SIDIS Dihadron Production at HERMES

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Argonne National Laboratory
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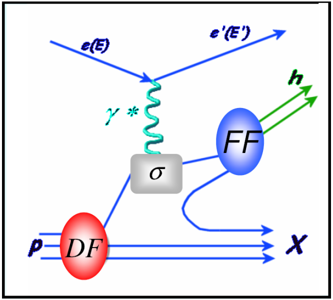
Outline

Motivation and Background

SIDIS Production of Hadrons



- The SIDIS hadron & dihadron processes

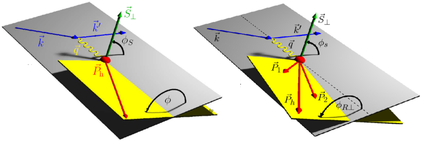
$$
\begin{aligned}
e + p &\rightarrow e' + h + X, \\
e + p &\rightarrow e' + h_1 + h_2 + X.
\end{aligned}
$$

- Factorization theorem implies

$$\sigma^{ep \rightarrow ehX} = \sum_q DF \otimes \sigma^{eq \rightarrow eq} \otimes FF$$

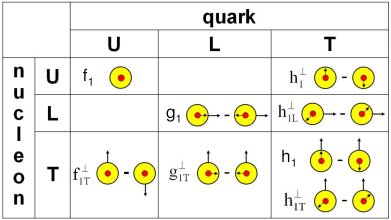
- Access integrals of DFs and FFs through Fourier moments of ϕ_h, ϕ_S, ϕ_R & Legendre polynomials in $\cos\vartheta$.

$$
\begin{aligned}
\phi_h &= \text{signum}\left[(\boldsymbol{k} \times \boldsymbol{P}_h) \cdot \boldsymbol{q}\right] \arccos \frac{(\boldsymbol{q} \times \boldsymbol{k}) \cdot (\boldsymbol{q} \times \boldsymbol{P}_h)}{|\boldsymbol{q} \times \boldsymbol{k}|\, |\boldsymbol{q} \times \boldsymbol{P}_h|}, \\
\phi_S &= \text{signum}\left[(\boldsymbol{k} \times \boldsymbol{S}) \cdot \boldsymbol{q}\right] \arccos \frac{(\boldsymbol{q} \times \boldsymbol{k}) \cdot (\boldsymbol{q} \times \boldsymbol{S})}{|\boldsymbol{q} \times \boldsymbol{k}|\, |\boldsymbol{q} \times \boldsymbol{S}|}, \\
\phi_R &= \text{signum}\left[(\boldsymbol{R} \times \boldsymbol{P}_h) \cdot \boldsymbol{n}\right] \arccos \frac{(\boldsymbol{q} \times \boldsymbol{k}) \cdot (\boldsymbol{P}_h \times \boldsymbol{R})}{|\boldsymbol{q} \times \boldsymbol{k}|\, |\boldsymbol{P}_h \times \boldsymbol{R}|}.
\end{aligned}
$$



Distribution Functions (DFs)

- Distribution functions are categorized according to quark and target nucleon polarization.
- Each function depends on intrinsic transverse momenta and the flavor of the quark (anti-quark).



- Leading twist distribution functions include
  - Unpolarized f_1
  - Helicity distribution g_1 or Δq
  - Transversity h_1 or δq
  - Boer-Mulders $h_1^\perp$
    - Polarized quarks in unpolarized nucleon
  - Sivers function $f_{1T}^\perp$
    - For K^+K^- and ϕ production, $f_{1T}^\perp$ related to gluon orbital angular momentum.
  - Pretzelocity $h_{1T}^\perp$
    - Nonzero $h_{1T}^\perp$ equivalent to non-spherical proton.
  - Wormgear $g_{1T}^\perp$, $h_{1L}^\perp$
    - Quarks and nucleon polarized in different bases.

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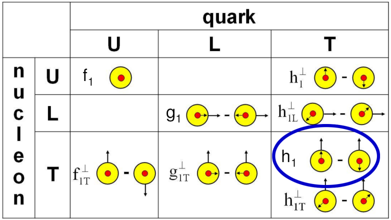

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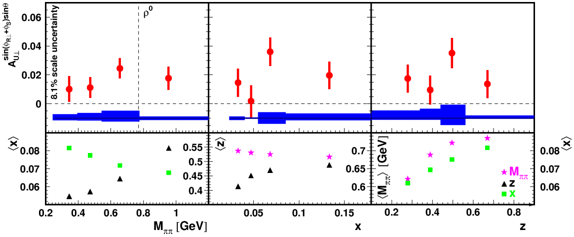
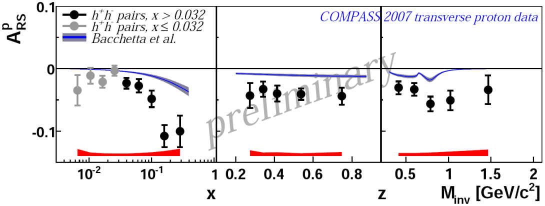
Collinear Dihadron Results

HERMES



COMPASS



- Measure asymmetry $2\,\langle\sin(\phi_{R\perp}+\phi_S)\sin\theta\rangle$ in $\pi^+\pi^-$ pair production.
- Related to h_1 DF (transversity) and *sp* interference FF $H_{1,UT}^{\sphericalangle sp}$.
- Model based on HERMES results by Bacchetta, *et al.* (PRD 74:114007, 2006)
- Prediction for COMPASS results yields too small of an asymmetry. (arXiv:0907.0961v1)
- Both experiments indicate non-zero h_1 and $H_{1,UT}^{\sphericalangle sp}$.

