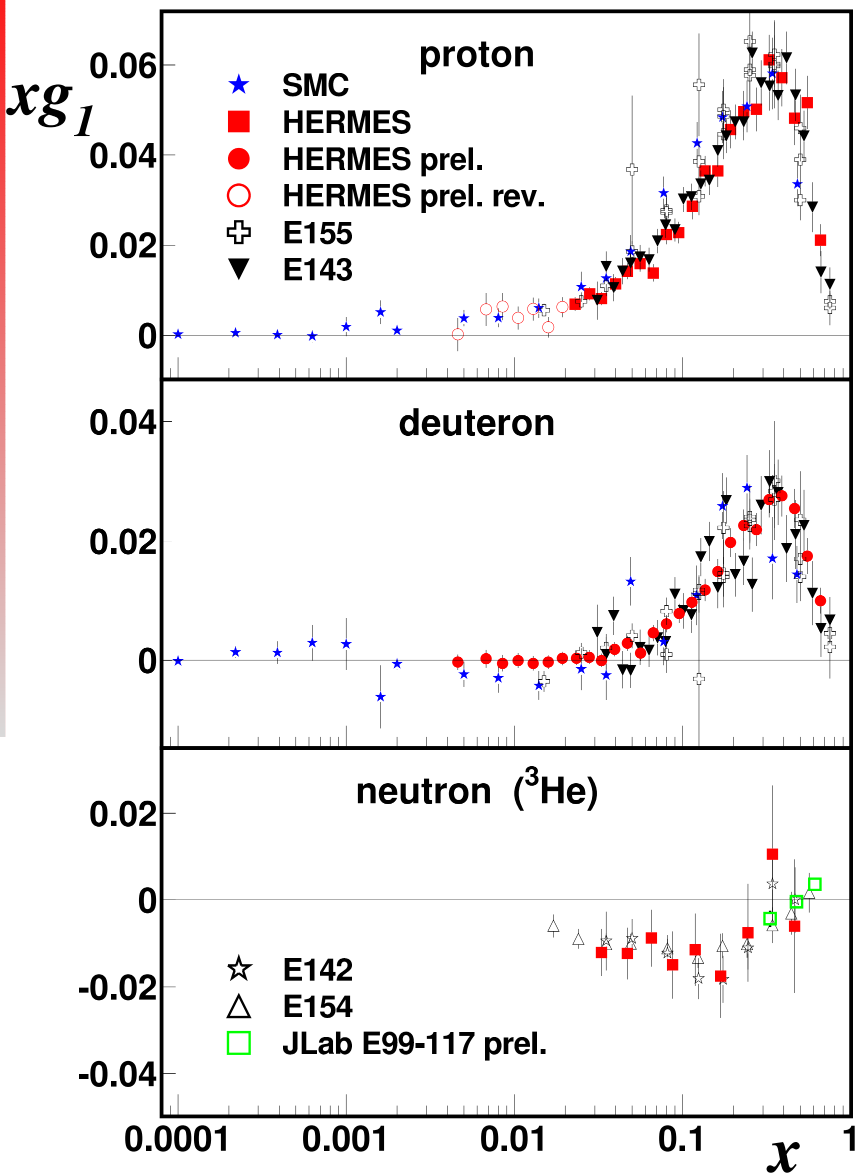
- $F_1 = \frac{(1+\gamma^2)}{2x(1+R)} F_2$
- F_2 from world data
- A_2: use $A_2^n = 0$, fit to A_2^p data or $A_2^d = A_2^{WW}$

Polarized Deuterium Data





World Data on $x \cdot g_1(x)$



"back-on-the-envelope"

$$g_1^p > g_1^d > g_1^n$$

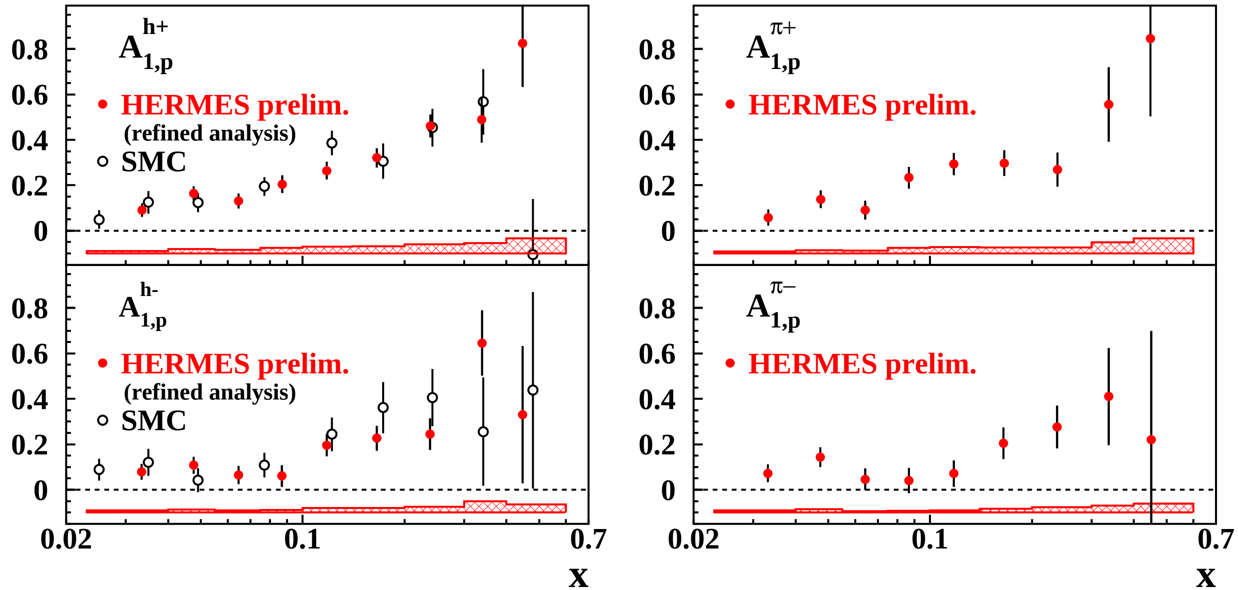
(neglecting sea quark contributions)

