



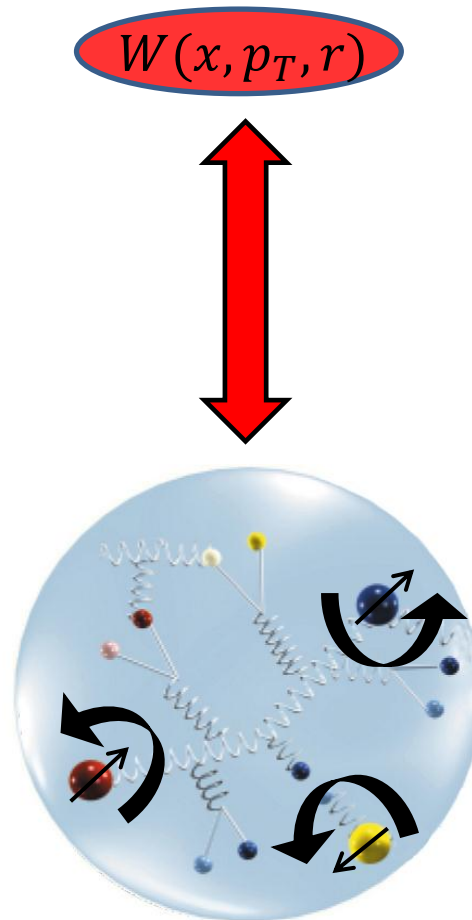
Selected TMD results from HERMES

Luciano L. Pappalardo

University of Ferrara

The phase-space distribution of partons

The full phase-space distribution of the partons encoded in the **Wigner function**

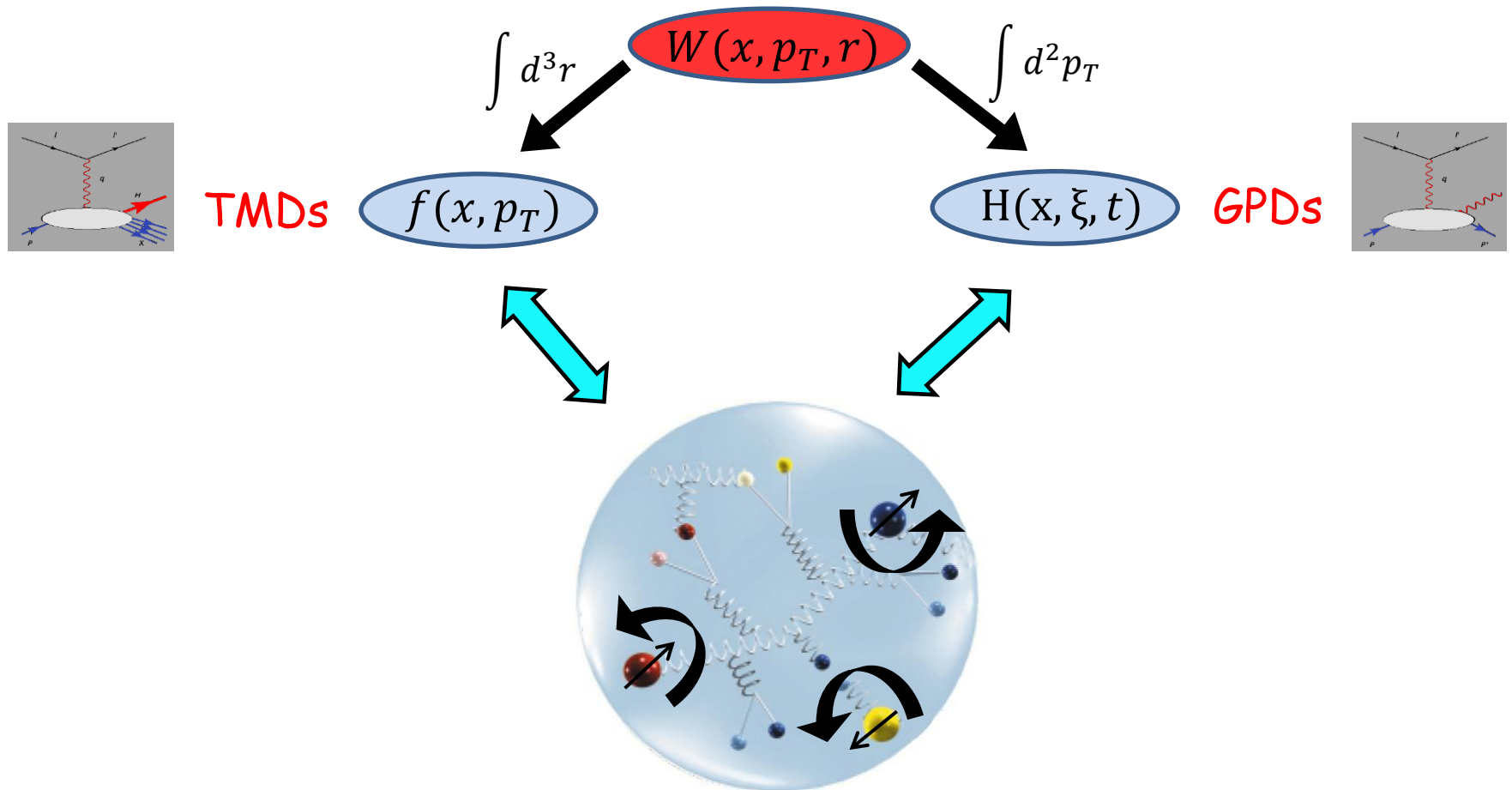


The phase-space distribution of partons

The full phase-space distribution of the partons encoded in the **Wigner function**

...but $\Delta x \Delta p \geq \frac{\hbar}{2} \rightarrow$ no simultaneous knowledge of momentum and position

cannot be directly accessed experimentally \rightarrow integrated quantities

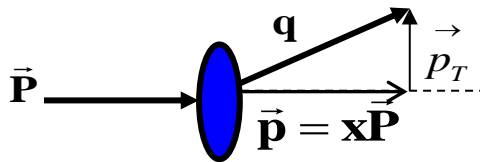
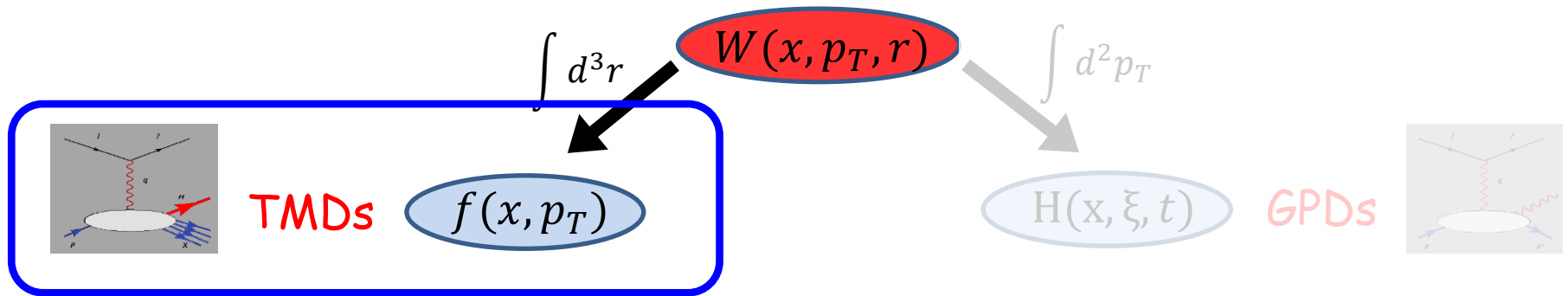


The non-collinear structure of the nucleon

The full phase-space distribution of the partons encoded in the **Wigner function**

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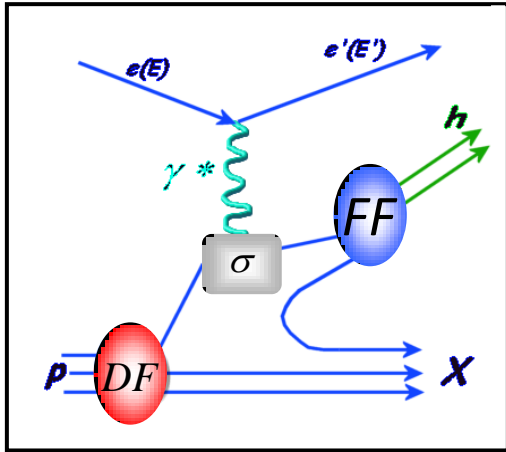
- TMDs depend on x and p_T
- Describe correlations between p_T and quark or nucleon spin (**spin-orbit correlations**)
- Provide a **3-dim picture** of the nucleon in momentum space (**nucleon tomography**)

		momentum	helicity	Boer-Mulders
		quark		
		U	L	T
nucleon	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_{1T}^\perp

Additional callouts: "transversity" points to the h_{1L}^\perp and h_{1T}^\perp terms; "pretzelosity" points to the h_{1T}^\perp term; "Sivers" points to f_{1T}^\perp ; "worm-gears" points to g_{1T}^\perp .

The non-collinear structure of the nucleon

Mostly investigated in **SIDIS**: detection of transverse momentum of produced hadrons gives access to p_T



Fragmentation Functions (FF)

		quark		
		U	L	T
h a d r o n	U	D_1		H_1^\perp
	L		G_{1L}	H_{1L}^\perp
	T	D_{1T}^\perp	G_{1T}^\perp	H_{1T}^\perp

Collins FF
chiral-odd

unpol. FF
chiral-even

$$\sigma^{ep \rightarrow ehX} = \sum_q DF \otimes \sigma^{eq \rightarrow eq} \otimes FF$$

- TMDs depend on x and p_T
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momentum helicity Boer-Mulders

		quark		
		U	L	T
n u c l e o n	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_{1T}^\perp

transversity

pretzelosity

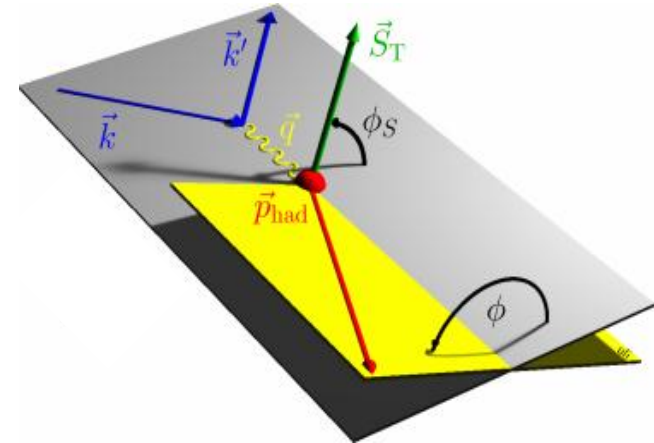
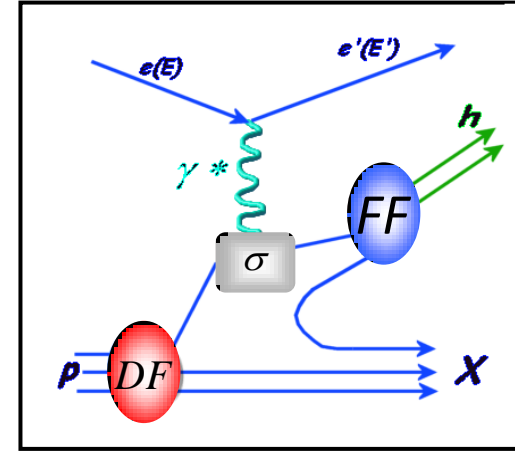
Sivers

worm-gears

The SIDIS cross-section

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ & + \lambda_L \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\ & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\ & + S_L \lambda_L \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\ & + S_T \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & \left. + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \\ & + S_T \lambda_L \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \end{aligned} \right\}$$



The SIDIS cross-section

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{b\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right)$$

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unpolarized

$$+ \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

beam polarization

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

target polarization

$$+ S_T \left[\begin{array}{l} \sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \\ + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \end{array} \right]$$

beam and target polarization

$$+ S_T \lambda_l \left[\begin{array}{l} \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \end{array} \right] \left. \right\}$$

The SIDIS cross-section

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \boxed{F_{UU,T}} + \epsilon F_{UU,L} \\ & + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) \boxed{F_{UU}^{\cos(\phi)}} + \epsilon \cos(2\phi) \boxed{F_{UU}^{\cos(2\phi)}} \end{aligned} \right.$$

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$$+ S_T \lambda_l \left\{ \begin{aligned} & \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) \boxed{F_{LT}^{\cos(\phi - \phi_S)}} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) \boxed{F_{LT}^{\cos(\phi_S)}} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) \boxed{F_{LT}^{\cos(2\phi - \phi_S)}} \end{aligned} \right\}$$

18 Structure Functions

Leading twist

Sub-leading Twist

Fragmentation Functions				
		quark		
		U	L	T
h	U	D_1		H_1^\perp -

$$F_{XY} \propto DF \otimes FF$$

Distribution Functions				
		quark		
		U	L	T
n u c l e o n	U	f_1		h_1^\perp -
	L		g_1 -	h_{1L}^\perp -
	T	f_{1T}^\perp -	g_{1T}^\perp -	h_{1T}^\perp -

Selected **twist-2** and **twist-3** 1-hadron SIDIS results

Sivers function

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2 y^2}{xyQ^2 2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

$$+ \lambda_L \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

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$$+ S_L \lambda_L \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$
















$$\left. \begin{aligned} + S_T & \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \end{aligned} \right\}$$

$$\left. \begin{aligned} + S_T \lambda_L & \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \end{aligned} \right\}$$




$$F_{UT,T}^{\sin(\phi_h - \phi_S)} = C \left[-\frac{\hat{h} \cdot \mathbf{p}_T}{M} f_{1T}^\perp D_1 \right]$$

Describes correlation between quark transverse momentum and nucleon transverse polarization

Distribution Functions

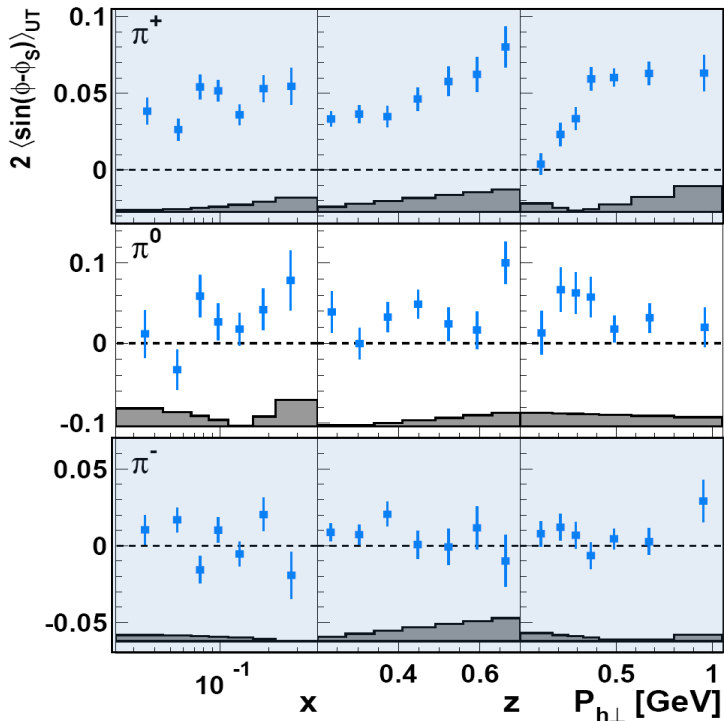
		quark		
		U	L	T
nucleon	U	f_1 		h_1^\perp  - 
	L		g_1  - 	h_{1L}^\perp  - 
	T	f_{1T}^\perp  - 	g_{1T}^\perp  - 	h_1  -  h_{1T}^\perp  - 

Fragmentation Functions

		quark		
		U	L	T
h	U	D_1 		H_1^\perp  - 

Sivers amplitudes $\propto f_{1T}^\perp \otimes D_1$

[Airapetian *et al.*, Phys. Rev. Lett. 103 (2009) 152002]

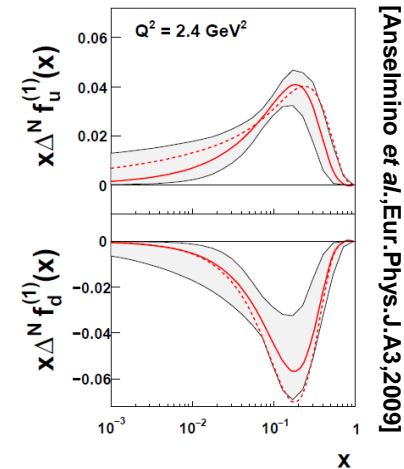


☞ significantly positive

☞ slightly positive
(isospin-symmetry)

☞ consistent with zero

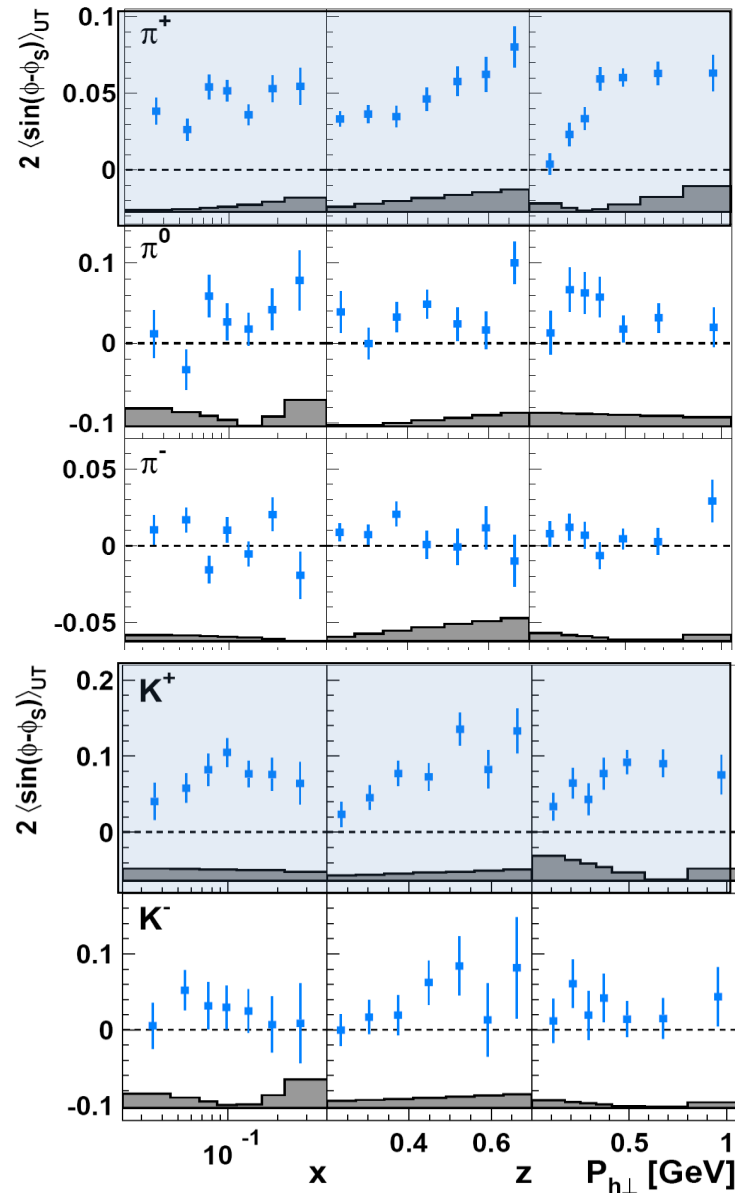
consistent with Sivers func. of opposite sign for u and d quarks



