# Helicity and Invariant Amplitudes for Exclusive Vector-Meson Electroproduction

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# Physics Motivation

- Process $\gamma^*(q,\ \lambda_\gamma) + N(p_1,\ \lambda_1) \to V(v,\ \lambda_V) + N(p_2,\ \lambda_2)$ at high $Q^2$ is a perfect reaction to study both vector-meson ($V = \rho^0,\ \phi,\ \ \omega$ etc.) production mechanism and hadron structure (Generalized Parton Distributions).
- Consideration based on Spin Density Matrix Elements (SDMEs) formalism ignores the interference between amplitudes of vector-meson production (VMP) and its decay with background process.
  Direct extraction of VMP amplitudes permits to take into account the interference.
- Application of invariant amplitudes permits to calculate amplitudes of process under consideration in any Lorentz system.

# Basic Formulas for Vector-Meson Production on Spinless Target

- "Materials": 16 Dirac matrices, invariant tensors $g_{\mu\tau}$, $\epsilon_{\mu\tau\alpha\beta}$, and kinematic vectors: $q,\ v,\ p_1,\ p_2,\ q+p_1=v+p_2$
  Pseudo-vector d: $d_\mu=\epsilon_{\mu\nu\lambda\beta}q^\nu v^\lambda p^\beta, \quad p=(p_1+p_2)/2.$
  Virtual-photon polarization vector $e^\tau(\lambda_\gamma)$ orthogonal to its momentum $q^\mu$ ($q_\tau e^\tau(\lambda_\gamma)=0$); $\lambda_\gamma=\pm1$ transverse, $\lambda_\gamma=0$ longitudinal polarization.
  Vector-meson polarization vector $\varepsilon^\mu(\lambda_V)$ orthogonal to its momentum $v^\mu$ ($\varepsilon^{*\mu}(\lambda_V)v_\mu=0$); $\lambda_V=\pm1$ transverse, $\lambda_V=0$ longitudinal polarization.
  Bispinors $u_1\equiv u(p_1,\lambda_1)$ and $u_2\equiv u(p_2,\lambda_2)$ for initial and final nucleon.

- Poincare group. $C$, $P$, $T$ invariance of strong and electromagnetic interactions.

# Basic Formulas for Vector-Meson Production on Spinless Target

- Basic Relation between Invariant and Physical (Helicity) Amplitudes of Vector-Meson Production with Heavy Photon

$$\mathcal{T}_{\lambda_V \lambda_\gamma} = \varepsilon^{*\mu}(\lambda_V) T_{\mu\tau} e^{\tau}(\lambda_\gamma),$$

$$T_{\mu\tau} = \sum_{m=1}^{M} F_m(Q^2, W, t, m_V) \mathcal{K}^{(m)}_{\mu\tau},$$

$T_{\mu\tau}$ is the fundamental tensor obeying relations $v^{\mu} T_{\mu\tau} = T_{\mu\tau} q^{\tau} = 0$.
$F_m$ is invariant amplitude, $\mathcal{K}^{(m)}_{\mu\tau}$ is particular kinematic tensor, $1 \le m \le M = 5$.

- Tensor $T_{\mu\tau}$ is a simple function of kinematic variables without singularities.
- Representation of $T_{\mu\tau}$ through unit kinematic four-vectors

$$T_{\mu\tau} = F_1 (h_3)_\mu (g_0)_\tau + F_2 (h_3)_\mu (g_1)_\tau + F_3 (g_1)_\mu (g_0)_\tau$$
$$+ F_4[(g_0)_\mu (g_0)_\tau - (g_3)_\mu (g_3)_\tau - g_{\mu\tau}] + F_5[-(g_1)_\mu (g_1)_\tau + (g_2)_\mu (g_2)_\tau]$$

# Basic Formulas for Vector-Meson Production on Spinless Target

Unit mutually orthogonal kinematic vectors $g_0$, $g_1$, $g_2$, $g_3$ and unit vector $h_3$:

$$g_0 = \frac{Q^2 v + (qv)q}{Qz},$$

$$g_1 = \frac{[\nu m_N - (m_V^2 + Q^2 - t)/4][v(m_V^2 + Q^2 - t)/2 - q(m_V^2 + Q^2 + t)/2] - pz^2}{m_N z v_T \sqrt{\nu^2 + Q^2}},$$

$$g_2 = \frac{d}{v_T m_N \sqrt{\nu^2 + Q^2}},$$

$$g_3 = \frac{q}{Q},$$

$$h_3 = \frac{q m_V^2 - v(qv)}{z m_V},$$

$$\nu = (qp_1)/M_N,\ Q^2 = -q^2,\ W^2 = 2M_N\nu + M_N^2 - Q^2,\ t = (p_1 - p_2)^2,$$

$$(qv) = (m_V^2 - Q^2 - t)/2,\ z^2 = (qv)^2 + Q^2 m_V^2,$$

$v_T$ is transverse part of vector-meson three-momentum in center-of-mass system.