$$p: \ 2 \cdot \frac{4}{p} \Delta u_p + \frac{1}{9} \Delta d_p$$
$$d: \ p + n$$
$$n: \ 2 \cdot \frac{1}{9} \Delta d_n + \frac{4}{9} \Delta u_n$$
$$= 2 \cdot \frac{1}{p} \Delta u_p + \frac{4}{9} \Delta d_p$$

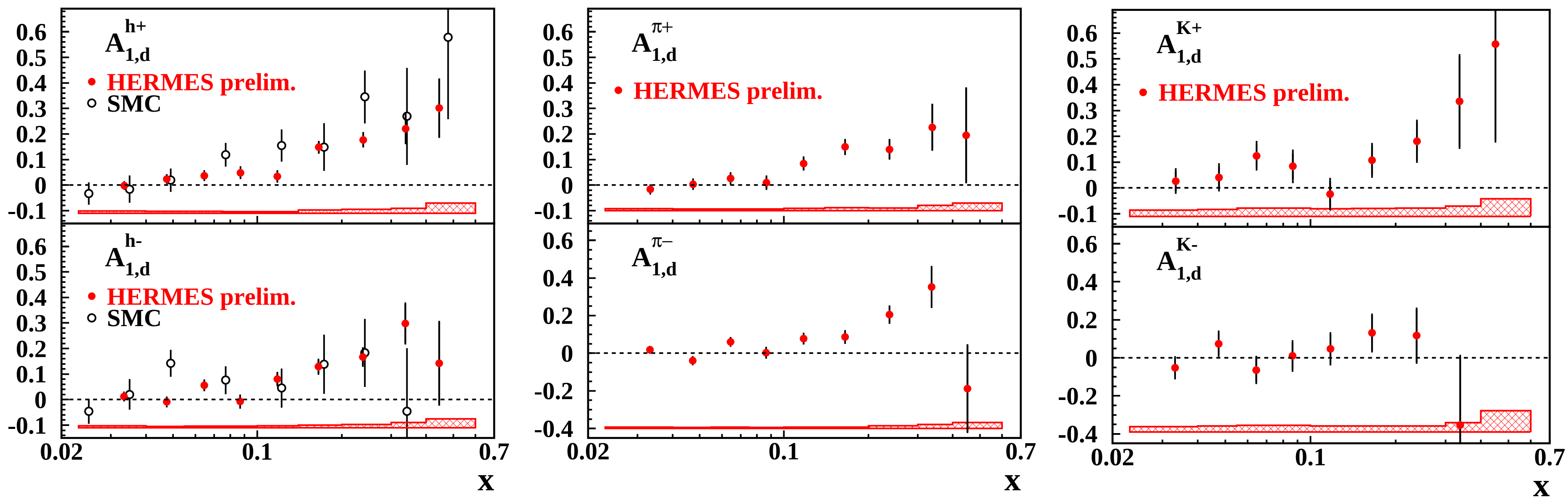
$\Delta u_p > 0 \qquad \Delta d_p < 0$

$\Gamma_1^{p,d} > 0$ **vs.** $I_{GDH}(0) < 0$ **!!!**

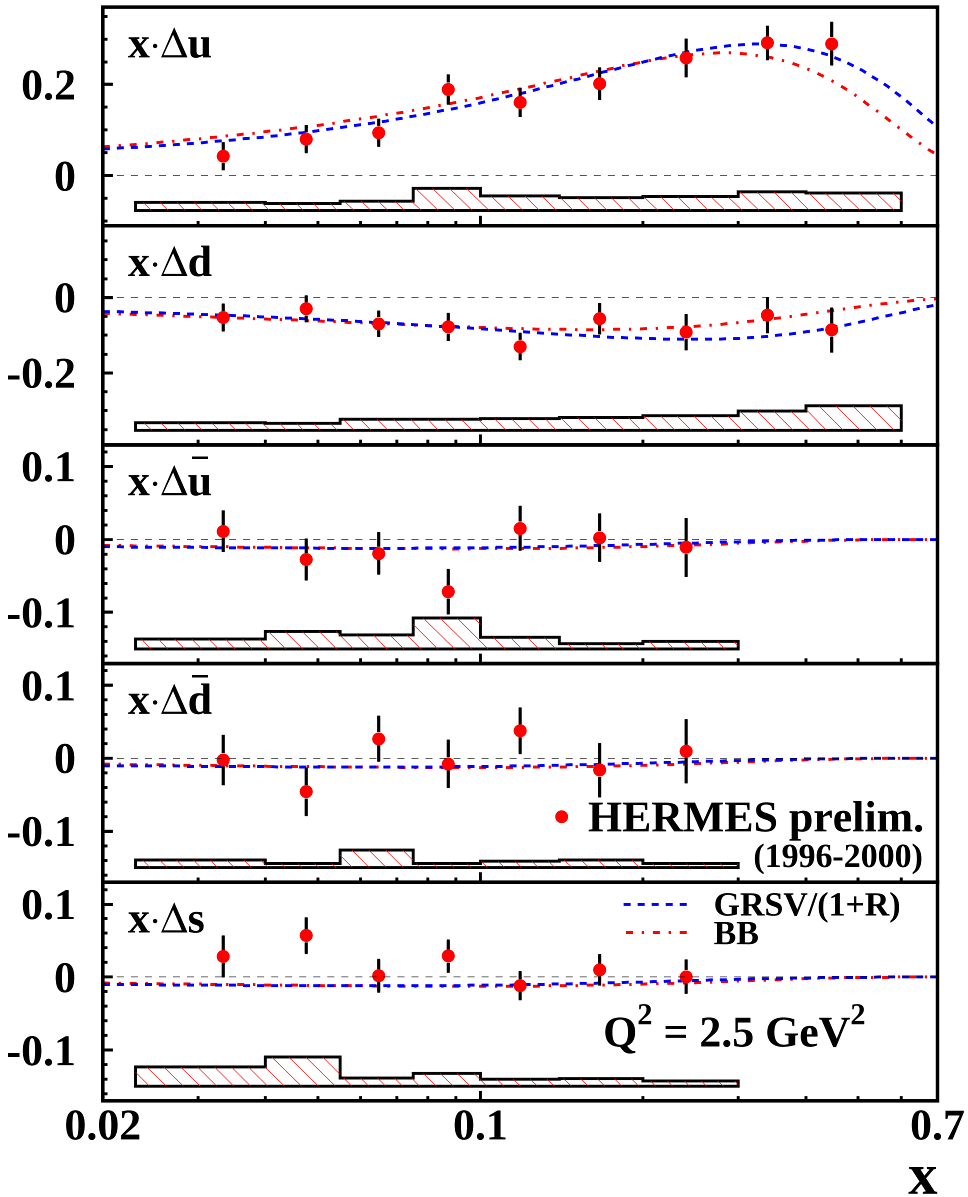
Hadron Asymmetries



$$0.023 \leq x \leq 0.6$$
$$0.2 \leq z \leq 0.8$$
$$10 \leq W^2$$
$$1 \leq Q^2$$



Polarized Quark Distributions



- 5-param. flavor decomposition
- assume symmetric quark sea
- u-quark strongly polarized
- d-quark strongly anti-polarized
- sea quark polarizations consistent with zero
- good agreement with LO-QCD fits

