GMC_TRANS

- A Monte Carlo Generator for TMDs -

PKU-RBRC Workshop on Transverse Spin Physics
Beijing, June 30 - July 5, 2008

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Prelude: The Role of Acceptance in Experiments

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"
  - obvious for detectors with gaps/holes
  - but also for "4π", especially when looking at complicated final states

An unfortunate Lemma

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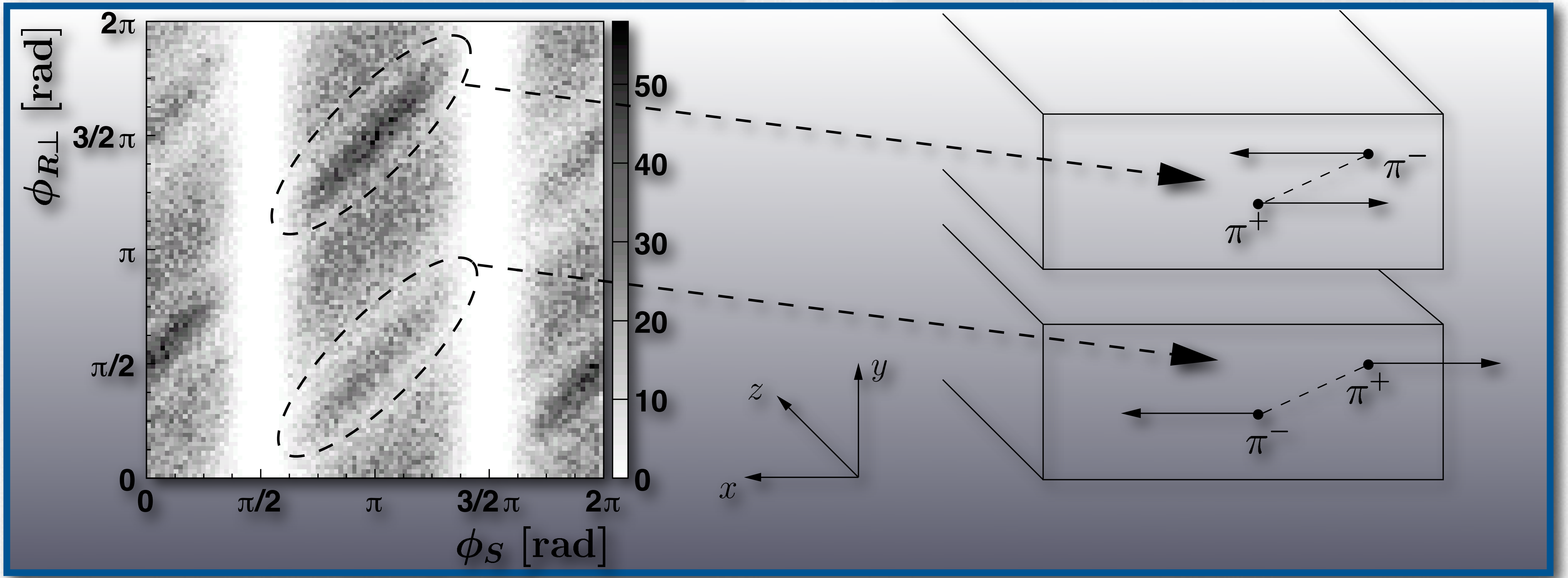

HERMES azimuthal acceptance for 2-hadron production

[P. van der Nat, Ph.D. thesis, Vrije Universiteit (2007)]

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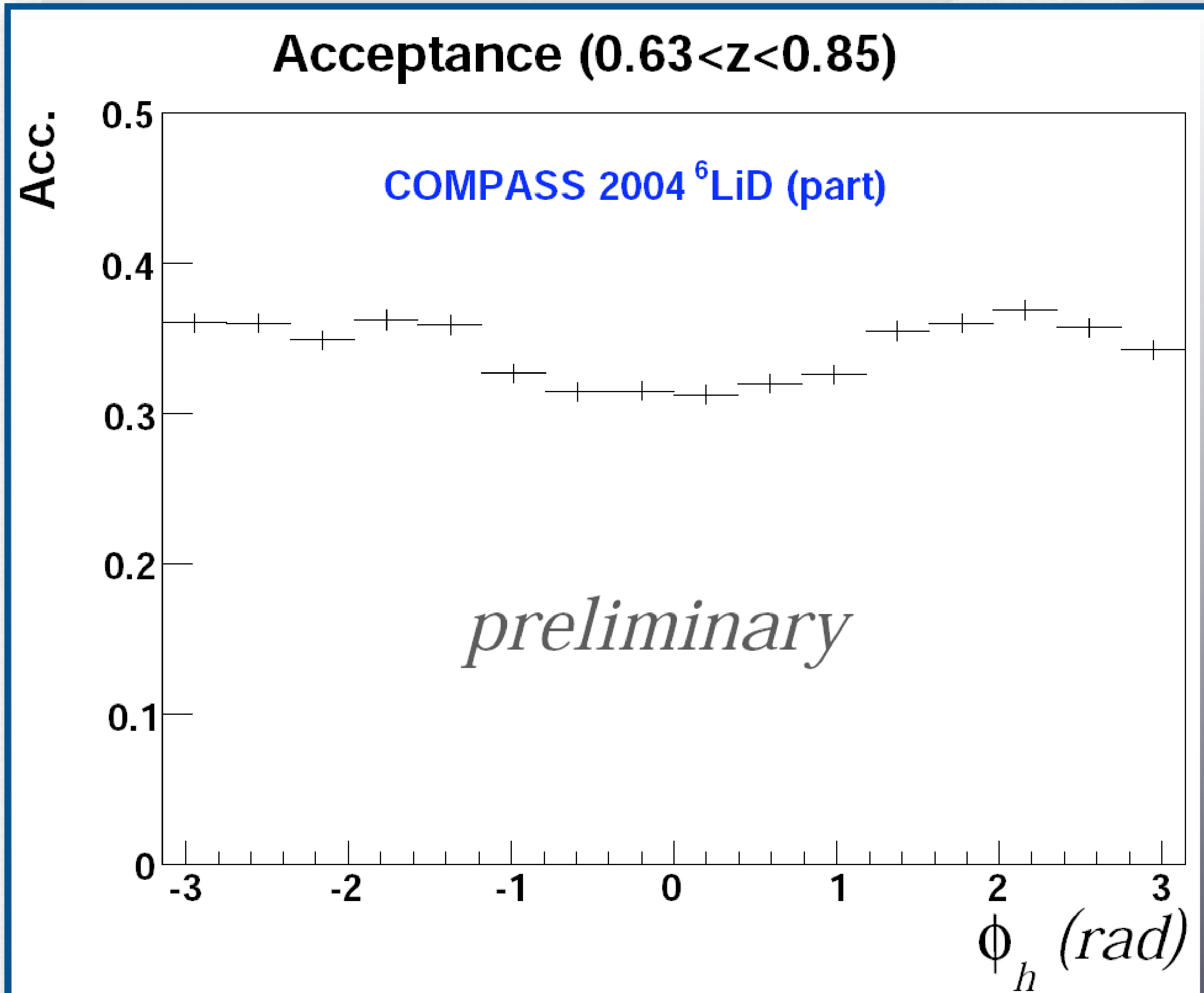

maybe “4π” around beam axis, but not around virtual-photon axis because of lower limit on *θ*

[W. Käfer, Transversity 2008, Ferrara]

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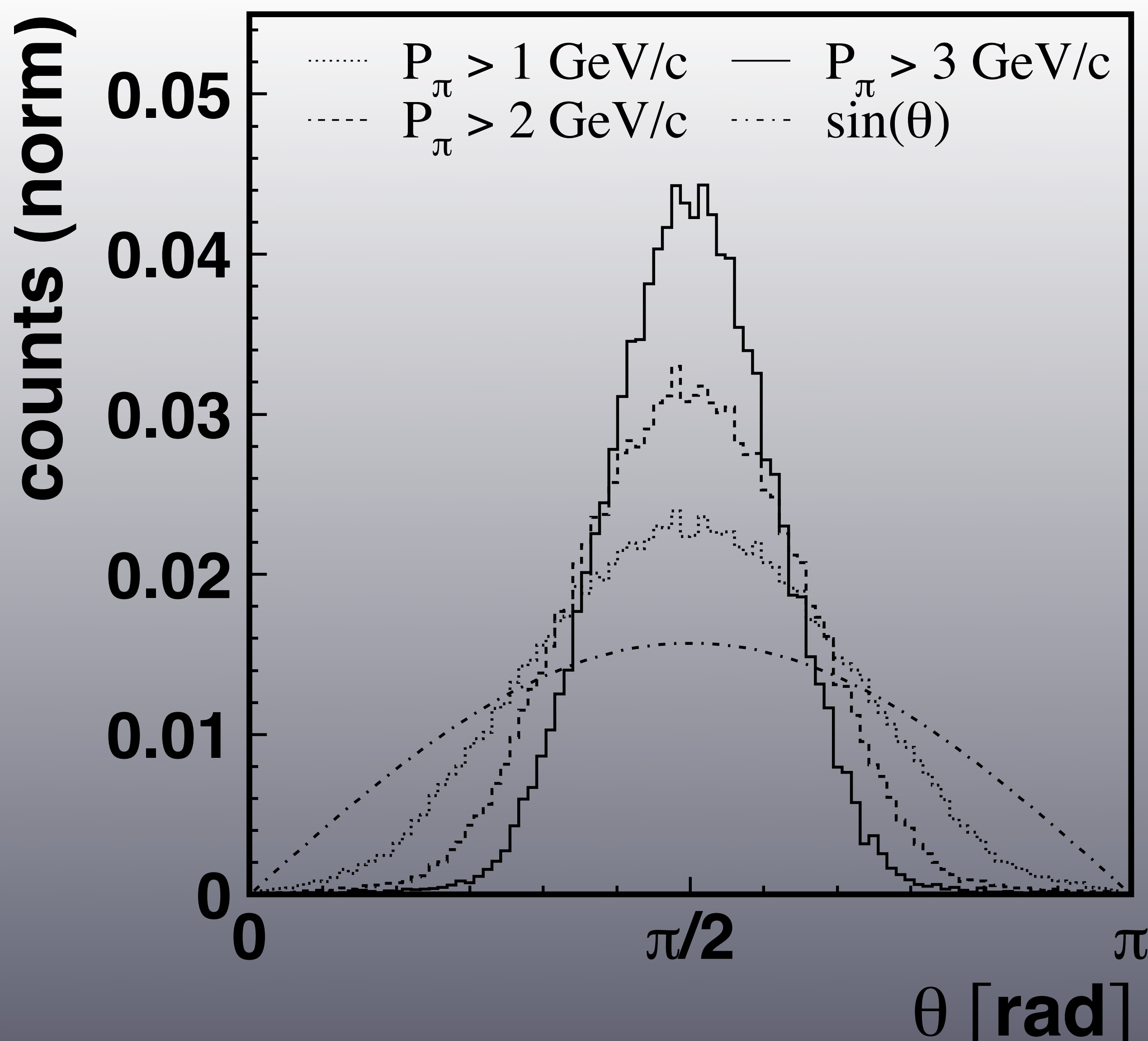

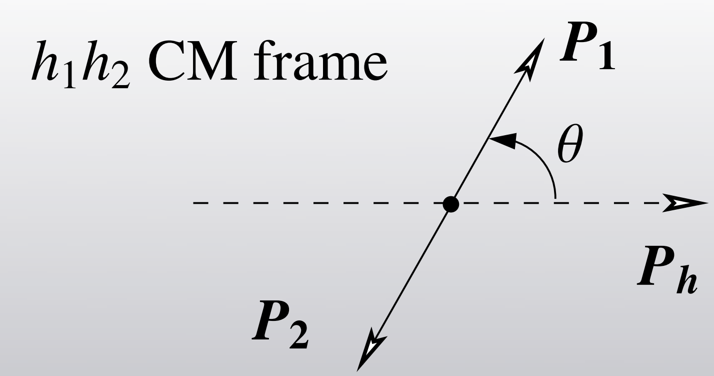

momentum cuts strongly distort kinematic distributions even for "4π" acceptance

[P. van der Nat, Ph.D. thesis, Vrije Universiteit (2007)]

An unfortunate Lemma

- “No particle-physics experiment has a perfect acceptance!”
  - obvious for detectors with gaps/holes
  - but also for “4π”, especially when looking at complicated final states
- How acceptance effects are handled is one of the essential questions in experiments!