## Lorentz systems with collinear three-momenta of photon and vector meson

- Definition of systems with collinear three-momenta of photon and vector meson (CMPVM): any boost along three-momentum of photon in the rest system of vector meson gives such a system

- Relation between invariant and helicity amplitudes in CMPVM systems

$$\mathcal{T}_{00} = -F_1,$$
$$\mathcal{T}_{11} = -F_4,$$
$$\mathcal{T}_{01} = -\frac{1}{\sqrt{2}}F_2,$$
$$\mathcal{T}_{10} = \frac{1}{\sqrt{2}}F_3,$$
$$\mathcal{T}_{1-1} = F_5.$$

- Helicity amplitudes $\mathcal{T}_{\lambda_v \lambda_\gamma}$ are physical (observable). They are regular. Hence invariant amplitudes $F_m$ are also non-singular.

## Asymptotic behaviour at low $Q$, $v_T$, and $m_V$

- Tensor $T_{\mu\tau}$ is regular everywhere.
- Small $Q$ limit
  Since $g_0 \propto 1/Q$ at $Q \to 0$,

$$\mathcal{T}_{00} = -F_1 \propto Q, \;\; \mathcal{T}_{10} = \frac{1}{\sqrt{2}} F_3 \propto Q.$$

- Small $v_T$ limit
  Since $g_1 \propto 1/v_T$, $g_2 \propto 1/v_T$ at $v_T \to 0$,

$$\mathcal{T}_{01} = -\frac{1}{\sqrt{2}} F_2 \propto v_T, \;\; \mathcal{T}_{10} = \frac{1}{\sqrt{2}} F_3 \propto v_T, \;\; \mathcal{T}_{1-1} = F_5 \propto v_T^2.$$

Hierarchy at $v_T \to 0$: $\mathcal{T}_{00} \sim \mathcal{T}_{11} \gg \mathcal{T}_{01} \sim \mathcal{T}_{10} \gg \mathcal{T}_{1-1}$.

- Small $m_V$ limit
  Since $h_3 \propto 1/m_V$, at $m_V \to 0$,
  $\mathcal{T}_{00} = -F_1 \propto m_V, \;\; \mathcal{T}_{01} = -\frac{1}{\sqrt{2}} F_2 \propto m_V.$
  It is interesting to study dependence of $\mathcal{T}_{00}$ and $\mathcal{T}_{01}$ on $m_V$ at small mass.

# Center-of-Mass System of Photon-Nucleon System

- Relation between helicity amplitudes in CMPVM and center-of-mass (CM) systems

$$\mathcal{T}^C_+ = \mathcal{T}_+, \;\; \mathcal{T}_\pm = \mathcal{T}_{11} \pm \mathcal{T}_{1-1}, \;\; \mathcal{T}^C_\pm = \mathcal{T}^C_{11} \pm \mathcal{T}^C_{1-1},$$

$$\mathcal{T}^C_{00} = \mathcal{T}_{00} - \frac{4Qm_V v_T^2}{(Q^2+m_V^2)^2}\mathcal{T}_- - \frac{\sqrt{8}Qv_T}{Q^2+m_V^2}\mathcal{T}_{01} + \frac{\sqrt{8}m_V v_T}{Q^2+m_V^2}\mathcal{T}_{10},$$

$$\mathcal{T}^C_- = \frac{4Qm_V v_T^2}{(Q^2+m_V^2)^2}\mathcal{T}_{00} + \mathcal{T}_- - \frac{\sqrt{8}m_V v_T}{Q^2+m_V^2}\mathcal{T}_{01} - \frac{\sqrt{8}Qv_T}{Q^2+m_V^2}\mathcal{T}_{10},$$