Analysis of Resonance Region

$$g_1^{Res,DIS}(x,Q^2) = A_1(x) \cdot F_1(x,Q^2)$$

Resonance Region:

- $A_1 = \frac{A_\parallel}{D} - \eta A_2$
- $F_1 = \frac{F_2}{2x(1+R)}$
- $R = 0.18; \quad F_2 = F_2^{Res}$
  Bodek et al.'79
- $A_2 = 0.06 \pm 0.16 \; (Q^2 = 3)$
  E143 '98

DIS Region:

- A_1 from parametrization (i.e. x^α with $\alpha = 0.7$) or from data
- $F_1 = \frac{F_2}{2x(1+R)}$
- $R = R(x,Q^2); \quad F_2 = F_2^{DIS}$
  Whitlow et al. '90; NMC '95

g_1-Integrals

want to study $R_i = I_i/S_i$ with

$$I_i(Q^2) = \int_{x_{min_i}}^{x_{max_i}} g_1^{Res}(x)dx$$

$$S_i(Q^2) = \int_{x_{min_i}}^{x_{max_i}} g_1^{DIS}(x)dx$$

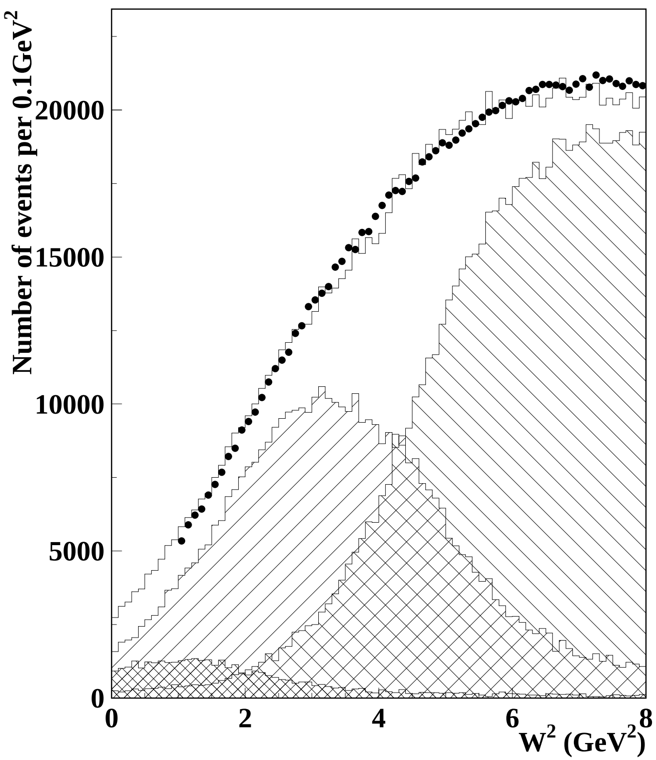
Δx_i delimited by ΔW^2 region ($1 \leq W^2 \leq 4.0$ GeV^2):