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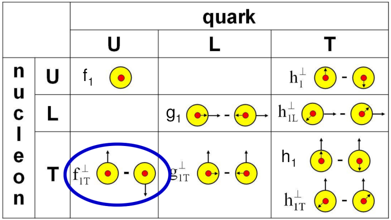

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Brodsky, Gardner arXiv:hep-ph/0608219v2

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Evidence for the Absence of Gluon Orbital Angular Momentum in the Nucleon *

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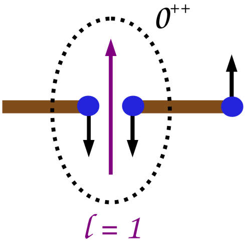
We have argued that the L_g mechanism is small and cannot always produce a leading hadron, so that one is left to ponder how current empirical constraints can be bettered. It strikes us as efficacious to study SSAs associated with produced hadrons of non-valence quark content. The $\gamma^* g \to s\bar{s} \to K^- K^+ + X$ reaction is one such possibility. In principle, one can trace the SSA of the $K^- K^+$ to the gluon's orbital angular momentum L_g. One can also consider the $\gamma^* g \to s\bar{s} \to \phi + X$ reaction: the SSA in ϕ production. Both reactions are important tests for the L_g mechanism, since the gluon contributions of the two nucleons to the SSA add. One can consider these

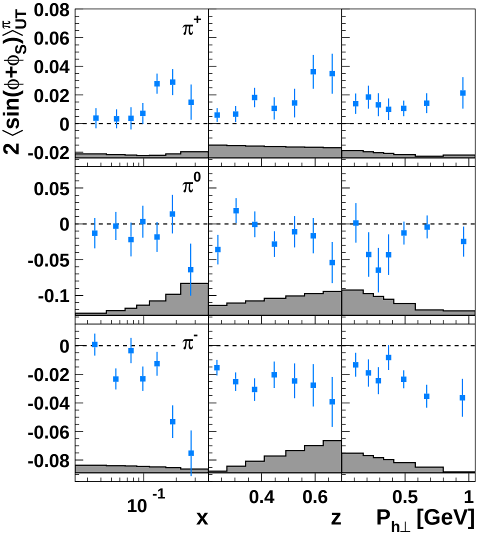
Lund/Artru String Fragmentation Model



- Model fragmentation as the breaking of a gluon flux tube between the struck quark and the remnant.
- Assume that the flux tube breaks into a $q\bar{q}$ pair with quantum numbers equal to the vacuum.

- Expect mesons overlapping with $|\frac{1}{2},\frac{1}{2}\rangle|\frac{1}{2},-\frac{1}{2}\rangle$, $|\frac{1}{2},-\frac{1}{2}\rangle|\frac{1}{2},\frac{1}{2}\rangle$ states to prefer "quark left".
  - $|0,0\rangle =$ pseudo-scalar mesons.
  - $|1,0\rangle =$ longitudinally polarized vector mesons.
- Expect mesons overlapping with $|\frac{1}{2},\frac{1}{2}\rangle|\frac{1}{2},\frac{1}{2}\rangle$, $|\frac{1}{2},-\frac{1}{2}\rangle|\frac{1}{2},-\frac{1}{2}\rangle$ states to prefer "quark right".
  - $|1,\pm 1\rangle =$ transversely polarized vector mesons.
- For the two ρ_T's, "the Collins function" should have opposite sign to that for π
- For ρ_L, "the Collins function" is zero.

HERMES Collins Moments for Pions



- Final result published in January A. Airapetian et al, Phys. Lett. B 693 (2010) 11-16. arXiv:1006.4221 (hep-ex)
- Significant π^- asymmetry implies $H_1^{\perp,disf} \approx -H_1^{\perp,fav}$
- Pions have small, but non-zero asymmetry
- Expect Collins moments negative for $\rho^{\pm}$.
- Would like uncertainties on dihadron moments on the order of 0.02.

Items Which Required Additional Development

- Acceptance/smearing correction method
  - Standard methods cannot handle 4 angular and 5 kinematic variables.
- Testing acceptance correction requires non-collinear SIDIS Monte Carlo generator at sub-leading twist.
  - Must simulate azimuthal dependence of cross section for systematic studies.
  - Cannot use polynomial fits to the data as was done for pseudo-scalar analysis.
- Generator requires
  - Non-collinear cross section at sub-leading twist.
  - Non-collinear fragmentation models.
- Would also like to understand "Which term in the cross section includes 'the Collins function' for ρ_L, ρ_T?"
  - Use alternate partial wave expansion