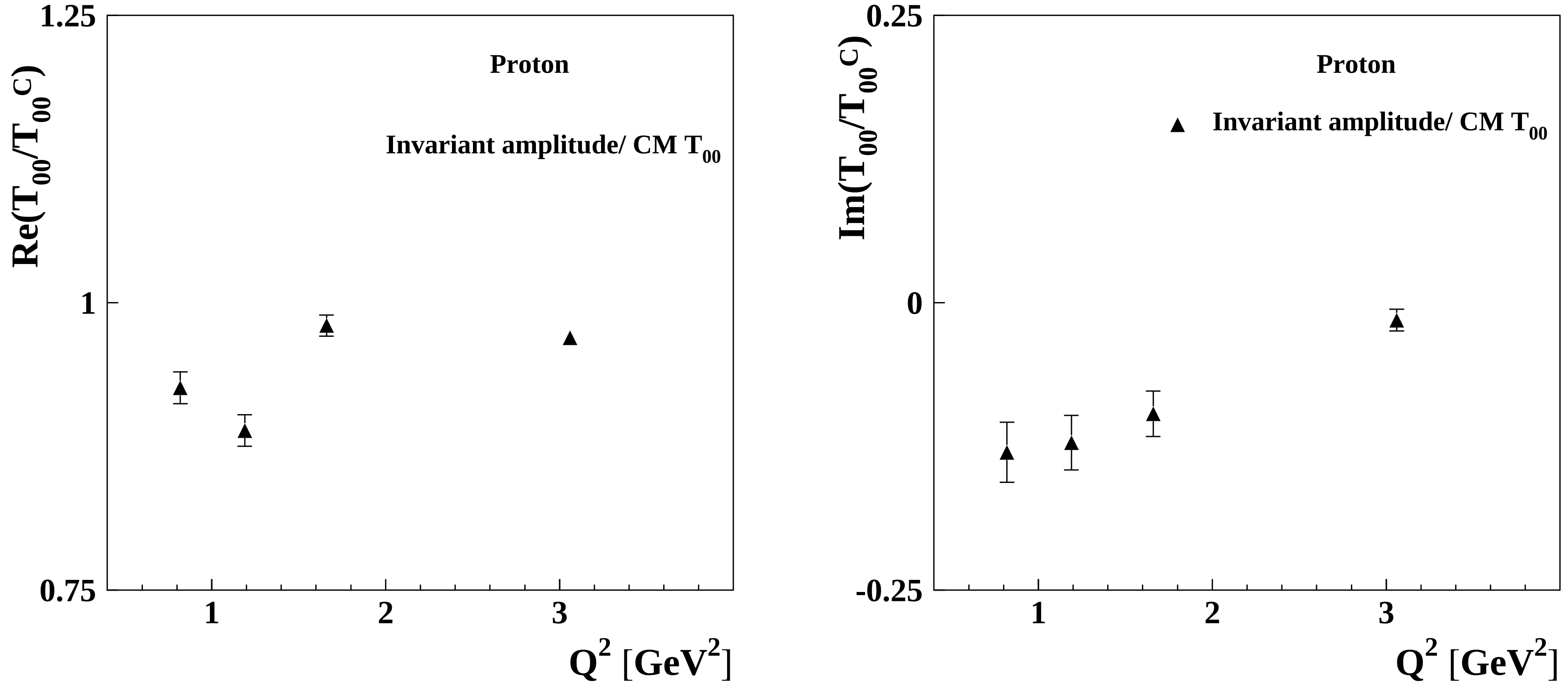
$$\mathcal{T}^C_{01} = -\frac{\sqrt{2}Qv_T}{Q^2+m_V^2}\mathcal{T}_{00} + \frac{\sqrt{2}m_V v_T}{Q^2+m_V^2}\mathcal{T}_- + \mathcal{T}_{01} - \frac{4Qm_V v_T^2}{(Q^2+m_V^2)^2}\mathcal{T}_{10},$$

$$\mathcal{T}^C_{10} = -\frac{\sqrt{2}m_V v_T}{Q^2+m_V^2}\mathcal{T}_{00} - \frac{\sqrt{2}Qv_T}{Q^2+m_V^2}\mathcal{T}_- + \frac{4Qm_V v_T^2}{(Q^2+m_V^2)^2}\mathcal{T}_{01} + \mathcal{T}_{10}.$$

Inverse relation are obtained by transformation $v_T \to -v_T$.

- CM amplitudes $\mathcal{T}^C_{\lambda_V\lambda_\gamma}$ have the same bevaviour at small $Q$, $v_T$,and $m_V$ as CMPVM amplitudes $\mathcal{T}_{\lambda_V\lambda_\gamma}$.