Q_i^2 (GeV^2)	Δx_i
1.2 - 2.4	0.35 - 0.93
2.4 - 4.0	0.49 - 0.96
4.0 - 12.0	0.63 - 0.98

Background Contributions

$$A_{\parallel}^{meas} = f_{el} A_{\parallel}^{el} + f_{Res} A_{\parallel}^{Res} + f_{DIS} A_{\parallel}^{DIS}$$

$$A_{\parallel}^{Res} = (A_{\parallel}^{meas} - f_{el} A_{\parallel}^{el} - f_{DIS} A_{\parallel}^{DIS}) / f_{Res}$$

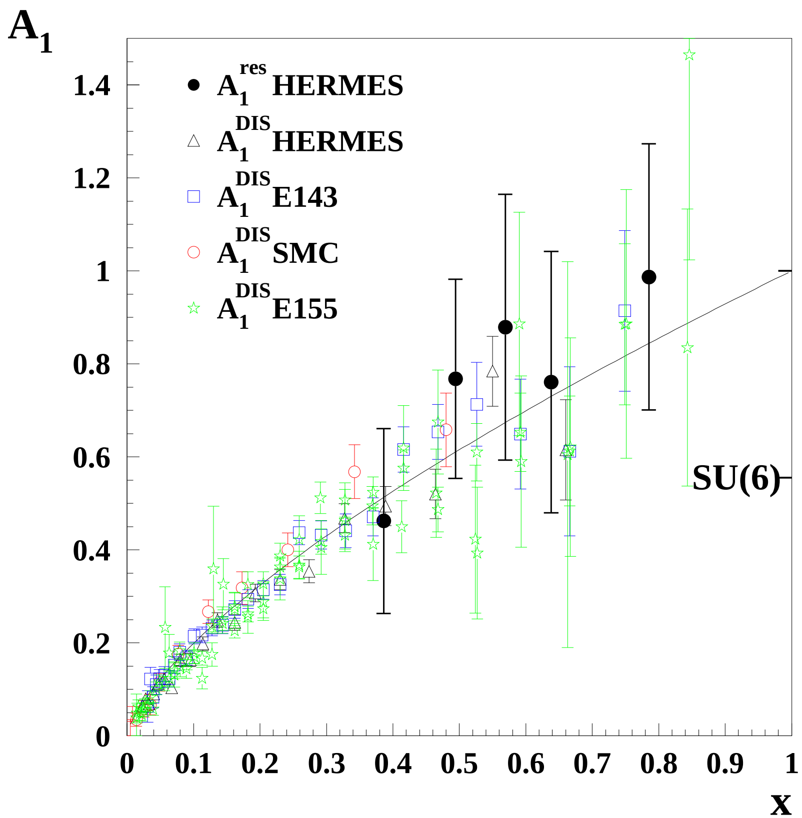


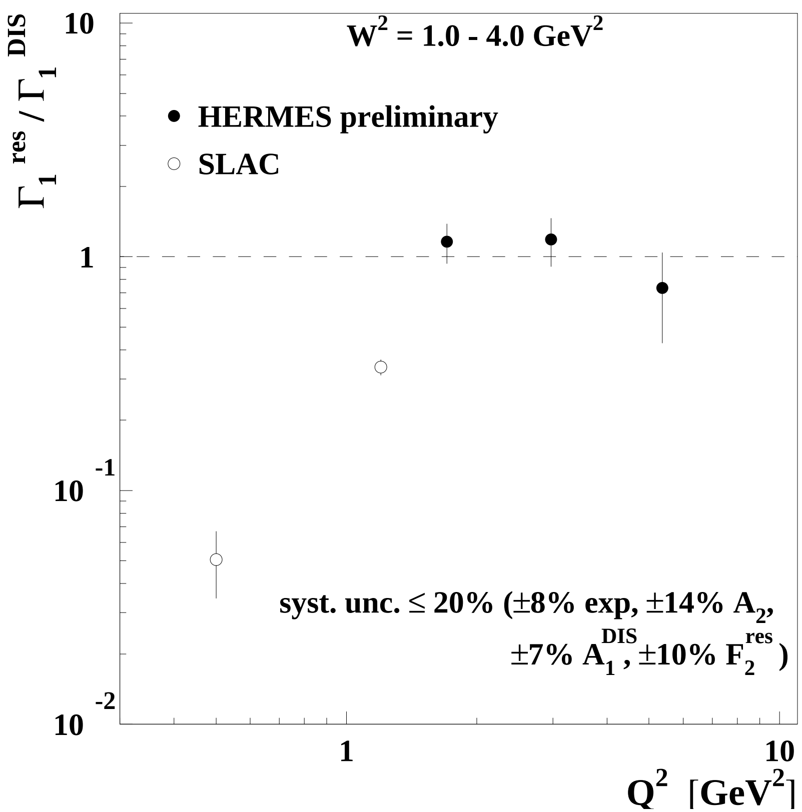
Contributions to Resonance Region:

Q^2-bin	from elastic peak	from DIS
1	0.091	0.099
2	0.054	0.135
3	0.038	0.184

small contribution from elastic tail

Duality for Polarized Proton Scattering





- $\langle A_1^{Res}/A_1^{DIS} \rangle = 1.11 \pm 0.16$ **(stat.)** ± 0.18 **(syst.)**
- $\Gamma_1^{Res}/\Gamma_1^{DIS}$ **independent of** Q^2 **in HERMES region**
- **Duality holds for** $Q^2 \geq 1.6\, GeV^2$
- **First observation of duality in polarized lepton scattering**