The TMDGen Generator

New TMDGen Generator

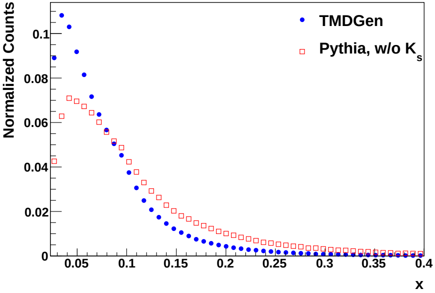
- No previous Monte Carlo generator has TMD dihadron production with full angular dependence
- Method
  - Integrates cross section per flavor to determine "quark branching ratios"
  - Throw a flavor type according to ratios
  - Throw kinematic/angular variables by evaluating cross section
    - Can use weights or acceptance rejection
  - Full TMD simulation: each event has specific $|\boldsymbol{p}_T|$, ϕ_p, $|\boldsymbol{k}_T|$, ϕ_k values
  - Includes both pseudo-scalar and dihadron SIDIS cross sections
- Guiding plans
  - Extreme flexibility
    - Allow many models for fragmentation and distribution functions
    - Various final states: pseudo-scalars, vector mesons, hadron pairs, etc.
    - Output options & connecting to analysis chains of various experiments
    - Minimize dependencies on other libraries
  - Full flavor and transverse momentum dependence.
- Current C++ package considered stable and allows further expansion
- Can be useful for both experimentalists and theorists.

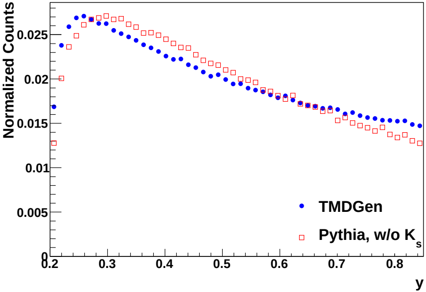
Available Models

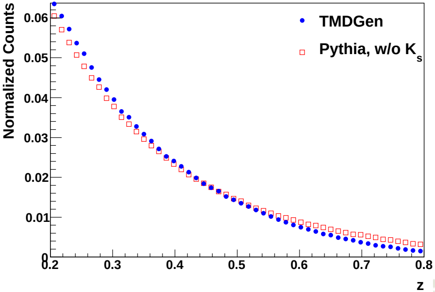
Distribution Functions	Model Identifier
f_1	CTEQ
f_1	LHAPDF
f_1	BCR08
f_1	GRV98
g_1	GRSV2000
$f_{1T}, h_{1T}^{\perp}, h_1$	Torino Group
$f_1, g_1, g_{1L}, g_{1T}, f_{1T}, h_1, h_1^{\perp}, h_{1T}^{\perp}$	Pavia Spectator Model

Frag. Functions	Final State	Model Identifier
D_1	pseudo-scalar	fDSS
D_1	pseudo-scalar	Kretzer
$D_1, H_1^{\perp}$	dihadron	Spectator Model
$D_1, H_1^{\perp}$	dihadron	Set given partial wave proportional to any other partial wave

$\pi^+\pi^0$ Kinematic Distributions, p.1

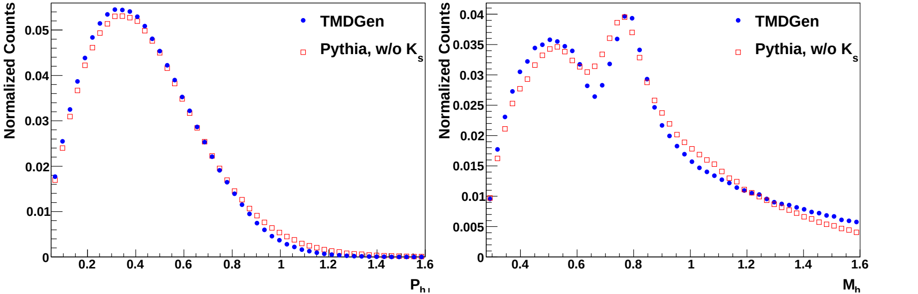






- Close agreement for x, y, z distributions.
- Main discrepancy in x—may be due to imbalance in the flavor contributions, or Q^2 effects.
- Similar results for other $\pi\pi$ and KK dihadrons.

$\pi^+\pi^0$ Kinematic Distributions, p.2



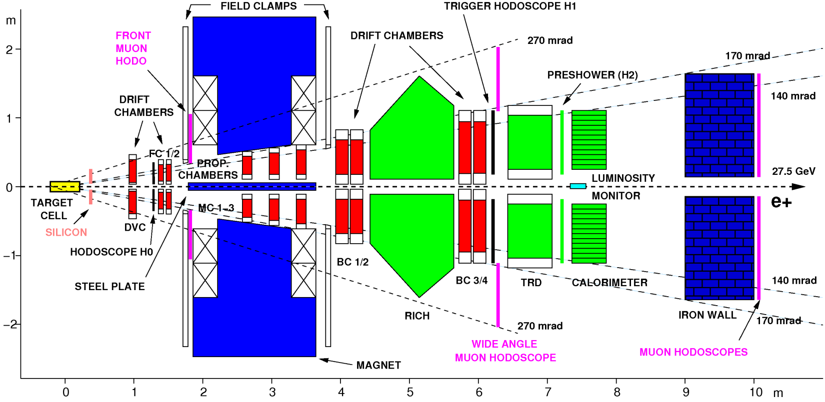
- Fairly good agreement in both $P_{h\perp}$ and M_h distributions.
- Note: some discrepancies in full $5D$ kinematic, but PYTHIA also doesn't match data in full $5D$

HERMES Detector

The HERA Storage Ring

- 6.3 km (3.9 mi) circumference
- Protons were at 920 GeV and electrons/positrons at 27.6 GeV
- Electrons and positrons were polarized due to the Sokolov-Ternov effect
- Operational from 1990 to 2007.

