

Direct Extraction of Helicity Amplitude Ratios in Exclusive ρ^0 Electroproduction

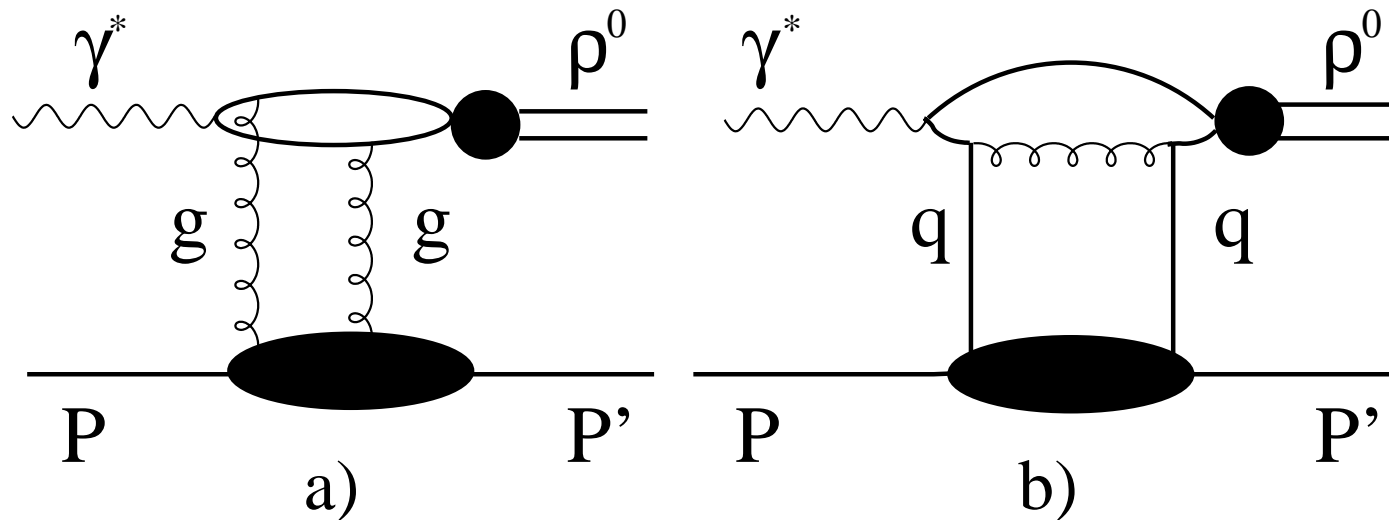
S. I. Manaenkov, Petersburg Nuclear Physics Institute,
on behalf of the HERMES Collaboration

International Workshop
Hadron Structure and QCD

Gatchina, Russia, July 5 - 9, 2010

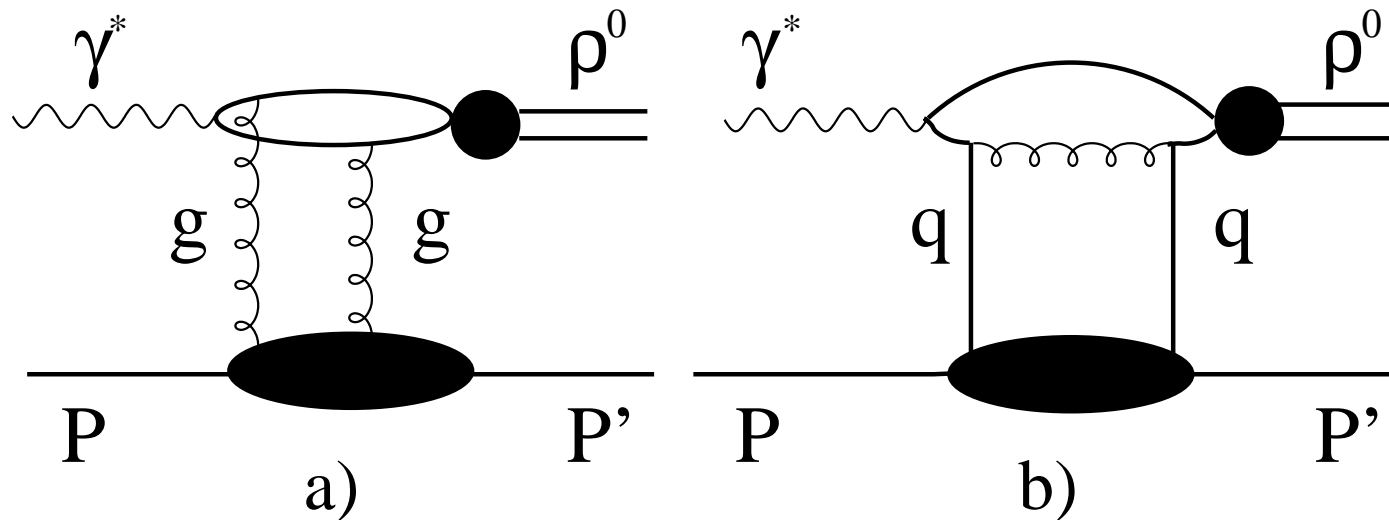
- Physics Motivation
- Data Processing
- Amplitude Method and Spin-Density Matrix Element Method
- Kinematic Dependences of Ratios of Helicity Amplitudes
- World Results on Ratios of Helicity Amplitudes
- Summary and Outlook

Physics Motivation



- $\gamma^* + N \rightarrow V + N$ is a perfect reaction to study both vector-meson production mechanism and hadron structure. Spin Density Matrix Elements (SDMEs) of ρ^0 at HERMES: EPJ C62 (2009) 659. SDMEs are expressible in terms of ratios of helicity amplitudes, hence ratios can be extracted from angular distribution of decay $\pi^+\pi^-$.
- Data on $d\sigma/dt = \sum |F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}|^2$ additional to SDMEs gives a possibility to extract moduli of all the helicity amplitudes and phase differences between them.
- Generalized Parton Distributions (GPDs) of the nucleon can be obtained from the amplitude $F_{00} \equiv F_{0\frac{1}{2}0\frac{1}{2}} (\gamma_L \rightarrow V_L)$ for which factorization theorem is proved. Extraction of amplitude ratios is a first step to get F_{00} and GPDs.
- Difference between proton and deuteron results would point out contribution of $q\bar{q}$ -exchange with isospin $I = 1$ and natural parity $P = (-1)^J$ (ρ, a_0, a_2 reggeons).