[Anselmino *et al.*, Eur. Phys. J. A3, 2009]

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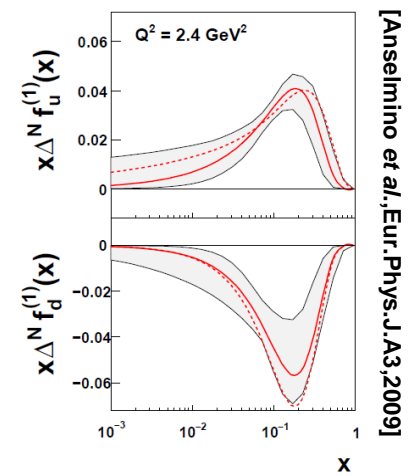
☞ consistent with zero

☞ significantly positive

Similar kinematic dependence of π^+

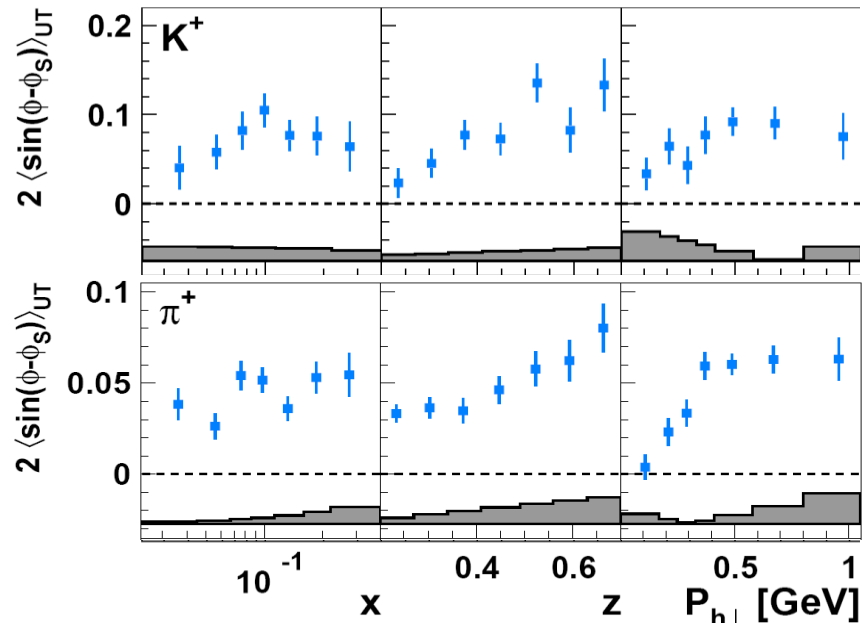
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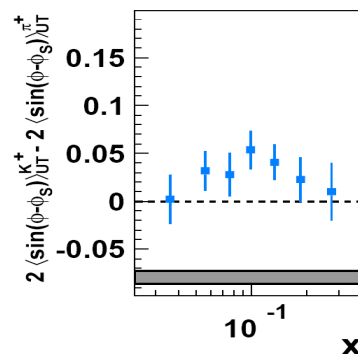
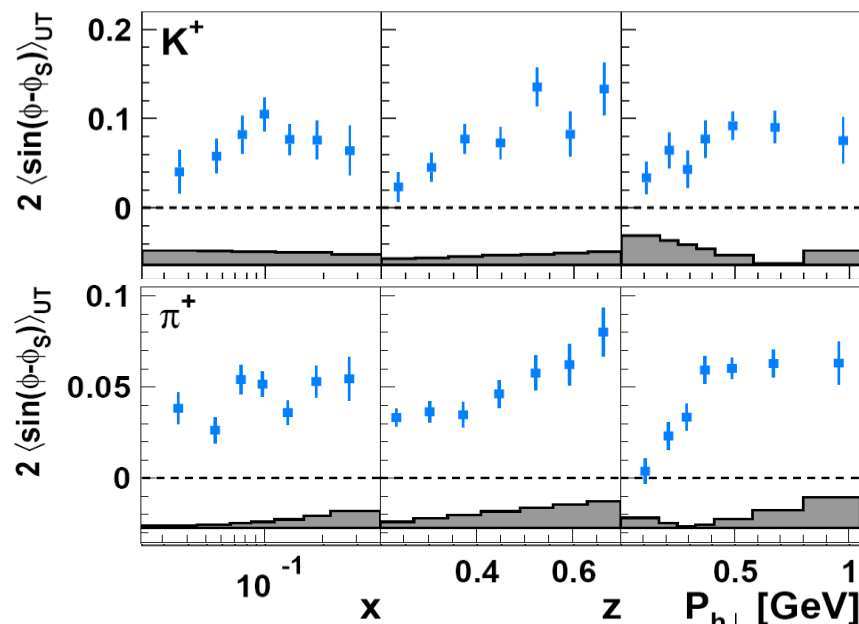
Sivers kaons amplitudes: open questions

π^+/K^+ production dominated by u-quarks, but:



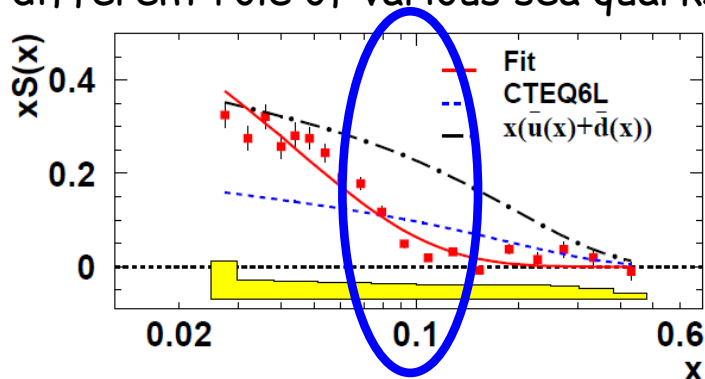
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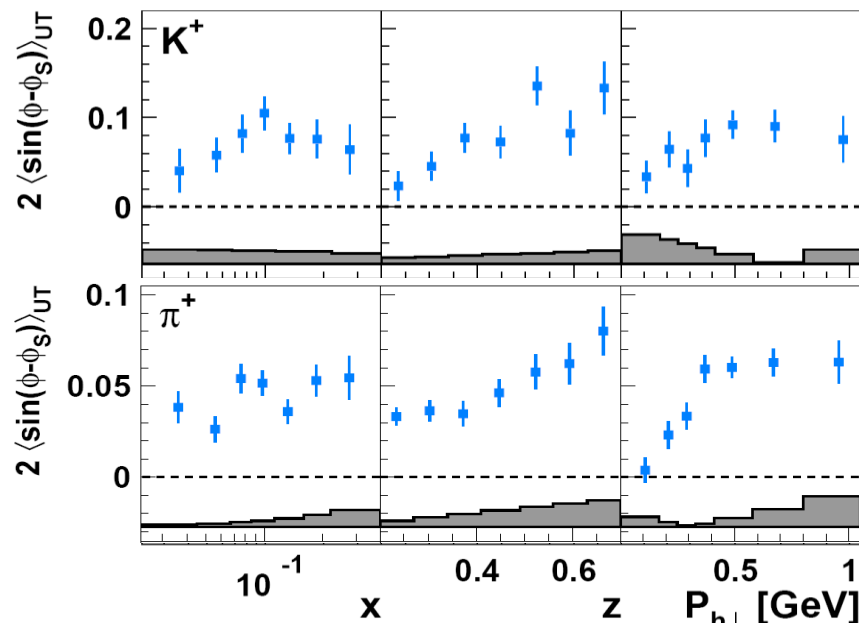
$$\pi^+ \equiv |u\bar{d}\rangle, K^+ \equiv |u\bar{s}\rangle \rightarrow$$

different role of various sea quarks ?

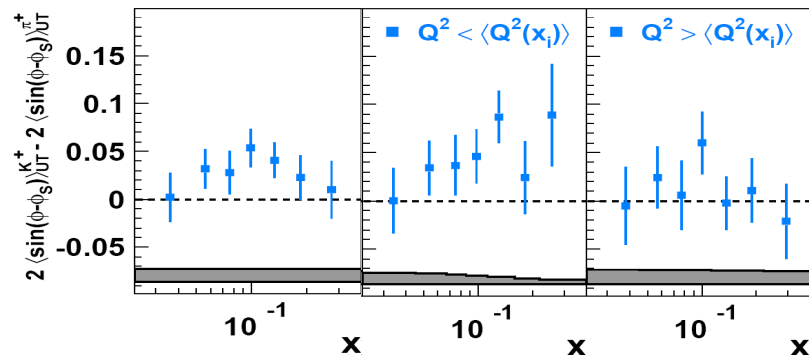


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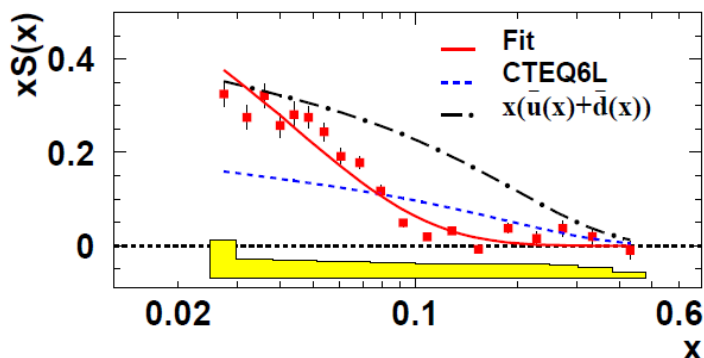


only in low- Q^2 region significant deviation

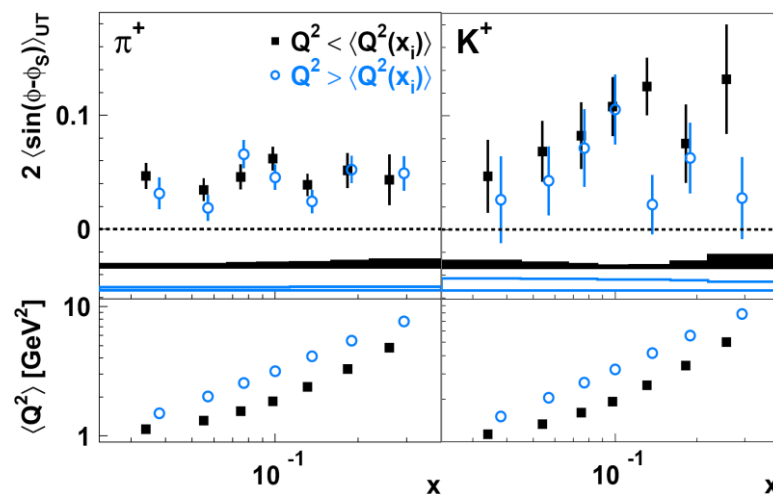


$$\pi^+ \equiv |u\bar{d}\rangle, K^+ \equiv |u\bar{s}\rangle \rightarrow$$

different role of various sea quarks ?



each x-bin divided into two Q^2 bins



no effect for pions, but hint of a systematic shifts for kaons



Higher-twist contrib. for Kaons

Pretzelosity

$$F_{UT}^{\sin(3\phi_h - \phi_S)} = C \left[\frac{2(\hat{h} \cdot p_T)(p_T \cdot k_T) + p_T^2(\hat{h} \cdot k_T) - 4(\hat{h} \cdot p_T)^2(\hat{h} \cdot k_T)}{2M^2 M_h} h_{1T}^\perp H_1^\perp \right]$$

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi dP_{h\perp}^2} = \frac{\alpha^2 y^2}{xyQ^2 2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ & + \lambda_L \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\ & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\ & + S_L \lambda_L \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\ & + S_T \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \right. \\ & \left. + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \\ & + S_T \lambda_L \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \end{aligned} \right\}$$

Describes correlation between quark transverse momentum and transverse spin in a transversely pol. nucleon

➤ Sensitive to **non-spherical shape** of the nucleon

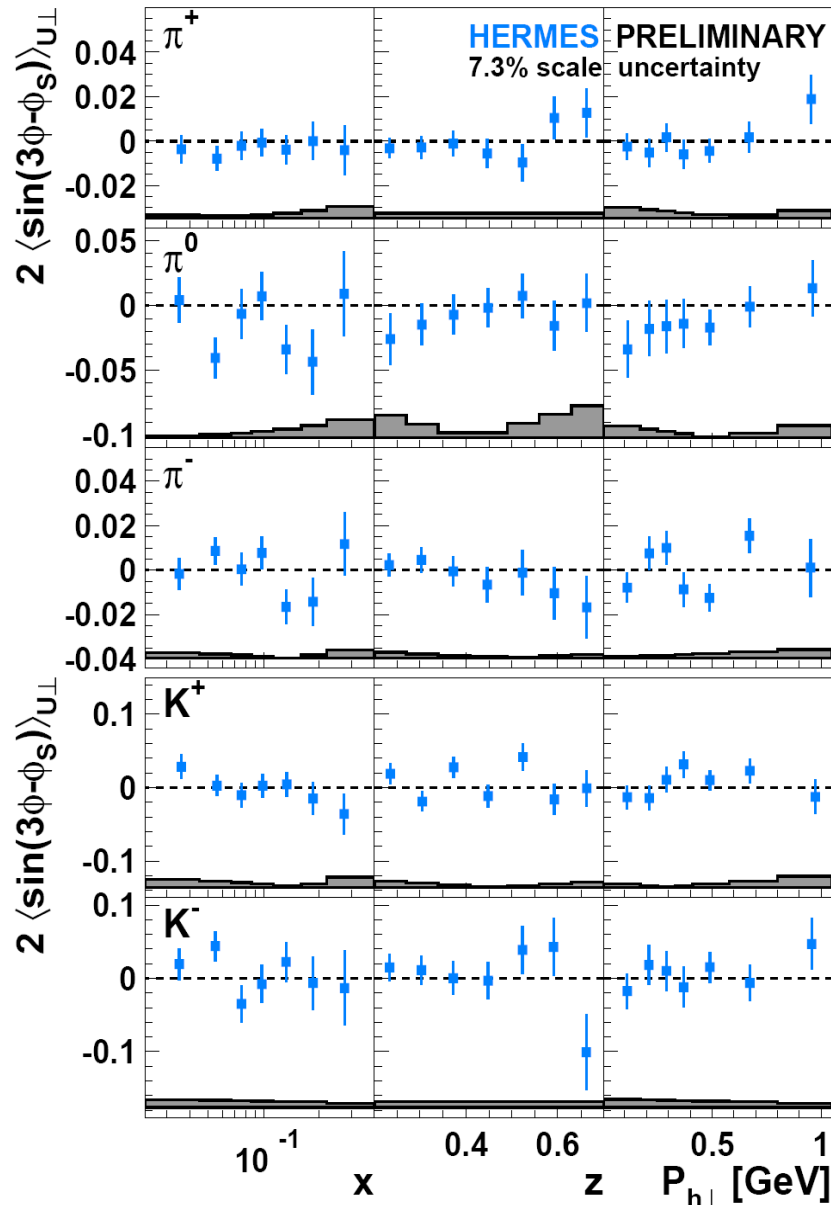
Distribution Functions

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		U	L	T
nucleon	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_{1T}^\perp

Fragmentation Functions

		quark		
		U	L	T
h	U	D_1		H_1^\perp

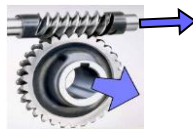
The $\sin(3\phi - \phi_S)_{LT}$ amplitudes $\propto h_{1T}^\perp \otimes H_1^\perp$



All amplitudes consistent with zero

...suppressed by two powers of $P_{h\perp}$
w.r.t. Sivers amplitudes

Worm-gear g_{1T}^\perp



$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ & + \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\ & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\ & + S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\ & + S_T \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & \left. + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \end{aligned} \right.$$

$$\left. \begin{aligned} & + S_T \lambda_l \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \end{aligned} \right\}$$

$$F_{LT}^{\cos(\phi_h - \phi_S)} = C \left[\frac{\hat{h} \cdot p_T}{M} g_{1T}^\perp D_1 \right]$$

Describes the probability to find longitudinally polarized quarks in a transversely polarized nucleon!