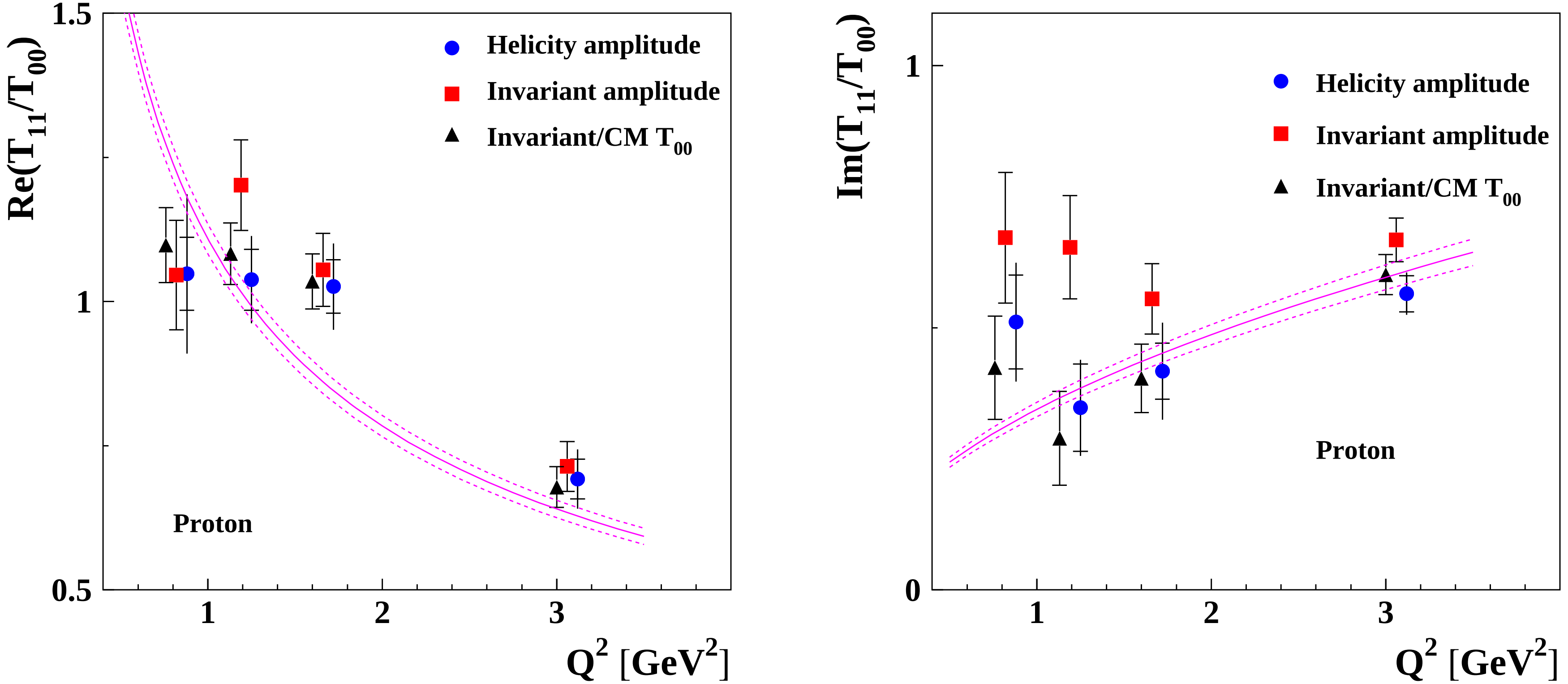
# Calculation of Invariant Amplitudes for HERMES Kinematics




Helicity amplitude ratios: HERMES Coll., Eur.Phys.J C71 (2011) 1609

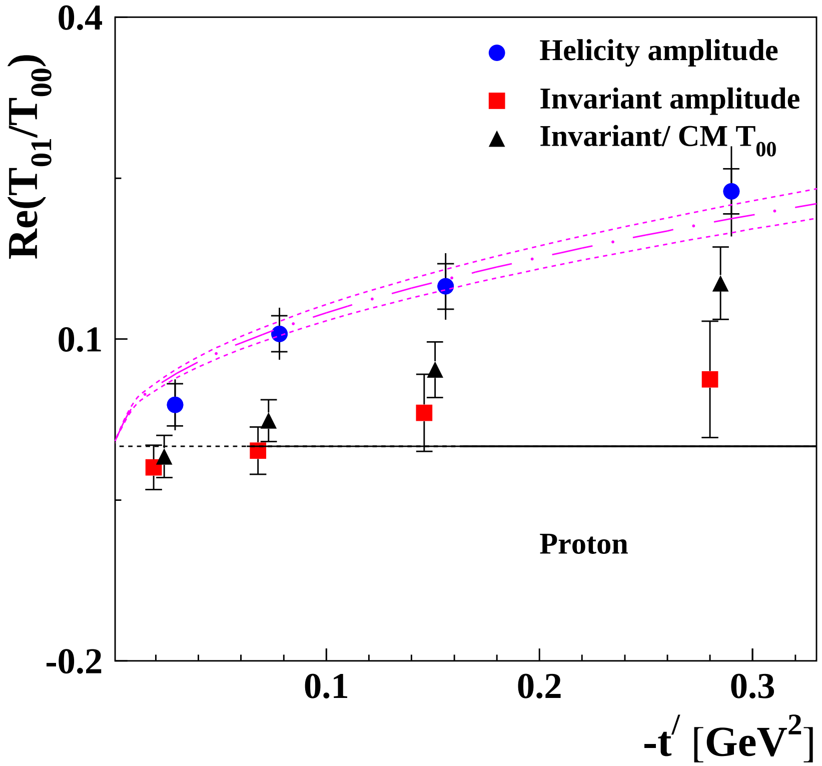
Ratio of amplitudes $\mathcal{T}_{00}/\mathcal{T}_{00}^{C}$ Statistical uncertainty only