The HERMES Spectrometer



Beam	Long. pol. $e^{\pm}$ at 27.6 GeV	**Lep.-Had. Sep.**	High efficiency $\approx 98\%$
Target	Trans. pol. H ($\approx 75\%$)		Low contamination (<2%)
	Log. pol. H ($\approx 85\%$)	**Hadron PID**	Separates $\pi^{\pm}$, $K^{\pm}$, p, $\bar{p}$
	Unpol. H,D,Ne,Kr,. . .		with momenta in 2-15 GeV

Acceptance Effects

- HERMES detector/experiment has many strengths
  - Excellent PID using a RICH
  - Many combinations of target and beam polarization
  - Both electron and positron beams
  - Accurate geometry for Monte Carlo simulations
- One major setback though...
  - Somewhat limited angular acceptance
  - Effect of angular acceptance can mimic physics signal, especially when combined with radiative effects.

Acceptance Correction Method

Smearing/Acceptance Effects

- Let $\boldsymbol{x}^{(T)}$ be true value of variables, $\boldsymbol{x}^{(R)}$ the reconstructed values
- A conditional probability $p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right)$ relates the true PDF $p\left(\boldsymbol{x}^{(T)}\right)$ with the PDF of the reconstructed variables, $p\left(\boldsymbol{x}^{(R)}\right)$.
- Specific relation given by Fredholm integral equation

$$
\begin{aligned}
p\left(\boldsymbol{x}^{(R)}\right) &= \eta \int d^D\boldsymbol{x}^{(T)}\, p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right) p\left(\boldsymbol{x}^{(T)}\right), \\
\frac{1}{\eta} &= \int d^D\boldsymbol{x}^{(R)} d^D\boldsymbol{x}^{(T)}\, p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right) p\left(\boldsymbol{x}^{(T)}\right).
\end{aligned}
$$

- Can rewrite in terms of a smearing operator

$$
S\left[g(\boldsymbol{x}^{(T)})\right] = \int d^D\boldsymbol{x}^{(T)}\, p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right) g\left(\boldsymbol{x}^{(T)}\right).
$$

- Fredholm equation is simply

$$
p\left(\boldsymbol{x}^{(R)}\right) = S\left[\eta p\left(\boldsymbol{x}^{(T)}\right)\right].
$$

Solution with Finite Basis and Integrated Squared Error

- Restrict to finite basis

$$\begin{aligned} \eta p\left(\boldsymbol{x}^{(T)}\right) &= \sum_i \alpha_i f_i\left(\boldsymbol{x}^{(T)}\right), \\ p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right) &= \sum_{i,j} \Gamma_{i,j} f_i\left(\boldsymbol{x}^{(R)}\right) f_j\left(\boldsymbol{x}^{(T)}\right). \end{aligned}$$

- Determine parameters by minimizing the integrated squared error (ISE)

$$\begin{aligned} ISE_1 &= \int d^D\boldsymbol{x}^{(R)} d^D\boldsymbol{x}^{(T)} \left[p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right) - \sum_{i,j} \Gamma_{i,j} f_i(\boldsymbol{x}^{(R)}) f_j(\boldsymbol{x}^{(T)})\right]^2, \\ ISE_2 &= \int d^D\boldsymbol{x}^{(R)} \left\{p\left(\boldsymbol{x}^{(R)}\right) - \sum_i \alpha_i S\left[f_i\left(\boldsymbol{x}^{(T)}\right)\right]\right\}^2. \end{aligned}$$

Analytic Solution

- Define/compute

$$
\begin{aligned}
F_{i,j} &= \int d^D\boldsymbol{x}^{(\boldsymbol{T})}\, f_i\left(\boldsymbol{x}^{(\boldsymbol{T})}\right) f_j\left(\boldsymbol{x}^{(\boldsymbol{T})}\right), \\
B_{i,j} &= \int d^D\boldsymbol{x}^{(\boldsymbol{R})} d^D\boldsymbol{x}^{(\boldsymbol{T})}\, p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right) f_i\left(\boldsymbol{x}^{(\boldsymbol{R})}\right) f_j\left(\boldsymbol{x}^{(\boldsymbol{T})}\right), \\
&= V\int d^D\boldsymbol{x}^{(\boldsymbol{R})} d^D\boldsymbol{x}^{(\boldsymbol{T})}\, p_{MC}\left(\boldsymbol{x}^{(T)}, \boldsymbol{x}^{(R)}\right) f_i\left(\boldsymbol{x}^{(\boldsymbol{R})}\right) f_j\left(\boldsymbol{x}^{(\boldsymbol{T})}\right), \\
b_i &= \int d^D\boldsymbol{x}^{(R)} p\left(\boldsymbol{x}^{(R)}\right) f_i\left(\boldsymbol{x}^{(R)}\right).
\end{aligned}
$$

- ISEs reduce to the matrix equation

$$
B^T F^{-1} B\boldsymbol{\alpha} = B^T F^{-1}\boldsymbol{b}.
$$

- Assuming B is invertible, this reduces to $B\boldsymbol{\alpha} = \boldsymbol{b}$.
- Note: the least squares solution, ignoring smearing, is $F\boldsymbol{\alpha} = \boldsymbol{b}$.