Physics Motivation

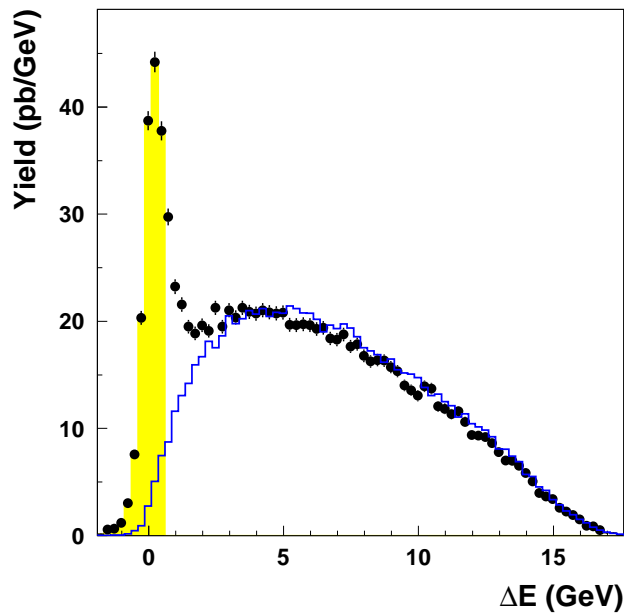


- Extraction of amplitude ratios provides a possibility to distinguish between contributions of Natural Parity Exchange (NPE, $J^P = 0^+, 1^-, \dots$) amplitudes $T_{\lambda_V \lambda_\gamma}$ (Pomeron = two-gluon exchange, ρ, ω, a_2, \dots reggeons = $q\bar{q}$ exchange) and Unnatural Parity Exchange (UPE, $J^P = 0^-, 1^+, \dots$) amplitudes (π, a_1, b_1, \dots reggeons = $q\bar{q}$ exchange) $U_{\lambda_V \lambda_\gamma}$ better than in SDME method.
- Violation of s -channel helicity ($\lambda_V \neq \lambda_\gamma$) can be studied more reliably on the language of amplitude ratios rather than in SDME analysis. Spin-flip amplitudes T_{01}, T_{10} provide information on vector-meson structure. They are zero in the absence of quark motion in vector mesons (if quark carries momentum fraction $z = \frac{1}{2}$).

Kinematics of Exclusive ρ^0 -Meson Production at HERMES

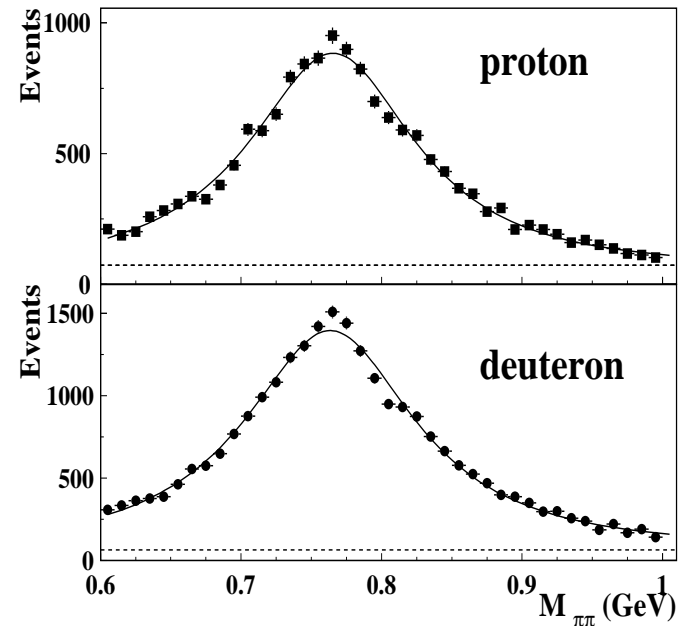
- $W = 3.0 \div 6.5$ GeV, $\langle W \rangle = 4.9$ GeV Total number of events (1996-2005)
- $Q^2 = 0.5 \div 7.0$ GeV², $\langle Q^2 \rangle = 1.95$ GeV² Deuteron: ρ^0 - 16388
- $x_B = 0.01 \div 0.35$, $\langle x_B \rangle = 0.08$ Hydrogen: ρ^0 - 9860
- $0 \leq -t' \leq 0.4$ GeV², $\langle -t' \rangle = 0.13$ GeV² with $t' = t - t_{min}$

$$\Delta E = \frac{M_X^2 - M_p^2}{2M_p} \text{ with } M_X^2 = (p + q - p_{\pi^+} - p_{\pi^-})^2 \text{ and } M_X \text{ being missing mass}$$