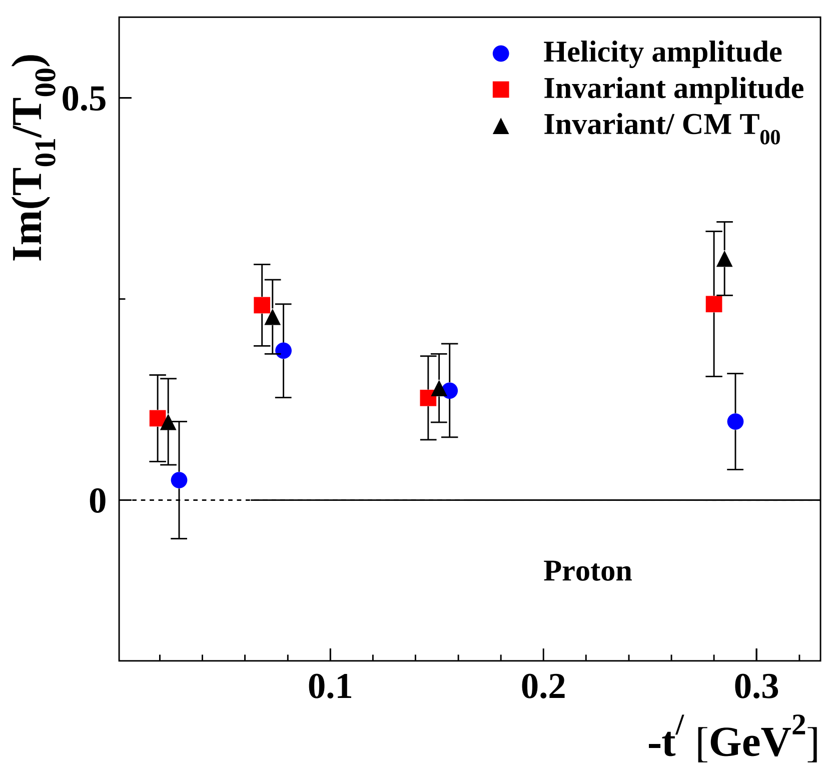
# Calculation of Invariant Amplitudes for HERMES Kinematics




Helicity Amplitudes: $\mathcal{T}_{11}^C/\mathcal{T}_{00}^C$ Invariant Amplitudes: $\mathcal{T}_{11}/\mathcal{T}_{00}$
Invariant Amplitude/ CM $T_{00}$: $\mathcal{T}_{11}/\mathcal{T}_{00}^C$ Statistical uncertainty only
Solid curve - best fit; Dashed curve shows total fit uncertainty

# Calculation of Invariant Amplitudes for HERMES Kinematics




Helicity Amplitudes: $\mathcal{T}^C_{01}/\mathcal{T}^C_{00}$ Invariant Amplitudes: $\mathcal{T}_{01}/\mathcal{T}_{00}$

Invariant Amplitude/ CM $T_{00}$: $\mathcal{T}_{11}/\mathcal{T}^C_{00}$ Statistical uncertainty only

Solid curve - best fit; Dashed curve shows total fit uncertainty.

Conservation of helicity in CMPVM systems?

# Comparison to Fraas-Schildknecht Representation

- Representation for $T_{\mu\tau}$ by Fraas and Schildknecht (1969)

$$T_{\mu\tau} = \sum_{m=1}^{M} \mathcal{F}_m S^{(m)}_{\mu\tau},$$

$$\mathcal{K}^{(m)}_{\mu\tau} \Rightarrow S^{(m)}_{\mu\tau}, \quad F_m \Rightarrow \mathcal{F}_m$$

$$S^{(1)}_{\mu\tau} = \{p_\mu p_\tau (vq) - q_\mu p_\tau (pq) - p_\mu v_\tau (pq) + g_{\mu\tau}(pq)^2\}/m_N^4,$$

$$S^{(2)}_{\mu\tau} = \{-Q^2[g_{\mu\tau}(pq) - p_\mu v_\tau] + [p_\mu (vq) - q_\mu (pq)]q_\tau\}/m_N^4,$$

$$S^{(3)}_{\mu\tau} = \{m_V^2[g_{\mu\tau}(pq) - q_\mu p_\tau] + v_\mu [p_\tau (vq) - v_\tau (pq)]\}/m_N^4,$$

$$S^{(4)}_{\mu\tau} = \{-Q^2 m_V^2 g_{\mu\tau} + v_\mu q_\tau (vq) - m_V^2 q_\mu q_\tau + Q^2 v_\mu v_\tau\}/m_N^4,$$

$$S^{(5)}_{\mu\tau} = \{(vq) g_{\mu\tau} - q_\mu v_\tau\}/m_N^2.$$

- Relation between Fraas-Schildknecht (FS) invariant amplitudes $\mathcal{F}_m$ and $\mathcal{T}_{\lambda_V \lambda_\gamma}$

$$\mathcal{F}_1 = -\frac{2z^2 m_N^2}{v_T^2(\nu^2+Q^2)(vq)}\mathcal{T}_{1-1},$$

$$\mathcal{F}_2 = -\frac{\sqrt{2} m_N^3}{Q v_T \sqrt{\nu^2+Q^2}}\mathcal{T}_{10} - \frac{(pq)(Q^2+m_V^2+t)m_N^2}{v_T^2(\nu^2+Q^2)(vq)}\mathcal{T}_{1-1},$$

$$\mathcal{F}_3 = -\frac{\sqrt{2} m_N^3}{m_V v_T \sqrt{\nu^2+Q^2}}\mathcal{T}_{01} + \frac{(pq)(Q^2+m_V^2-t)m_N^2}{v_T^2(\nu^2+Q^2)(vq)}\mathcal{T}_{1-1},$$