Numeric Solution

- The quantities can be computed as

$$
\begin{aligned}
b_i &= \frac{V}{N_R} \sum_{k=1}^{N_R} f_i\left(\boldsymbol{x}^{(R,k)}\right), \\
B_{i,j} &= \frac{V^3}{N_{MC}} \sum_{k=1}^{N_{MC}} f_i\left(\boldsymbol{x}^{(R,k)}\right) f_j\left(\boldsymbol{x}^{(T,k)}\right).
\end{aligned}
$$

- Use standard methods to solve $B\boldsymbol{\alpha} = \boldsymbol{b}$.
- One is simply unfolding in the parameter space.

Uncertainty Calculation

- Define

$$\left(C^{\boldsymbol{b}}\right)_{j,j'} = \frac{\delta_{j,j'}}{N_R - 1}\left[\frac{V^2}{N_R}\sum_{k=1}^{N_R} f_i^2\left(\boldsymbol{x}^{(R,k)}\right) - (b_i)^2\right],$$

$$\left(C^{B}\right)_{j,k;j',k'} = \frac{\delta_{j,j'}\delta_{k,k'}}{N_\epsilon - 1}\left[\frac{V^6}{N_\epsilon}\sum_{k=1}^{N_\epsilon} f_j^2\left(\boldsymbol{x}^{(M,k)}\right) f_k^2\left(\boldsymbol{x}^{(T,k)}\right) - (B_{j,k})^2\right],$$

$$C'^{(B)}_{i,i'} = \sum_{j,j'} C^{(B)}_{i,j;i',j'}\alpha_j\alpha_{j'}.$$

- The uncertainty on $\boldsymbol{\alpha}$ is then

$$C^{(\boldsymbol{\alpha})} = B^{-1}C^{(\boldsymbol{b})}B^{-T} + B^{-1}C'^{(B)}B^{-T}.$$

- One could consider a third term $\left(B^T F^{-1} B\right)^{-1}$, the Hessian of the matrix eq.
  - Numeric studies show this term is not a meaningful estimate of the uncertainty, and that it can be neglected.

Alternate Derivation

- Again, assume that $p\left(\boldsymbol{x}^{(R)} \middle| \boldsymbol{x}^{(T)}\right) = Vp(\boldsymbol{x}^{(R)}, \boldsymbol{x}^{(T)})$.
- Substitute $\eta p\left(\boldsymbol{x}^{(T)}\right) = \sum_i \alpha_i f_i\left(\boldsymbol{x}^{(T)}\right)$ into the Fredholm integral equation:

$$p\left(\boldsymbol{x}^{(R)}\right) = V\sum_i \alpha_i \int d^D\boldsymbol{x}^{(T)}\, p_{MC}\left(\boldsymbol{x}^{(T)}, \boldsymbol{x}^{(R)}\right) f_i\left(\boldsymbol{x}^{(T)}\right).$$

- Applying the operator $\int d^D\boldsymbol{x}^{(R)} f_j\left(\boldsymbol{x}^{(R)}\right)$ to both sides yields

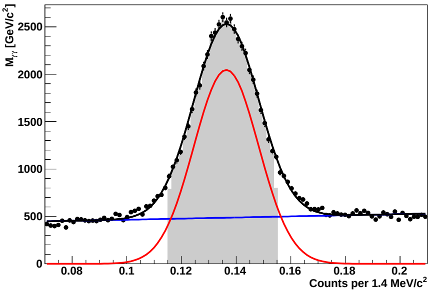
$$\int d^D\boldsymbol{x}^{(R)} f_j\left(\boldsymbol{x}^{(R)}\right) p\left(\boldsymbol{x}^{(R)}\right) = V\sum_i \alpha_i \int d^D\boldsymbol{x}^{(R)} d^D\boldsymbol{x}^{(T)}\, p_{MC}\left(\boldsymbol{x}^{(T)}, \boldsymbol{x}^{(R)}\right) f_i\left(\boldsymbol{x}^{(T)}\right),$$

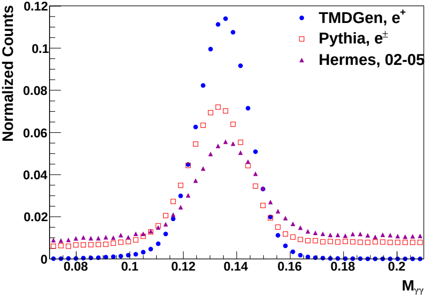
- Using the definitions of $\boldsymbol{b}$ and B, this reduces to