The Generalized GDH at HERMES

Combine data from DIS and Resonance regions to evaluate generalized GDH integral:

$$I_{GDH}(Q^2) = \frac{8\pi^2\alpha}{M} \int_0^{x_0} \frac{dx}{x} \frac{A_1(x,Q^2)F_1(x,Q^2)}{K}$$

$$I^n_{GDH} = \frac{I^d_{GDH}}{1-1.5\omega_d} - I^p_{GDH}$$

$$A_1 = \frac{A_\parallel}{D} - \eta A_2$$

$$A_\parallel = \frac{N^{\uparrow\downarrow}L^{\uparrow\uparrow} - N^{\uparrow\uparrow}L^{\uparrow\downarrow}}{N^{\uparrow\downarrow}L_P^{\uparrow\uparrow} + N^{\uparrow\uparrow}L_P^{\uparrow\downarrow}}$$

$$F_1 = F_2 \frac{1+\gamma^2}{2x(1+R)}$$

$$K = \nu\sqrt{1+\gamma^2}$$

$$\gamma^2 = Q^2/\nu^2$$

$$x_0 = Q^2/2M\nu_0$$

$F_2^{DIS} \rightarrow$ NMC fit

$R^{DIS} \rightarrow$ SLAC fit

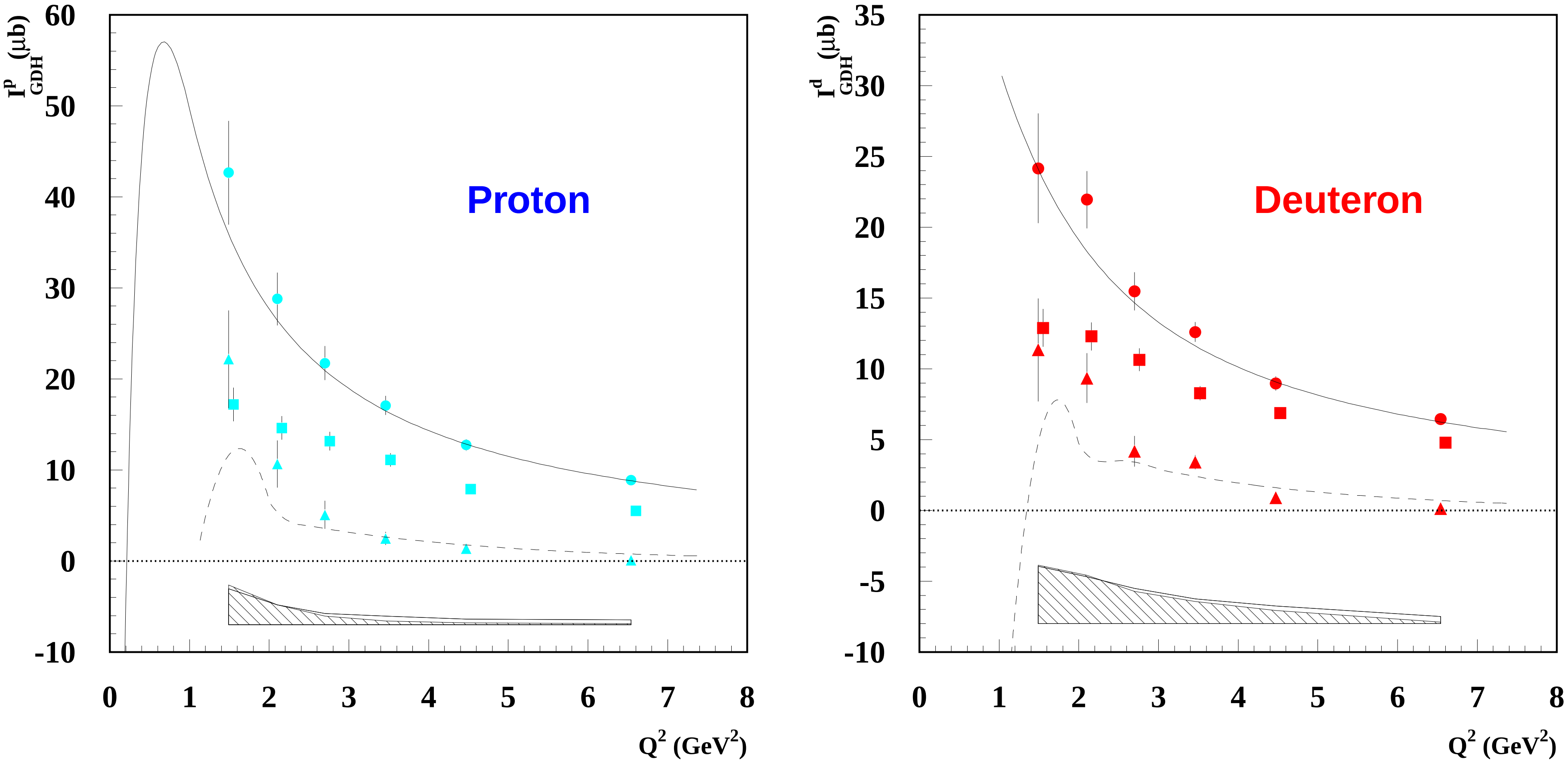
$$A_2^{DIS,d(p)} = \frac{0.2(0.53)Mx}{\sqrt{Q^2}}$$