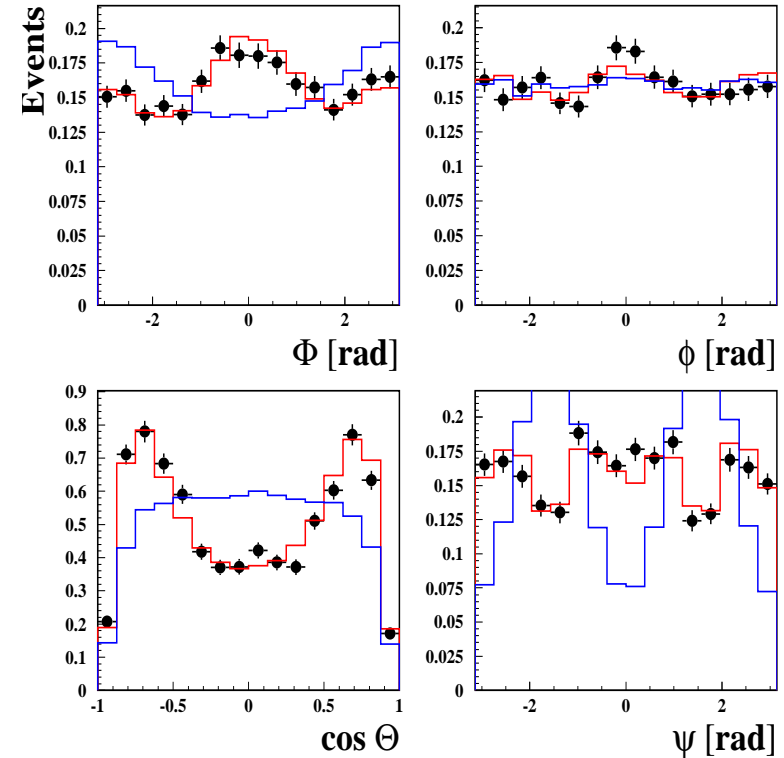
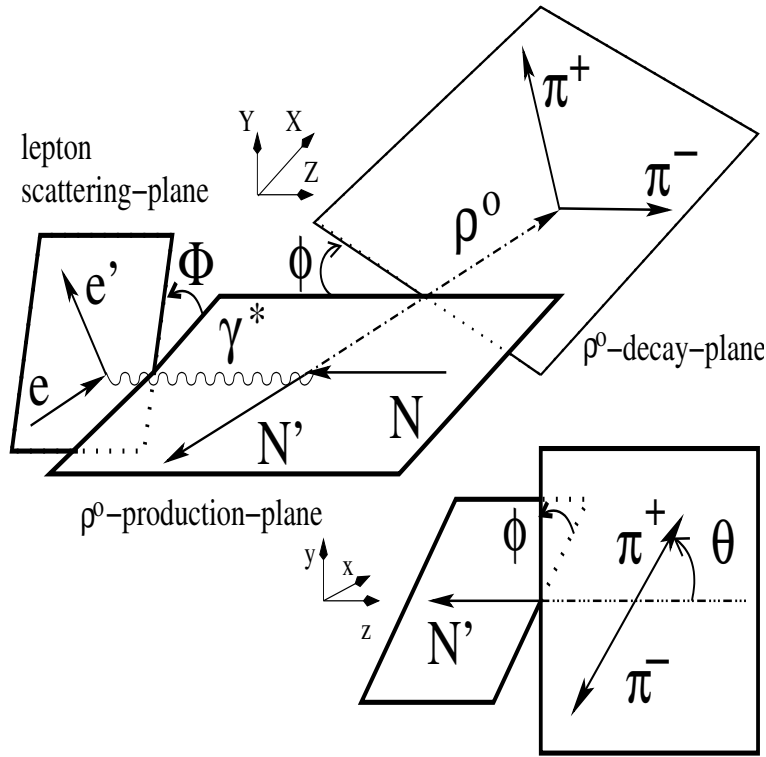
$$-1.0 < \Delta E < 0.6 \text{ GeV,}$$

SIDIS background is subtracted with the help of MC (PYTHIA)



$$0.6 < M_{\pi\pi} < 1 \text{ GeV,}$$

Data Processing using Maximum Likelihood Method in MINUIT



$$\Psi = \phi - \Phi \text{ (in } S\text{-Channel Helicity Conservation (SCHC) approximation)}$$

- **Monte Carlo Events:** 3-dimensional matrix of fully reconstructed MC events at initial uniform angular distribution.
- **Binned Maximum Likelihood (BML) Method:** $8 \times 8 \times 8$ bins of $\cos(\Theta), \phi, \Phi$. Simultaneous fit of 23 SDMEs (5 ratios of helicity amplitudes) for data with negative and positive beam helicity ($\langle P_b \rangle = \pm 53.5\%$) and unpolarized target.

Agreement of fitted angular distributions with the HERMES data

Amplitude Method and Spin-Density Matrix Element Method

- First: $e \rightarrow e + \gamma^*$ (QED)
Spin-Density Matrix (SDM) of the virtual photon $\rho(\Phi, \epsilon)$

- Second: $\gamma^* + N \rightarrow V + N$ (QCD)

Helicity amplitudes in CM system of $\gamma^* N$ $F_{\lambda_V \lambda'_V; \lambda_\gamma \lambda_N}(W, Q^2, t')$

Vector-meson spin-density matrix

$$r = \frac{1}{2N} \text{tr}_{\lambda_N \lambda'_N} \{F \rho F^+\},$$

$$N = \text{Tr}_{\lambda_V \lambda'_V \lambda_N \lambda'_N} \{F \rho F^+\}.$$

If SDM of γ^* is decomposed into set of nine matrices Σ^α then SDMEs are

$$r_{\lambda_V \lambda'_V}^\alpha = \frac{1}{2N} \text{tr}_{\lambda_N \lambda'_N} \{F \Sigma^\alpha F^+\}_{\lambda_V \lambda'_V}.$$

- Third: $\rho^0 \Rightarrow \pi^+ \pi^-$ (conservation of \vec{J})

$$|\rho^0; 1m\rangle \rightarrow |\pi^+ \pi^-; 1m\rangle \Rightarrow Y_{1m}(\theta, \phi)$$

Angular distribution $\mathcal{W}(\Phi, \phi, \cos \Theta)$

depends linearly on $r_{\lambda_V \lambda'_V}^\alpha$ and P_b .