Falsified Lemma I

- **"acceptance cancels in asymmetries"**

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$$A_{UT}(\phi,\Omega) = \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)}$$

$$\Omega = x, y, z, \ldots$$

$$= \frac{\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)}$$

ϵ : **detection efficiency**

$$\neq \frac{\int \mathrm{d}\Omega\, \sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\int \mathrm{d}\Omega\, \sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \quad \frac{\int \mathrm{d}\Omega\, \sigma_{UT}(\phi,\Omega)}{\int \mathrm{d}\Omega\, \sigma_{UU}(\phi,\Omega)} \equiv A_{UT}(\phi)$$

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Acceptance does **not cancel** in general when **integrating** numerator and denominator over (large) ranges in kinematic variables!

... possible ways out

- **for *linear* dependence on *all kinematic variables* of asymmetry, average asymmetry equal to asymmetry at average kinematics**
- **for all other cases: can one maybe use 1-D (*projected*) acceptance function, e.g. $\epsilon(\phi)$, to correct asymmetry $A_{UT}(\phi)$?**

Falsified Lemma II

- use Monte Carlo (physics generator * detector model) to extract acceptance function
- "projected acceptance function is independent from cross-section model"

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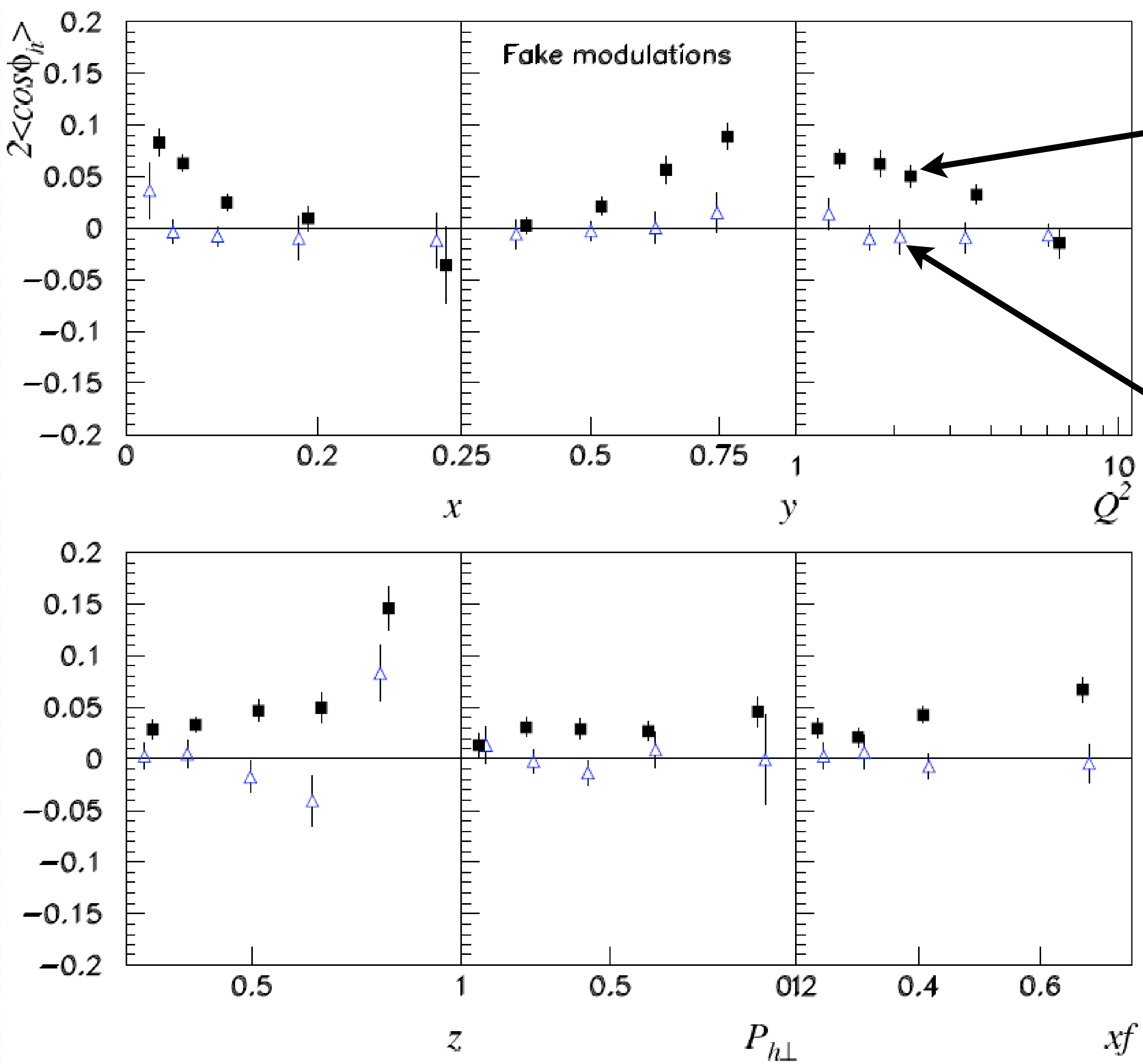
Cross-section model does **not cancel** in general when **integrating** numerator and denominator over (large) ranges in kinematic variables!

"Classique" Example: $\langle \cos\phi \rangle_{UU}$



1D correction

(input: MC without azimuthal modulation)

5D correction

[F. Giordano, Transversity 2008, Ferrara]

... one way out: multi-D unfolding

true yield (used to calculate azimuthal moments)

experimental yield

$$n_{\mathrm{corr}} = S_{\mathrm{MC}}^{-1} \left[n_{\mathrm{exp}} - BG_{\mathrm{MC}} \right]$$

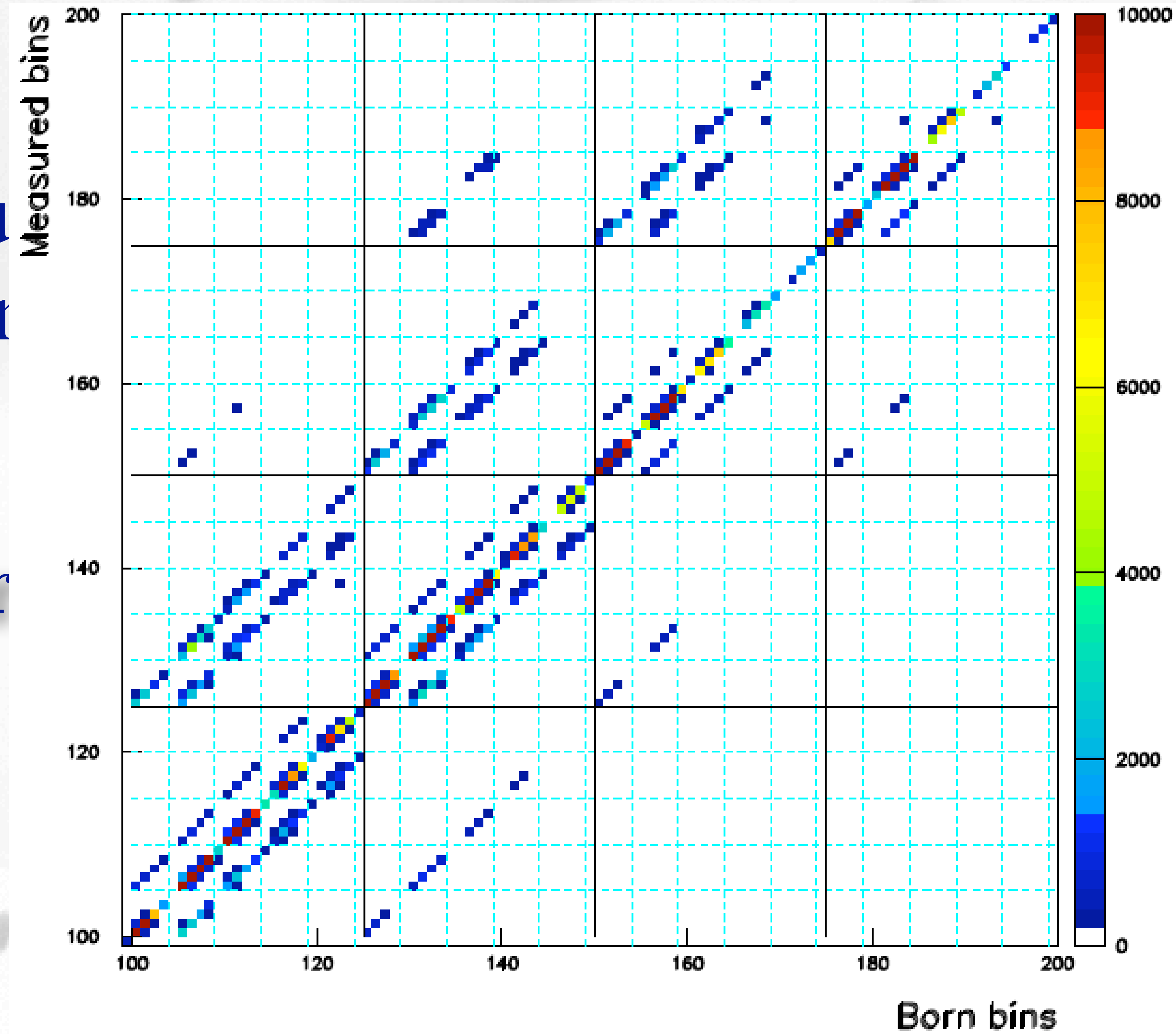
(inverted) multi-dimensional smearing matrix (depends on detector and radiative effects only!)

... one way out: multi-D unfolding

true

calcu

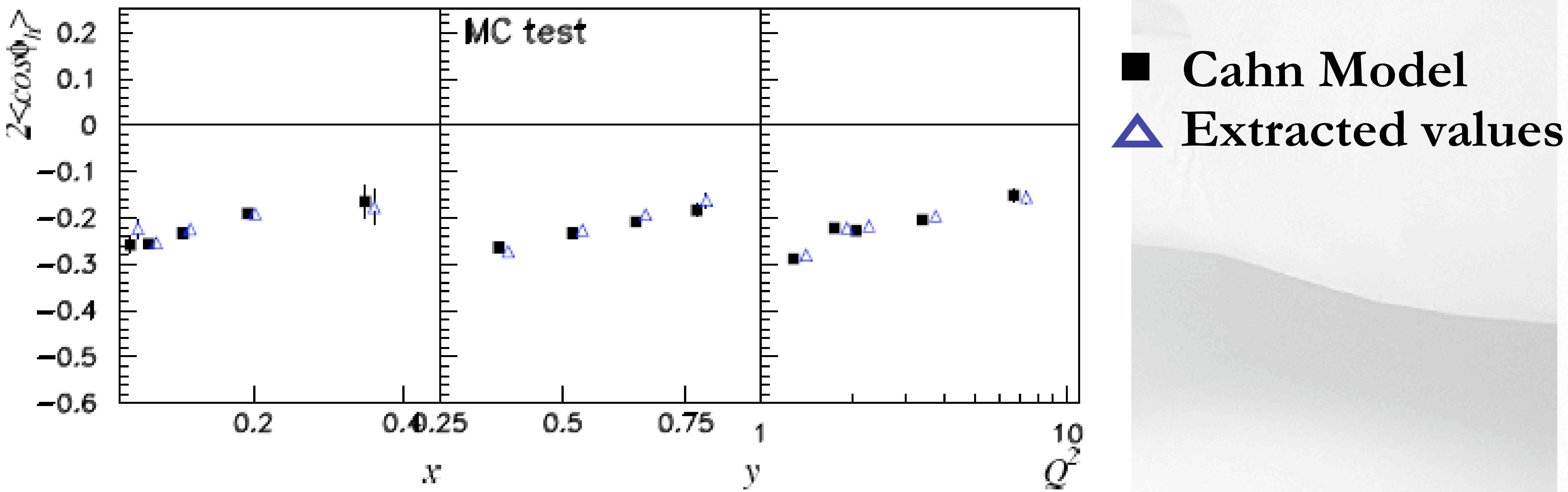
yield



n_{cor} ... $_{MC}]$

(depends on detector and radiative effects only!)