$$\mathcal{F}_4 = \frac{(vq)m_N^4}{z^2 Q m_V}\mathcal{T}_{00} - \frac{(pq)(Q^2+m_V^2+t)m_N^3}{\sqrt{2}z^2 m_V v_T\sqrt{\nu^2+Q^2}}\mathcal{T}_{01} + \frac{(pq)(Q^2+m_V^2-t)m_N^3}{\sqrt{2}z^2 Q v_T\sqrt{\nu^2+Q^2}}\mathcal{T}_{10}$$
$$+\frac{(pq)^2[(Q^2+m_V^2)^2-t^2)]m_N^2}{2z^2 v_T^2(\nu^2+Q^2)(vq)}\mathcal{T}_{1-1} + \frac{m_N^4}{z^2}(\mathcal{T}_{11}+\mathcal{T}_{1-1}),$$

$$\mathcal{F}_5 = \frac{m_V Q m_N^2}{z^2}\mathcal{T}_{00} + \frac{(pq)(Q^2+m_V^2-t)m_V m_N}{\sqrt{2}z v_T\sqrt{\nu^2+Q^2}}\mathcal{T}_{01} + \frac{(pq)(Q^2+m_V^2+t)Q m_N}{\sqrt{2}z v_T\sqrt{\nu^2+Q^2}}\mathcal{T}_{10}$$
$$-\frac{(pq)^2[(Q^2+m_V^2)^2-t^2)]}{2z^2 v_T^2(\nu^2+Q^2)}\mathcal{T}_{1-1} - \frac{m_N^2(qv)}{z^2}(\mathcal{T}_{11}+\mathcal{T}_{1-1}).$$

There are unphysical poles in the FS invariant amplitudes at $Q^2 = m_V^2 - t$.
When $(vq) = (m_V^2 - t - Q^2)/2 = 0$ amplitudes $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, and $\mathcal{F}_4$ are singular.

# Basic Formulas for Vector-Meson Production on Nucleon

- Basic Relation between Invariant and Physical (Helicity) Amplitudes

$$F_{\lambda_V \lambda_2 \lambda_\gamma \lambda_1} = \varepsilon^{*\mu}(\lambda_V)\bar{u}_2(p_2, \lambda_2)\hat{T}_{\mu\tau}u_1(p_1, \lambda_1)e^\tau(\lambda_\gamma), \quad (1)$$

$$\hat{T}_{\mu\tau} = \sum_{m=1}^{18} F_m \hat{\mathcal{K}}^{(m)}_{\mu\tau}, \quad (2)$$

$u_1(p_1, \lambda_1)$ is Dirac bispinor of the initial nucleon,
$p_1$ and $\lambda_1$ are its momentum and helicity, while $u_2(p_2, \lambda_2)$ describes final nucleon.
$F_m$ is invariant amplitude, $\hat{\mathcal{K}}^{(m)}_{\mu\tau}$ is particular kinematic tensor, $m = 1, 2, ..., 18$.

- $\hat{T}_{\mu\tau}$ is fundamental tensor being $4 \times 4$ matrix acting on bispinors $u_1$ and $\bar{u}_2$.
It is a simple function of kinematic variables without singularities.

- Natural Parity Exchange (NPE: exchange by $0^+$, $1^-$, $2^+$... states) and Unnatural Parity Exchange (UPE: exchange by $0^-$, $1^+$, $2^-$... states) Amplitudes

$$F_{\lambda_V \lambda_2 \lambda_\gamma \lambda_1} = T_{\lambda_V \lambda_2 \lambda_\gamma \lambda_1} + U_{\lambda_V \lambda_2 \lambda_\gamma \lambda_1}$$

$$T_{\lambda_V \lambda_2 \lambda_\gamma \lambda_1} = (-1)^{\lambda_V - \lambda_\gamma} T_{-\lambda_V \lambda_2 - \lambda_\gamma \lambda_1} = (-1)^{\lambda_2 - \lambda_1} T_{\lambda_V - \lambda_2 \lambda_\gamma - \lambda_1}.$$

$$U_{\lambda_V \lambda_2 \lambda_\gamma \lambda_1} = -(-1)^{\lambda_V - \lambda_\gamma} U_{-\lambda_V \lambda_2 - \lambda_\gamma \lambda_1} = -(-1)^{\lambda_2 - \lambda_1} U_{\lambda_V - \lambda_2 \lambda_\gamma - \lambda_1}.$$

- Fundamental Tensors for Natural and Unnatural Parity Exchange Amplitudes

$$\hat{T}_{\mu\tau} = \hat{N}_{\mu\tau} + \hat{U}_{\mu\tau}$$

Theorem: $\hat{N}_{\mu\tau}$ commutes while $\hat{U}_{\mu\tau}$ anti-commutes with $\hat{R} = \gamma_5 d_\mu \gamma^\mu / |d|$,

$$d_\mu = \epsilon_{\mu\nu\lambda\beta} q^\nu v^\lambda p^\beta, \quad p = (p_1 + p_2)/2$$