SDME method

- 23 LU SDMEs (for Longitudinally polarized beam and Unpolarized target) are considered as free parameters in fit of angular distribution of pions from decay $\rho^0 \rightarrow \pi^+ + \pi^-$ in any small bin of kinematic variables (Q^2, t' etc.).
- Relation of SDMEs and helicity amplitudes is ignored.

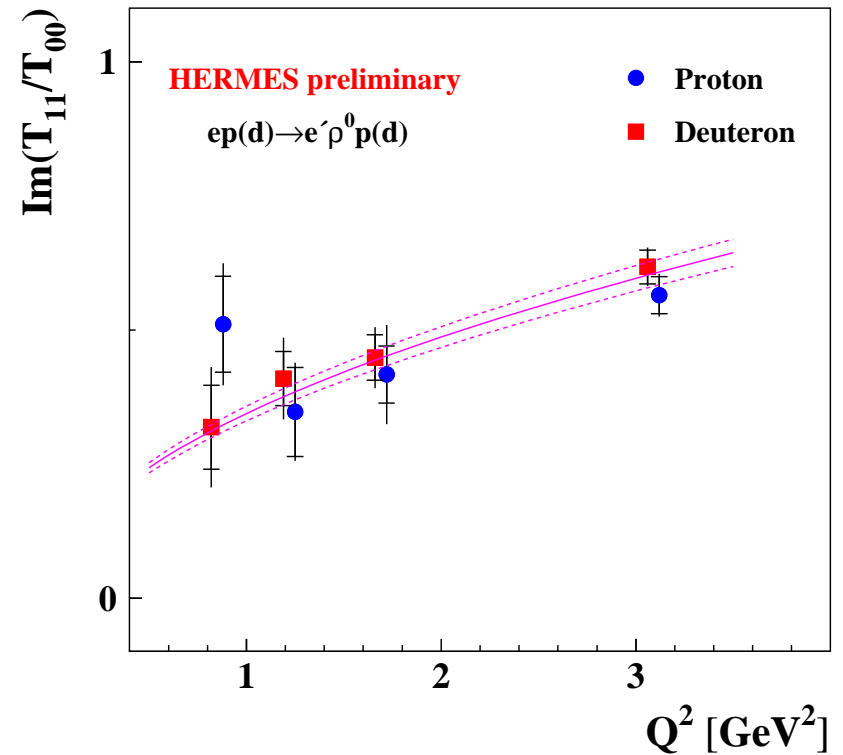
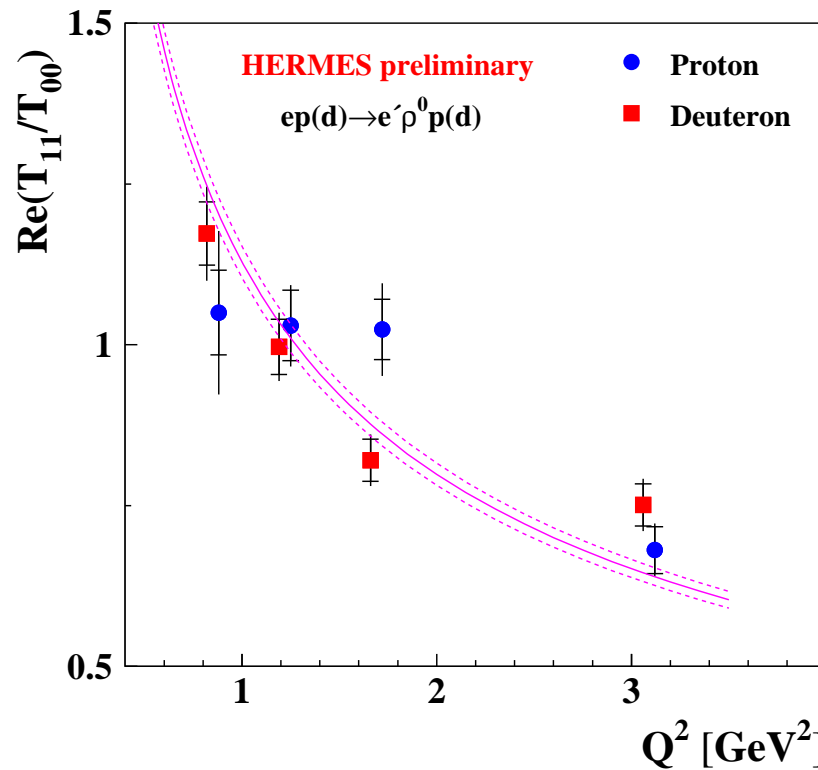
Amplitude method

- SDMEs are expressed in terms of ratios of helicity amplitudes.
- Helicity amplitude ratios are free parameters in fit of angular distribution in any small bin.
- Binning 4×4 of Q^2 and $-t'$.