$$\boldsymbol{b} = B\boldsymbol{\alpha}.$$

HERMES Dihadron Analysis

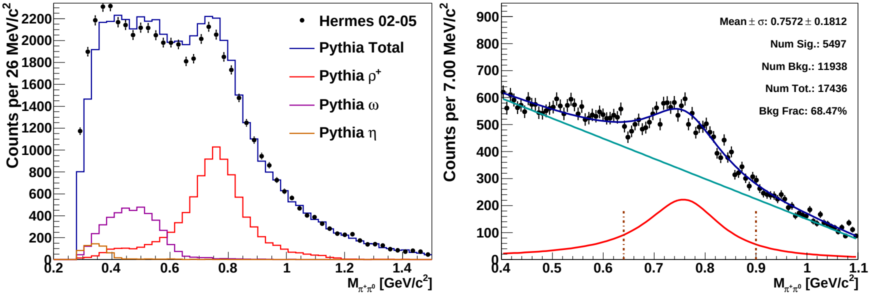
Neutral Pion Reconstruction





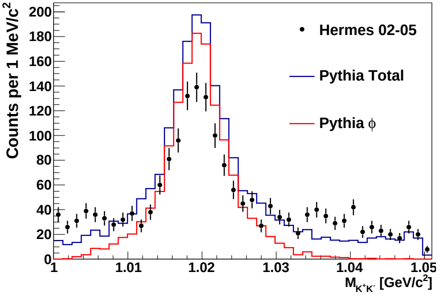
- Invariant mass spectrum of $\gamma\gamma$-system for $\pi^+\gamma\gamma$ events.
- $E_{\text{clus.}} = \alpha E_\gamma$, with α equal to 0.97, 0.9255 and 0.95 for HERMES, PYTHIA, and TMDGEN data, respectively.
- Central value of the peak is sufficiently close to the accepted value.
- Width of the peak is reflection of the resolution of the spectrometer for the π^0 mass.

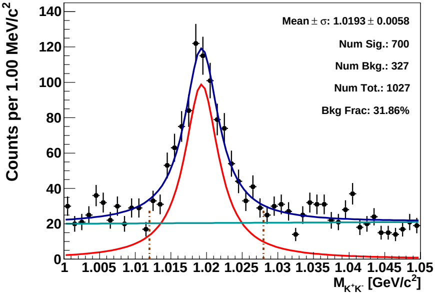
Mass Distribution: $\pi^+\pi^0$



- Left panel: comparison with PYTHIA, highlighting various process decaying into $\pi^+\pi^-$ pair.
- Right panel: Hermes 02-05 data, fit to Breit-Wigner plus linear background to estimate background fraction.
- High background fraction, but hope only VMs in *pp*-wave.
- Distributions for other $\pi\pi$ dihadron effectively the same.

Mass Distribution: K^+K^-





- Lower signal, but much lower background fraction.
- No other mesons decaying into K^+K^- within mass window.
- Clean access to strange quark distribution and fragmentation functions.

Fitting Functions

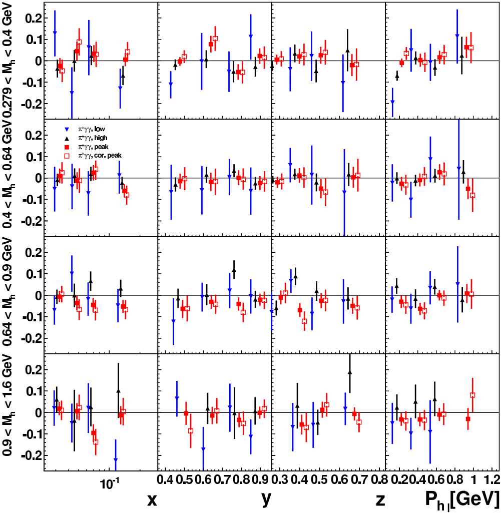
- Perform angular fit in each kinematic bin
- Main focus is on transverse target Collins and Sivers moments
- Fit function includes 41 angular moments plus constant term
  - Unpolarized moments, twist-2 and twist-3 (24 moments)
  - The transverse target Collins and Sivers moments (18 moments)

$$
\begin{aligned}
f(\cos\vartheta, \phi_h, \phi_R, \phi_S) \quad = \quad & \sum_{\ell=0}^{2} \Bigg[\sum_{m=0}^{\ell} a_1^{|\ell,m\rangle} P_{\ell,m} \cos(m\phi_h - m\phi_R) \\
& + \sum_{m=-\ell}^{\ell} \Big(a_2^{|\ell,m\rangle} P_{\ell,m} \cos((2-m)\phi_h + m\phi_R) + a_3^{|\ell,m\rangle} P_{\ell,m} \cos((1-m)\phi_h + m\phi_R) \Big) \\
& + \sum_{m=-\ell}^{\ell} \Big(b_1^{|\ell,m\rangle} P_{\ell,m} \sin((m+1)\phi_h - m\phi_R - \phi_S) + b_2^{|\ell,m\rangle} P_{\ell,m} \sin((1-m)\phi_h + m\phi_R + \phi_S) \Big) \Bigg]
\end{aligned}
$$

- Constrain $a_1^{|0,0\rangle} = 1$.
- Fit parameters are integrals of structure functions, which are integrals of distribution and fragmentation functions

$$
\begin{aligned}
a_1^{|\ell,m\rangle} &\propto f_1 D_1^{|\ell,m\rangle} & a_3^{|\ell,m\rangle} &\propto f_1 D_1^{|\ell,m\rangle}, h_1^{\perp} H_1^{\perp|\ell,m\rangle} & b_1^{|\ell,m\rangle} &\propto f_{1T}^{\perp} D_1^{|\ell,m\rangle} \\
a_2^{|\ell,m\rangle} &\propto h_1^{\perp} H_1^{\perp|\ell,m\rangle} & & & b_2^{|\ell,m\rangle} &\propto h_1 H_1^{\perp|\ell,m\rangle}
\end{aligned}
$$