➤ Can be accessed in **LT DSAs**

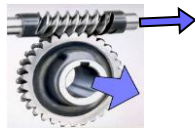
Distribution Functions

		quark		
		U	L	T
nucleon	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_1 h_{1T}^\perp

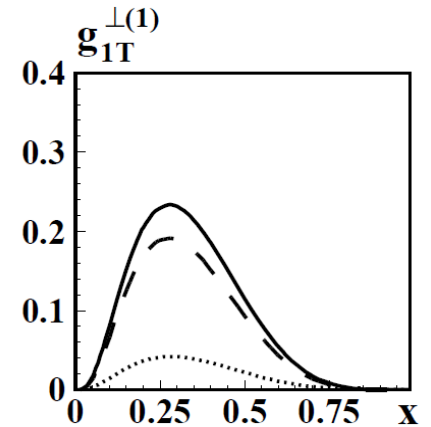
Fragmentation Functions

		quark		
		U	L	T
h	U	D_1		H_1^\perp

Worm-gear g_{1T}^\perp



- The only TMD that is both **chiral-even** and **naïve-T-even**
- requires interference between wave function components that differ by 1 unit of OAM \Rightarrow **quark orbital motion inside nucleons**

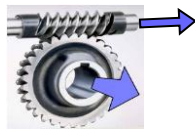


S. Boffi et al. (2009)
Phys. Rev. D 79 094012

**Light-cone constituent
quark model**

dashed line: interf. L=0, L=1
dotted line: interf L=1, L=2

Worm-gear g_{1T}^\perp

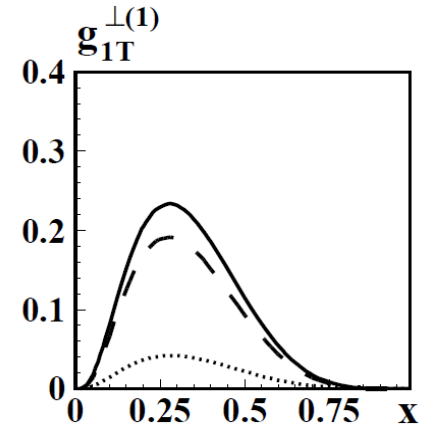


- The only TMD that is both **chiral-even** and **naïve-T-even**
- requires interference between wave function components that differ by 1 unit of OAM \Rightarrow **quark orbital motion inside nucleons**
- **Accessible in LT DSAs:**

$$F_{LT}^{\cos(\phi_h - \phi_S)} = C \left[\frac{\hat{h} \cdot \mathbf{p}_T}{M} g_{1T} D_1 \right]$$

$$F_{LT}^{\cos \phi_S} = \frac{2M}{Q} C \left\{ - \left(x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z} \right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^\perp}{z} \right) + \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z} \right) \right] \right\}$$

$$F_{LT}^{\cos(2\phi_h - \phi_S)} = \frac{2M}{Q} C \left\{ - \frac{2(\hat{h} \cdot \mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2} \left(x g_T^\perp D_1 + \frac{M_h}{M} h_{1T}^\perp \frac{\tilde{E}}{z} \right) + \frac{2(\hat{h} \cdot \mathbf{k}_T)(\hat{h} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^\perp}{z} \right) - \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z} \right) \right] \right\}$$

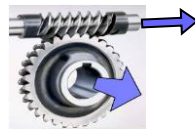


S. Boffi et al. (2009)
Phys. Rev. D 79 094012

Light-cone constituent quark model

dashed line: interf. L=0, L=1
dotted line: interf L=1, L=2

Worm-gear g_{1T}^\perp



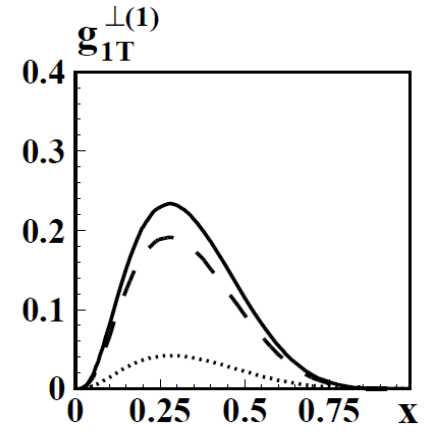
- The only TMD that is both **chiral-even** and **naïve-T-even**
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- **Accessible in LT DSAs:**

$$F_{LT}^{\cos(\phi_h - \phi_S)} = C \left[\frac{\hat{h} \cdot \mathbf{p}_T}{M} g_{1T} D_1 \right]$$

\rightarrow Simplest way to probe g_{1T}^\perp

$$F_{LT}^{\cos \phi_S} = \frac{2M}{Q} C \left\{ - \left(x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z} \right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^\perp}{z} \right) + \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z} \right) \right] \right\}$$

$$F_{LT}^{\cos(2\phi_h - \phi_S)} = \frac{2M}{Q} C \left\{ - \frac{2(\hat{h} \cdot \mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2} \left(x g_T^\perp D_1 + \frac{M_h}{M} h_{1T}^\perp \frac{\tilde{E}}{z} \right) + \frac{2(\hat{h} \cdot \mathbf{k}_T)(\hat{h} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{2MM_h} \left[\left(x e_T H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^\perp}{z} \right) - \left(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z} \right) \right] \right\}$$

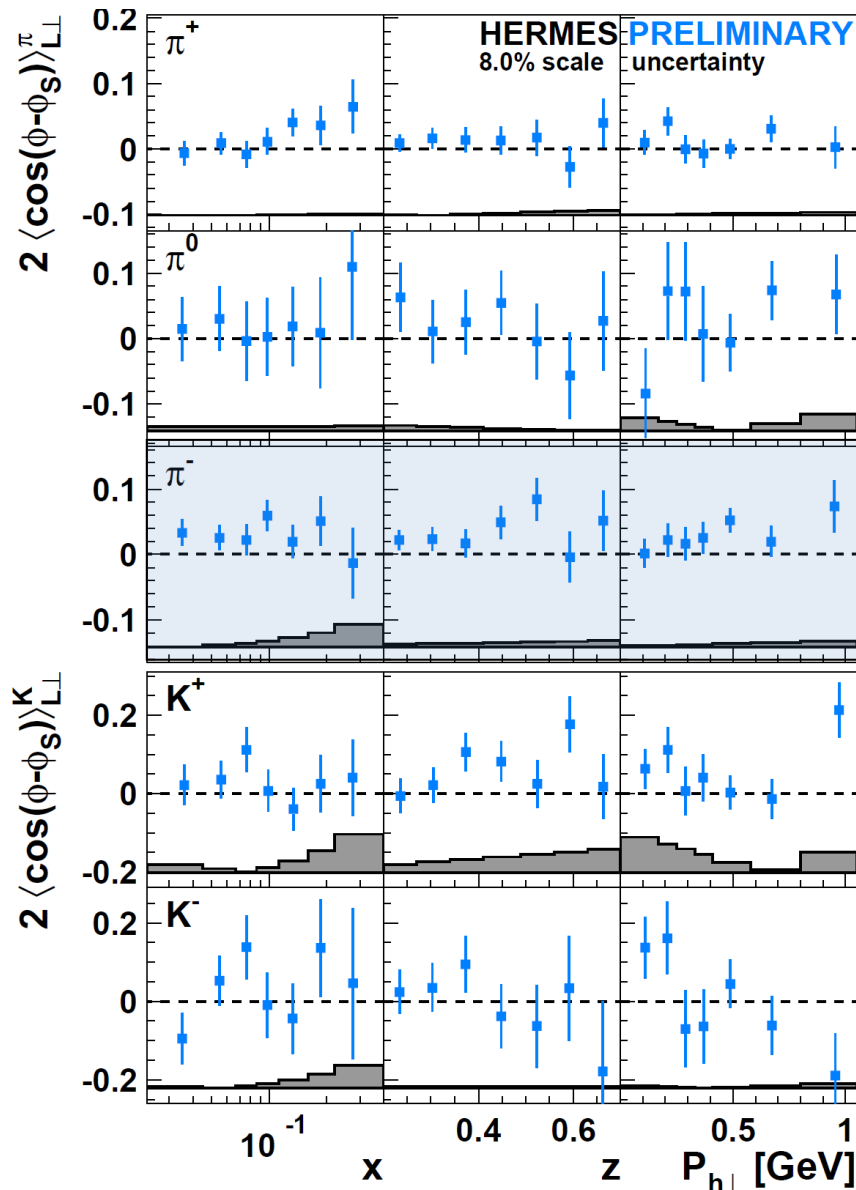


S. Boffi et al. (2009)
Phys. Rev. D 79 094012

Light-cone constituent quark model

dashed line: interf. L=0, L=1
dotted line: interf L=1, L=2

The $\cos(\phi - \phi_S)_{LT}$ amplitudes $\propto g_{1T}^\perp \otimes D_1$



☞ slightly positive ?

☞ consistent with zero

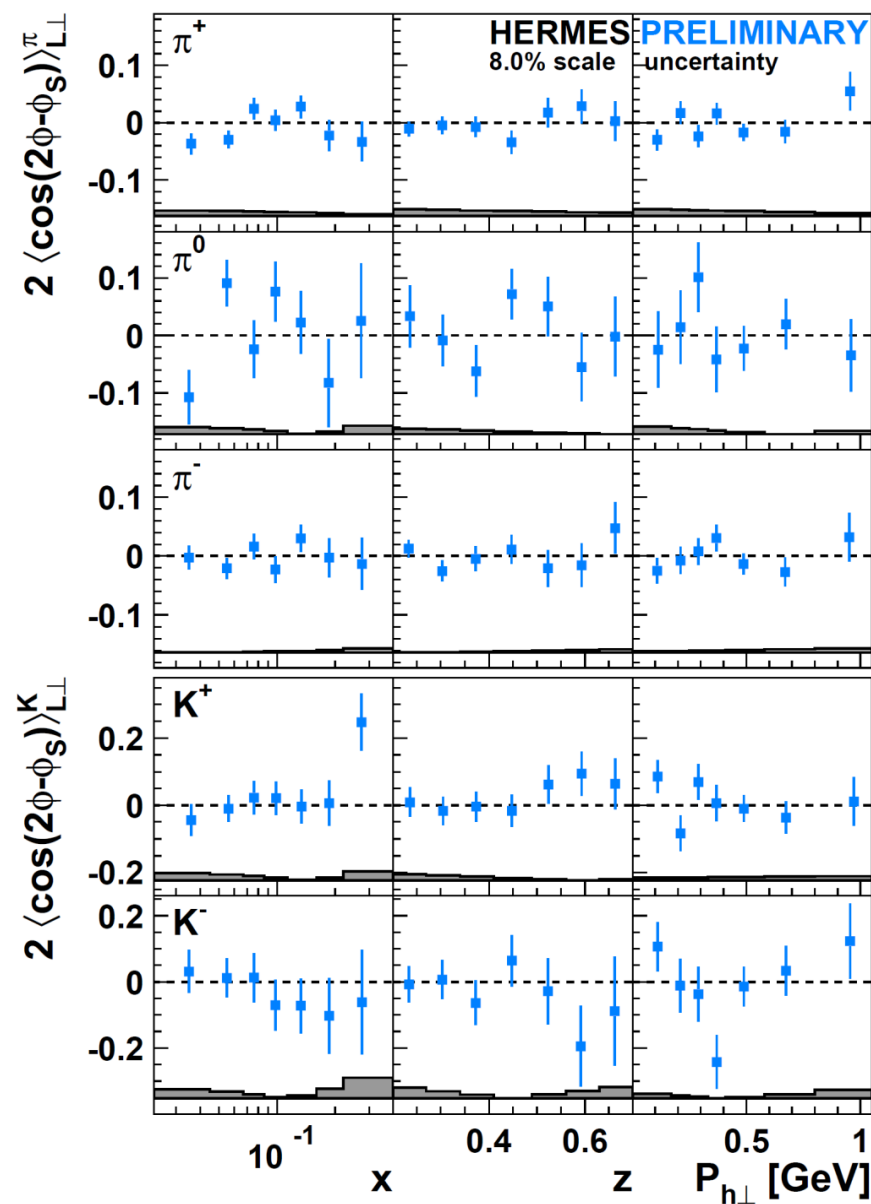
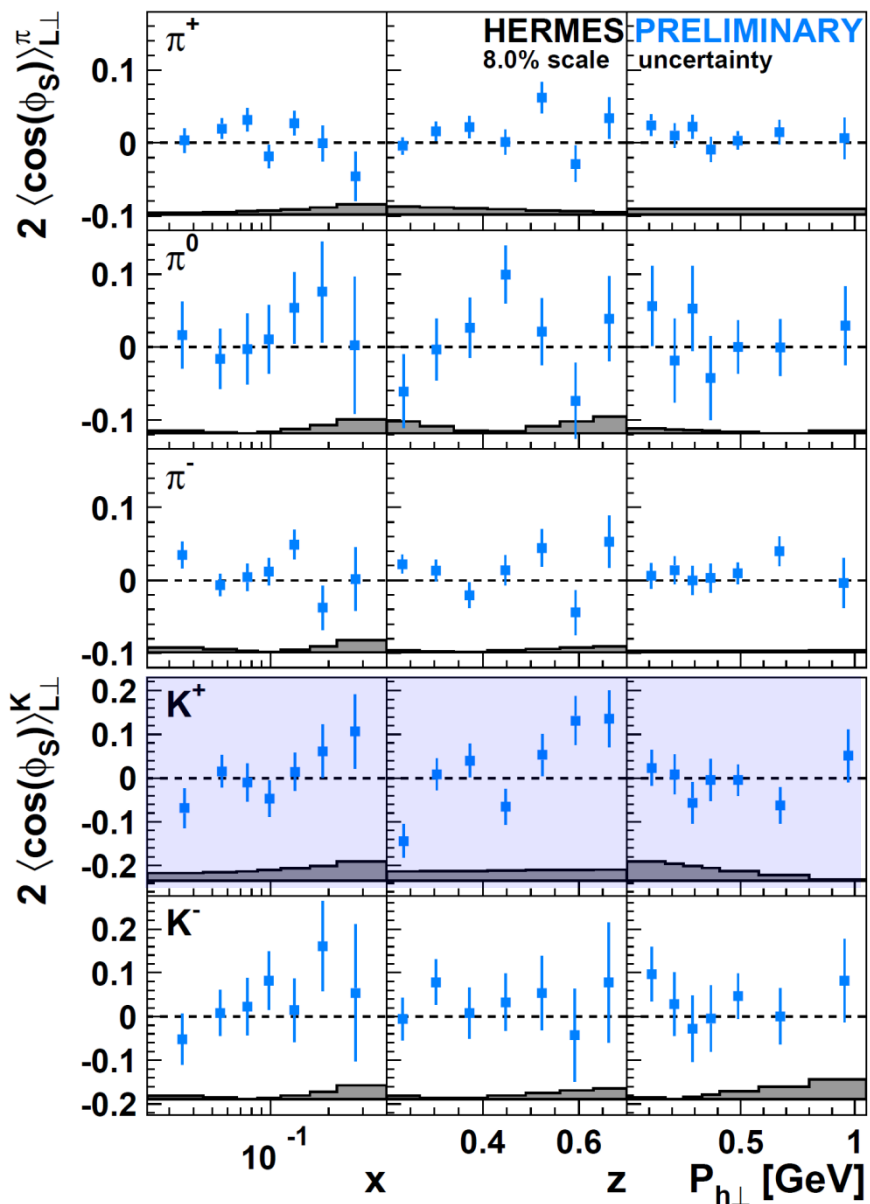
☞ positive!!

similar observations from
Hall-A and COMPASS

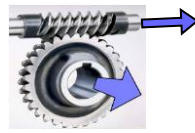
☞ slightly positive ?

☞ consistent with zero

The $\cos(\phi_S)_{LT}$ and $\cos(2\phi - \phi_S)_{LT}$ amplitudes



Worm-gear h^\perp_{1L}



$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2 y^2}{xyQ^2 2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right)$$

$$\left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

$$+ \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$+ S_T \left[\begin{aligned} & \sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \end{aligned} \right]$$

$$+ S_T \lambda_l \left[\begin{aligned} & \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \end{aligned} \right] \left. \right\}$$

$$F_{UL}^{\sin 2\phi_h} = C \left[-\frac{2(\hat{h} \cdot \mathbf{k}_T)(\hat{h} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_{1L}^\perp H_1^\perp \right]$$

Describes the probability to find transversely polarized quarks in a longitudinally polarized nucleon

➤ some models support the simple relation

$$g_{1T}^q = -h_{1L}^{\perp q}$$

Distribution Functions

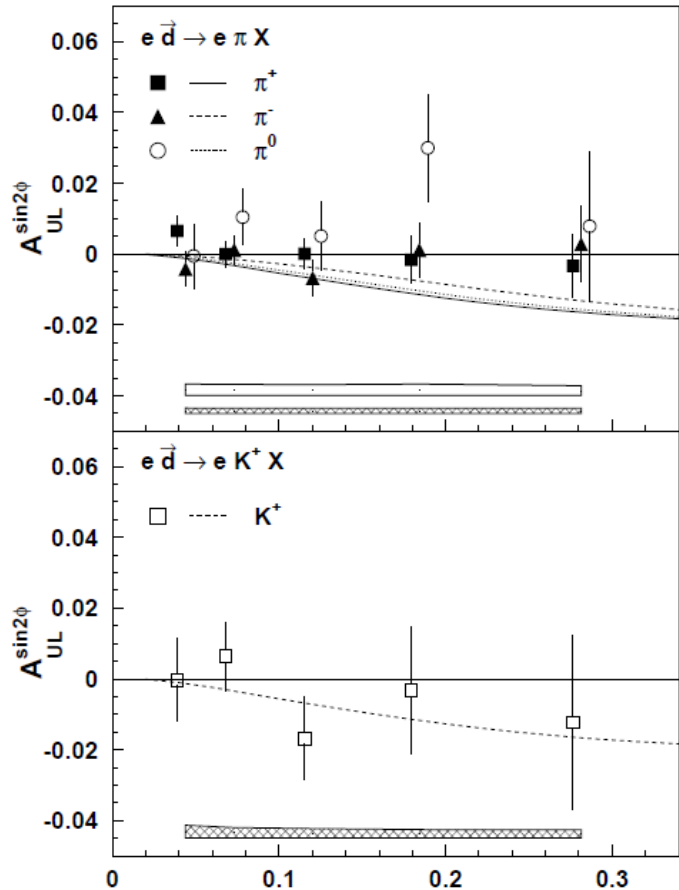
		quark		
		U	L	T
nucleon	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_{1T}^\perp