Amplitude Method

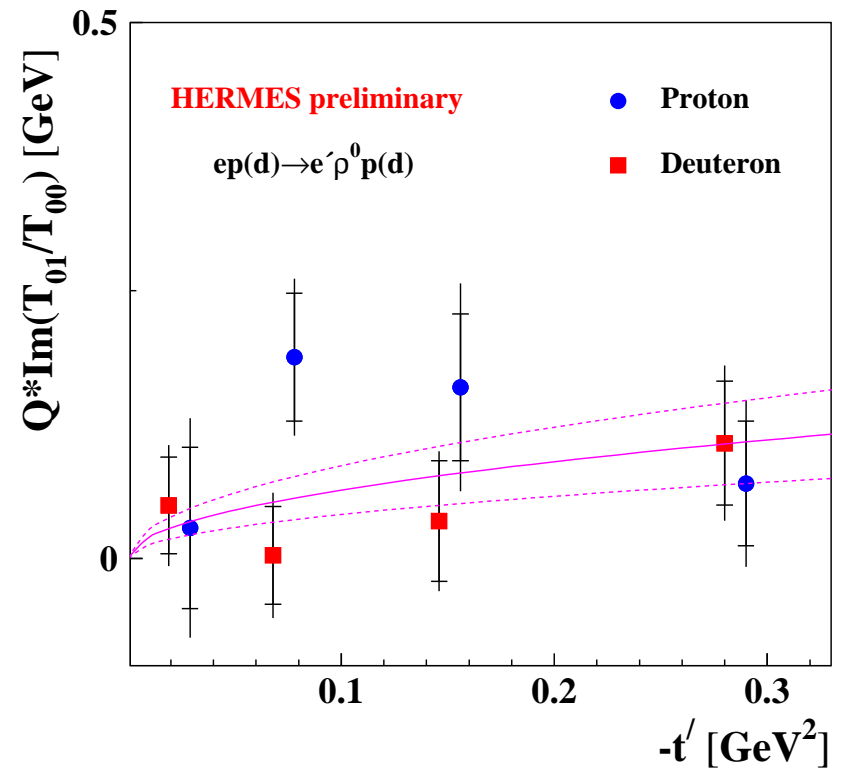
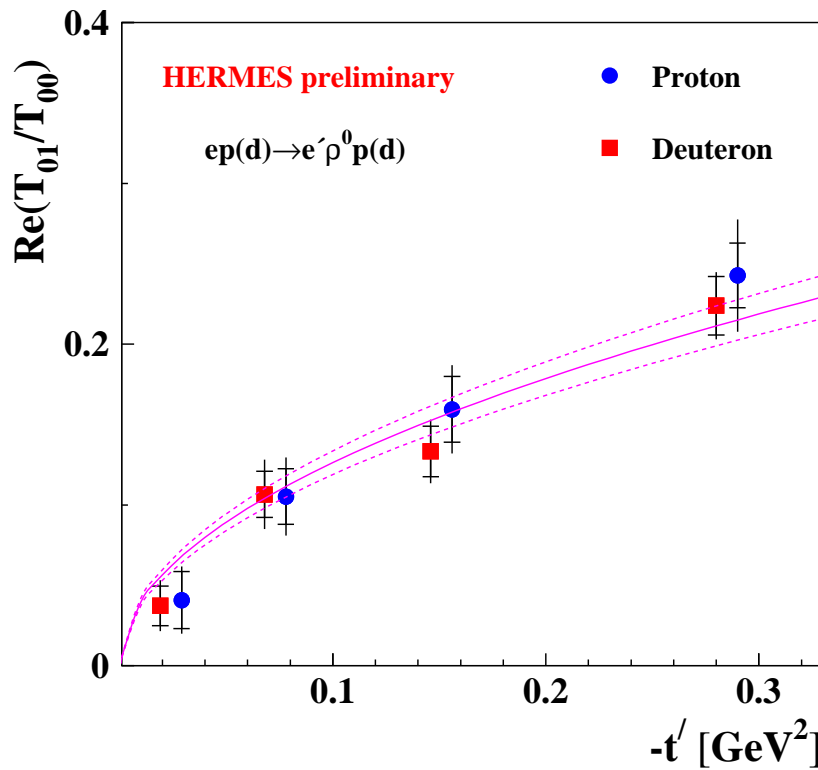
- 18 independent amplitudes
34 real free parameters (functions)
- 23 LU SDMEs (< 34)
- Hierarchy of amplitudes at small t' and high Q^2 .
Neglect small amplitudes.
- NPE ($T_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$) and UPE ($U_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N}$) helicity amplitudes.
 $F = T + U$,
 $T/U_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} = \frac{1}{2}(F_{\lambda_V \lambda'_N \lambda_\gamma \lambda_N} \pm (-1)^{\lambda_N - \lambda'_N} F_{\lambda_V - \lambda'_N \lambda_\gamma - \lambda_N})$
Shorthand notation:
 $T_{\lambda_V \lambda_\gamma} = T_{\lambda_V \frac{1}{2} \lambda_\gamma \frac{1}{2}}$
- No interference between NPE and UPE amplitudes for LU SDMEs
- UPE are suppressed at high W .
Neglect all UPE amplitudes?
- No interference between amplitudes with and without nucleon spin flip.
- $T_{\lambda'_N \neq \lambda_N} / T_{\lambda'_N = \lambda_N} \sim \alpha = v_T / (2M)$.
Fractional contribution of NPE amplitudes with $\lambda'_N \neq \lambda_N$ to LU SDMEs $\sim \alpha^2 < \text{experimental uncertainty}$.
- Neglect with NPE nucleon spin-flip amplitudes retains T_{11}/T_{00} , T_{01}/T_{00} , T_{10}/T_{00} , T_{1-1}/T_{00} (8 parameters).
- SDME analysis: S -channel helicity conservation (SCHC) at small t' .
 $|U_{01}|, |U_{10}|, |U_{1-1}| \ll |U_{11}|$ retains only $|U_{11}| = \sqrt{|U_{1\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1-\frac{1}{2}1\frac{1}{2}}|^2}$
9th parameter: $|U_{11}|/|T_{00}|$.
- Hierarchy of extracted amplitudes at HERMES kinematic region
 $|T_{00}|^2 \sim |T_{11}|^2 \gg |U_{11}|^2 > |T_{01}|^2 > |T_{10}|^2 \sim |T_{1-1}|^2$