Non-resonant Photon Pairs



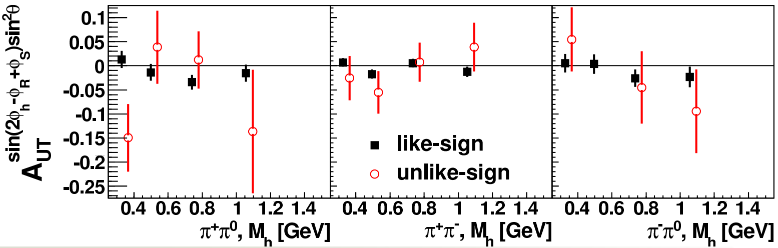
The Collins $|2,2\rangle$ moments for $\pi^+\pi^0$ dihadrons

- Exists about a 25% background of non-resonant photon pairs
- Data from higher and lower $M_{\gamma\gamma}$ regions are fit to interpolate the background asymmetry in the π^0 peak region.
- The effect of subtracting this background is not large

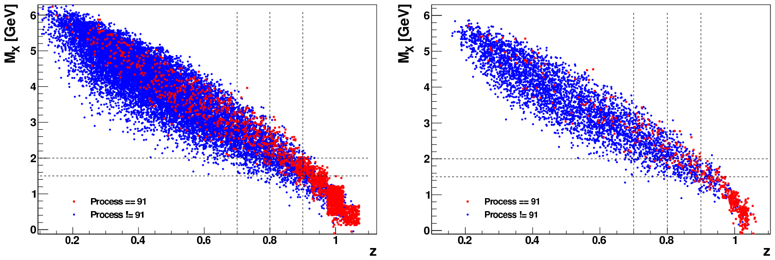
Charge Symmetric Background

- Some candidate DIS leptons actually from processes generating e^+e^- pairs.
- Can use data where lepton has opposite charge as the beam to estimate bkg.
- Little data, but within statistical uncertainty, the background asymmetry appears negligible.

Year	$\pi^+\pi^0$		$\pi^+\pi^-$		$\pi^-\pi^0$		K^+K^-	
2002	222	5.0%	827	3.8%	145	4.3%	2	1.1%
2003	120	4.9%	477	3.9%	74	3.8%	1	1.0%
2004	762	5.0%	2849	3.9%	487	4.2%	4	0.7%
2005	1608	4.7%	7346	4.5%	1667	6.4%	18	1.4%
Total	2712	4.9%	11499	4.3%	2373	5.5%	25	1.2%



Exclusive Background



- Left panel, $\pi^+\pi^-$; right panel, K^+K^-.
- Comparison based on PYTHIA production tuned to HERMES kinematics, within HERMES acceptance
- Good background suppression ($\approx 3.5\%$) by limiting $z < 0.8$
- Missing mass (M_X) cut reduces statistics, but does not reduce background fraction

Vector Meson Fraction

Dihadron	Est. VM Stats.	Bkg. Frac.
$\pi^+\pi^0$	5497	68.5%
$\pi^+\pi^-$	10846	85.4%
$\pi^-\pi^0$	2774	77.9%
K^+K^-	700	31.9%

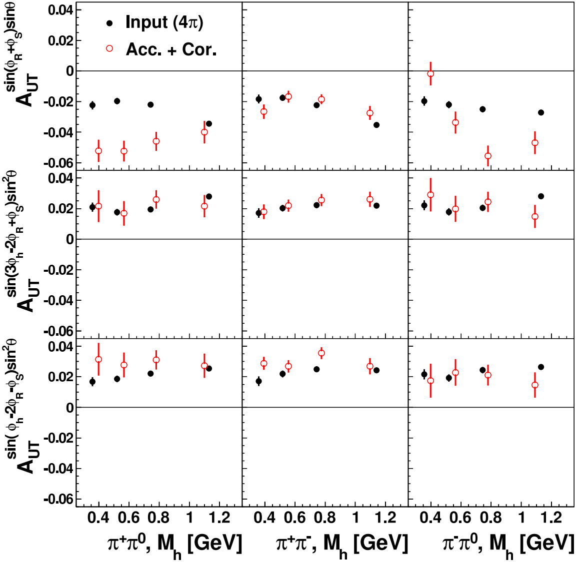
- Some partial waves, such as $|2, \pm 2\rangle$, are a simple sum of vector meson plus non-vector meson contributions
- Other partial waves involving interference, such as $|1, 1\rangle$, cannot be separated into a sum.
- For the $|2, \pm 2\rangle$ partial waves, one can consider isolating the vector meson contribution using the estimated background fractions.
- The fractions are determined by a Breit-Wigner plus linear background fit to the M_h distribution.

Systematic Studies

Monte Carlo Challenge

- TMDGEN, with no angular moments, is used to estimate the acceptance (matrix B).
- PYTHIA in acceptance plus RADGEN is used to act as data (vector $\boldsymbol{b}$).
  - Weights are introduced using the angular portion of the cross section from TMDGEN.
- Also weight born, 4π TMDGEN in the same manner, to determine true parameter values.
- The use of different generators insures no "lucky" cancellations due to the cross sections being identical
- Systematic uncertainty is estimated as half the difference between the results from MLE fit of weighted PYTHIA 4π data and from acceptance correction fit of weighted PYTHIA (w/ RADGEN) within acceptance.

MC Challenge Results: Collins $|2, 2\rangle$ $\pi^+\pi^0$



- As representative moments, above is shown the $|1, 1\rangle$, $|2, -2\rangle$ and $|2, 2\rangle$ Collins moment for pion-pair dihadrons versus M_h.