Unfolding Example



- **extracted Cahn amplitudes in good agreement with model amplitudes**
- **apparent: need Cahn model for Monte Carlo simulation to test procedure**
- **also needed to extrapolate into unmeasured region**

1st Entrée: gmc_trans ingredients

Initial Goals

- physics generator for SIDIS pion production
- include transverse-momentum dependence, in particular simulate Collins and Sivers effects
- be fast
- allow comparison of input model and reconstructed amplitudes
- to be used with standard HERMES Monte Carlo
- be extendable (e.g., open for new models)

Basic workings

- use cross section that can (almost) be calculated analytically
- start from 1-hadron SIDIS expressions of Mulders & Tangerman (Nucl.Phys.B461:197-237,1996) and others
- use Gaussian Ansatz for all transverse-momentum dependencies of DFs and FFs
- unpolarized DFs (as well as helicity distribution) and FFs from fits/parametrizations (e.g., Kretzer FFs etc.)
- "polarized" DFs and FFs either related to unpolarized ones (e.g., saturation of Soffer bound for transversity) or some parametrizations used

SIDIS Cross Section incl. TMDs

$$d\sigma_{UT} \equiv d\sigma_{UT}^{\text{Collins}} \cdot \sin(\phi+\phi_S) + d\sigma_{UT}^{\text{Sivers}} \cdot \sin(\phi-\phi_S)$$

$$d\sigma_{UT}^{\text{Collins}}(x,y,z,\phi_S,P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} B(y) \sum_q e_q^2\, \mathcal{I}\left[\left(\frac{k_T \cdot \hat{P}_{h\perp}}{M_h}\right) \cdot h_1^q H_1^{\perp q}\right]$$

$$d\sigma_{UT}^{\text{Sivers}}(x,y,z,\phi_S,P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2\, \mathcal{I}\left[\left(\frac{p_T \cdot \hat{P}_{h\perp}}{M_N}\right) \cdot f_{1T}^{\perp q} D_1^q\right]$$

$$d\sigma_{UU}(x,y,z,\phi_S,P_{h\perp}) \equiv \frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2\, \mathcal{I}\left[f_1^q D_1^q\right]$$

where

$$\mathcal{I}\left[\mathcal{W}\, f\, D\right] \equiv \int d^2p_T\, d^2k_T\, \delta^{(2)}\left(p_T - \frac{P_{h\perp}}{z} - k_T\right)\left[\mathcal{W}\, f(x,p_T)\, D(z,k_T)\right]$$

Gaussian Ansatz

- **want to deconvolve convolution integral over transverse momenta**
- **easy Ansatz: Gaussian dependencies of DFs and FFs on intrinsic (quark) transverse momentum:**

$$\mathcal{I}[f_1(x,\boldsymbol{p_T}^2)D_1(z,z^2\boldsymbol{k_T}^2)] = f_1(x)\cdot D_1(z)\cdot \frac{R^2}{\pi z^2}\cdot e^{-R^2\frac{P_{h\perp}^2}{z^2}}$$

with $$\boldsymbol{f_1(x,p_T^2) \;=\; f_1(x)\frac{1}{\pi\langle p_T^2\rangle}e^{-\frac{p_T^2}{\langle p_T^2\rangle}}}$$ $$\frac{1}{R^2} \equiv \langle k_T^2\rangle + \langle p_T^2\rangle = \frac{\langle P_{h\perp}^2\rangle}{z^2}$$

(similar: $D_1(z,z^2\boldsymbol{k_T}^2)$)

Caution: different notations for intrinsic transverse momentum exist! (Here: Amsterdam notation)

Positivity Constraints

- **DFs (FFs) have to fulfill various positivity constraints (resulting cross section has to be positive!)**
- **based on probability considerations can derive positivity limits for leading-twist functions: Bacchetta et al., Phys.Rev.Lett.85:712-715, 2000**

➡ **transversity: e.g., Soffer bound**

➡ **Sivers and Collins functions: e.g., loose bounds:**

$$\frac{|p_T|}{2M_N} f_{1T}^{\perp}(x, p_T^2) \equiv \quad f_{1T}^{\perp(1/2)}(x, p_T^2) \quad \leq \frac{1}{2} f_1(x, p_T^2)$$

$$\frac{|k_T|}{2M_h} H_1^{\perp}(z, z^2 k_T^2) \equiv \quad H_1^{\perp(1/2)}(z, z^2 k_T^2) \quad \leq \frac{1}{2} D_1(z, z^2 k_T^2)$$

Positivity and the Gaussian Ansatz

$$\frac{|\boldsymbol{p_T}|}{2M_N} f_{1T}^{\perp}(x, \boldsymbol{p_T^2}) \leq \frac{1}{2} f_1(x, \boldsymbol{p_T^2})$$

$$\text{with} \quad \boldsymbol{f_1(x, p_T^2)} = \boldsymbol{f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}}$$

$$\boldsymbol{f_{1T}^{\perp}(x, p_T^2)} = \boldsymbol{f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}}$$

$$\Rightarrow \quad \boldsymbol{|p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)}$$

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$$f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$\Rightarrow \quad |p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)$$

No (useful) solution for non-zero Sivers fctn!

Modify Gaussian Width

$$f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x)\,\frac{1}{(1-C)\pi\langle p_T^2\rangle}\,e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}}$$

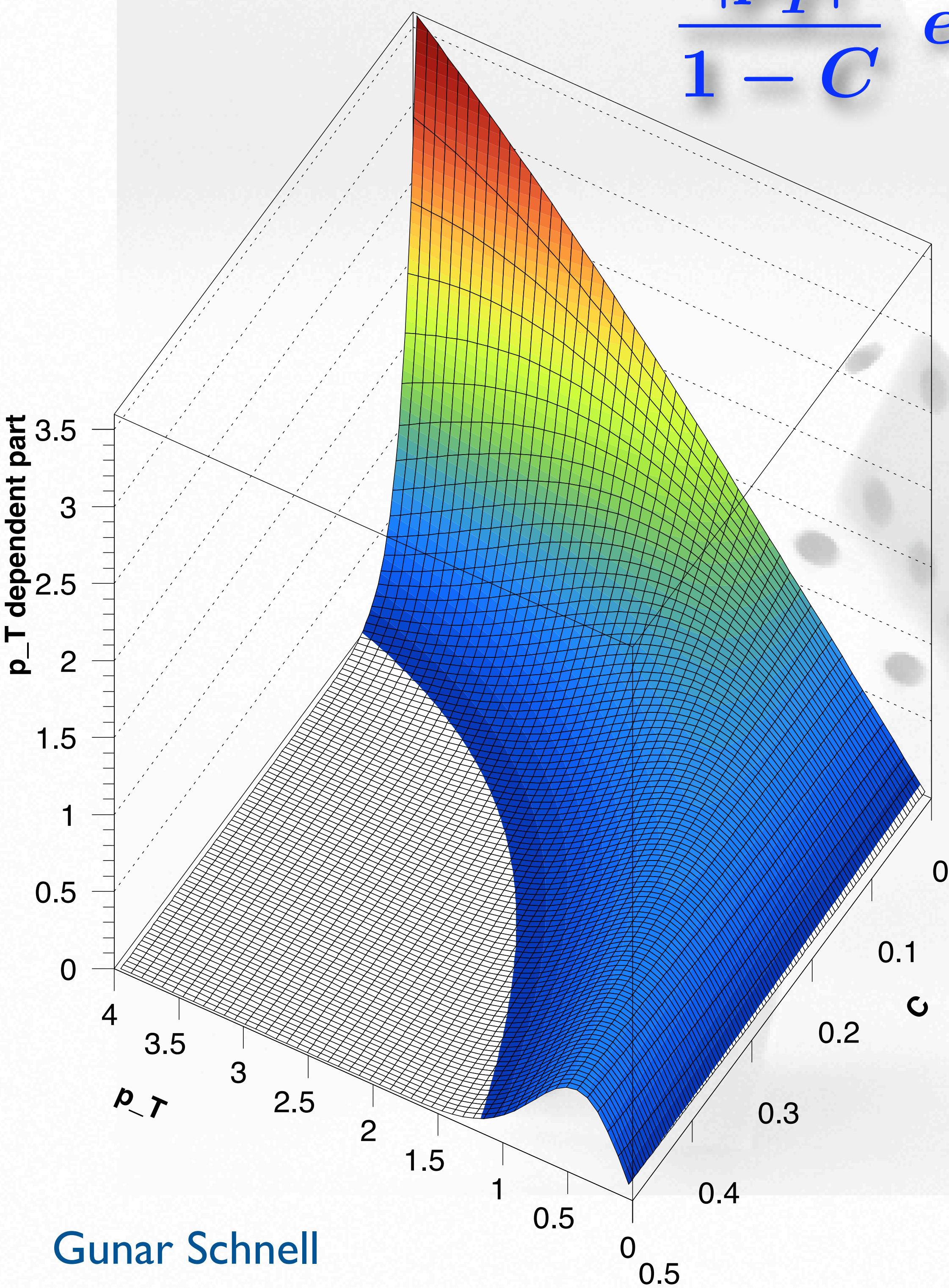
➡ **positivity limit:**

$$f_{1T}^{\perp}(x)\,\frac{|p_T|}{2M_N}\,\frac{1}{\pi(1-C)\langle p_T^2\rangle}\,e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}} \leq 1/2\, f_1(x)\,\frac{1}{\pi\langle p_T^2\rangle}\,e^{-\frac{p_T^2}{\langle p_T^2\rangle}}$$

$$\Rightarrow \frac{|p_T|}{1-C}\,e^{-\frac{C}{1-C}\frac{p_T^2}{\langle p_T^2\rangle}} \leq M_N\,\frac{f_1(x)}{f_{1T}^{\perp}(x)}$$

Reevaluation of positivity constraint

$$\frac{|p_T|}{1-C}\, e^{-\frac{C}{1-C}\frac{p_T^2}{\langle p_T^2\rangle}} \le M_N \frac{f_1(x)}{f_{1T}^{\perp}(x)}$$



Minimum at $p_T = \sqrt{\frac{\langle p_T^2\rangle}{2C}}$

thus $\frac{f_{1T}^{\perp}(x)}{f_1(x)} \le M_N \sqrt{\frac{2eC(1-C)}{\langle p_T^2\rangle}}$

or $\frac{f_{1T}^{\perp(1/2)}(x)}{f_1(x)} \le \frac{1}{2}\sqrt{\frac{e\pi C}{2}}(1-C) \le 0.4$

SIDIS Cross Section incl. TMDs

$$\sum_q \frac{e_q^2}{4\pi} \frac{\alpha^2}{(MExyz)^2} \left[X_{UU} + |S_T| X_{SIV} \sin(\phi_h - \phi_s) + |S_T| X_{COL} \sin(\phi_h + \phi_s) \right]$$

using Gaussian Ansatz for transverse-momentum dependence of DFs and FFs:

$$X_{UU} = R^2 e^{-R^2 P_{h\perp}^2/z^2} \left(1 - y + \frac{y^2}{2}\right) f_1(x) \cdot D_1(z)$$

$$X_{COL} = +\frac{|P_{h\perp}|}{M_\pi z} \frac{(1-C)\langle k_T^2\rangle}{\left[\langle p_T^2\rangle + (1-C)\langle k_T^2\rangle\right]^2} \exp\left[-\frac{P_{h\perp}^2/z^2}{\langle p_T^2\rangle + (1-C)\langle k_T^2\rangle}\right] \times (1-y) \cdot h_1(x) \cdot H_1^\perp(z)$$

$$X_{SIV} = -\frac{|P_{h\perp}|}{M_p z} \frac{(1-C')\langle p_T^2\rangle}{\left[\langle k_T^2\rangle + (1-C')\langle p_T^2\rangle\right]^2} \exp\left[-\frac{P_{h\perp}^2/z^2}{\langle k_T^2\rangle + (1-C')\langle p_T^2\rangle}\right] \times \left(1 - y + \frac{y^2}{2}\right) f_{1T}^\perp(x) \cdot D_1(z)$$

Sivers (azimuthal) Moments

use cross section expressions to evaluate azimuthal moments:

$$-\langle\sin(\phi-\phi_s)\rangle_{UT} = \frac{\sqrt{(1-C)\langle p_T^2\rangle}}{\sqrt{(1-C)\langle p_T^2\rangle+\langle k_T^2\rangle}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

$$-\langle\sin(\phi-\phi_s)\rangle_{UT} = \frac{M_N\sqrt{\pi}}{2\sqrt{(1-C)\langle p_T^2\rangle+\langle k_T^2\rangle}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

$$-\langle\frac{|P_{h\perp}|}{zM_N}\sin(\phi-\phi_s)\rangle_{UT} = \frac{2\sqrt{(1-C)\langle p_T^2\rangle}}{M_N\sqrt{\pi}} \frac{A(y)\frac{1}{xy^2}\sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{xy^2}\sum e_q^2 f_1(x)D_1(z)}$$

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model-dependence on transverse momenta "swallowed" by p_T^2- moment of Sivers fct.: $f_{1T}^{\perp(1)}$