Fragmentation Functions

		quark		
		U	L	T
h	U	D_1		H_1^\perp

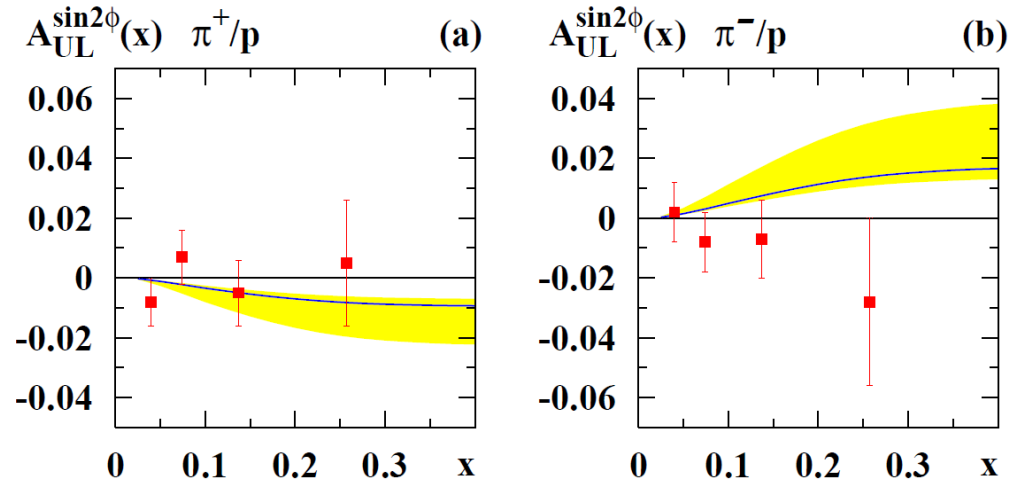
The $\sin(2\phi)_{UL}$ amplitude $\propto h_{1L}^\perp \otimes H_1^\perp$

Deuterium target



A. Airapetian et al, *Phys. Lett. B* 562 (2003)

Hydrogen target



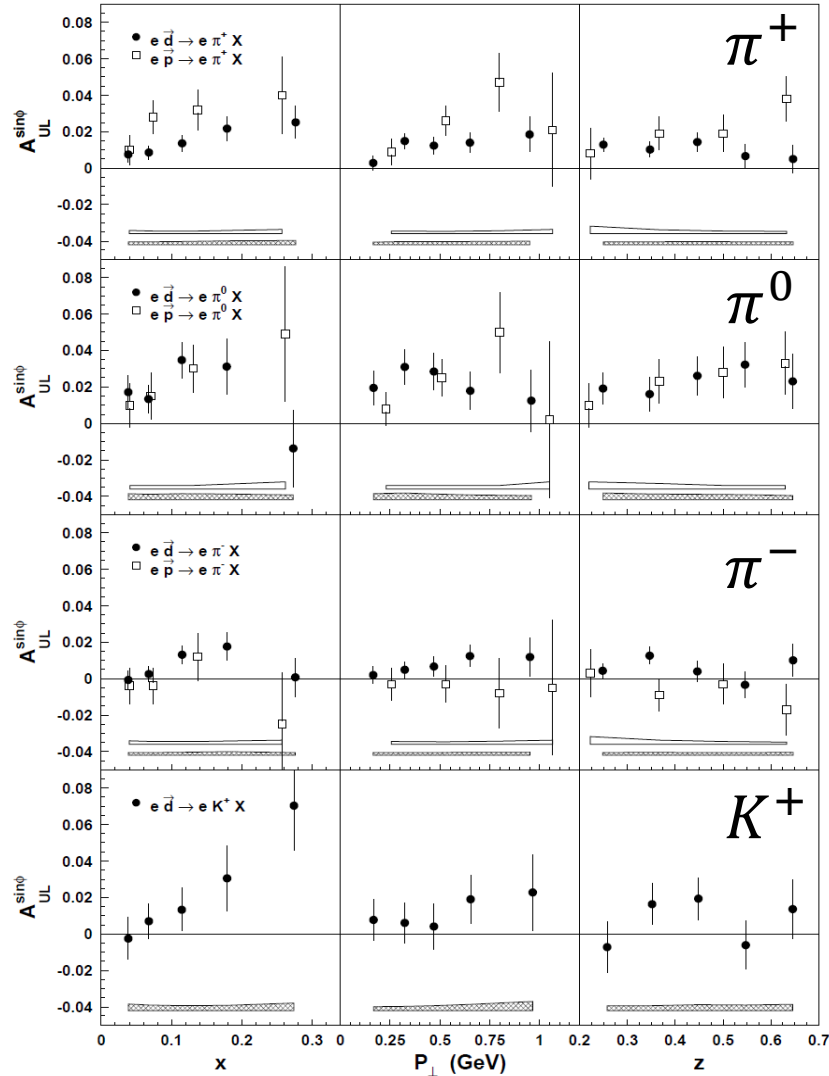
A. Airapetian et al, *Phys. Rev. Lett.* 84 (2000)

Amplitudes consistent with zero for all mesons and for both H and D targets

$\sin(\phi)_{UL}$ amplitude

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q} c \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x h_L H_1^\perp + \frac{M_h}{M} g_{1L} \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x f_L^\perp D_1 - \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{H}}{z} \right) \right]$$

A. Airapetian et al, Phys. Lett. B562 (2003)



Positive: Hydrogen results larger than Deuteron (u-quark dominance)

Positive: Hydrogen and Deuteron of same size

Deuteron positive, Hydrogen ≈ 0

Positive and consistent with π^+ (u-quark dominance)

Subleading twist

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2 y^2}{xyQ^2 2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

$$+ \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$\left. \begin{aligned} + S_T & \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \end{aligned} \right\}$$

$$\left. \begin{aligned} + S_T \lambda_l & \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \end{aligned} \right\}$$

$$F_{UT}^{\sin\phi_S} = \frac{2M}{Q} \mathcal{C} \left\{ \left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z} \right) - \frac{k_T \cdot p_T}{2MM_h} \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) - \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right] \right\}$$

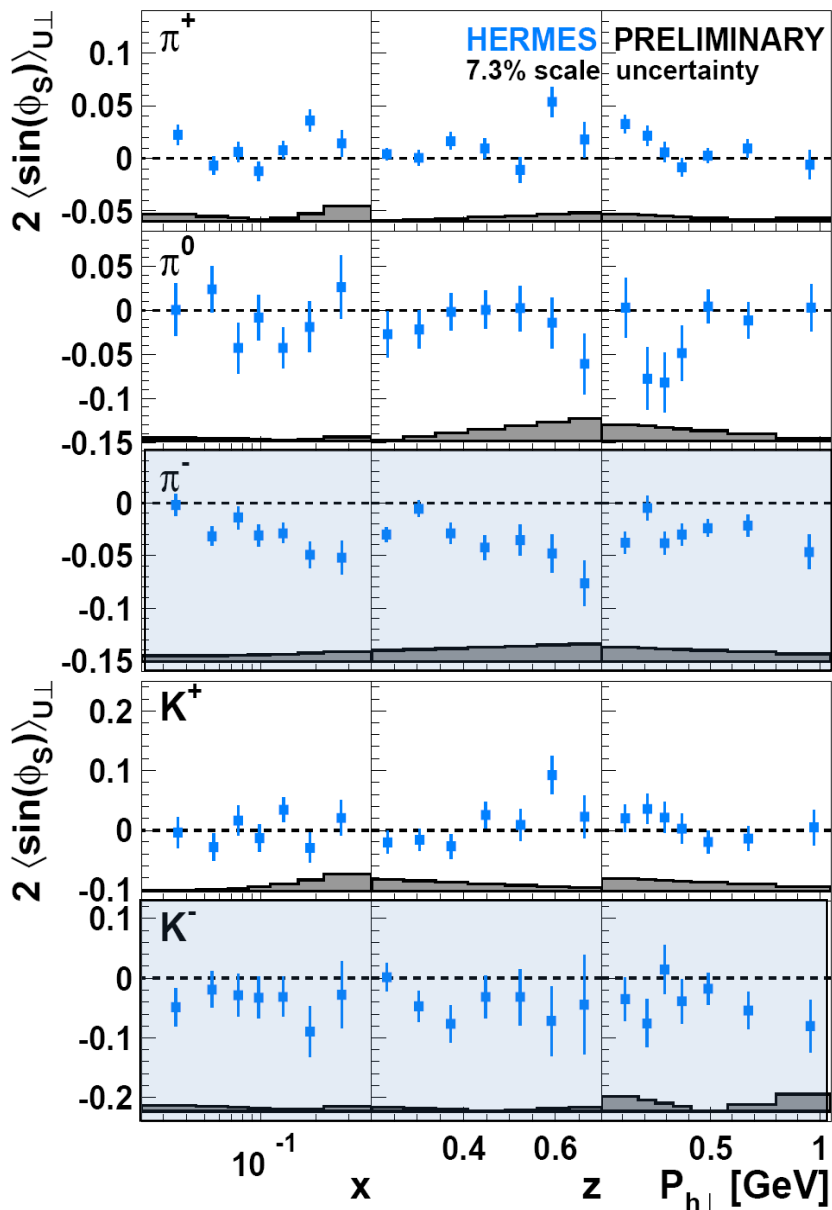
Sensitive to worm-gear g_{1T}^\perp , sivers, transversity + higher-twist DF and FF

Distribution Functions

		quark		
		U	L	T
nucleon	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_{1T}^\perp

Subleading-twist $\sin(\phi_S)$ Fourier component

- sensitive to **worm-gear** g_{1T}^\perp , **Sivers function**, **Transversity**, etc
- **significant non-zero signal for π^- and K^- !**

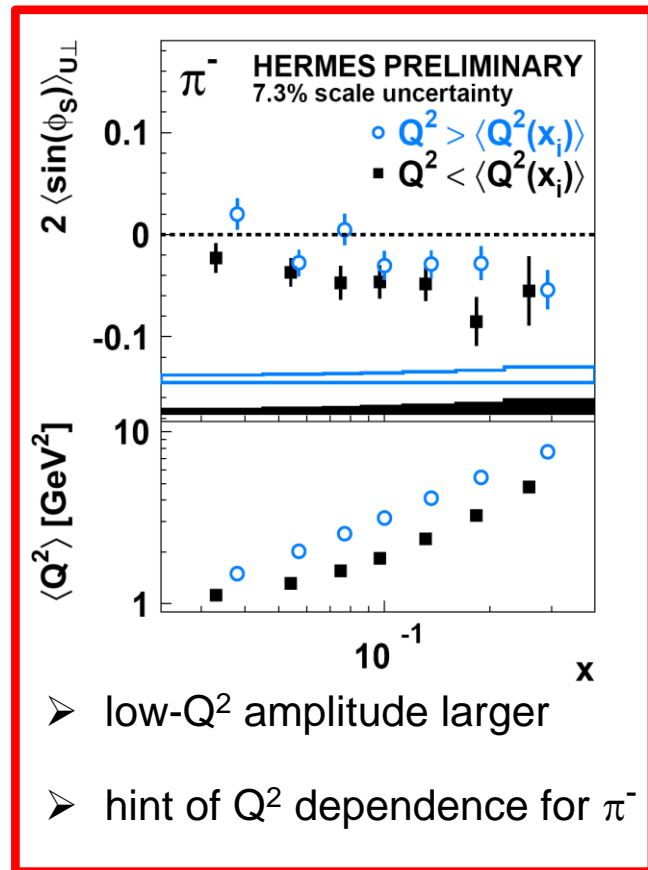
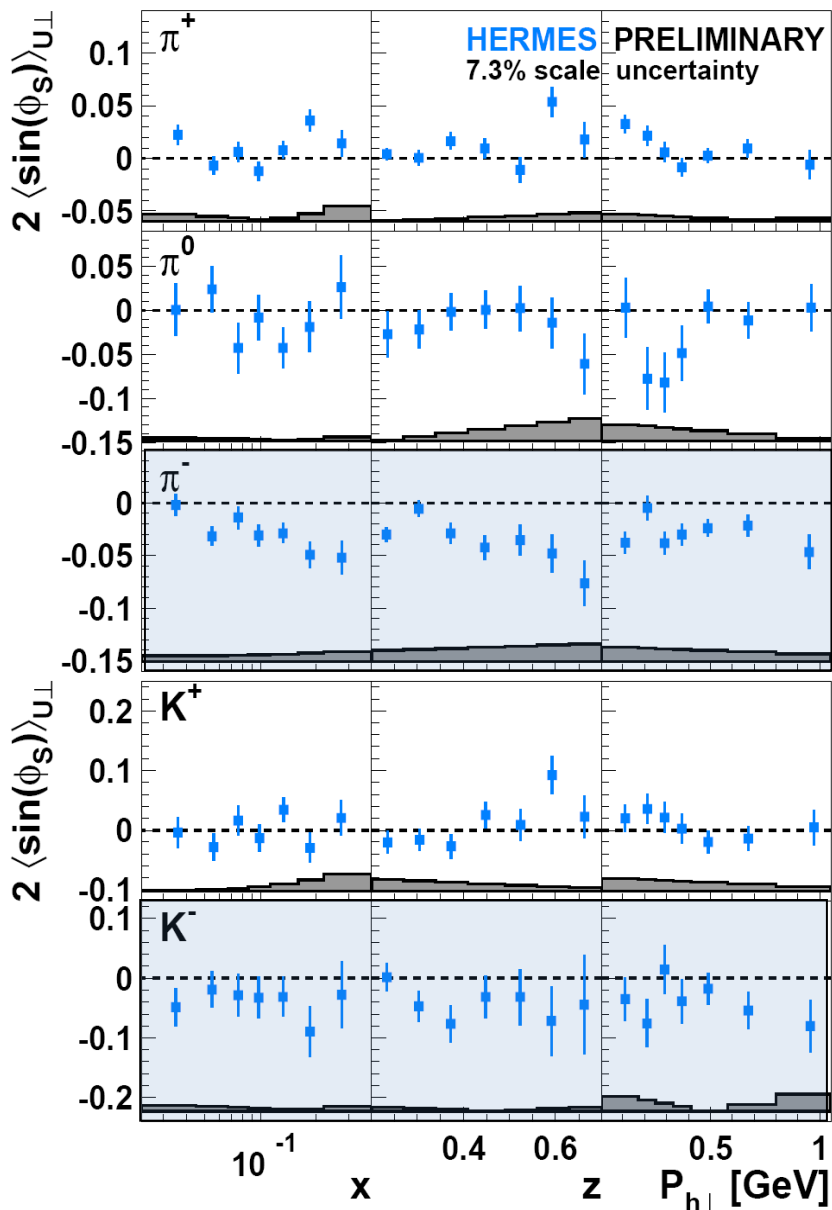


Large and negative

negative

Subleading-twist $\sin(\phi_S)$ Fourier component

- sensitive to **worm-gear** g_{1T}^\perp , **Sivers function**, **Transversity**, etc
- **significant non-zero signal for π^- and K^- !**



2-hadron SIDIS results

Following formalism developed by **Steve Gliske**

Find details in

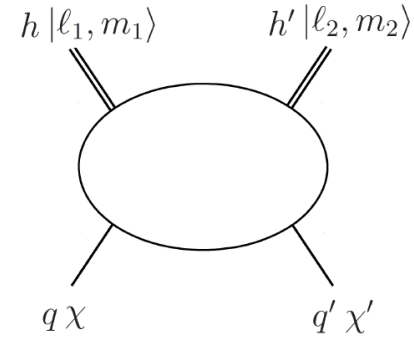
Transverse Target Moments of Dihadron Production in Semi-inclusive Deep Inelastic Scattering at HERMES
S. Gliske, PhD thesis, University of Michigan, 2011

<http://www-personal.umich.edu/~lorenzon/research/HERMES/PHDs/Gliske-PhD.pdf>

A short digression on di-hadron fragmentation functions

Standard definition of di-hadron FF assume no polarization of final state hadrons (pseudo-scalar mesons) or define mixtures of certain partial waves as new FFs

In the **new formalism** there are only two di-hadron FFs. Names and symbols are entirely associated with the quark spin, whereas the partial waves of the produced hadrons ($|l_1 m_1\rangle, |l_2 m_2\rangle$) are associated with partial waves of FFs.



$$\chi = \chi' \quad \longrightarrow \quad D_1 = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos \vartheta) e^{im(\phi_R - \phi_k)} D_1^{|\ell,m\rangle}(z, M_h, |\mathbf{k}_T|)$$

$$\chi \neq \chi' \quad \longrightarrow \quad H_1^\perp = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} P_{\ell,m}(\cos \vartheta) e^{im(\phi_R - \phi_k)} H_1^{\perp|\ell,m\rangle}(z, M_h, |\mathbf{k}_T|)$$

The cross-section is identical to the ones in literature, the only difference is the interpretation of the FFs:

$$\begin{aligned} D_1^{|0,0\rangle} &= D_{1,OO} = \left(\frac{1}{4} D_{1,OO}^s + \frac{3}{4} D_{1,OO}^p \right) & H_1^{\perp|0,0\rangle} &= H_{1,OO}^\perp = \frac{1}{4} H_{1,OO}^{\perp s} + \frac{3}{4} H_{1,OO}^{\perp p}, & H_1^{\perp|2,0\rangle} &= \frac{1}{2} H_{1,LL}^\perp, \\ D_1^{|1,0\rangle} &= D_{1,OL}, & H_1^{\perp|1,1\rangle} &= H_{1,OT}^\perp + \frac{|\mathbf{R}|}{|\mathbf{k}_T|} \bar{H}_{1,OT}^\perp = \frac{|\mathbf{R}|}{|\mathbf{k}_T|} H_{1,OT}^\perp, & H_1^{\perp|2,-1\rangle} &= \frac{1}{2} H_{1,LT}^\perp, \\ D_1^{|1,\pm 1\rangle} &= D_{1,OT} \mp \frac{|\mathbf{k}_T| |\mathbf{R}|}{M_h^2} G_{1,OT}^\perp, & H_1^{\perp|1,0\rangle} &= H_{1,OL}^\perp, & H_1^{\perp|2,-2\rangle} &= H_{1,TT}^\perp, \\ D_1^{|2,0\rangle} &= \frac{1}{2} D_{1,LL}, & H_1^{\perp|1,-1\rangle} &= H_{1,OT}^\perp, \\ D_1^{|2,\pm 1\rangle} &= \frac{1}{2} \left(D_{1,LT} \mp \frac{|\mathbf{k}_T| |\mathbf{R}|}{M_h^2} G_{1,LT}^\perp \right), & H_1^{\perp|2,2\rangle} &= H_{1,TT}^\perp + \frac{|\mathbf{R}|}{|\mathbf{k}_T|} \bar{H}_{1,TT}^\perp = \frac{|\mathbf{R}|}{|\mathbf{k}_T|} H_{1,TT}^\perp, \\ D_1^{|2,\pm 2\rangle} &= D_{1,TT} \mp \frac{1}{2} \frac{|\mathbf{k}_T| |\mathbf{R}|}{M_h^2} G_{1,TT}^\perp, & H_1^{\perp|2,1\rangle} &= \frac{1}{2} H_{1,LT}^\perp + \frac{1}{2} \frac{|\mathbf{R}|}{|\mathbf{k}_T|} \bar{H}_{1,LT}^\perp = \frac{1}{2} \frac{|\mathbf{R}|}{|\mathbf{k}_T|} H_{1,LT}^\perp, \end{aligned}$$