Kinematic Dependences of Ratios of Helicity Amplitudes



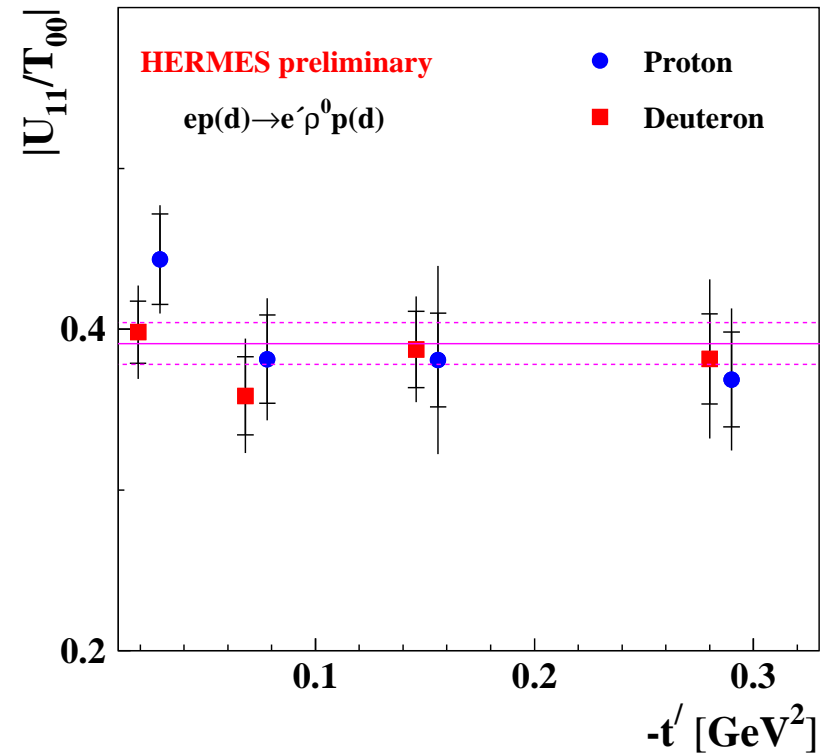
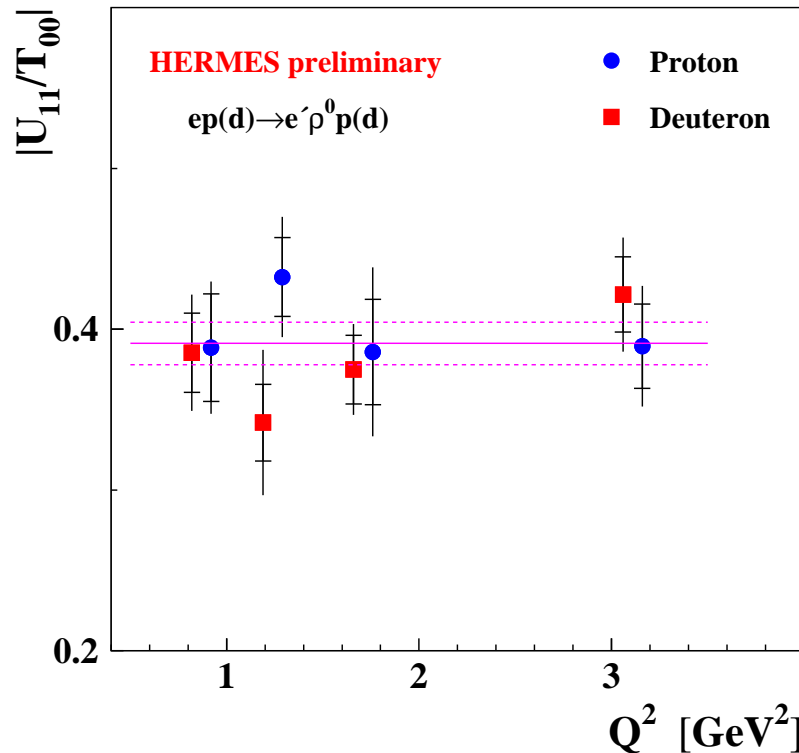
- No difference between proton and deuteron results for amplitude ratio T_{11}/T_{00} .
- pQCD prediction (Ivanov, Kirshner; Kuraev, Nikolaev, Zakharov): $T_{11}/T_{00} \propto M_\rho/Q$.
- Fit of Q dependence: $\text{Re}(T_{11}/T_{00}) = a/Q$, $\text{Im}(T_{11}/T_{00}) = b \cdot Q$.
Combined data on proton and deuteron: $a = 1.129 \pm 0.024 \text{ GeV}$, $\chi^2/N_{df} = 1.02$;
 $b = 0.344 \pm 0.014 \text{ GeV}^{-1}$, $\chi^2/N_{df} = 0.87$.
- Behaviour of $\text{Im}(T_{11}/T_{00})$ is in a contradiction with high- Q asymptotic in pQCD.
Phase difference $\delta_{11} \sim 30^\circ$ and grows with Q^2 in disagreement with pQCD calculation
- No t dependence: difference of slopes $\beta_L - \beta_T = -0.6 \pm 0.4 \text{ GeV}^{-2}$.