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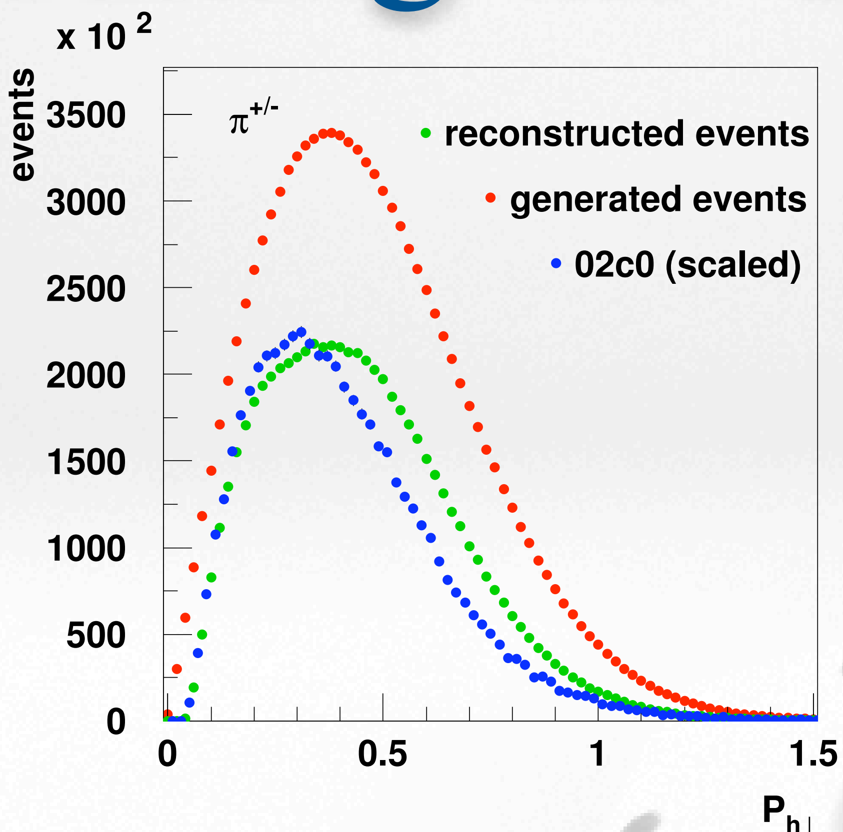
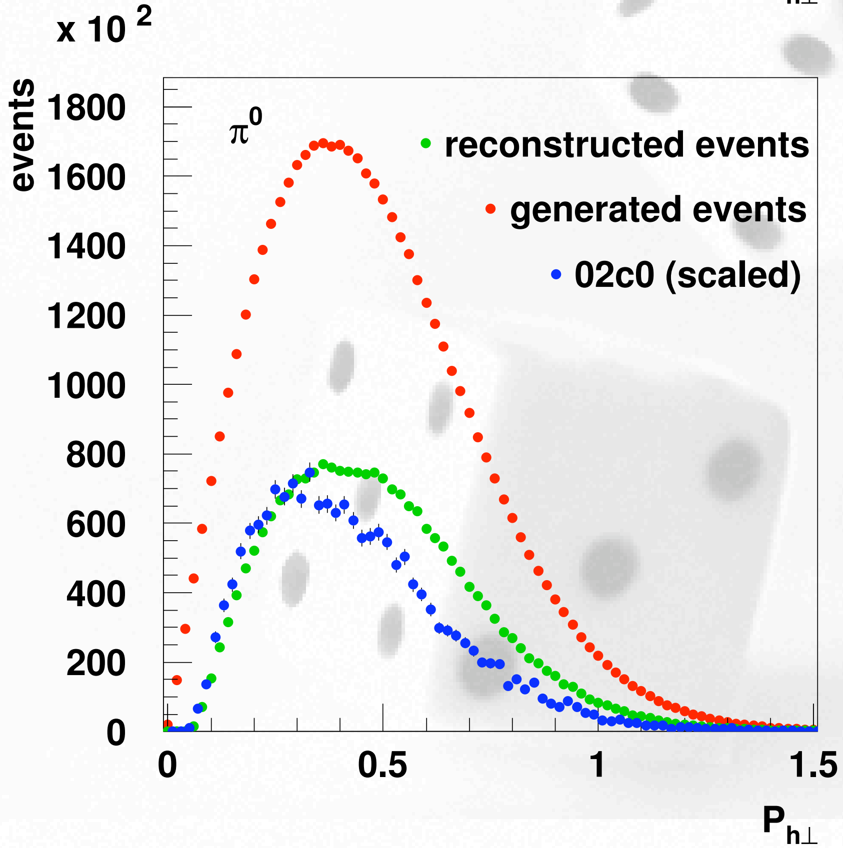
(similar for Collins moments)

2nd Entrée: Selected Results

Tuning the Gaussians in gmc_trans





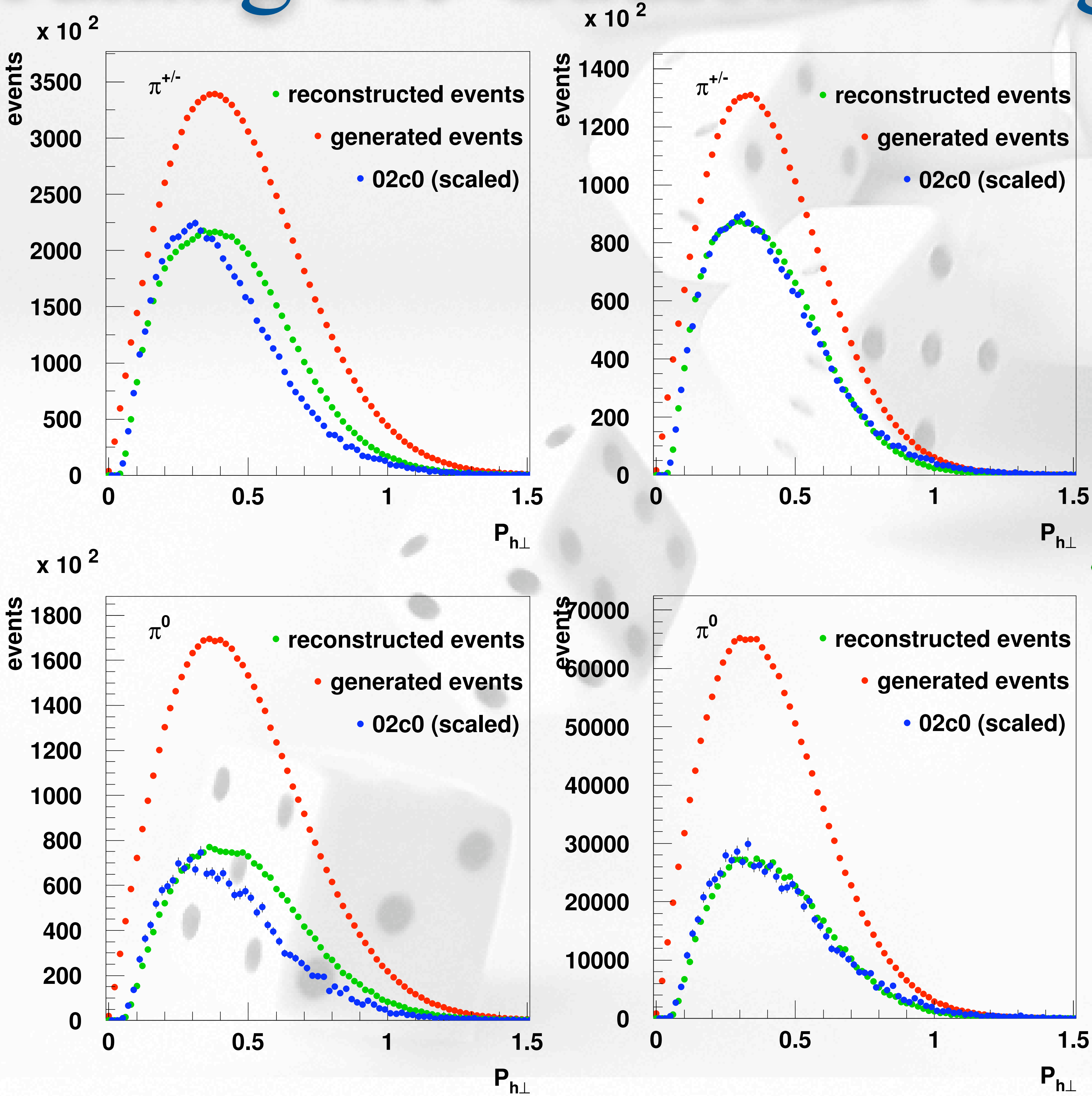
- **constant Gaussian widths, i.e., no dependence on x or z:**

$$\langle p_T \rangle = 0.44$$

$$\langle K_T \rangle = 0.44$$

- **tune to data integrated over whole kinematic range**

Tuning the Gaussians in gmc_trans

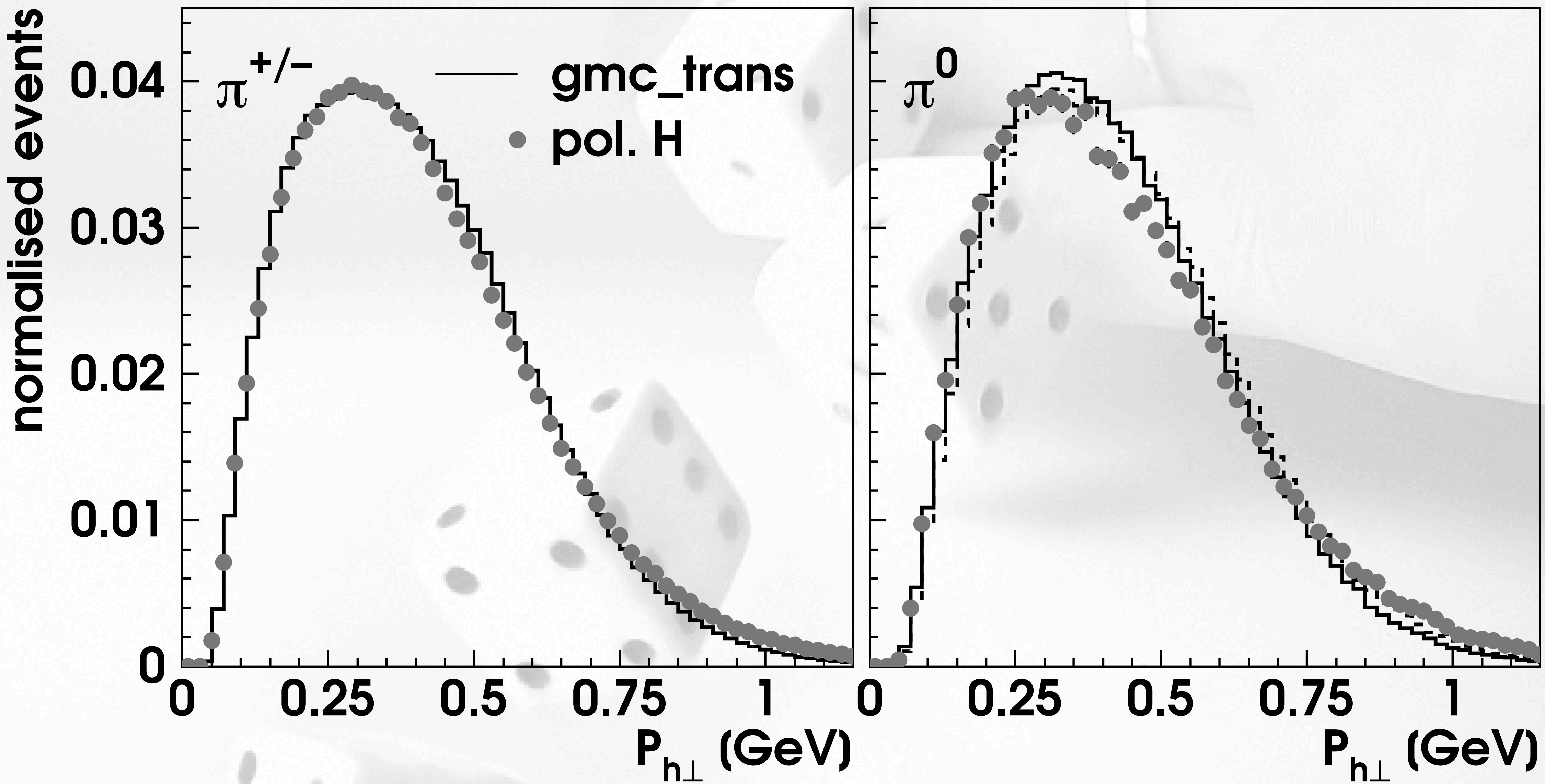


Better:

$$\langle p_T \rangle = 0.38$$

$$\langle K_T \rangle = 0.38$$

Comparison Data-MC:



$\langle p_T^2 \rangle = \langle K_T^2 \rangle = 0.18\,\mathrm{GeV}^2$ $(\langle|\vec{p}_T|\rangle = \langle|\vec{K}_T|\rangle = 0.38\,\mathrm{GeV})$

where: $\langle K_T^2 \rangle = z^2 \langle k_T^2 \rangle$

Tuning the Gaussians in gmc_trans

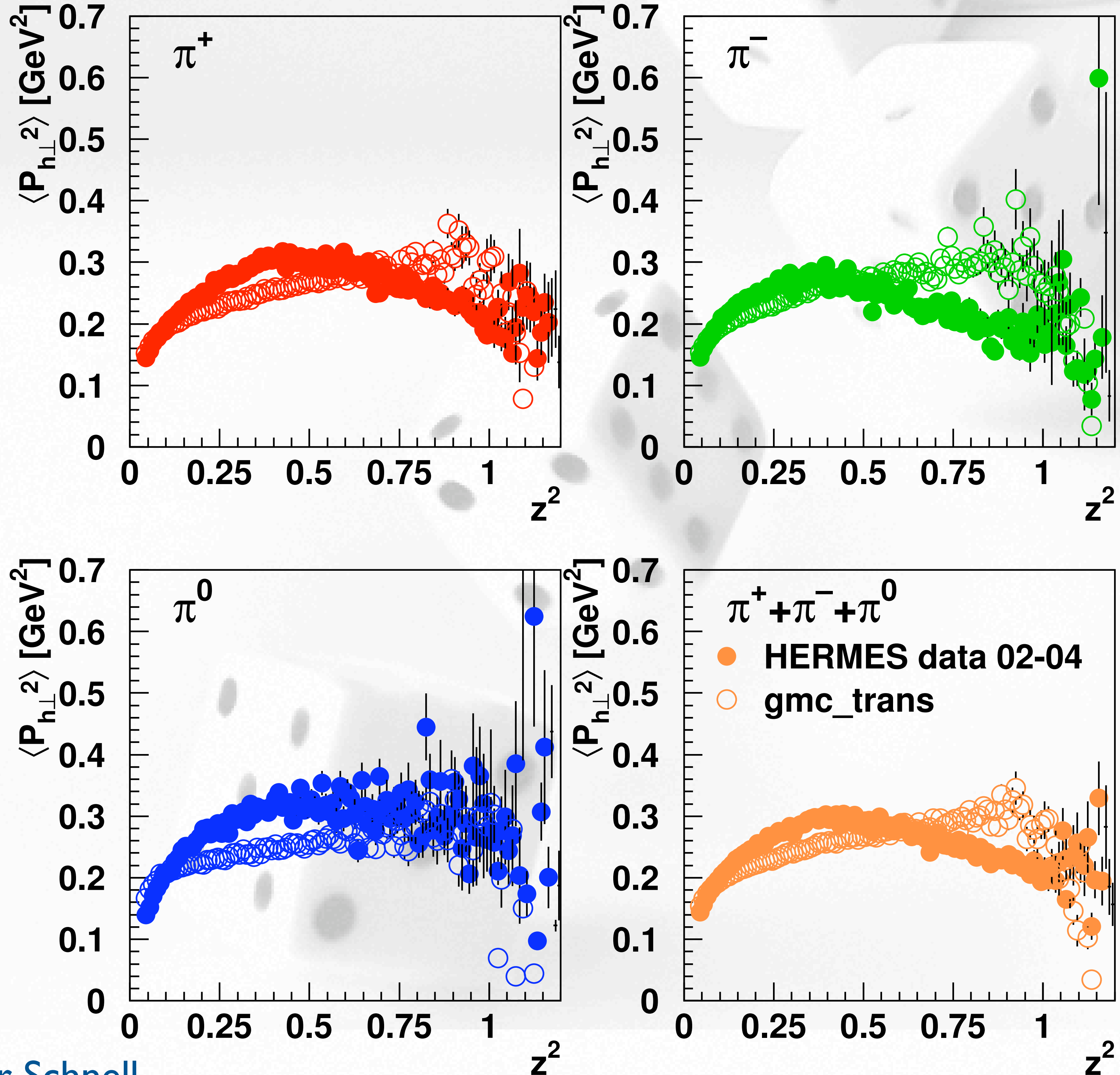
in general: $\langle P_{h\perp}^2(x,z)\rangle = z^2 \langle p_T^2(x)\rangle + \langle K_T^2(z)\rangle$

so far: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2\rangle$

constant!

Tuning the Gaussians in gmc_trans

so far: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2\rangle$



$\langle p_T\rangle = 0.38$

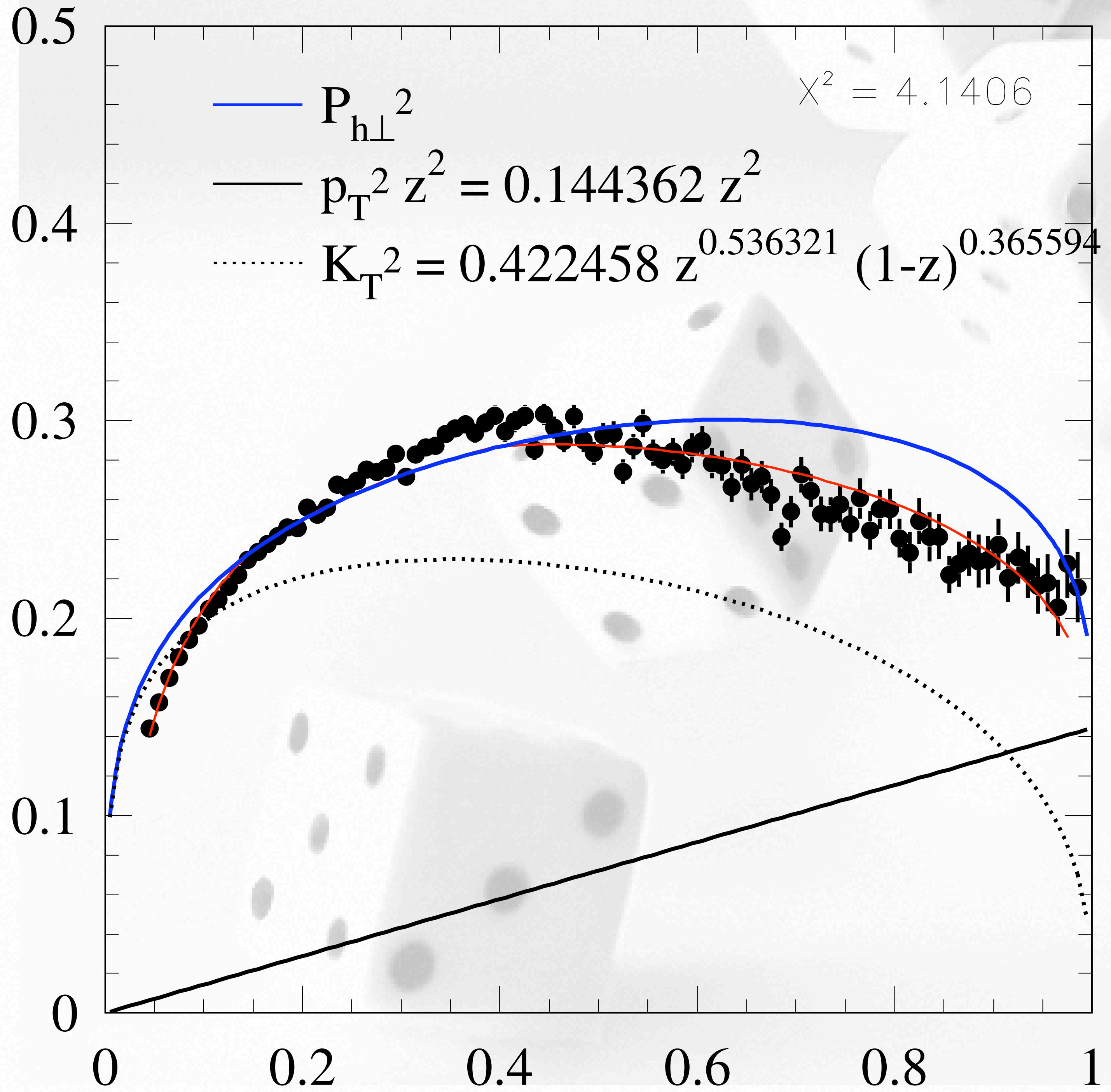
$\langle K_T\rangle = 0.38$

$\langle p_T^2\rangle \simeq 0.185$

$\langle K_T^2\rangle \simeq 0.185$

Tuning the Gaussians in gmc_trans

now: $\langle P_{h\perp}^2(z)\rangle = z^2\,\langle p_T^2\rangle + \langle K_T^2(z)\rangle$



now z-dependent!

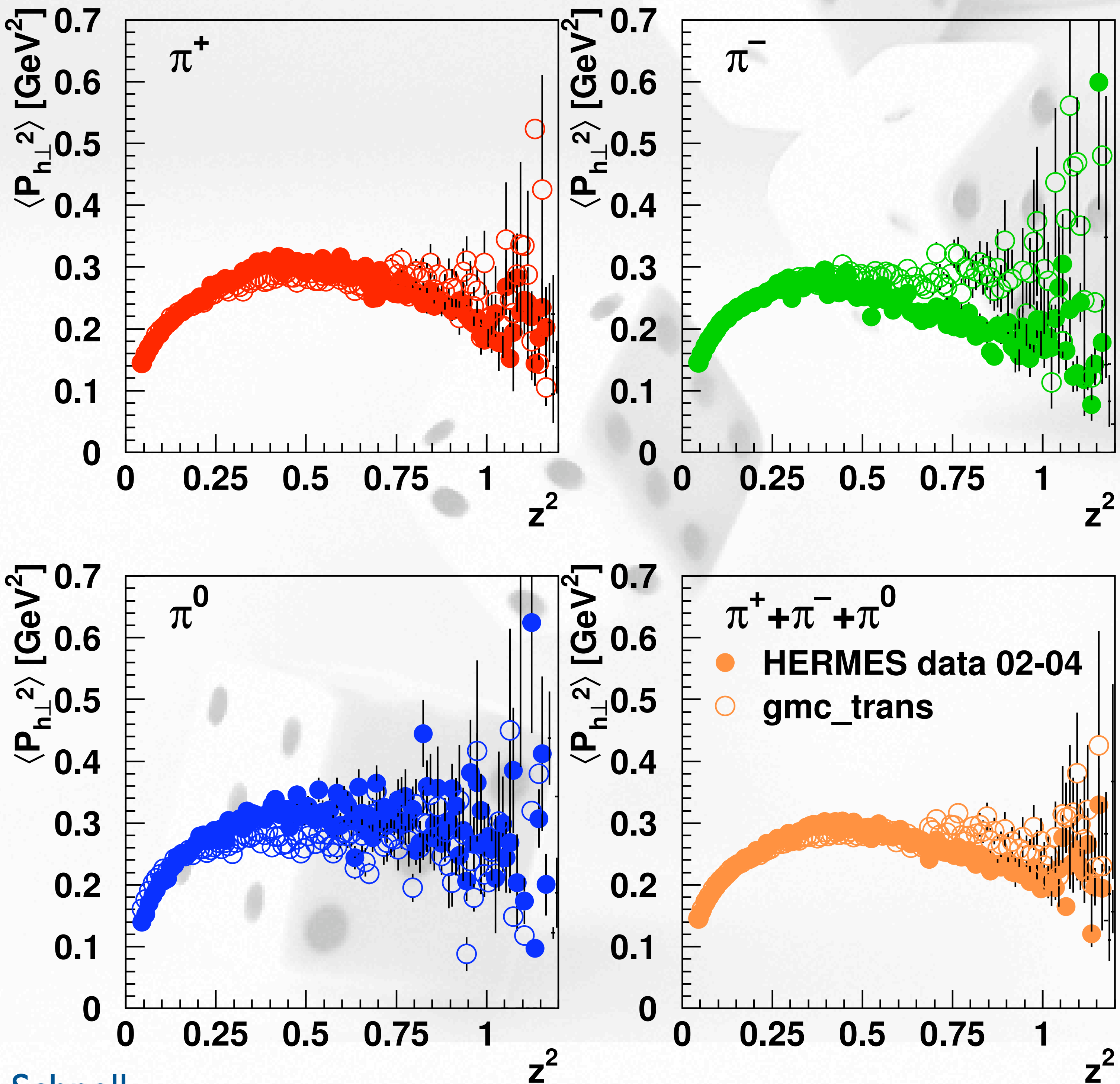
tuned to HERMES data in acceptance

"Hashi set"

Tuning the Gaussians in gmc_trans

$$\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2(z)\rangle$$

z-dependent!