The di-hadron SIDIS cross-section

$$\begin{aligned}
 d\sigma_{UT} = & \frac{\alpha^2 M_h P_{h\perp}}{2\pi xy Q^2} \left(1 + \frac{\gamma^2}{2x}\right) |\mathbf{S}_\perp| \\
 & \times \sum_{\ell=0}^2 \sum_{m=-\ell}^{\ell} \left\{ A(x, y) \left[P_{\ell, m} \sin((m+1)\phi_h - m\phi_R - \phi_S) \right. \right. \\
 & \quad \times \left. \left(F_{UT,T}^{P_{\ell, m} \sin((m+1)\phi_h - m\phi_R - \phi_S)} + \epsilon F_{UT,L}^{P_{\ell, m} \sin((m+1)\phi_h - m\phi_R - \phi_S)} \right) \right] \\
 & + B(x, y) \left[P_{\ell, m} \sin((1-m)\phi_h + m\phi_R + \phi_S) F_{UT}^{P_{\ell, m} \sin((1-m)\phi_h + m\phi_R + \phi_S)} \right. \\
 & \quad \left. + P_{\ell, m} \sin((3-m)\phi_h + m\phi_R - \phi_S) F_{UT}^{P_{\ell, m} \sin((3-m)\phi_h + m\phi_R - \phi_S)} \right] \\
 & + V(x, y) \left[P_{\ell, m} \sin(-m\phi_h + m\phi_R + \phi_S) F_{UT}^{P_{\ell, m} \sin(-m\phi_h + m\phi_R + \phi_S)} \right. \\
 & \quad \left. + P_{\ell, m} \sin((2-m)\phi_h + m\phi_R - \phi_S) F_{UT}^{P_{\ell, m} \sin((2-m)\phi_h + m\phi_R - \phi_S)} \right] \left. \right\}.
 \end{aligned}$$

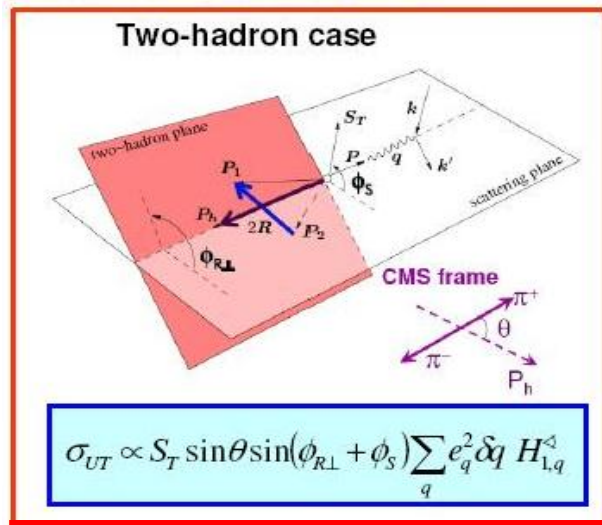
l and m correspond to $|lm\rangle$ angular momentum state of the hadron

Considering all terms ($d\sigma_{UU}, d\sigma_{LU}, d\sigma_{UL}, d\sigma_{LL}, d\sigma_{UT}, d\sigma_{LT}$) there are **144 non-zero structure functions** at twist-3 level. The most known is

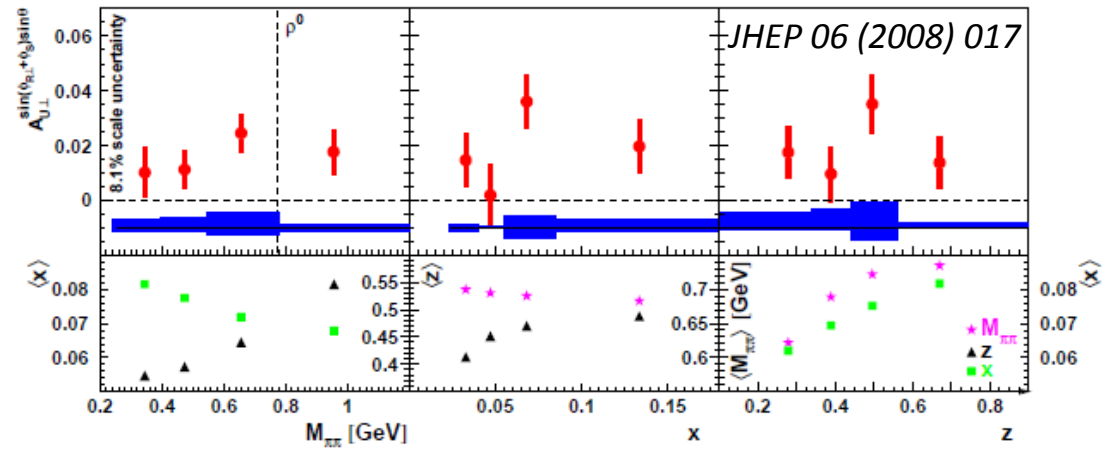
$$F_{UT}^{P_{\ell, m} \sin((1-m)\phi_h + m\phi_R + \phi_S)} = -\mathcal{I} \left[\frac{|\mathbf{k}_T|}{M_h} \cos((m-1)\phi_h - \phi_p - m\phi_k) h_1 H_1^{\perp|l, m} \right]$$

which for $l = 1$ and $m = 1$ reduces to the well known collinear $F_{UT}^{\sin \vartheta \sin(\phi_R + \phi_S)}$ related to transversity

The di-hadron SIDIS cross-section

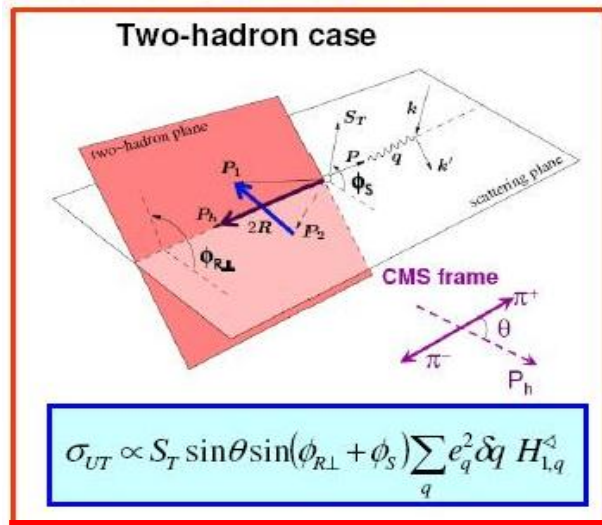


Published $\pi^+\pi^-$ Results

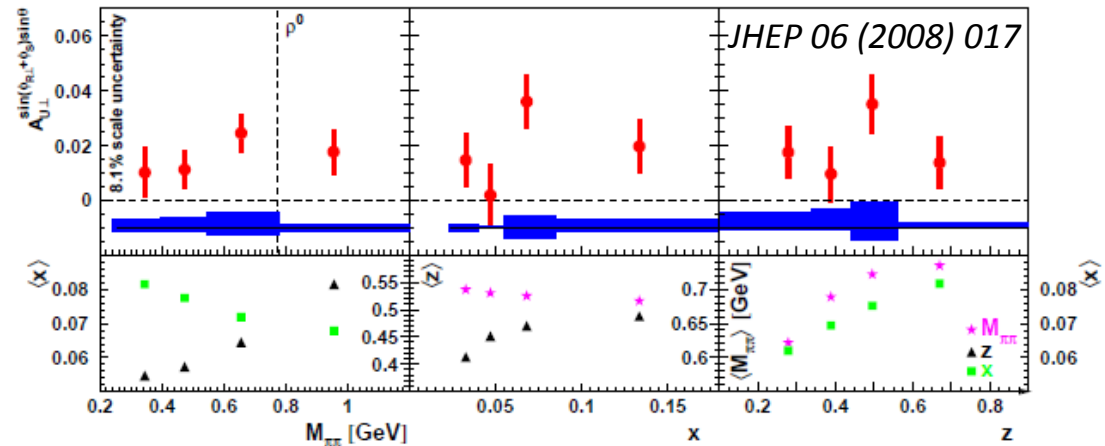


- independent way to access transversity
- Collinear \rightarrow no convolution integral
- significantly positive amplitudes
- 1st evidence of non zero dihadron FF
- limited statistical power (v.r.t. 1 hadron)

The di-hadron SIDIS cross-section

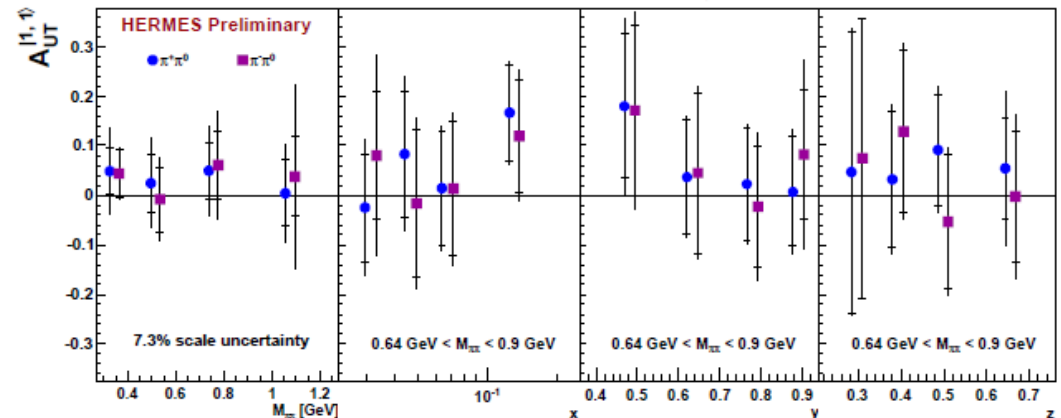


Published $\pi^+\pi^-$ Results



- independent way to access transversity
- Collinear \rightarrow no convolution integral
- significantly positive amplitudes
- 1st evidence of non zero dihadron FF
- limited statistical power (v.r.t. 1 hadron)
- signs are consistent for all $\pi\pi$ species
- statistics much more limited for $\pi^\pm\pi^0$
- despite uncertainties may still help to constrain global fits and may assist in $u - d$ flavor separation

New $\pi^\pm\pi^0$ Results



- New tracking, new PID, use of ϕ_R rather than $\phi_{R\perp}$
- Different fitting procedure and function
- Acceptance correction

Conclusions

A rich phenomenology and surprising effects arise when parton transverse momentum is not integrated out!

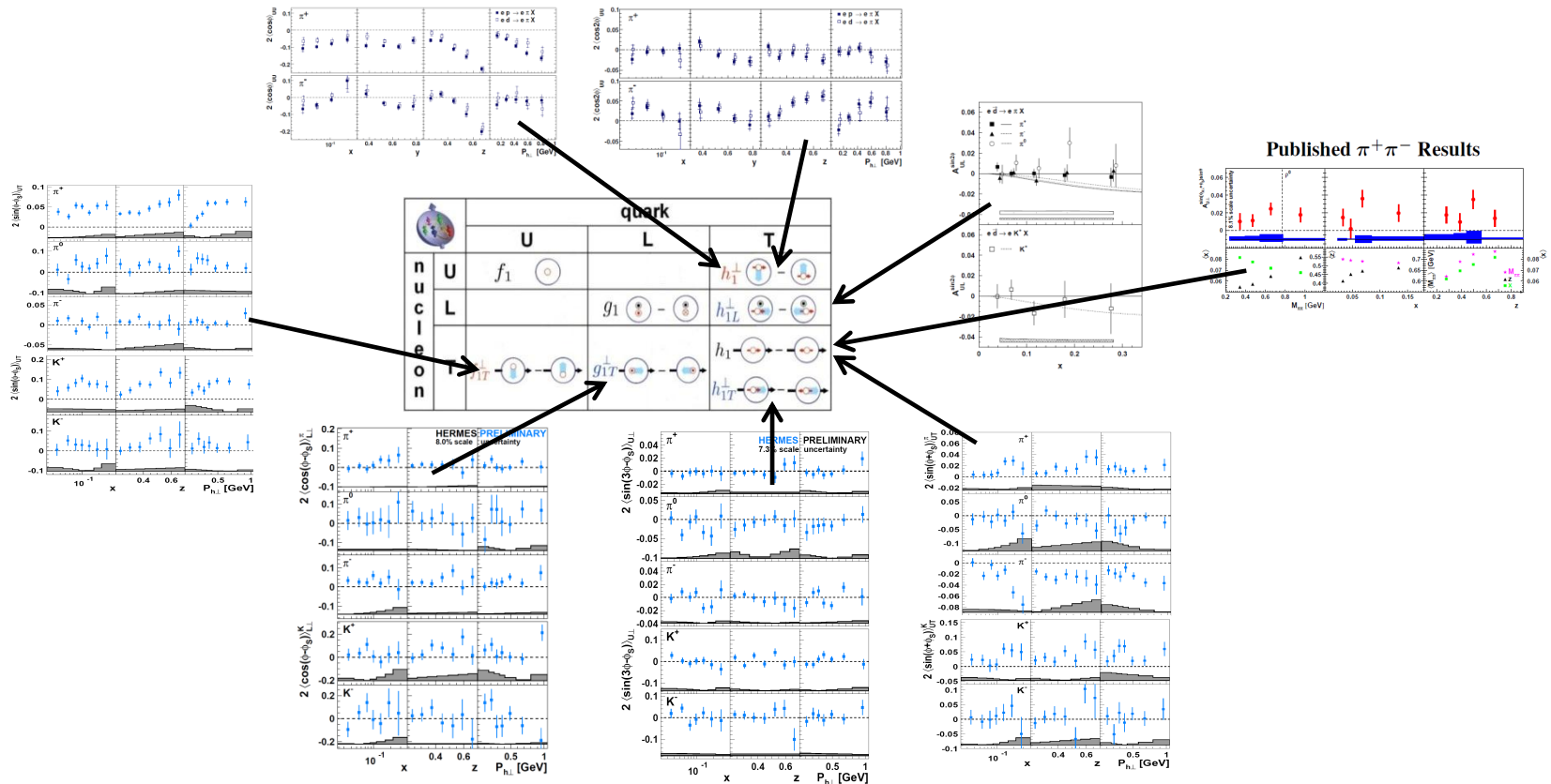
Transverse effects and orbital motion of partons are now established as key ingredients of the nucleon internal dynamics

Conclusions

A rich phenomenology and surprising effects arise when parton transverse momentum is not integrated out!

Transverse effects and orbital motion of partons are now established as key ingredients of the nucleon internal dynamics

The HERMES experiment has played a pioneering role in these studies:



Back-up

Boer-Mulders function

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2 y^2}{xyQ^2 2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right)$$

$$\left\{ \begin{aligned} & F_{UU,T} + \epsilon F_{UU,L} \\ & + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \end{aligned} \right\}$$

$$+ \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$+ S_T \left[\begin{aligned} & \sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \end{aligned} \right]$$

$$+ S_T \lambda_l \left[\begin{aligned} & \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \end{aligned} \right] \left. \right\}$$

$$F_{UU}^{\cos 2\phi_h} = C \left[-\frac{2(\hat{h} \cdot \mathbf{k}_T)(\hat{h} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_1^\perp H_1^\perp \right]$$

Describes correlation between quark transverse momentum and transverse spin in unpolarized nucleon

Distribution Functions

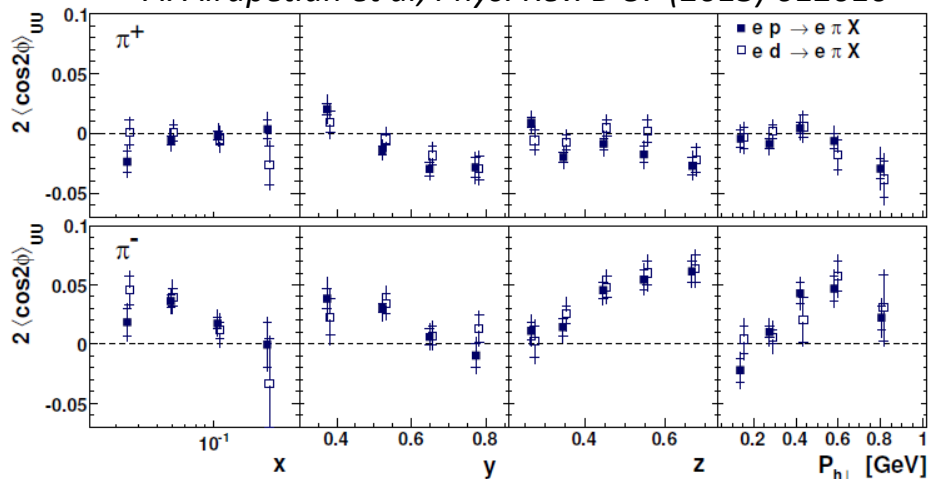
		quark		
		U	L	T
nucleon	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_{1T}^\perp

Fragmentation Functions

		quark		
		U	L	T
h	U	D_1		H_1^\perp

The $\cos 2\phi$ amplitudes $\propto h_1^\perp(x, p_T^2) \otimes H_1^\perp(z, k_T^2)$