$F_2^{Res} \rightarrow$ Bodek fit

$$R^{DIS} = \sigma_L/\sigma_T = 0.18$$

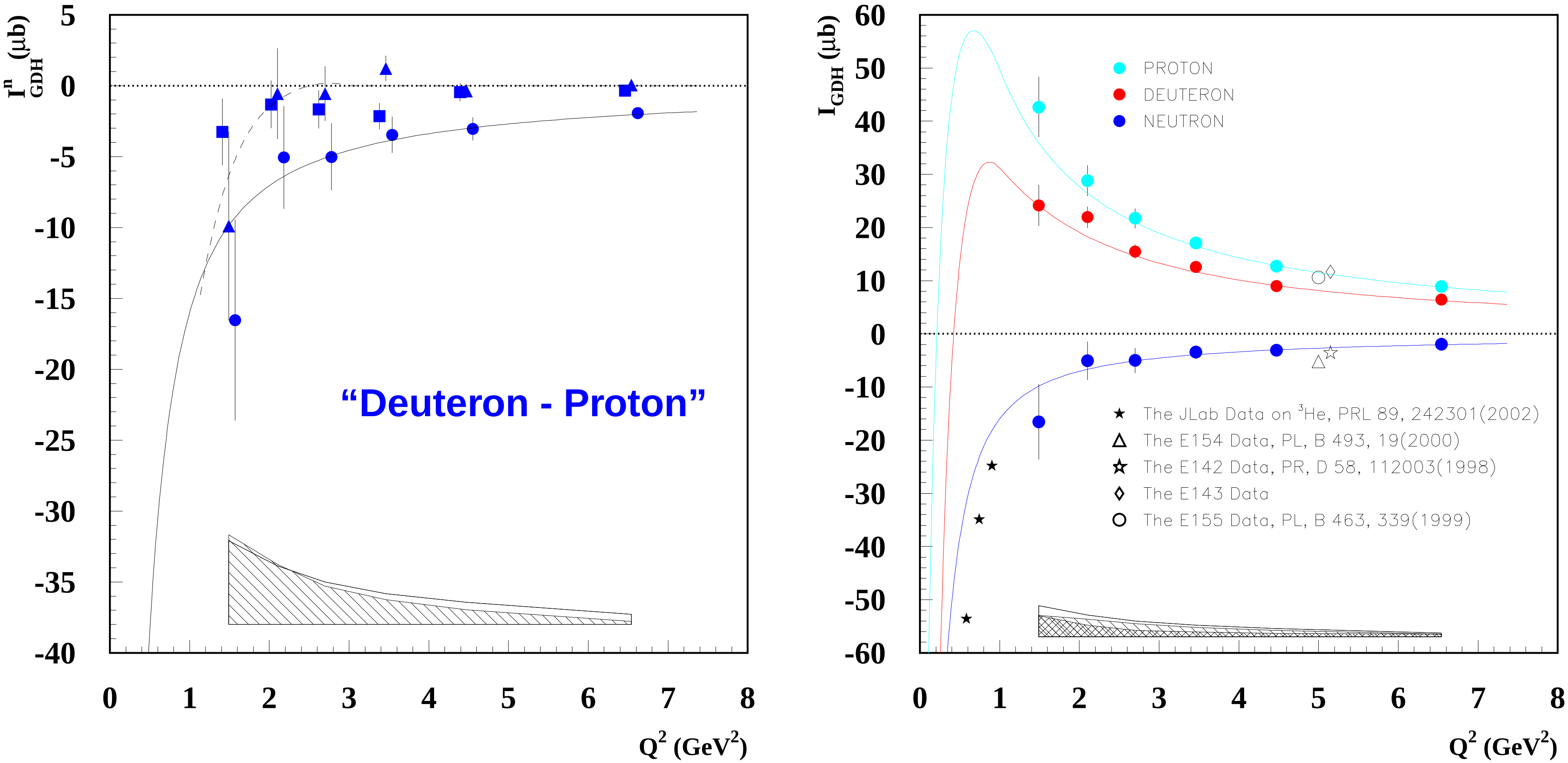
$$A_2^{Res,d(p)} = 0 \; (0.06 \pm 0.16)$$

GDH Integral for Proton and Deuteron



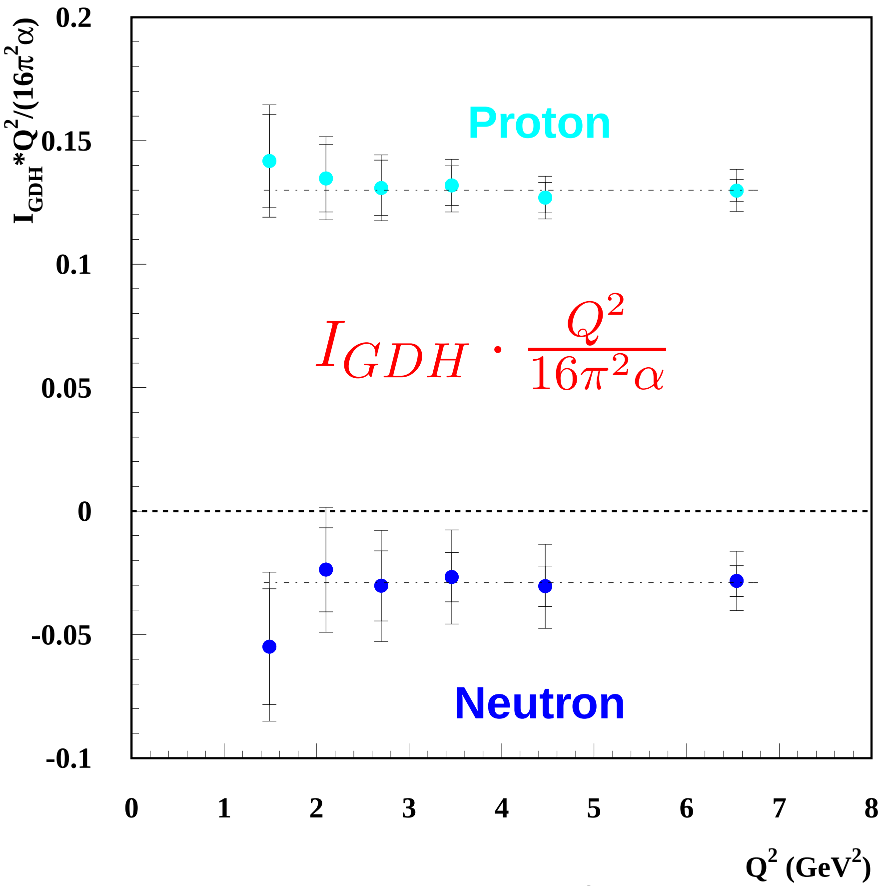
- **Parametrization by Soffer and Teryaev (solid) describe both data sets well**
- **Model for resonance contribution (dashed) by Aznauryan falls off to early at low Q^2**
- **Q^2- behaviour consistent with pQCD evolution of integral**
- **DIS contribution dominates at higher Q^2**