Kinematic Dependences of Ratios of Helicity Amplitudes



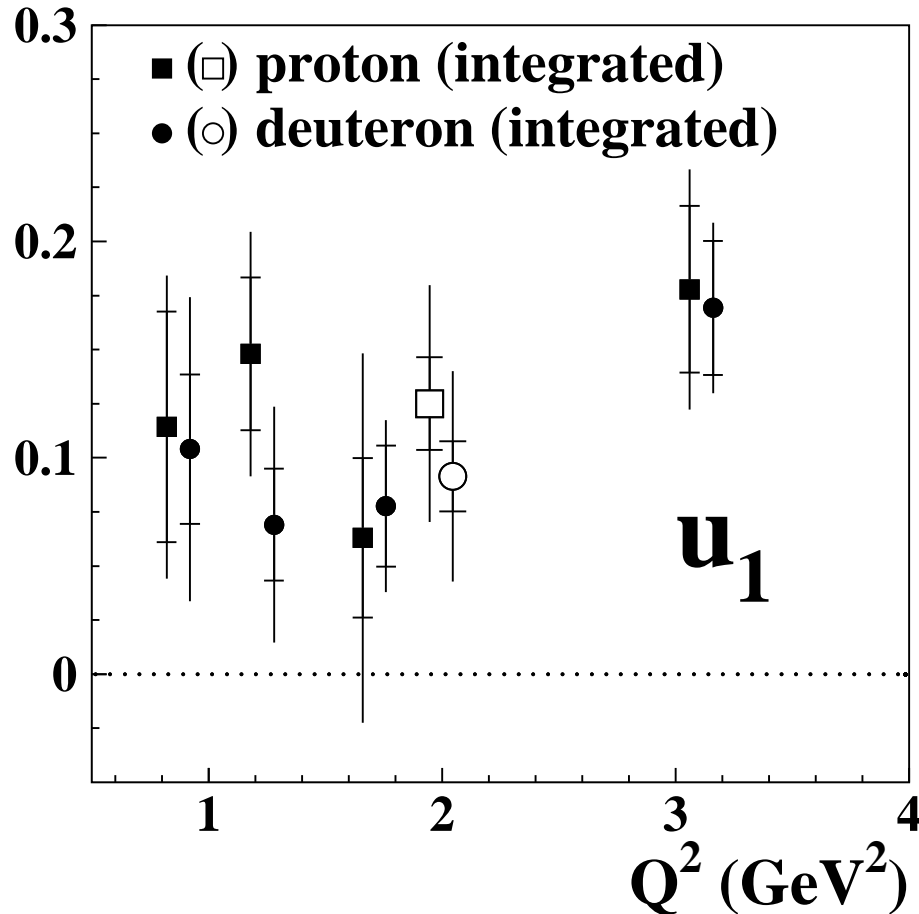
- Violation of S -Channel Helicity ($\lambda_V \neq \lambda_\gamma$): $T_{01} \neq 0$.
- No difference between proton and deuteron results for amplitude ratio T_{01}/T_{00} .
- pQCD prediction (Ivanov, Kirshner; Kuraev, Nikolaev, Zakharov): $\frac{T_{01}}{T_{00}} \propto \frac{\sqrt{-t'}}{Q}$.
- Fit of t' dependence: $\text{Re}(T_{01}/T_{00}) = a\sqrt{-t'}$, $\text{Im}(T_{01}/T_{00}) = b\sqrt{-t'}/Q$.
 Combined proton and deuteron data: $a = 0.399 \pm 0.023 \text{ GeV}^{-1}$, $\chi^2/N_{df} = 0.72$;
 $b = 0.20 \pm 0.07$, $\chi^2/N_{df} = 1.09$.

Kinematic Dependences of Ratios of Helicity Amplitudes



- No difference between proton and deuteron results for amplitude ratio $|U_{11}/T_{00}|$.
- pQCD prediction: $U_{11}/T_{00} \propto M_\rho/Q$.
- Neither Q^2 nor t' dependence: $|U_{11}|/|T_{00}| = a$, $a = 0.391 \pm 0.013$, $\chi^2/N_{df} = 0.44$ where $|U_{11}|^2 \equiv |U_{1\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1\frac{1}{2}1-\frac{1}{2}}|^2$.
- Unnatural Parity Exchange is seen much better than in SDME method.
- Contradiction both with high-Q asymptotic and one-pion-exchange dominance.

Test of Unnatural-Parity Exchange for ρ^0 Meson



$$u_1 = 0.125 \pm 0.021_{stat} \pm 0.050_{syst} \text{ (H)},$$

$$u_1 = 0.091 \pm 0.016_{stat} \pm 0.046_{syst} \text{ (D)}$$

$$u_1 = 0.106 \pm 0.036_{tot} \text{ (H+D)}$$

HERMES, Eur. Phys. J. C62 (09) 659.

- Natural and Unnatural Parity Exchanges in the t -channel
 NPE: GPD H, E ; $T_{\lambda\rho\lambda\gamma}$
 UPE: GPD \tilde{H}, \tilde{E} ; $U_{\lambda\rho\lambda\gamma}$
 NPE (Pomeron, $\rho, \omega, f_2, a_2, \dots$) dominate and UPE (π, a_1, b_1, \dots) are suppressed at high energies
- Signal of UPE in SDME method

$$u_1 = 1 - r_{00}^{04} + 2r_{1-1}^{04} - 2r_{11}^1 - 2r_{1-1}^1,$$

$$u_1 = \sum_{\lambda_N \lambda'_N} \frac{2\epsilon |U_{10}|^2 + |U_{11} + U_{-11}|^2}{N}$$