A. Airapetian et al, Phys. Rev. D 87 (2013) 012010



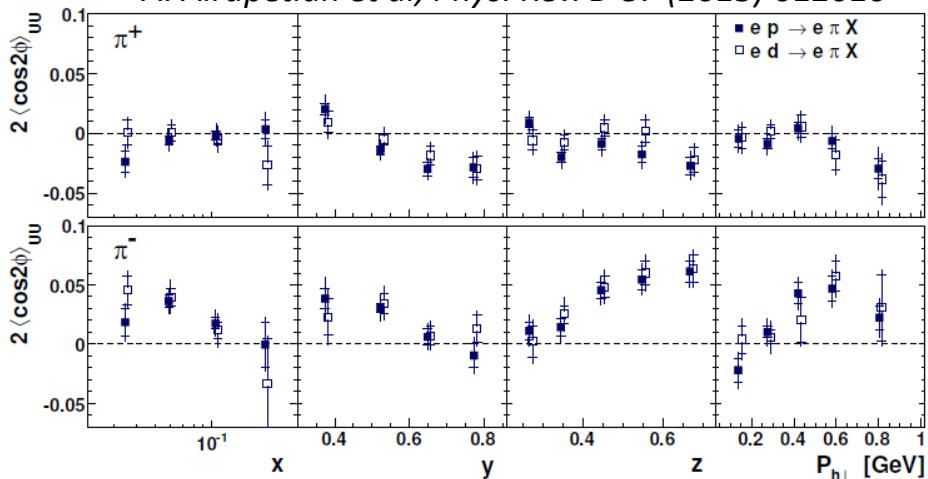
negative

positive

- Amplitudes are significant
→ clear evidence of BM effect
- similar results for H & D indicate $h_1^{\perp,u} \approx h_1^{\perp,d}$
- Opposite sign for π^+/π^- consistent with opposite signs of fav/unfav Collins

The $\cos 2\phi$ amplitudes $\propto h_1^\perp(x, p_T^2) \otimes H_1^\perp(z, k_T^2)$

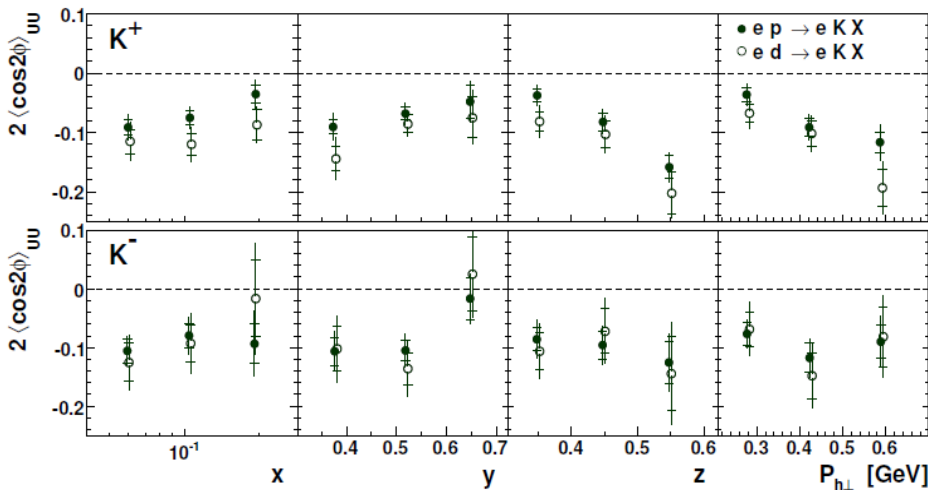
A. Airapetian et al, Phys. Rev. D 87 (2013) 012010



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Large and negative

Large and negative

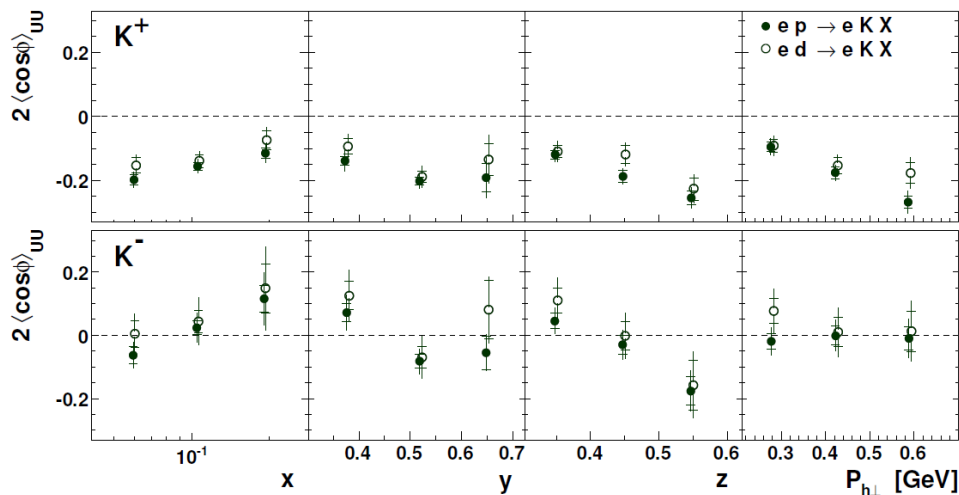
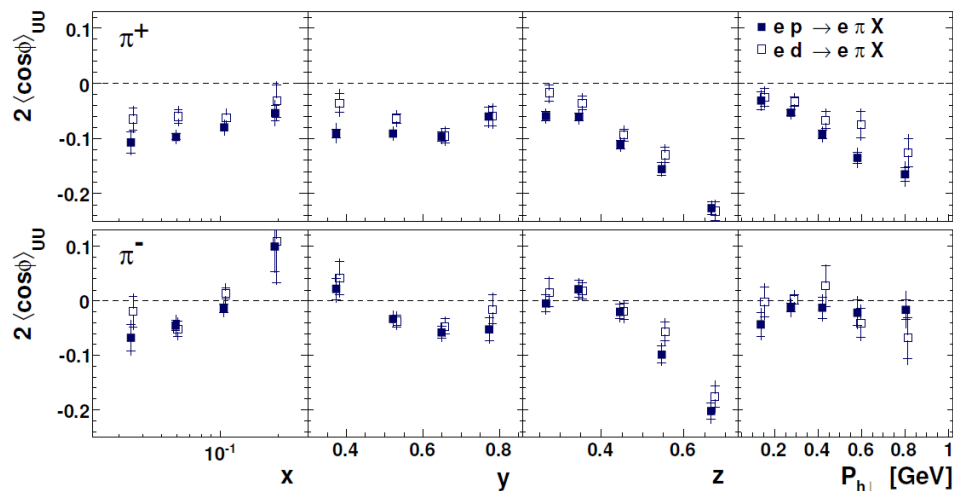
- K^+/K^- amplitudes are larger than for pions, have different kinematic dependencies than pions and have same sign
- different role of Collins FF for pions and kaons?
- Significant contribution from scattering off strange quarks?

Analysis multi-dimensional in $x, y, z,$ and P_t

Create your own projections of results through: <http://www-hermes.desy.de/cosnphi/>

The $\cos\phi$ amplitudes

A. Airapetian et al, Phys. Rev. D 87 (2013) 012010



Analysis multi-dimensional in x , y , z , and P_t

Create your own projections of results through: <http://www-hermes.desy.de/cosnphi/>

Transversity

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

$$+ \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$\left. \begin{aligned} + S_T & \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \right. \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \end{aligned} \right\}$$

$$\left. \begin{aligned} + S_T \lambda_l & \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} \right. \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \end{aligned} \right\}$$

$$F_{UT}^{\sin(\phi_h + \phi_S)} = C \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} h_1 H_1^\perp \right]$$

Describes probability to find transversely polarized quarks in a transversely polarized nucleon

Distribution Functions

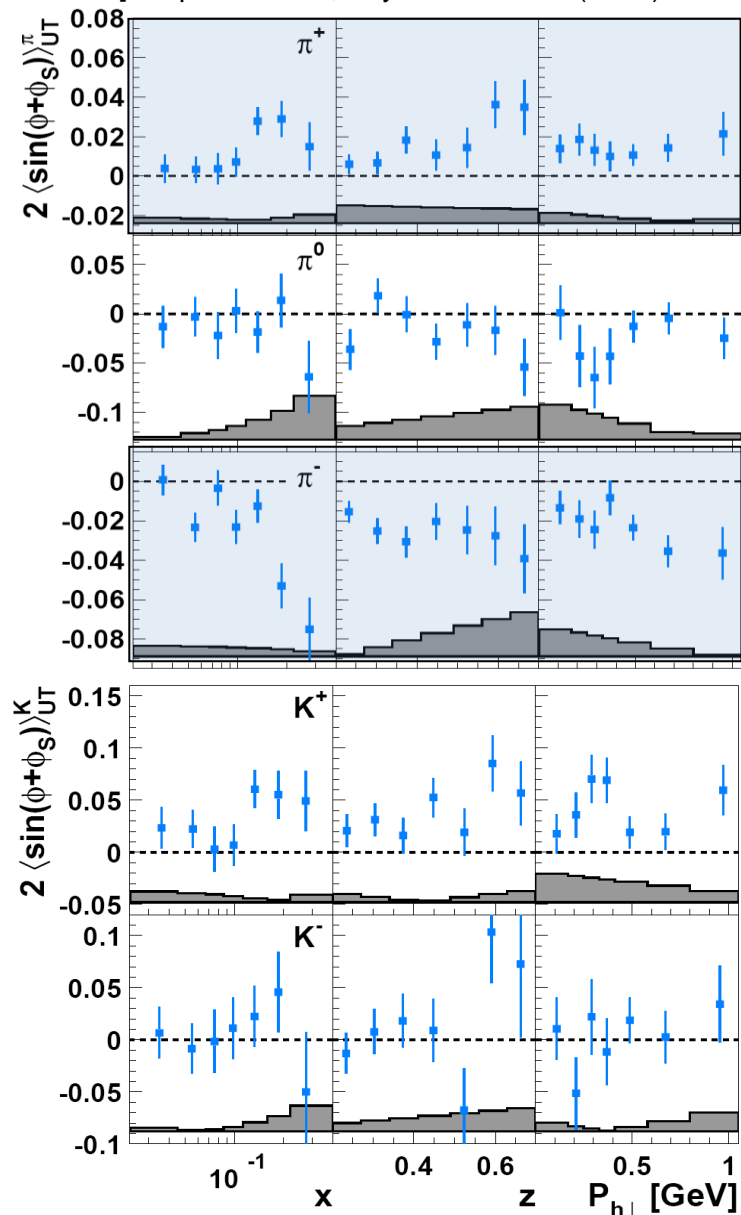
		quark		
		U	L	T
nucleon	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_1 h_{1T}^\perp

Fragmentation Functions

		quark		
		U	L	T
h	U	D_1		H_1^\perp

Collins amplitudes $\propto h_1(x, p_T^2) \otimes H_1^\perp(z, k_T^2)$

[Airapetian et al., Phys. Lett. B 693 (2010) 11-16]



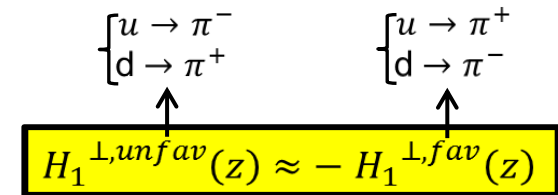
positive

consistent with zero
(isospin-symmetry)

large and negative!

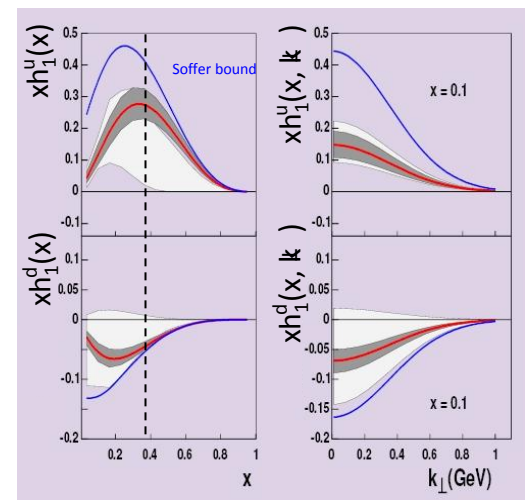
significantly positive

consistent with zero



Consistent with Belle/BaBar measurements in e^+e^-

$$e^+e^- \rightarrow \pi_{jet1}^+ \pi_{jet2}^- X$$



Anselmino et al. Phys. Rev. D 75 (2007)



$F_{LU}^{\sin \phi}$

$$F_{LU}^{\sin \phi_h} = \frac{2M}{Q} c \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right)$$

$$\left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \end{aligned} \right.$$

$$+ \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right]$$

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right]$$

$$+ S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right]$$

$$+ S_T \left[\begin{aligned} & \sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \\ & + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \\ & + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \end{aligned} \right]$$

$$+ S_T \lambda_l \left[\begin{aligned} & \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \\ & + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \end{aligned} \right] \left. \right\}$$

Sensitive to f_1 , Boer-Mulders + higher-twist DF and FF

Distribution Functions

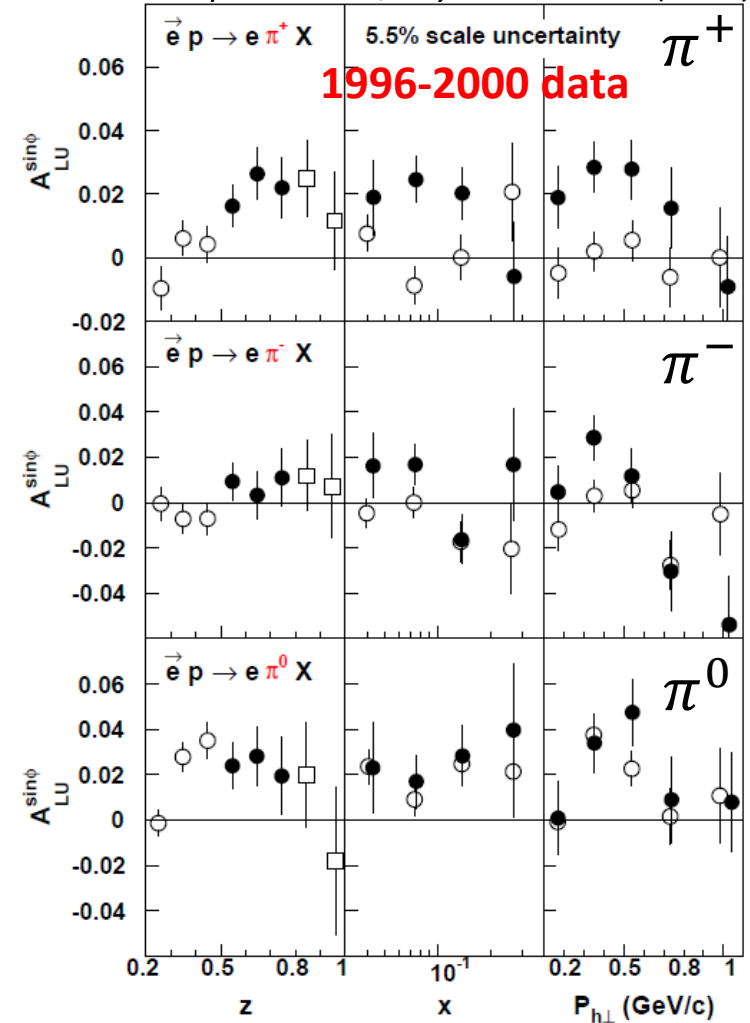
		quark		
		U	L	T
n u c l e o n	U	f_1		h_1^\perp
	L		g_1	h_{1L}^\perp
	T	f_{1T}^\perp	g_{1T}^\perp	h_{1T}^\perp

$F_{LU} \sin \phi$

$$F_{LU}^{\sin \phi_h} = \frac{2M}{Q} c \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

$$\frac{d\sigma^h}{dx dy d\phi_S dz d\phi d\mathbf{P}_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \left\{ \begin{aligned} & \left[F_{UU,T} + \epsilon F_{UU,L} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \cos(\phi) F_{UU}^{\cos(\phi)} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} \right] \\ & + \lambda_l \left[\sqrt{2\epsilon(1-\epsilon)} \sin(\phi) F_{LU}^{\sin(\phi)} \right] \\ & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin(\phi) F_{UL}^{\sin(\phi)} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] \\ & + S_L \lambda_l \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi) F_{LL}^{\cos(\phi)} \right] \\ & + S_T \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) \right. \\ & \left. + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(\phi_S) F_{UT}^{\sin(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \\ & + S_T \lambda_l \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(\phi_S) F_{LT}^{\cos(\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \end{aligned} \right\}$$

A. Airapetian et al, Phys. Lett. B 648 (2007)

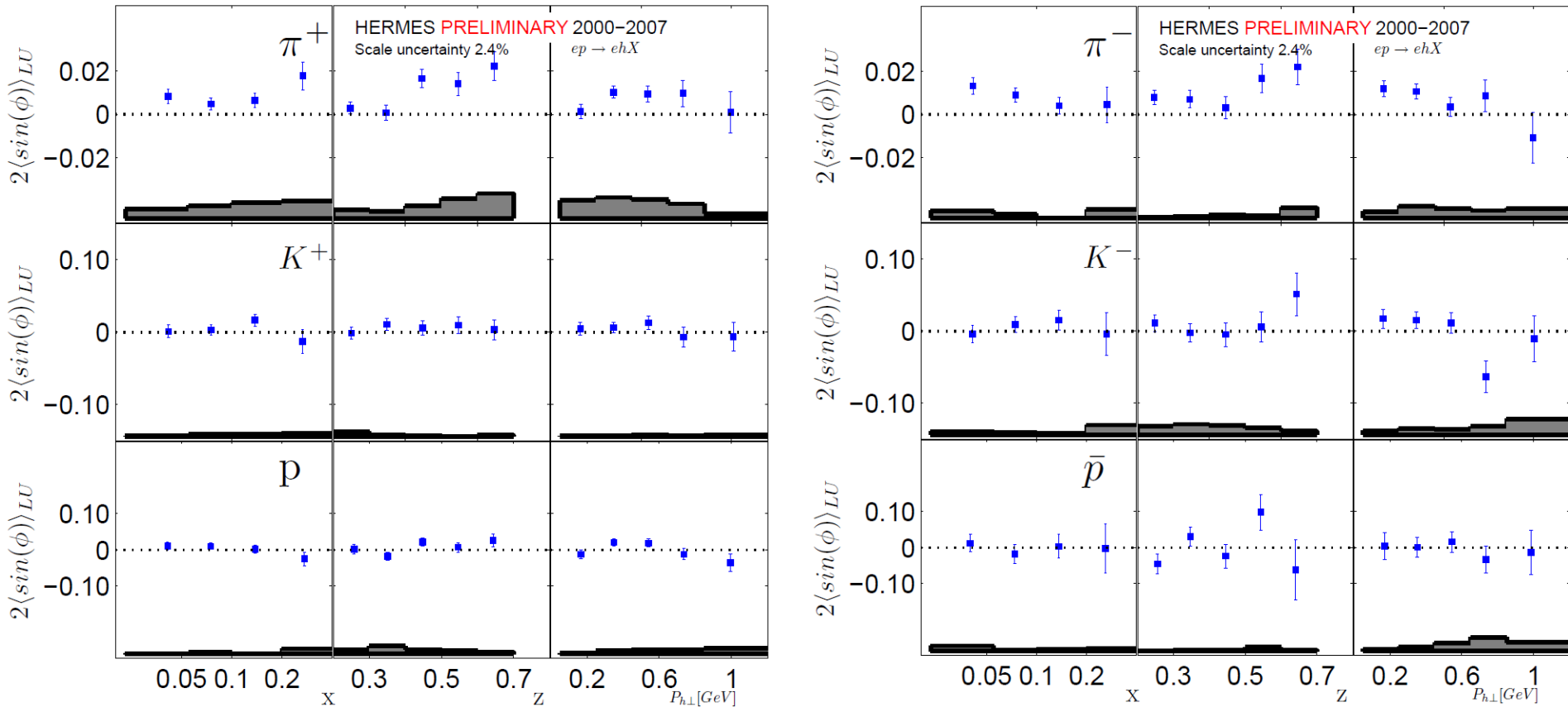


open circles $0.2 < z < 0.5$
 full circles $0.5 < z < 0.8$
 open squares: $0.8 < z < 1.0$