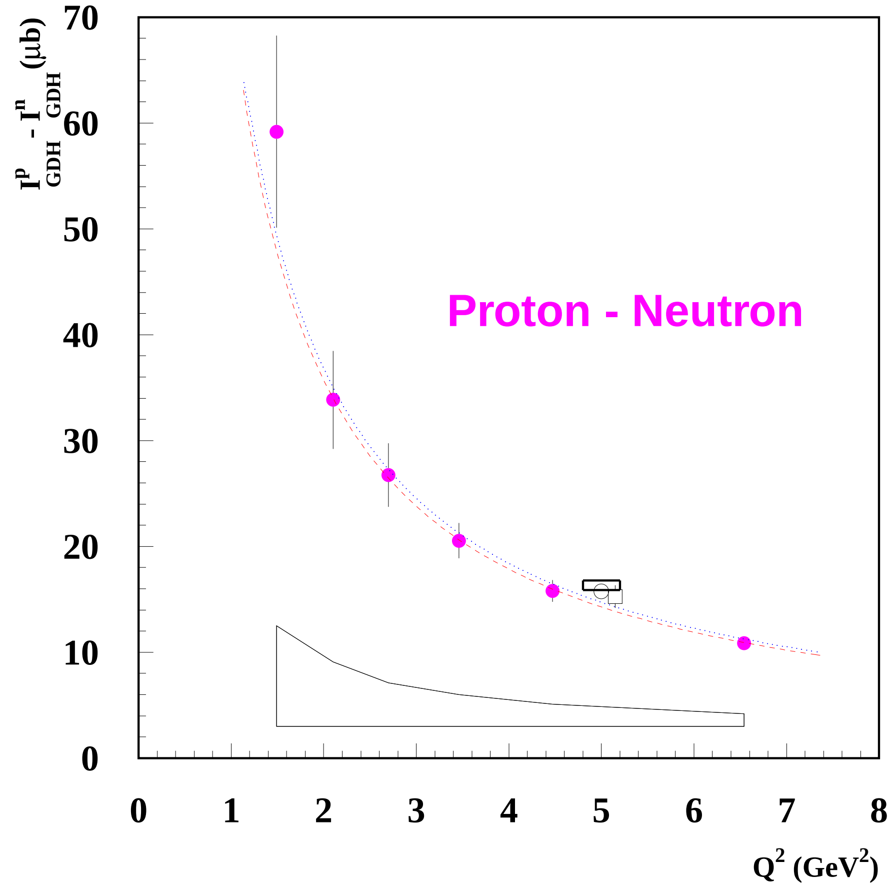
GDH Integral



- I_{GDH} **for neutron is negativ and smaller than for proton**
- I_{GDH} **for deuteron is positiv and smaller than for proton**
- **Agreement with SLAC and JLab data**
- **Turn-over to meet real-photon prediction not seen (**$I^{p,n,d}_{GDH}$**=-204, -233, -0.65)**