where $N = N_T + \epsilon N_L$,

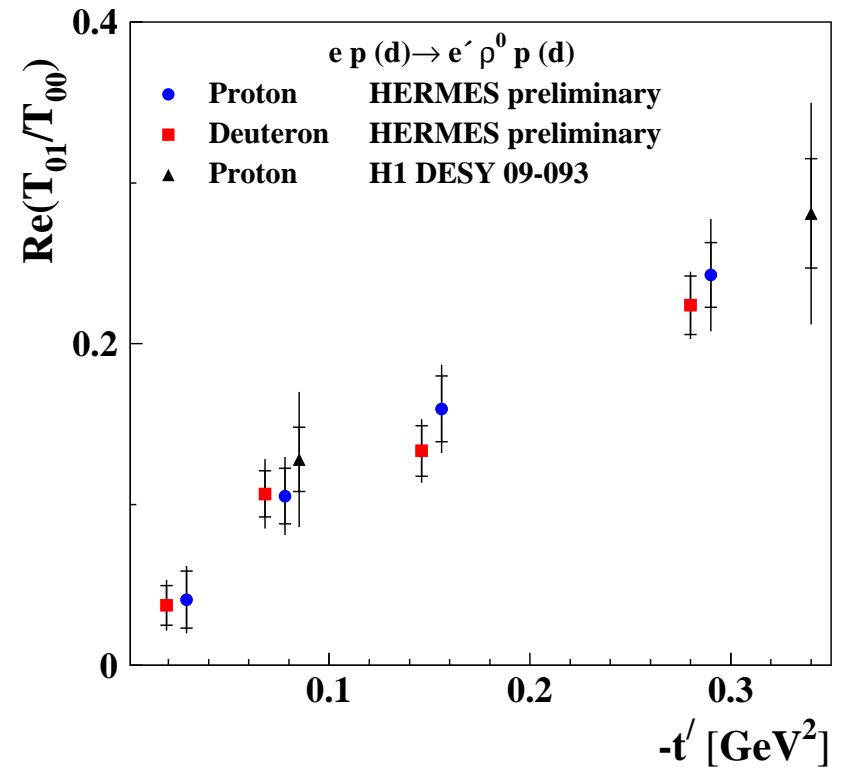
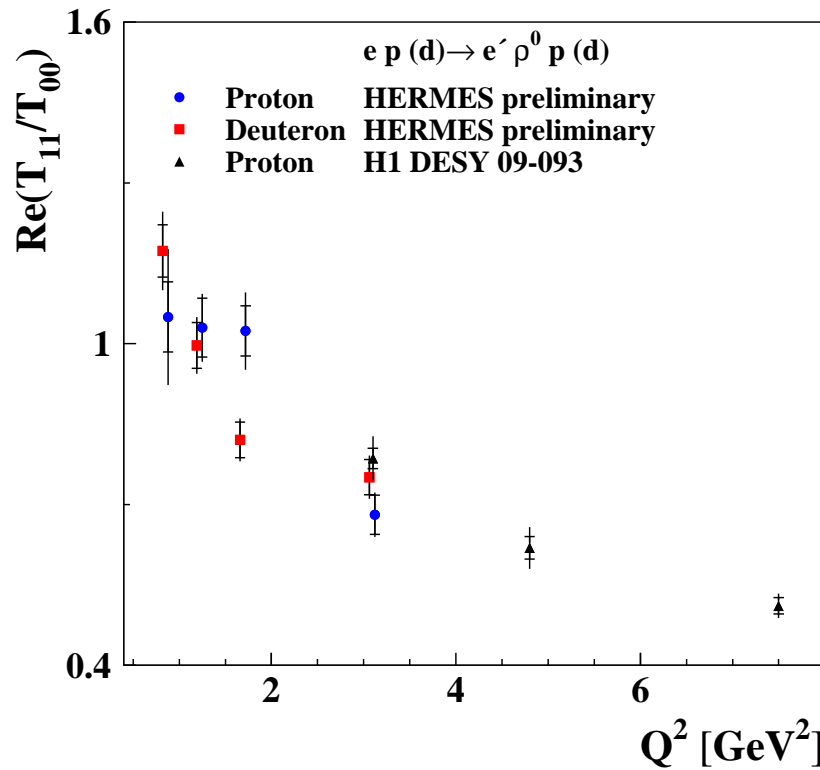
$$N_T = \sum_{\lambda_N \lambda'_N} (|T_{11}|^2 + |T_{01}|^2 +$$

$$|T_{-11}|^2 + |U_{11}|^2 + |U_{01}|^2 + |U_{-11}|^2),$$

$$N_L = \sum_{\lambda_N \lambda'_N} (|T_{00}|^2 + |T_{10}|^2 +$$

$$|T_{-10}|^2 + |U_{10}|^2 + |U_{-10}|^2).$$

World Results on Ratios of Helicity Amplitudes



- H1: Unpolarized beam and unpolarized target (15 SDMEs), $\langle Q^2 \rangle = 3.3$ GeV².
- Additional assumption: all amplitudes are imaginary, all amplitude ratios are real.
- HERMES: Longitudinally polarized beam and unpolarized target (23 SDMEs). Both real and imaginary parts of ratios of helicity amplitudes are extracted.
- Excellent agreement of amplitude ratios extracted by H1 and HERMES.

Summary

- Measurement of ρ^0 -meson production by longitudinally polarized electron/positron beam on unpolarized proton and deuteron in the HERMES experiment permits to extract both real and imaginary parts of T_{11}/T_{00} , T_{01}/T_{00} , T_{10}/T_{00} , T_{1-1}/T_{00} , and $|U_{11}/T_{00}|$.
- Dependences of the most reliably obtained ratios T_{11}/T_{00} , T_{01}/T_{00} , $|U_{11}/T_{00}|$ on Q^2 and t' is studied. The observed dependences of $\text{Im}(T_{11}/T_{00})$ and $|U_{11}/T_{00}|$ are in contradiction with high- Q asymptotic behaviour predicted in pQCD while dependences of $\text{Re}(T_{11}/T_{00})$ and $\text{Im}(T_{01}/T_{00})$ are in agreement with pQCD prediction.
- No statistically significant difference between proton and deuteron results for amplitude ratios T_{11}/T_{00} , T_{01}/T_{00} , $|U_{11}/T_{00}|$ is found.
- Violation of S -channel helicity is observed in amplitude method with higher accuracy than in SDME method.
- Contribution of unnatural parity exchange amplitude U_{11} of ρ^0 -meson production is found in amplitude method with much higher accuracy than in SDME analysis.

Outlook

- To decrease background contribution by measuring recoil nucleon.
- Include data on transversely polarized target into amplitude analysis.