$F_{LU}^{\sin \phi}$

$$F_{LU}^{\sin \phi} = \frac{2M}{Q} c \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

H target, 2000-2007 data $0.2 < z < 0.7$

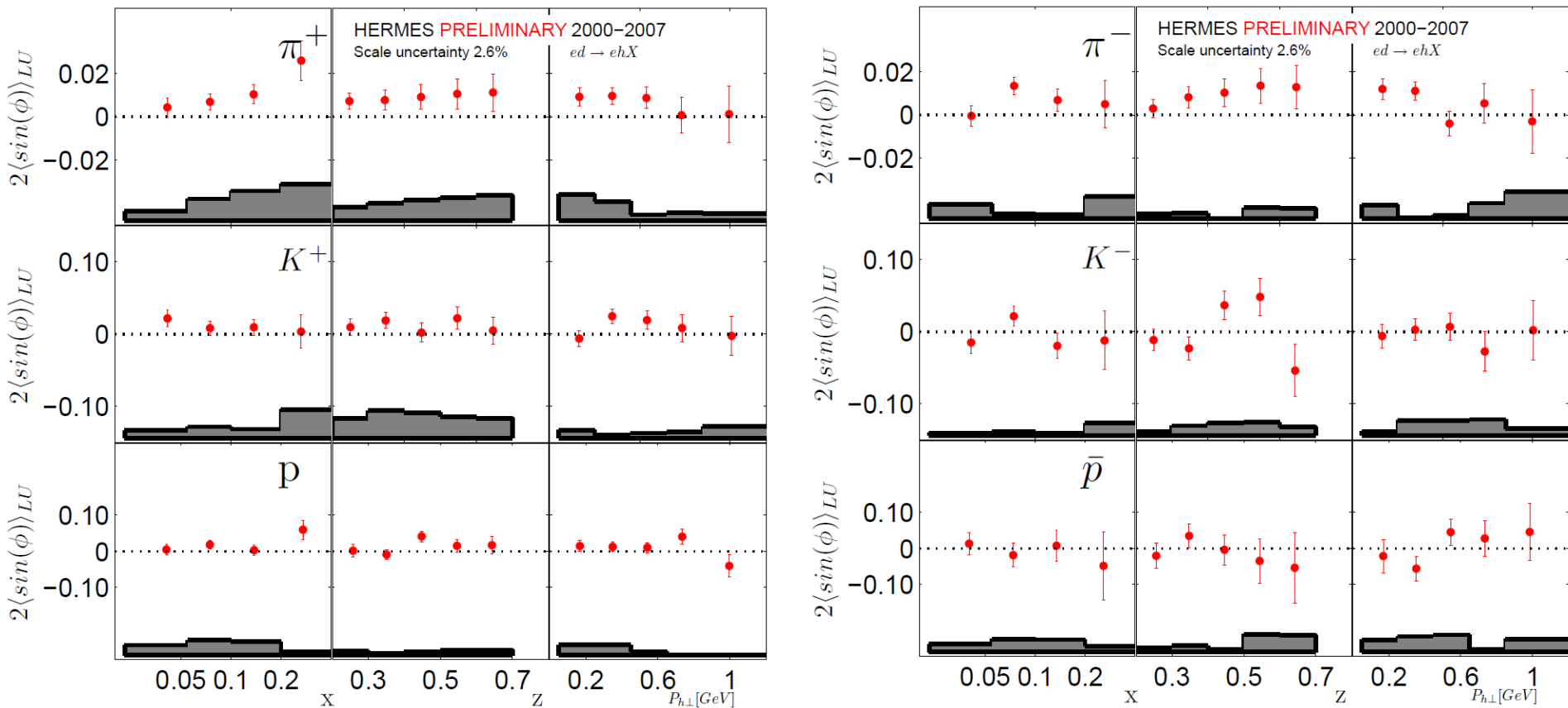


Released yesterday!!

$F_{LU} \sin \phi$

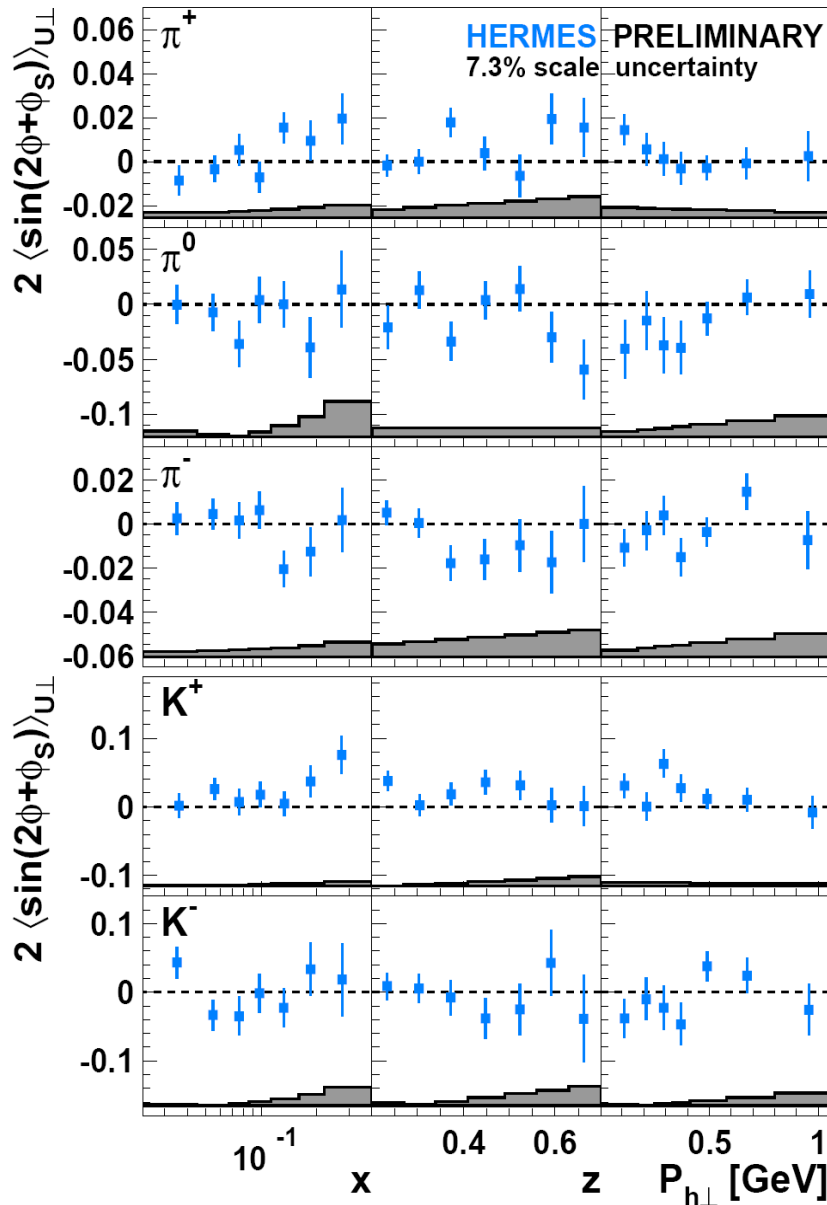
$$F_{LU}^{\sin \phi_h} = \frac{2M}{Q} c \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

D target, 2000-2007 data $0.2 < z < 0.7$

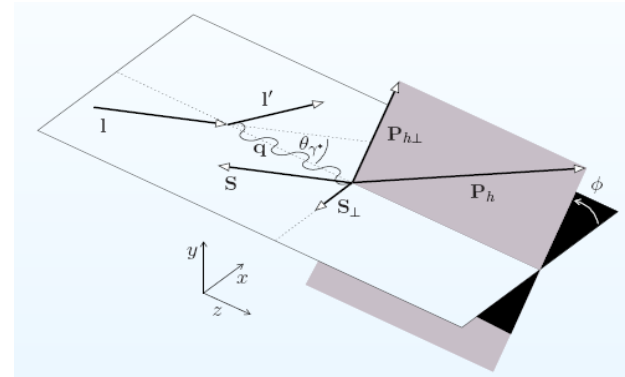


Released yesterday!!

The $\sin(2\phi + \phi_S)$ Fourier component



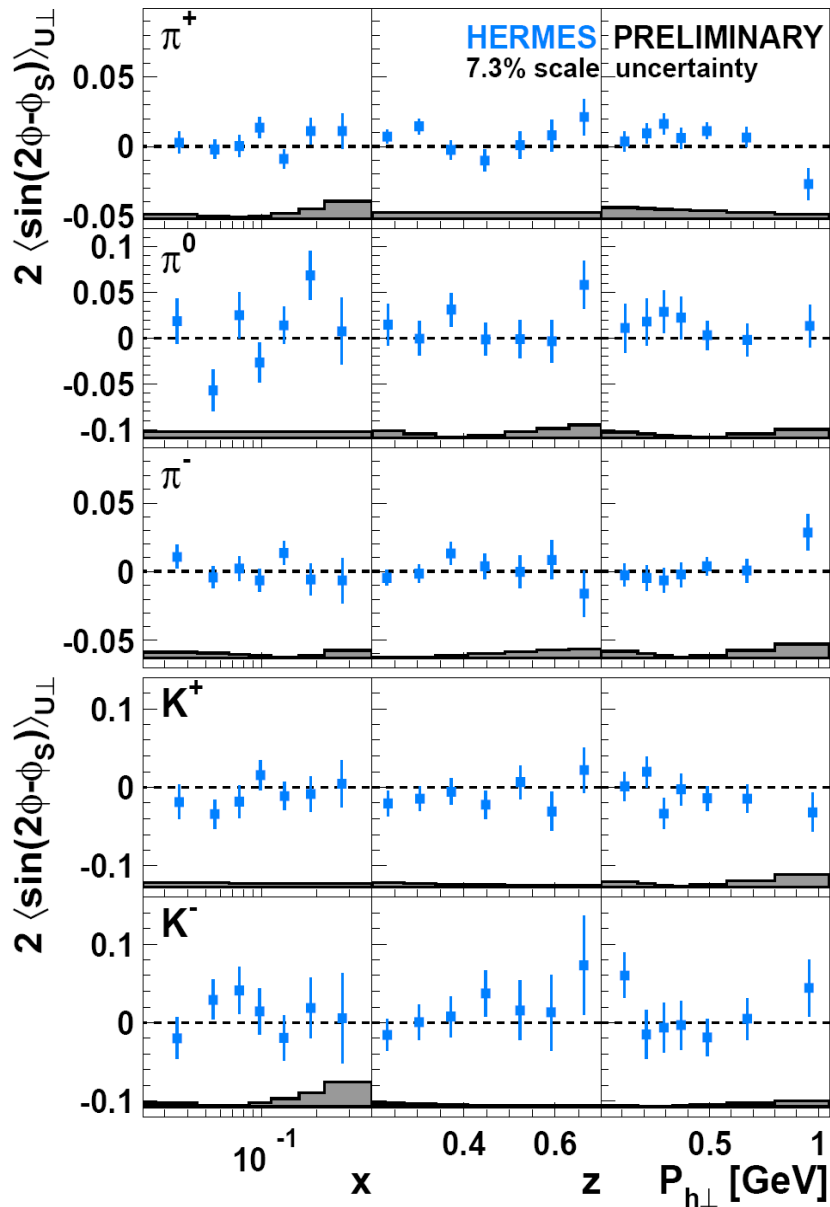
- arises solely from longitudinal (w.r.t. virtual photon direction) component of the target spin



- related to $\langle \sin(2\phi) \rangle_{UL}$ Fourier comp:

$$2\langle \sin(2\phi + \phi_S) \rangle_{UT}^h \propto \frac{1}{2} \sin(\mathcal{G}_{l\gamma^*}) 2\langle \sin(2\phi) \rangle_{UL}^h$$
- sensitive to **worm-gear** h_{1L}^\perp
- suppressed by one power of $P_{h\perp}$ w.r.t. Collins and Sivers amplitudes
- **no significant signal observed (except maybe for K⁺)**

The subleading-twist $\sin(2\phi-\phi_S)$ Fourier component



- sensitive to **worm-gear** g_{1T}^\perp , **Pretzelosity** and **Sivers function**:

$$\propto W_1(p_T, k_T, P_{h\perp}) \left(x f_T^\perp D_1 - \frac{M_h}{M} h_{1T}^\perp \frac{\tilde{H}}{z} \right) - W_2(p_T, k_T, P_{h\perp}) \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) + \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right]$$

- suppressed by one power of $P_{h\perp}$ w.r.t. Collins and Sivers amplitudes

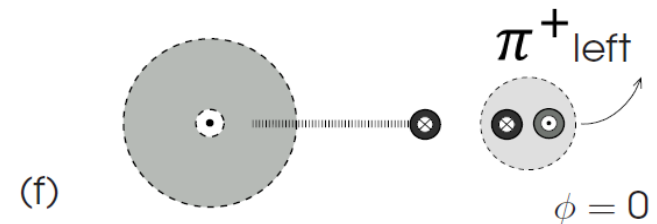
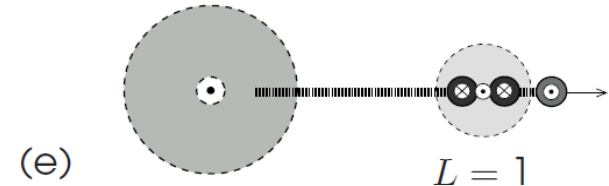
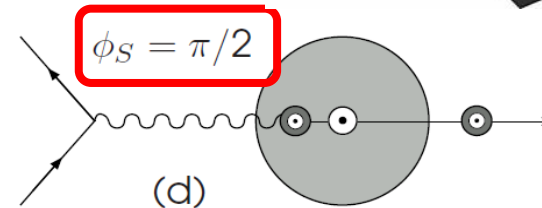
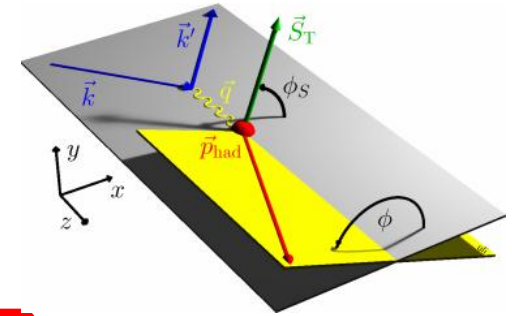
- **no significant non-zero signal observed**

A short digression on the Lund/Artru string fragmentation model

(a phenomenological explanation of the Collins effect)

In the cross-section the Collins FF is always paired with a distrib. function involving a transv. pol. quark.

1. Assume u quark and proton have (transverse) spin aligned in the direction $\phi_S = \pi/2$. The model assumes that the struck quark is initially connected with the remnant via a gluon-flux tube (string)
2. When the string breaks, a $q\bar{q}$ pair is created with vacuum quantum numbers $J^P = 0^+$. The positive parity requires that the spins of q and \bar{q} are aligned, thus an OAM $L = 1$ has to compensate the spins
3. This OAM generates a transverse momentum of the produced pseudo-scalar meson (e.g. π^+) and deflects the meson to the **left side** w.r.t. the struck quark direction, generating left-right azimuthal asymmetries



A short digression on the Lund/Artru string fragmentation model

Relative to the proton transv. spin, the fragmenting quark can have spin parallel or antiparallel to $\left| \frac{1}{2}, \pm \frac{1}{2} \right\rangle$

Then combining the spins of the formed di-quark systems one can get:

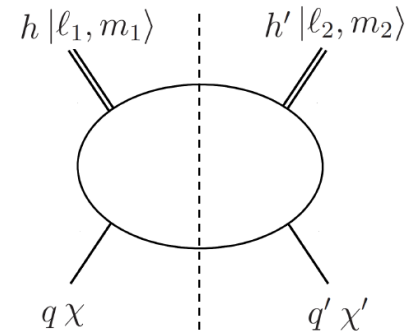
$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0 \Rightarrow \begin{cases} 1 \text{ spin 0 state } |0, 0\rangle & 1 \text{ pseudo-scalar meson (PSM)} \\ 3 \text{ spin 1 states } \begin{cases} |1, 0\rangle & 1 \text{ Longitudinal VM} \\ |1, \pm 1\rangle & 2 \text{ transvrse VM} \end{cases} \end{cases}$$

Lund/Artru prediction at the amplitude level: the asymmetry for PSM has opposite sign to that for transversely polarized VM (left vs. right side), and the amplitude for $|1, 0\rangle$ is 0

Lund/Artru model makes predictions for the individual di-hadrons, but the Collins function includes pairs of di-hadrons

→ to make predictions for the Collins function one needs to consider the cross-section level, i.e. the sum of contributing amplitudes times their complex conjugate

Using the Clebsch-Gordan algebra one obtains: $|1, \pm 1\rangle |1, \pm 1\rangle \equiv |2, \pm 2\rangle$

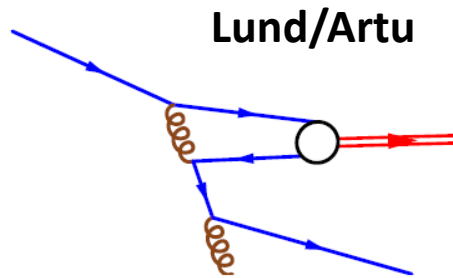


Lund/Artru prediction at the cross-section level: the $|2, \pm 2\rangle$ partial waves of the Collins func. for SIDIS VM production have the opposite sign as the respective PS Collins func.