- Representation of $\hat{N}_{\mu\tau}$ and $\hat{U}_{\mu\tau}$ through unit kinematic vectors

$$\hat{N}_{\mu\tau} = \hat{N}^{(1)}_{\mu\tau} + \hat{N}^{(2)}_{\mu\tau}, \quad \hat{U}_{\mu\tau} = \hat{U}^{(1)}_{\mu\tau} + \hat{U}^{(2)}_{\mu\tau}$$

$$\hat{N}^{(j)}_{\mu\tau} = \{F^{(j)}_1 (h_3)_\mu (g_0)_\tau + F^{(j)}_2 (h_3)_\mu (g_1)_\tau + F^{(j)}_3 (g_1)_\mu (g_0)_\tau$$
$$+F^{(j)}_4 [(g_0)_\mu (g_0)_\tau - (g_3)_\mu (g_3)_\tau - g_{\mu\tau}] + F^{(j)}_5 [-(g_1)_\mu (g_1)_\tau + (g_2)_\mu (g_2)_\tau]\}\hat{A}_j.$$

with $j = 1, 2$, $\hat{A}_1 = I$, $\hat{A}_2 = \gamma_5 \hat{g}_2 \equiv \gamma_5 (g_2)_\mu \gamma^\mu$ (alternatively $\hat{A}'_1 = I$, $\hat{A}'_2 = \hat{q}/m_N$ ).

$$\hat{U}^{(j)}_{\mu\tau} = \{G^{(j)}_1 (h_3)_\mu (g_2)_\tau + G^{(j)}_2 \frac{(g_1)_\mu (g_2)_\tau + (g_2)_\mu (g_1)_\tau}{2}$$
$$+G^{(j)}_3 (g_2)_\mu (g_0)_\tau + G^{(j)}_4 \frac{\epsilon_{\mu\nu\alpha\beta} q^\alpha v^\beta}{z\sqrt{2}}\}\hat{B}_j$$

with $j = 1, 2$, $\hat{B}_1 = \gamma_5$, $\hat{B}_2 = \hat{g}_2$ (alternatively $\hat{B}'_1 = \gamma_5$, $\hat{B}'_2 = \gamma_5 \hat{q}/m_N$).
Two independent 4×4 matrices both for $\hat{N}_{\mu\tau}$ ($\hat{A}_1$, $\hat{A}_2$) and $\hat{U}_{\mu\tau}$ ($\hat{B}_1$, $\hat{B}_2$).
Natural parity amplitudes: $5 \times 2 = 10$. Unnatural parity amplitudes: $4 \times 2 = 8$.

# Behaviour at low $Q$, $v_T$, and $m_V$ of NPE amplitudes

- Tensor $\hat{T}^{(j)}_{\mu\tau}$ for $j = 1, 2$ is regular everywhere.
- Since $\hat{T}^{(1)}_{\mu\tau} \propto I$ while $\hat{T}^{(2)}_{\mu\tau} \propto \hat{g}_2$ and $g_2 \sim 1/v_T$ then $F^{(1)}_n$ and $F^{(2)}_n/v_T$ behaves at low $Q$, $v_T$ and $m_V$ as amplitudes $F_n$ for the same $n = 1, 2, 3, 4$ or 5 for scalar target.
- Hierarchy at small $v_T/M_N$
  A: $F^{(1)}_1$, $F^{(1)}_4 \propto (v_T)^0$,
  B: $F^{(1)}_2$, $F^{(1)}_3$, $F^{(2)}_1$, $F^{(2)}_4 \propto v_T$,
  C: $F^{(1)}_5$, $F^{(2)}_2$, $F^{(2)}_3 \propto (v_T)^2$,
  D: $F^{(2)}_5 \propto (v_T)^3$
  A $\gg$ B $\gg$ C $\gg$ D

## Behaviour at low $Q$, $v_T$, and $m_V$ of UPE amplitudes

- Tensor $\hat{U}^{(j)}_{\mu\tau}$ for $j = 1, 2$ is regular everywhere.

- Small $Q$ limit
  Since $g_0 \propto 1/Q$ at $Q \to 0$, then for $j = 1, 2$
  $G^{(j)}_3 \propto Q.$

- Small $v_T$ limit
  Since $g_1 \propto 1/v_T$, $g_2 \propto 1/v_T$ at $v_T \to 0$, then
  $G^{(1)}_4 \propto (v_T)^0, \;\; G^{(1)}_1 \propto v_T, \;\; G^{(1)}_3 \propto v_T, \;\; G^{(1)}_2 \propto v_T^2; \quad G^{(2)}_n/G^{(1)}_n \propto v_T, \;\; n = 1, ..., 4$