MC Challenge Results: $\pi^+\pi^0$ χ^2/ndf Statistics

Moment	χ^2/ndf per Binning Option				
	M_h	M_h-x	M_h-y	M_h-z	M_h-$P_{h\perp}$
Sivers $\|0,0\rangle$	10.257	6.154	5.898	6.199	6.886
Sivers $\|1,-1\rangle$	8.649	2.064	1.872	2.681	3.024
Sivers $\|1,0\rangle$	38.928	48.047	27.105	59.303	16.620
Sivers $\|1,1\rangle$	1.072	1.729	2.029	1.393	1.549
Sivers $\|2,-2\rangle$	8.710	1.312	2.256	1.948	2.242
Sivers $\|2,-1\rangle$	14.156	7.346	5.586	11.712	5.233
Sivers $\|2,0\rangle$	191.392	81.096	46.959	106.730	80.811
Sivers $\|2,1\rangle$	9.984	1.987	6.877	4.140	4.155
Sivers $\|2,2\rangle$	1.746	0.987	0.993	1.409	1.403
Collins $\|0,0\rangle$	12.917	5.923	9.475	24.251	6.392
Collins $\|1,-1\rangle$	0.806	1.851	1.135	2.099	2.088
Collins $\|1,0\rangle$	47.455	31.840	37.332	45.703	20.431
Collins $\|1,1\rangle$	16.554	2.497	3.843	4.319	3.131
Collins $\|2,-2\rangle$	0.605	1.011	0.465	0.569	1.363
Collins $\|2,-1\rangle$	12.480	2.694	2.772	14.441	3.673
Collins $\|2,0\rangle$	33.781	32.088	28.132	174.760	16.624
Collins $\|2,1\rangle$	3.693	2.127	2.664	10.043	1.161
Collins $\|2,2\rangle$	1.596	0.740	1.227	1.364	1.048

- As a representative case, above are the χ^2/ndf statistics for $\pi^+\pi^0$ dihadrons.
- Some moments reconstructed well, such as the Collins $|2,\pm 2\rangle$
- Moments with $m = 0$ generally reconstructed quite poorly.
  - Acceptance is worse for $|\cos\vartheta|$ near 1.
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Year Dependence

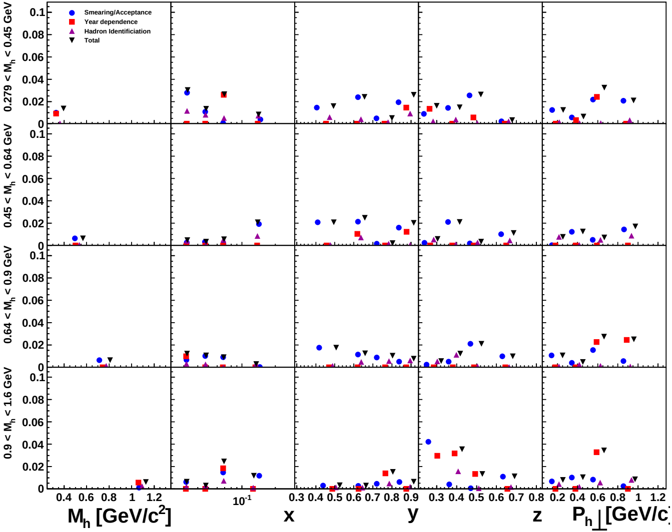
- Data with both positron (2002-2004) and electron (2005) beams is combined for final sample
- Though SIDIS cross section invariant with respect to beam charge, systematic effects are not.
- Study 1
  - Compare the results from the combined fit versus results from combining the separate fit results
  - Very close agreement
- Study 2
  - Compare corrected 2002-2004 results with corrected 2005 results
  - Agreement not as good.
  - Systematic uncertainty is estimated as half the uncertainty needed to reduce the χ^2 per moment per bin to 1.

$$\delta A_{year} = \frac{1}{4}\sqrt{(A_e - A_p)^2 - \delta^2 A_e - \delta^2 A_p} \approx \frac{1}{4}\left|A_e - A_p\right|.$$

Particle Identification Procedure

- Two methods exist for assigning particle identification
  1. Assign the identification with the highest probability
  2. Assign all identifications to each event, but with varying weights
- Weights for second method computed according to
  - Define $P_{i,j} = p(ID_i|ID_j)$, where ID_j is the true identification and ID_i is the identification given by method 1.
  - For a given event, let ID_{i^*} be the identification given by method 1.
  - Weight for the event being ID_j is then P^{-1}_{j,i^*}.
- These methods both part of standard HERMES procedures.
- A systematic uncertainty is assigned, equal to half the difference between the two methods.

Combined Systematic Uncertainty



- Representative plot for the comparison of sources and combined systematic uncertainties: the $|2,2\rangle$ Collins moment for $\pi^+\pi^0$ dihadrons.

Conclusions and Outlook

Conclusions and Outlook

- TMDGen Monte Carlo generator–stage 1 complete
  - Fully ready for this analysis
  - Ready to begin use by others
- Acceptance Correction Method
  - Methodology complete
  - Numerical studies suggest it works fairly well
- Analysis and Systematics
  - Results not yet released
  - Approaching final state
  - Only need a cross check and possibly some fine tuning
- Can also extend the analysis
  - Analyze the full mass range for K^+K^-
  - Analyze the four K^*s