“gluon radiation model” vs. Lund/Artru model

The Lund/Artru model only accounts for favored Collins fragmentation. An extension of the model (the **gluon radiation model**), elaborated by **S. Gliske** accounts for the disfavored case

1. Struck quark emits a gluon in such a way that most of its momentum is transferred to the gluon
2. The struck quark then becomes part of the remnant
3. The radiated gluon produces a $q\bar{q}$ pair that eventually converts into a meson
4. For PSM the di-quark must interact further with the remnant to get the PSM quantum numbers. In case of VM the di-quark directly forms the meson

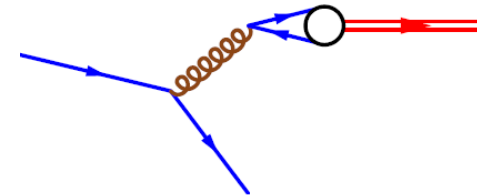


Lund/Artru

- Di-quark has q.n. of vacuum
- **Struck quark** joins the anti-quark in the final state → **favored fragment.**

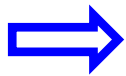
Prediction: the $|2, \pm 2\rangle$ partial wave of the Collins funct. for SIDIS VM production have the opposite sign as the respective PS Collins function

Gluon radiation



- Di-quark has q.n. of observed final state
- **Produced quark** joins the anti-quark in the final state → **disfavored fragment.**

Prediction: the disfavored $|2, \pm 2\rangle$ Collins frag. also is expected to have opposite sign as the respective PS Collins function.



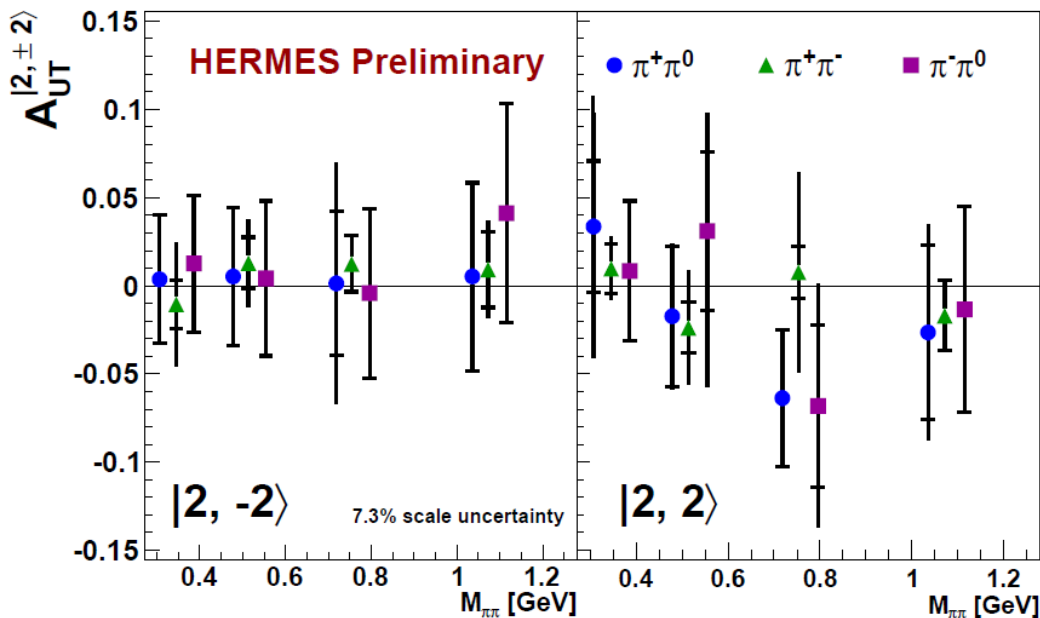
Models predict: fav = disfav for VM
Data say: fav \cong - disfav for PSM (Collins π^+ vs. π^-)

...and now let's look at the results

	Fragment. process	Fav/disfav	Deflection	Sign of amplitude	
u dominance	$u \rightarrow \pi^+$	fav PSM	left ($\phi_h \rightarrow 0$)	> 0 (Collins π^+)	from data
	$u \rightarrow \pi^-$	disfav PSM	right ($\phi_h \rightarrow \pi$)	< 0 (Collins π^-)	
	$u \rightarrow \rho^+ \rightarrow \pi^+\pi^0$	fav VM	right ($\phi_h \rightarrow \pi$)	< 0	from models
	$u \rightarrow \rho^- \rightarrow \pi^-\pi^0$	disfav VM	right ($\phi_h \rightarrow \pi$)	< 0	
	$u \rightarrow \rho^0 \rightarrow \pi^+\pi^-$	mixed VM	right ($\phi_h \rightarrow \pi$)	0 or < 0	

...and now let's look at the results

	Fragment. process	Fav/disfav	Deflection	Sign of amplitude	
}	$u \rightarrow \pi^+$	fav PSM	left ($\phi_h \rightarrow 0$)	> 0 (Collins π^+)	} from data
	$u \rightarrow \pi^-$	disfav PSM	right ($\phi_h \rightarrow \pi$)	< 0 (Collins π^-)	
	$u \rightarrow \rho^+ \rightarrow \pi^+\pi^0$	fav VM	right ($\phi_h \rightarrow \pi$)	< 0	} from models
	$u \rightarrow \rho^- \rightarrow \pi^-\pi^0$	disfav VM	right ($\phi_h \rightarrow \pi$)	< 0	
	$u \rightarrow \rho^0 \rightarrow \pi^+\pi^-$	mixed VM	right ($\phi_h \rightarrow \pi$)	0 or < 0	

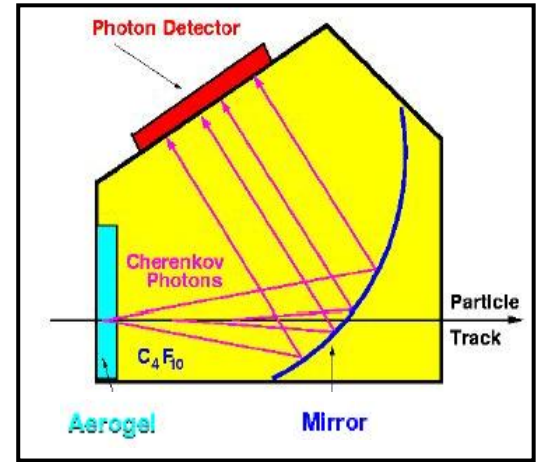
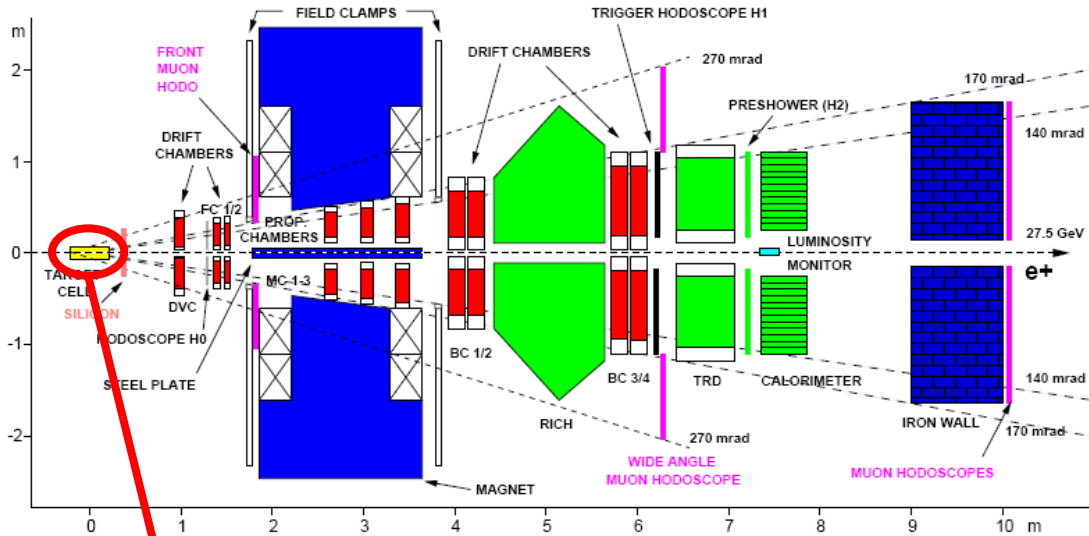


$[2, -2]$ consistent with zero for all flavors
 Not in contradiction with models: if the transversity function causes the fragmenting quark to have positive polarization than Collins $[2, -2]$ must be zero as this partial wave requires fragmenting quark with negative polarization

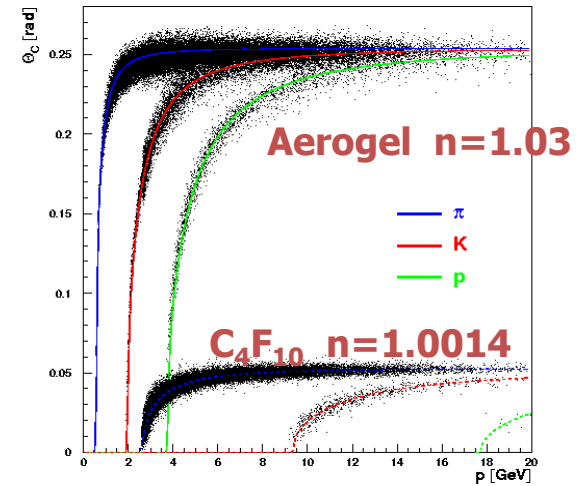
$[2, +2]$ consistent with model expect:

- No signal outside ρ -mass bin
 \rightarrow no non-resonant pion-pairs in p-wave
- Negative for ρ^\pm (model predictions)
- very small for ρ^0 (consistent with small Collins π^0)

The HERMES experiment at HERA

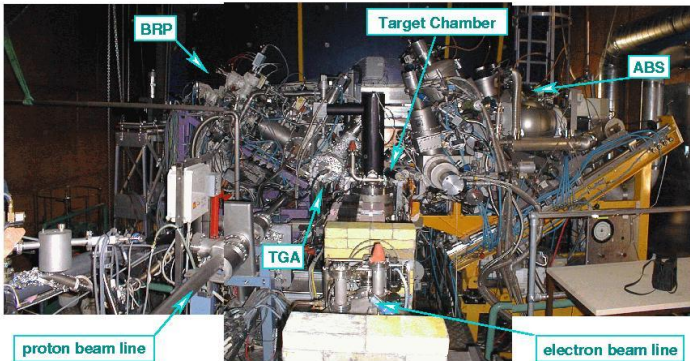
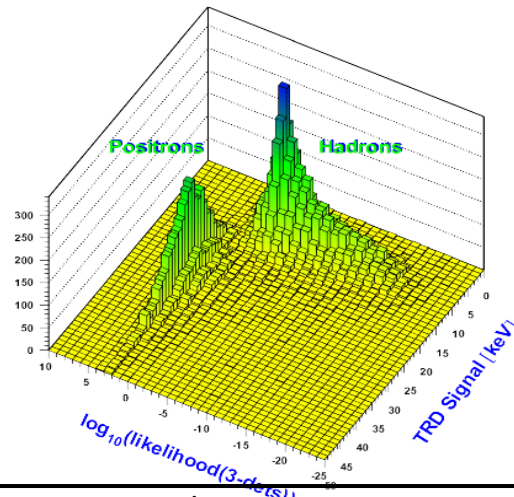


hadron separation

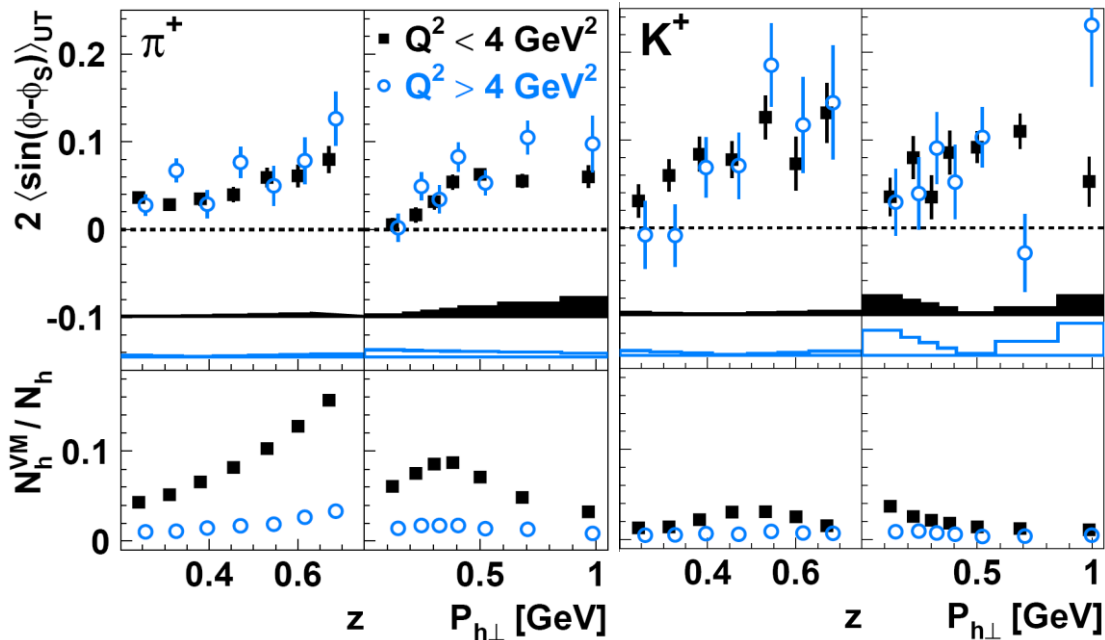


$\pi \sim 98\%$, $K \sim 88\%$, $P \sim 85\%$

TRD, Calorimeter,
preshower, RICH:
lepton-hadron > 98%



Siver amplitudes: additional studies

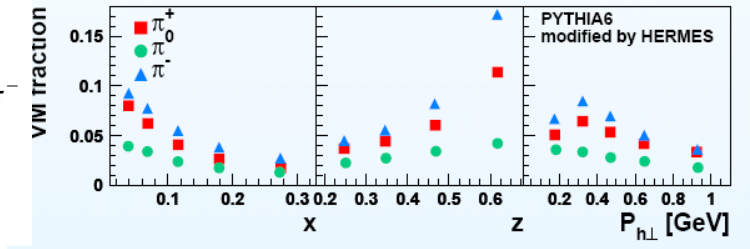
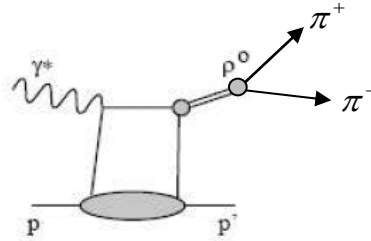


No systematic shifts observed between high and low Q^2 amplitudes for both π^+ and K^+

No indication of important contributions from exclusive VM

The pion-difference asymmetry

Contribution by decay of exclusively produced vector mesons (ρ^0, ω, ϕ) is not negligible (6-7% for pions and 2-3% for kaons), though substantially limited by the requirement $z < 0.7$.

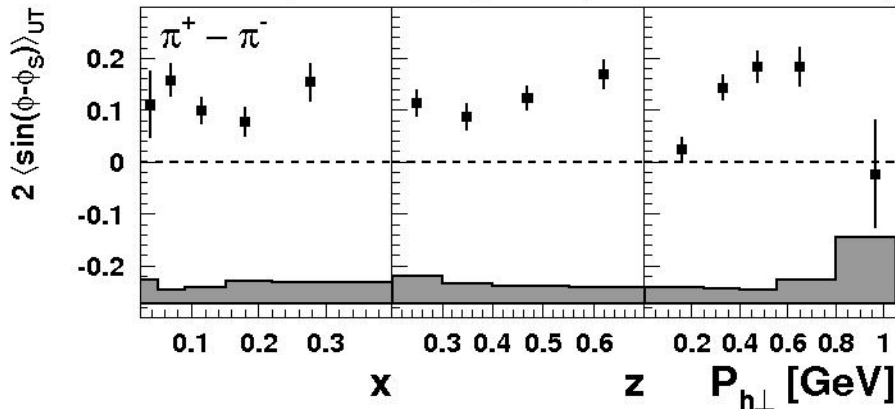


a new observable

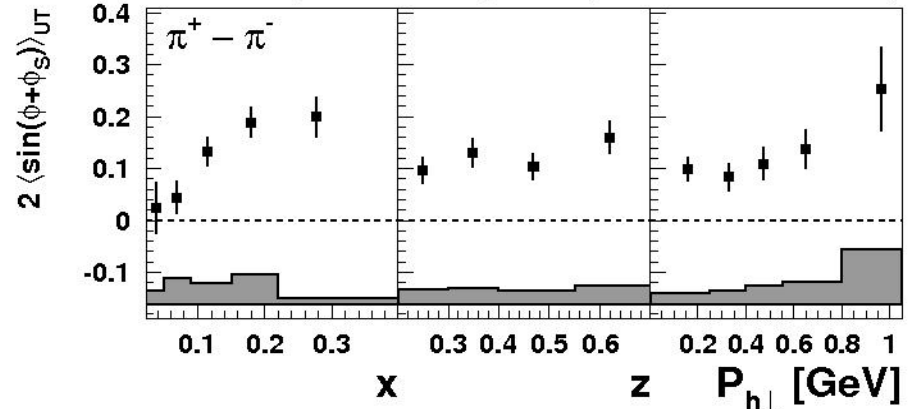
$$A_{UT}^{\pi^+ - \pi^-}(\phi, \phi_S) \equiv \frac{1}{P_T} \frac{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-}) - (\sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-})}{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^+}) + (\sigma_{U\uparrow}^{\pi^-} - \sigma_{U\downarrow}^{\pi^-})}$$

Contribution from exclusive ρ^0 largely cancels out!

HERMES PRELIMINARY 2002-2005
lepton beam amplitudes, 8.1% scale uncertainty



HERMES PRELIMINARY 2002-2005
lepton beam amplitudes, 8.1% scale uncertainty



- significantly positive Sivers and Collins amplitudes are obtained
- measured amplitudes are not generated by exclusive VM contribution