Some Simple Models for Transversity & Friends

$$\delta u(x) = 0.7 \cdot \Delta u(x) \qquad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x)$$

$$\delta d(x) = 0.7 \cdot \Delta d(x) \qquad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x)$$

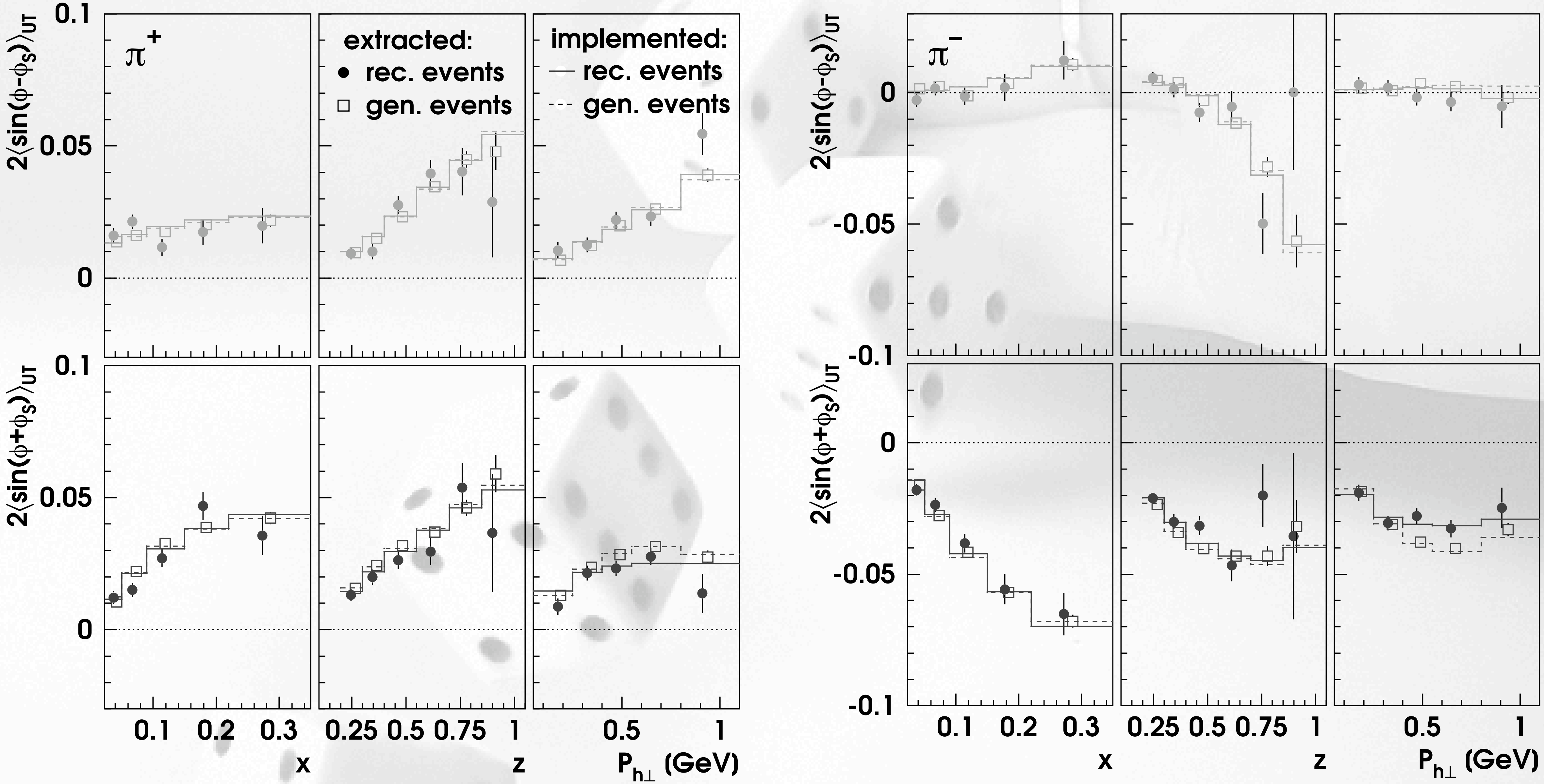
$$\delta q(x) = 0.3 \cdot \Delta q(x) \qquad f_{1T}^{\perp q}(x) = 0.0 \qquad q = \bar{u}, \bar{d}, s, \bar{s}$$

$$H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$$

$$H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$$

GRSV for PDFs and Kretzer FF for D_1

Generated vs. Extracted Amplitudes



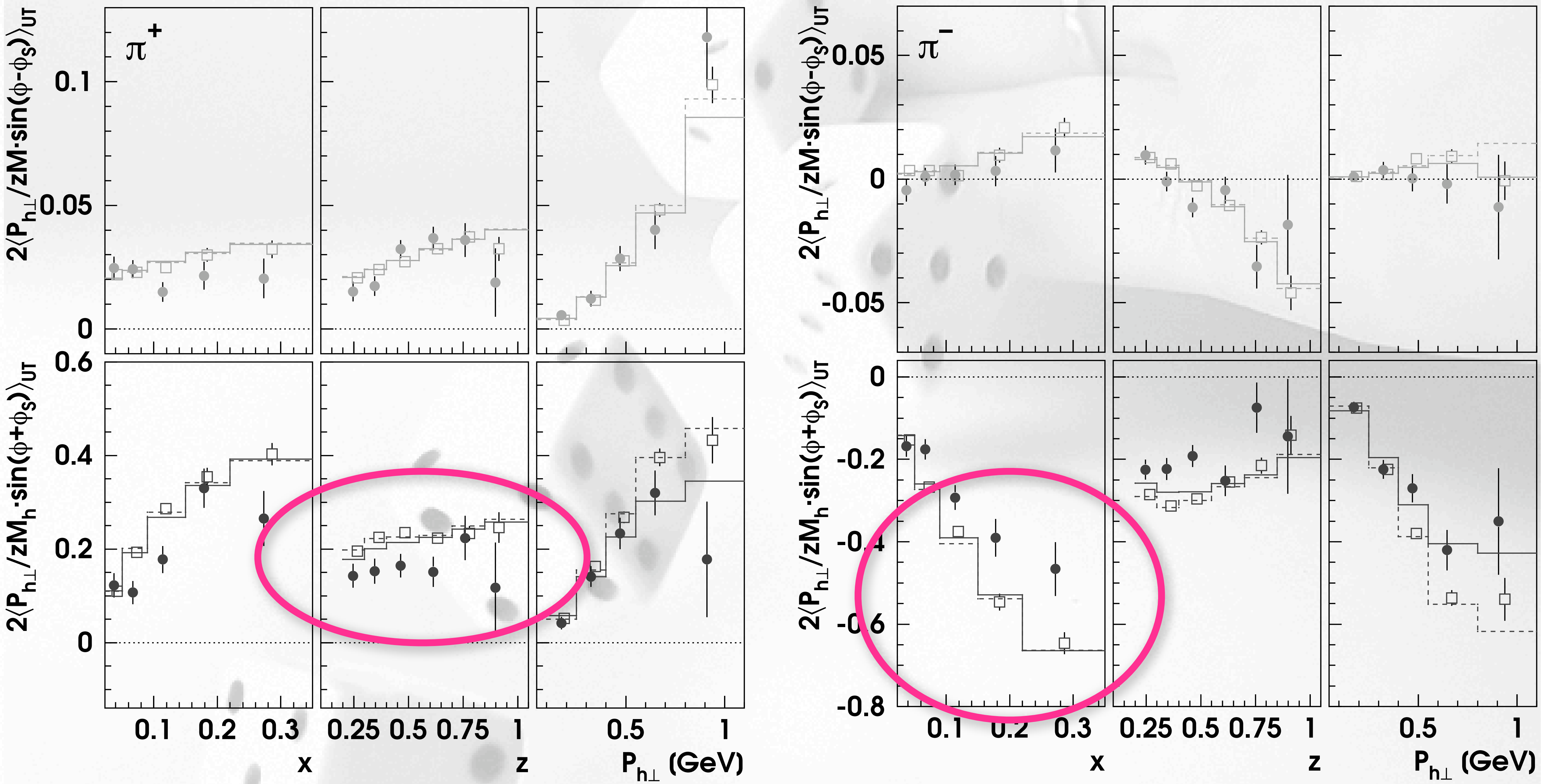
$$\delta u(x) = 0.7 \cdot \Delta u(x) \quad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x) \qquad H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$$

$$\delta d(x) = 0.7 \cdot \Delta d(x) \quad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x) \qquad H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$$

$$\delta q(x) = 0.3 \cdot \Delta q(x) \quad f_{1T}^{\perp q}(x) = 0.0 \qquad q = \bar{u}, \bar{d}, s, \bar{s}$$