  Hierarchy at small $v_T/m_N$:

$$G^{(1)}_4 \gg G^{(1)}_1 \sim G^{(1)}_3 \sim G^{(2)}_4 \gg G^{(1)}_2 \sim G^{(2)}_1 \sim G^{(2)}_3 \gg G^{(2)}_2$$

- Small $m_V$ limit
  Since $h_3 \propto 1/m_V$, at $m_V \to 0$, then for $j = 1, 2$
  $G^{(j)}_1 \propto m_V.$

# Angular Distribution of Final Particles

- Angular Distribution ( Symbolic $W = \mathcal{F}\varrho(\gamma^*)\rho(N)\mathcal{F}^+/\mathrm{tr}\{\mathcal{F}\varrho(\gamma^*)\rho(\mathrm{N})\mathcal{F}^+\}$ )
  $W(\Phi,\theta,\varphi) =$
  $\frac{1}{\mathcal{N}}\sum_{\lambda_N\lambda'_N\mu_N\lambda_\gamma\mu_\gamma}\mathcal{F}_{\lambda'_N\lambda_\gamma\lambda_N}(\Phi,\theta,\varphi)\varrho(\gamma^*)_{\lambda_\gamma\mu_\gamma}\rho(N)_{\lambda_N\mu_N}\mathcal{F}^*_{\lambda'_N\mu'_\gamma\mu_N}(\Phi,\theta,\varphi).$
  $\mathcal{N}$ is normalization factor: $\int W(\Phi,\theta,\varphi)d\Omega = 1.$
- Full Amplitude of Two Pion Production
  Process: $\gamma^* + N \to \rho^0 + N \to \pi^+ + \pi^- + N$. Background: $\gamma^* + N \to \pi^+ + \pi^- + N$.
  $\mathcal{F}_{\lambda'_N\lambda_\gamma\lambda_N}(\Phi,\theta,\varphi) = F^{BG}_{00\lambda'_N\lambda_\gamma\lambda_N}Y_{00}(\theta,\varphi)$
  $+(F_{\lambda_V\lambda'_N\lambda_\gamma\lambda_N}\ f^D + F^{BG}_{1\lambda_V\lambda'_N\lambda_\gamma\lambda_N})Y_{1\lambda_V}(\theta,\varphi) + F^{BG}_{2m\lambda'_N\lambda_\gamma\lambda_N}Y_{2m}(\theta,\varphi) + \ldots,$
  $\theta$ and $\varphi$ are polar and azimuthal angles of $\vec{n} = (\vec{p}_{\pi^+} - \vec{p}_{\pi^-})/|\vec{p}_{\pi^+} - \vec{p}_{\pi^-}|$.
  $\Phi$ - angle between lepton-scattering plane and vector-meson-production plane.
  Amplitude of $\rho^0$ decay $f^D$ is constant while $F_{\lambda_V\lambda'_N\lambda_\gamma\lambda_N}$ depends on $W$, $Q^2$, $t$, $m_V$.
  Resonance factor: $F_{\lambda_V\lambda'_N\lambda_\gamma\lambda_N} = f_{\lambda_V\lambda'_N\lambda_\gamma\lambda_N}/(M_{\pi^+\pi^-} - m_\rho + i\Gamma_\rho)$.
- Full Amplitude of Three Pion Production
  Process: $\gamma^* + N \to \omega + N \to \pi^+ + \pi^- + \pi^0 + N$.
  Background: Direct pion production $\gamma^* + N \to \pi^+ + \pi^- + \pi^0 + N$.
  $\vec{n} = (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})/|\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}|, \quad (\theta,\ \varphi) \leftrightarrow \vec{n}.$
  Amplitudes of $\omega$ decay $f^D$ and $F^{BG}_{1\lambda_V\lambda'_N\lambda_\gamma\lambda_N}$ are functions of $(p_{\pi^+} - p_{\pi^-})^2$,
  $(p_{\pi^+} - p_{\pi^0})^2$, and $(p_{\pi^-} - p_{\pi^0})^2$.
  Interference is more complicated to take into account.

- Relations between invariant and helicity amplitudes of vector-meson production by virtual photon on nucleon are established.
- It is shown that there are two independent $4 \times 4$ matrices for natural-parity-exchange and other two matrices for unnatural-parity-exchange contributions to $\hat{T}_{\mu\tau}$.
- It is shown that invariant amplitudes in the Fraas-Schildknecht representation are singular at $Q^2 = m_V^2 - t$ and are not convenient for extraction of amplitude ratios from experimental data.
- Asymptotic behaviour of amplitudes at small $Q$, $v_T$, and $m_V$ is predicted both for natural-parity-exchange and unnatural-parity-exchange amplitudes. Hierarchy of amplitudes at small $v_T$ is established. It may be used in extraction of amplitude ratios from angular distribution of final particles in vector-meson electroproduction.
- Amplitude method takes into account interference between vector-meson-production and background processes while SDME method ignores the interference.

# Outlook

- To extract helicity and invariant amplitude ratios from the HERMES data on $\rho^0$-meson production on unpolarized and transversely polarized targets.