Q^2-Dependence of the GDH Integral

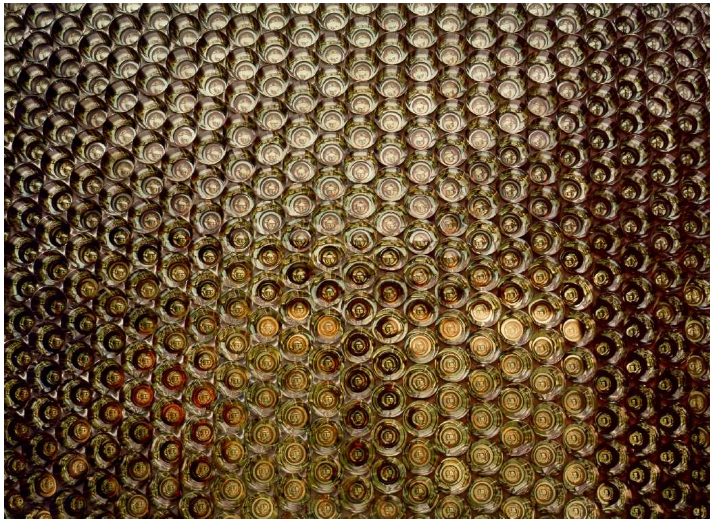


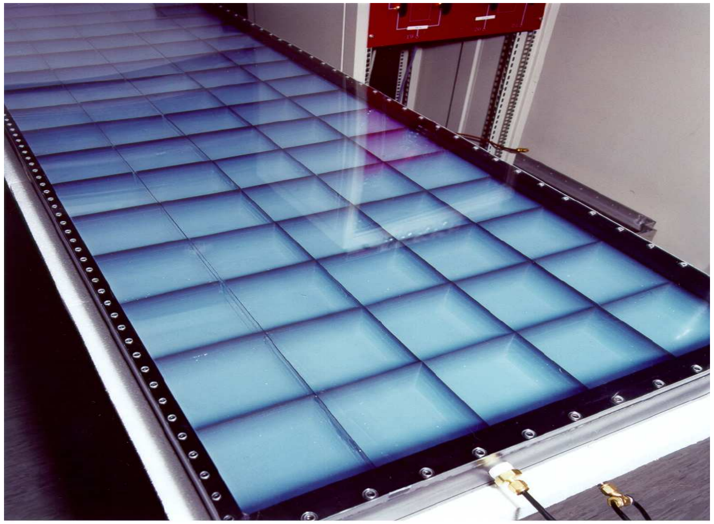


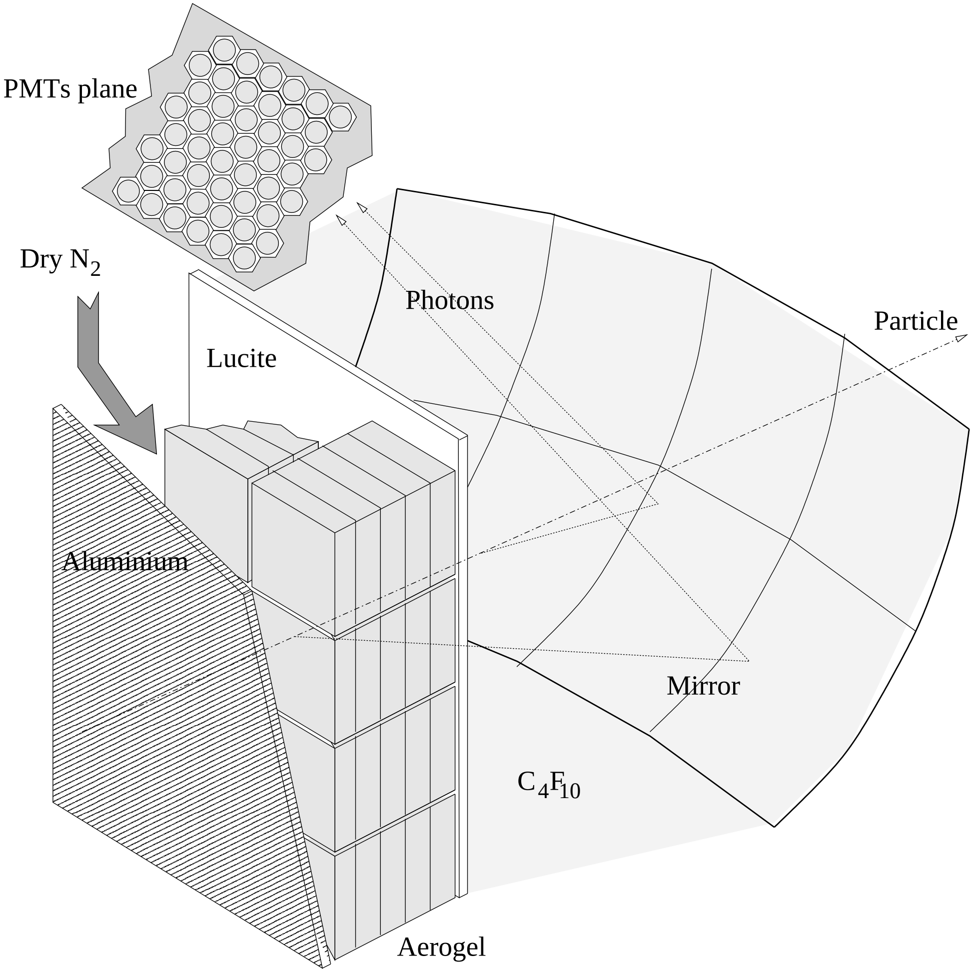
- **No deviation from $1/Q^2$-dependence seen**
- **No indication of higher-twist or resonance form factor contributions**
- **Agreement with Bjorken Sum Rule prediction and measurements for $Q^2 = 5$ GeV^2**
- **No turn-over seen to meet GDH prediction: $I^p_{GDH}(0) - I^n_{GDH}(0) = 29$**

Conclusions

- Precision measurement of polarized deuteron structure function presented
- First 5-parameter flavor decomposition of quark polarization
- First evidence of Quark-Hadron-Duality in spin structure function
- First extensive measurement of the generalized GDH integral for proton, deuteron and neutron
- DIS contribution to generalized GDH integral dominates for $Q^2 > 3$ GeV^2
- Turn-over of proton and deuteron GDH integral towards (negative) real photon prediction NOT observed at HERMES
- GDH difference Proton-Neutron in good agreement with Bjorken Sum Rule prediction and SLAC data
- Leading-Twist ($1/Q^2$-behaviour) for Q^2 down to 1.5 GeV^2







PMTs plane
Dry N_2
Photons
Particle
Lucite
Aluminium
Mirror
C_4F_{10}
Aerogel