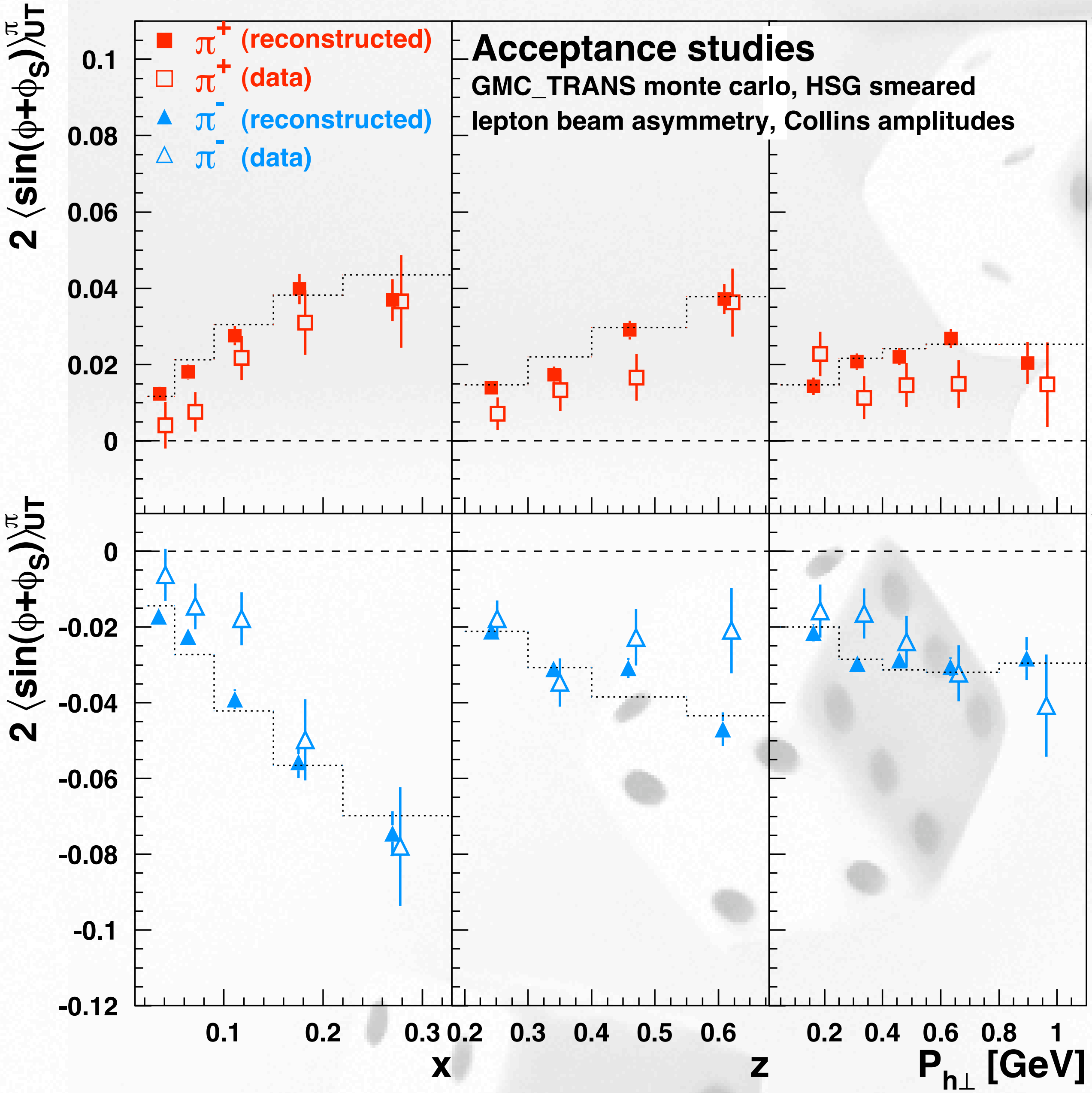
$$C_S = C_C = 0.25$$

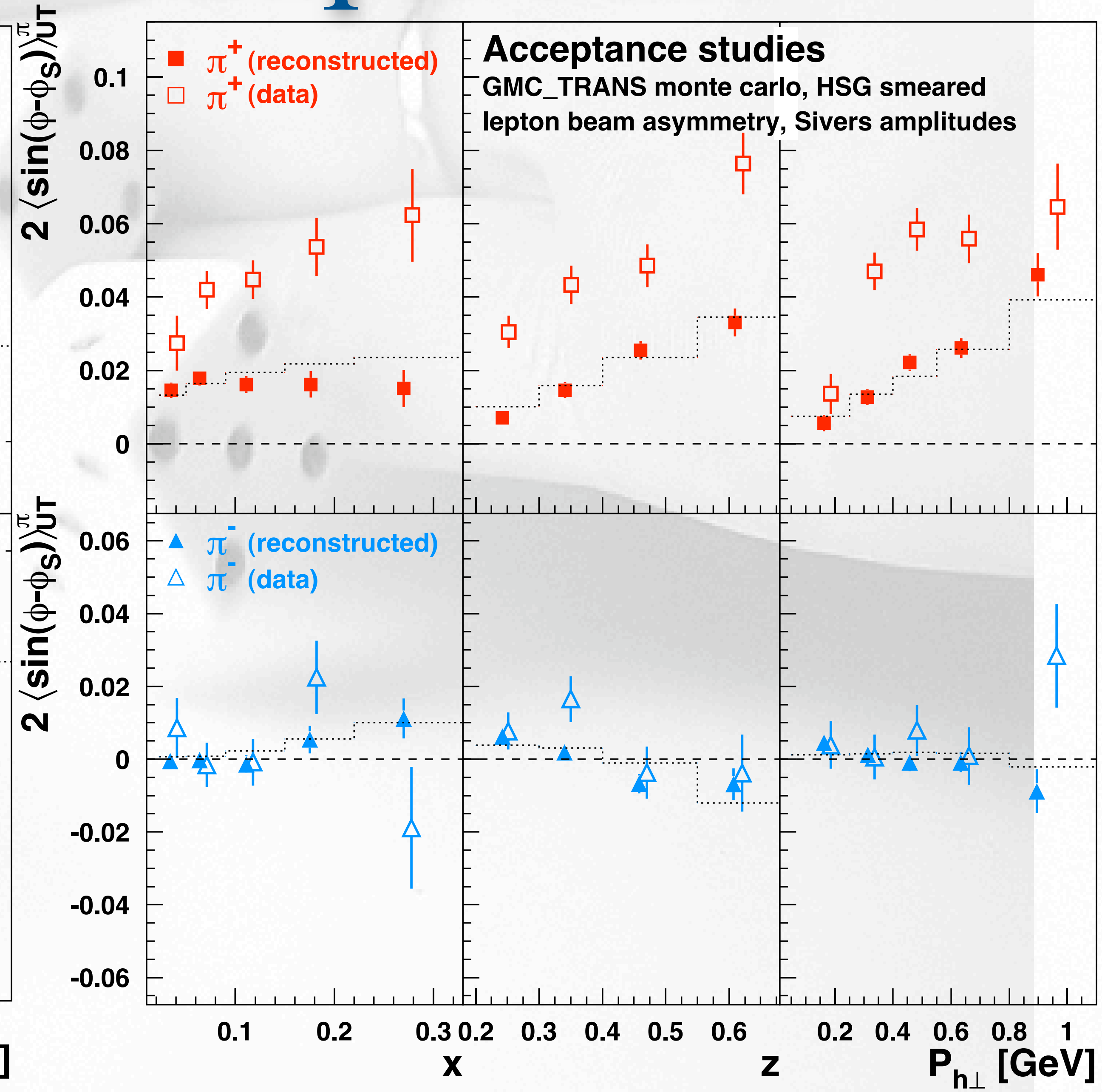
Comparison for Weighted Moments



Not so good news for weighted moments

GMC vs. Data Amplitudes





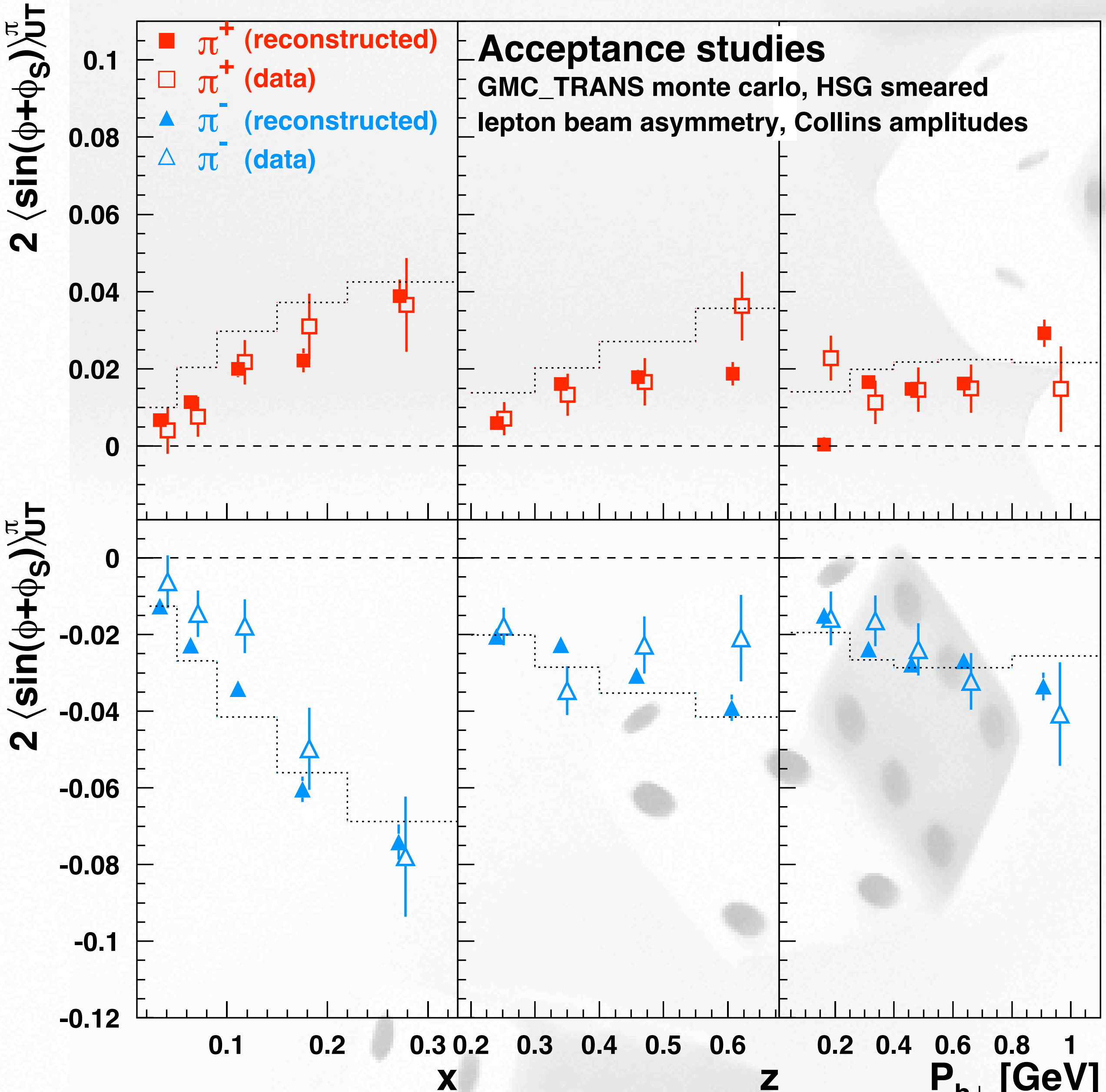
$$\delta u(x) = 0.7 \cdot \Delta u(x) \qquad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x) \qquad H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$$

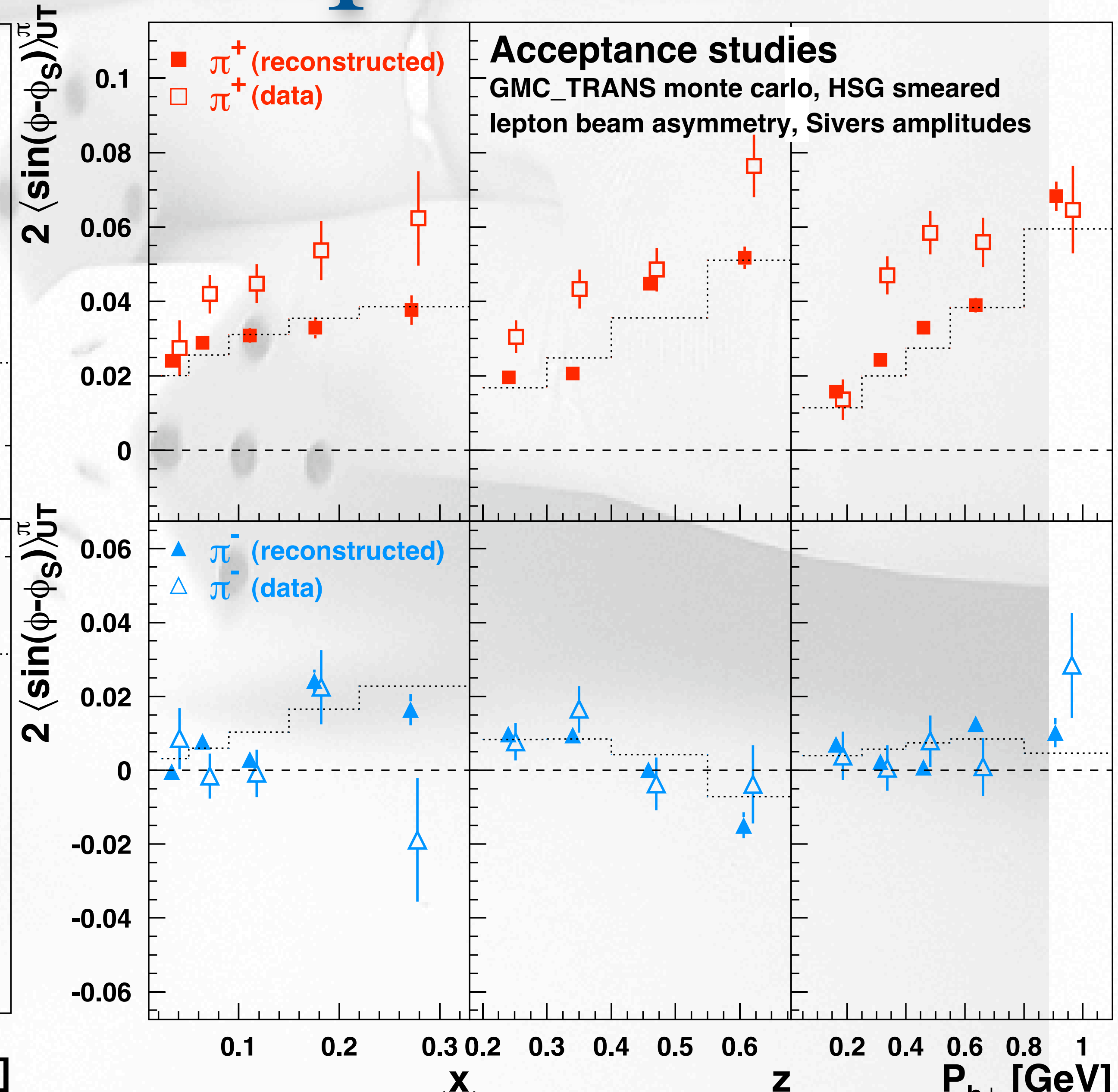
$$\delta d(x) = 0.7 \cdot \Delta d(x) \qquad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x) \qquad H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$$

$$\delta q(x) = 0.3 \cdot \Delta q(x) \qquad f_{1T}^{\perp q}(x) = 0.0 \qquad q = \bar{u}, d, s, \bar{s}$$

$$C_S = C_C = 0.25$$

GMC vs. Data Amplitudes





$\delta u(x) = 0.7 \cdot \Delta u(x)$ $\quad f_{1T}^{\perp u}(x) = -0.6 \cdot u(x)$ $\quad H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$

$\delta d(x) = 0.7 \cdot \Delta d(x)$ $\quad f_{1T}^{\perp d}(x) = 1.05 \cdot d(x)$ $\quad H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$

$\delta q(x) = 0.3 \cdot \Delta q(x)$ $\quad f_{1T}^{\perp q}(x) = 0.3 \cdot q(x)$ $\quad q = \bar{u}, \bar{d}, s, \bar{s}$

"hashi" set III for transverse momentum widths $\quad C_S = C_C = 0.25$

Positivity Limits Revisited

- **least stringent positivity constraints only involve considered 'polarized' function and the corresponding unpolarized function, e.g., Sivers vs. f_1**
- **more stringent relations arise from full density density matrix (e.g., Soffer relation for h_1 vs. g_1 and f_1):**

A. Bacchetta, M. Boglione, A. Henneman, and P. Mulders, Phys. Rev. Lett. **85**, 712 (2000).

$$\begin{pmatrix} f_1 + g_{1L} & \frac{|p_T|}{M} e^{i\phi}(g_{1T} + i f_{1T}^{\perp}) & \frac{|p_T|}{M} e^{-i\phi}(h_{1L}^{\perp} + i h_1^{\perp}) & 2h_1 \\ \frac{|p_T|}{M} e^{-i\phi}(g_{1T} - i f_{1T}^{\perp}) & f_1 - g_{1L} & \frac{|p_T|^2}{M^2} e^{-2i\phi} h_{1T}^{\perp} & -\frac{|p_T|}{M} e^{-i\phi}(h_{1L}^{\perp} - i h_1^{\perp}) \\ \frac{|p_T|}{M} e^{i\phi}(h_{1L}^{\perp} - i h_1^{\perp}) & \frac{|p_T|^2}{M^2} e^{2i\phi} h_{1T}^{\perp} & f_1 - g_{1L} & -\frac{|p_T|}{M} e^{i\phi}(g_{1T} - i f_{1T}^{\perp}) \\ 2h_1 & -\frac{|p_T|}{M} e^{i\phi}(h_{1L}^{\perp} + i h_1^{\perp}) & -\frac{|p_T|}{M} e^{-i\phi}(g_{1T} + i f_{1T}^{\perp}) & f_1 + g_{1L} \end{pmatrix}$$

has to be positive definite!

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A. Bacchetta, M. Boglione, A. Henneman, and P. Mulders, Phys. Rev. Lett. **85**, 712 (2000).

- **reanalysis yields a more complex positivity limit for Sivers:**

$$\frac{\boldsymbol{p_T}^2}{M^2}\left(f_{1T}^{\perp}(x,\boldsymbol{p_T}^2)\right)^2 \leq f_1(x,\boldsymbol{p_T}^2)\left(f_1(x,\boldsymbol{p_T}^2) - 2\,|h_1(x,\boldsymbol{p_T}^2)|\right)$$

(required setting all other DF to zero)

“Critique de Ferrara”

- **“positivity limit has to involve either only f_1 or (almost) all PDFs”**
- **implemented positivity limit in gmc_trans (self-consistently) involves only all in gmc_trans non-zero PDFs: f_1, h_1, Sivers (and if wanted BM)**
- **however, we know g_1 is not zero, thus at least g_1 has to be considered as well for gmc_trans to be realistic:**

$$\frac{\boldsymbol{p}_T^2}{M^2}\left(f_{1T}^{\perp}(x,\boldsymbol{p}_T^2)\right)^2 \le \left\{\left(f_1(x,\boldsymbol{p}_T^2)\right)^2 - \left(g_1(x,\boldsymbol{p}_T^2)\right)^2\right\}\left\{1 - \frac{2\,|h_1(x,\boldsymbol{p}_T^2)|}{f_1(x,\boldsymbol{p}_T^2)+g_1(x,\boldsymbol{p}_T^2)}\right\}$$

- **which positivity constraint is stronger and what about other PDFs?**

Positivity for $g_1 = 0$ vs. $g_1 \neq 0$

$$\Delta(\mathrm{P.L.}) = \left(g_1(x, \boldsymbol{p}_T^2)\right)^2 - 2\, g_1(x, \boldsymbol{p}_T^2)\, |h_1(x, \boldsymbol{p}_T^2)|$$

- **for g_1<0 (e.g., down quarks) g_1=0 limit is <u>less</u> strict**
- **for g_1>0 (e.g., u-quarks) it depends on size of h_1**
  ➡ so far in gmc_trans productions g_1=0 limit was <u>always less</u> strict
- **nevertheless, now implemented P.L. check that involves g_1 (and also BM function)**
  - **should check P.L. involving all functions, but too little known about other TMDs**

"Problem" with Sea Quarks

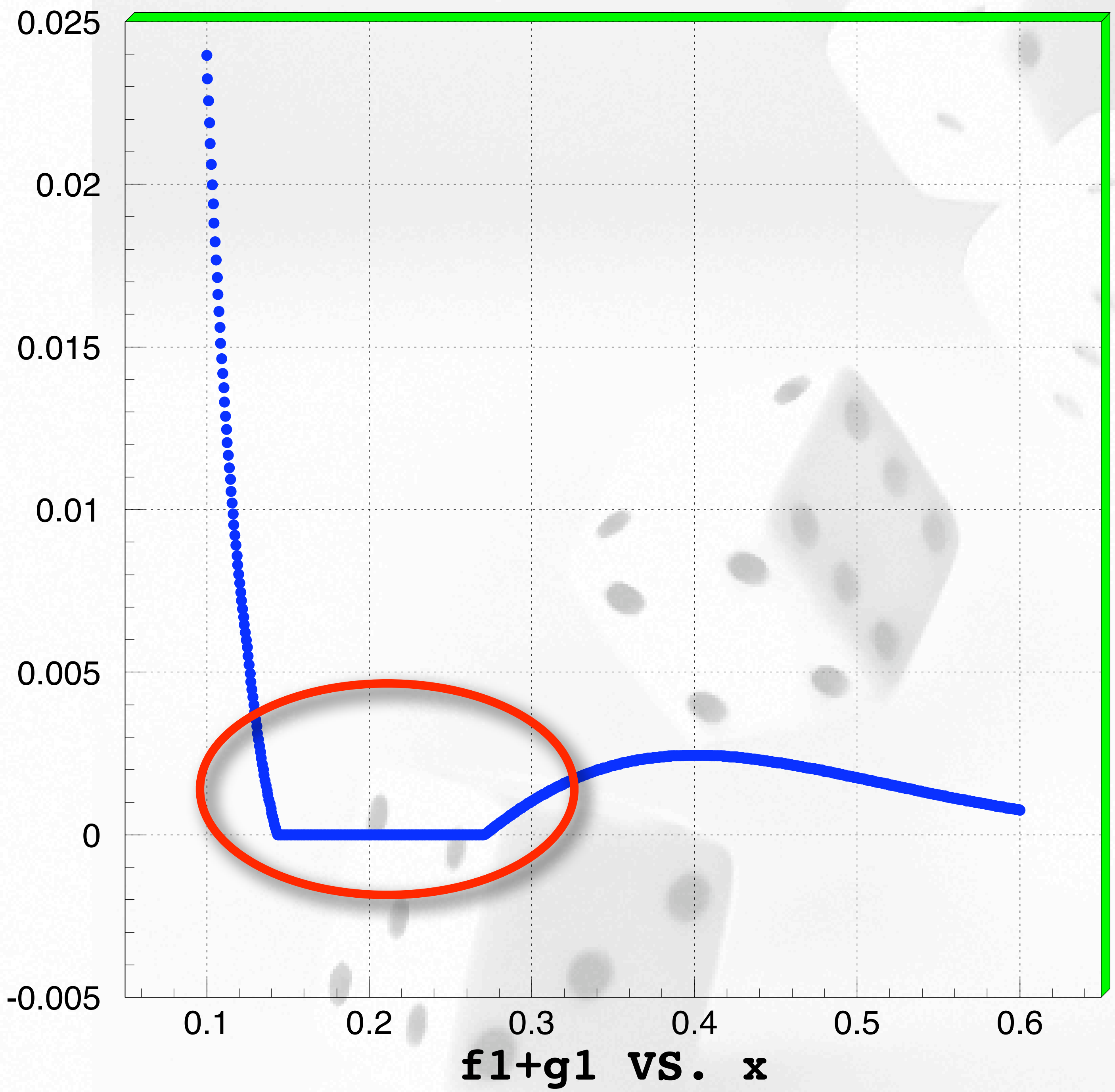
$\bar{s}$ quarks ➠

s quarks ➠

```
======   Check positivity limit for Transversity:
===========   positivity violation:
 iquark= -3: lhs=  24.5845945 > rhs=  0.5
 iquark= -2: lhs=  0.48761702 < rhs=  0.5
 iquark= -1: lhs=  0.08159421 < rhs=  0.5
 iquark=  1: lhs=  0.35142772 < rhs=  0.5
 iquark=  2: lhs=  0.25735636 < rhs=  0.5
===========   positivity violation:
 iquark=  3: lhs=  24.5845945 > rhs=  0.5
```

- **affected mainly (and strongly) *strange*-quark positivity limit**

The Strange(!) Sea



- $f_1(x) + g_1(x)$ for strange quarks
- LST(15) = 118; 'standard' scenario, leading order GRSV
- Q^2=1
- Soffer bound: $|h_1(x)| < 0.5\{f_1(x)+g_1(x)\}$ can’t be fulfilled for nonzero h_1

Sivers & Transversity Fits by Anselmino et al.

$g_1 = 0$

```
======   Check positivity limit for Transversity:
iquark= -3: lhs=  0. < rhs=  0.5
iquark= -2: lhs=  0. < rhs=  0.5
iquark= -1: lhs=  0. < rhs=  0.5
iquark= 1: lhs=  0.221290307 < rhs=  0.5
iquark= 2: lhs=  0.356384362 < rhs=  0.5
iquark= 3: lhs=  0. < rhs=  0.5

======   Check positivity limit for T-odd DFs:
iquark= -3: lhs=  0.38756798 < rhs=  0.38756827
iquark= -2: lhs=  0.01550272 < rhs=  0.38756827
iquark= -1: lhs=  0.15502719 < rhs=  0.38756827

==========   positivity violation!
iquark= 1: lhs=  0.460370198 > rhs=  0.38756827
iquark= 2: lhs=  0.250900224 < rhs=  0.38756827
iquark= 3: lhs=  0.093016313 < rhs=  0.38756827
```

$g_1 \neq 0$

```
======   Check positivity limit for Transversity:
iquark= -3: lhs=  0. < rhs=  0.5
iquark= -2: lhs=  0. < rhs=  0.5
iquark= -1: lhs=  0. < rhs=  0.5
iquark= 1: lhs=  0.309999922 < rhs=  0.5
iquark= 2: lhs=  0.239999932 < rhs=  0.5
iquark= 3: lhs=  0. < rhs=  0.5

======   Check positivity limit for T-odd DFs:
iquark= -3: lhs=  0.38756798 < rhs=  0.38756827
iquark= -2: lhs=  0.01550272 < rhs=  0.38756827
iquark= -1: lhs=  0.15502719 < rhs=  0.38756827

==========   positivity violation!
iquark= 1: lhs=  0.58568429 > rhs=  0.38756827
iquark= 2: lhs=  0.21341756 < rhs=  0.38756827
iquark= 3: lhs=  0.09301631 < rhs=  0.38756827
```

- **Sivers function rather close to positivity limit for anti-s**
- **Sivers function for d quarks 20% too large**

Sivers & Transversity Fits by Anselmino et al.

Non-Trivial Role of other TMDs!?

$g_1=0$

====== Check positivity limit for Transversity:
iquark= -3: lhs= 0. < rhs= 0.5
iquark= -2: lhs= 0. < rhs= 0.5
iquark= -1: lhs= 0. < rhs= 0.5
iquark= 1: lhs= 0.221290307 < rhs= 0.5
iquark= 2: lhs= 0.356384362 < rhs= 0.5
iquark= 3: lhs= 0. < rhs= 0.5

====== Check positivity limit for T-odd D
iquark= -3: lhs= 0.38756798 < rhs= 0.3
iquark= -2: lhs= 0.01550272 < rhs
iquark= -1: lhs= 0.15502719 < 27

========== positivi
iquark= 1: lhs= 0.4 s= 0.38756827
iquark= 2: lhs= rhs= 0.38756827
iquark= 3: l 13 < rhs= 0.38756827

0

====== Check positivi versity:
iquark= -3: lhs= 0. <
iquark= -2: lhs=
iquark= -1: l .5
iquark= 9922 < rhs= 0.5
iqua 9999932 < rhs= 0.5
0. < rhs= 0.5

Check positivity limit for T-odd DFs:
ark= -3: lhs= 0.38756798 < rhs= 0.38756827
iquark= -2: lhs= 0.01550272 < rhs= 0.38756827
iquark= -1: lhs= 0.15502719 < rhs= 0.38756827

========== positivity violation!
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iquark= 3: lhs= 0.09301631 < rhs= 0.38756827

- **Si nction rather close to positivity limit for anti-s**
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(finally) Dessert:
the leaf of mint on the cake

Beyond Collins and Sivers

- certainly would like to model all TMDs, e.g., Boer-Mulders function, to get full cross section
- even go to subleading-twist, e.g., Cahn effect
- first attempts to implement those have been made
- leading twist -- “straight forward” (just a few more convolution integrals)
- subleading twist -- “hmmmm...”
    - biggest problem there: positivity limits don’t exist on DF and FF level

Current ToDo and Done List

- finish leading-twist implementation
- implement newest results from fits and model calculations on transversity, Sivers & Collins, ...
- add radiative corrections (e.g., RADGEN)
- make it portable to other experiments

(since Ferrara meeting:)

✓ Charged kaons and protons

✓ DSS FFs and published fits by Anselmino et al.

✓ neutron target

➡ comparison of HERMES and COMPASS data possible (but not yet done)

Epilogue

- **Acceptance plays crucial part in analysis of multi-particle final states**
- **Acceptance studies and/or corrections (e.g., unfolding) require realistic Monte Carlo simulation of underlying physics**
- **gmc_trans provides Collins and Sivers amplitudes for pions and kaons based on Gaussian Ansatz for TMDs**
- **Positivity limits ➥ smaller Gaussian width for TMDs**
- **Comparison of unpolarized hadron yield suggests z-dependent average fragmentation K_T**
- **Don't fully trust GRSV strange polarization at low Q^2!**
- **Non-trivial role of unmeasured TMDs in fulfilling positivity of Sivers distribution**